Unfolding of Coloured Petri Nets
—
Comparing the Runtime of two Approaches

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This document summarizes some computational experiments comparing the runtime performance of the IDD unfolding and Gecode engines by help of the tool Marcie. The first two sections categorize the Coloured PNML models collected over the years for the Model Checking Contest (MCC) [3]. The models are called as on the MCC web site and are alphabetically ordered. The third section comprises some results obtained by case studies of our own collection, comprising $\mathcal{PN}^{\mathcal{C}}$, $\mathcal{CPN}^{\mathcal{C}}$, $\mathcal{SPN}^{\mathcal{C}}$, and $\mathcal{HPN}^{\mathcal{C}}$.

All experiments were done with Marcie [2] running scripts on a linux machine in our computer lab (2.83 GHz, 8.0 GB RAM running x_86_64 Bit linux Ubuntu 18.04 LTS).

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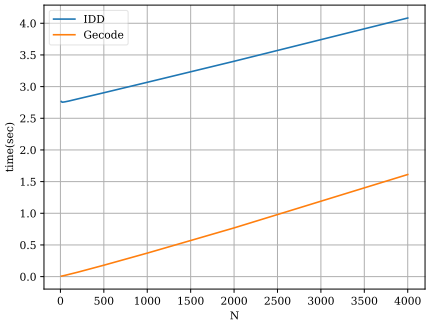
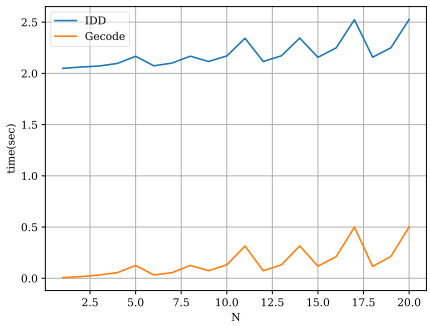
1 MCC models – No substantial unfolding time



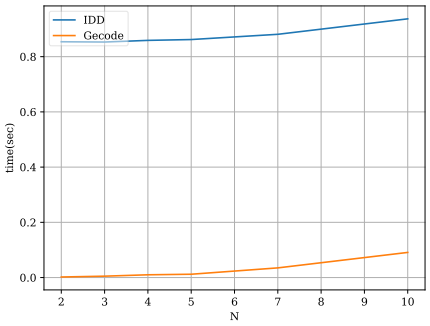
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
10	89	88	333
20	159	168	638
50	369	408	1553
100	719	808	3078
200	1419	1608	6128
500	3519	4008	15278
1000	7019	8008	30528
2000	14019	16008	61028
4000	28019	32008	122028

Fig. 1: Airplane system; N – a value from wich the maximum speed, the maximum altitude, and the associated thresholds are computed



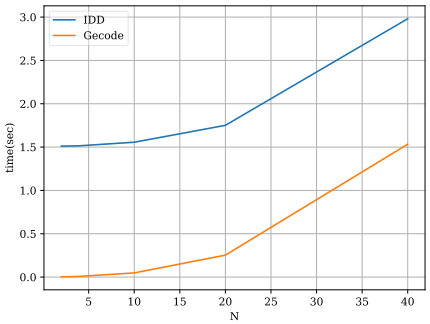
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
1	28	52	326
2	48	288	2090
4	78	968	7350
5	108	2228	17 190
9	128	1328	10 010
10	138	2348	18 090
11	168	5408	42 330
15	188	2108	15 950
16	198	3728	28 830
20	228	8588	67 470

Fig. 2: Bridges and vehicles; N – sequential model version number.



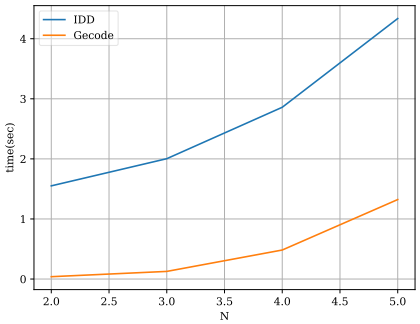
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
2	23	28	92
3	58	81	279
4	117	176	624
5	206	325	1175
7	498	833	3087
10	1311	2300	8700

Fig. 3: CS Repetitions; N – number of Clients, Servers and Buffer Size.



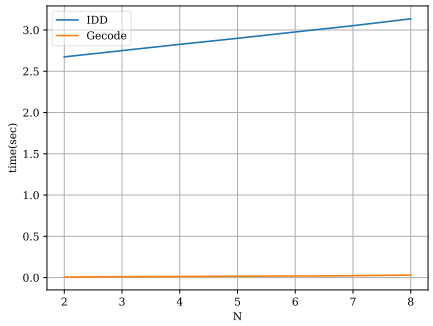
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
2	38	32	88
4	140	128	416
10	830	800	3800
20	3260	3200	23200
40	12920	12800	156800

Fig. 4: Database With Mutex.; N – number of servers on which the database is distributed and the number of files in the database.



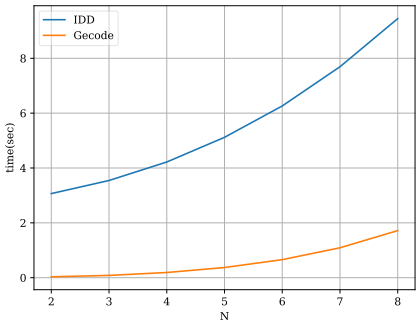
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
2	62	164	1616
3	132	1096	12064
4	230	3956	45200
5	356	10472	121760

Fig. 5: Dot and Boxes; N – number of Dots Per each grid line .



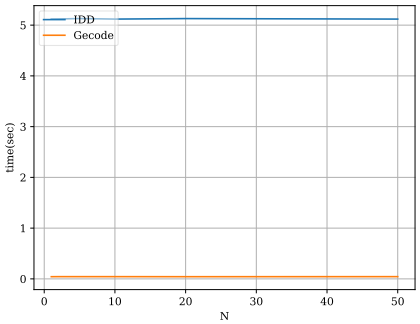
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
2	69	96	402
3	100	156	664
4	135	230	990
5	174	318	1380
6	217	420	1834
7	264	536	2352
8	315	666	2934

Fig. 6: Lamport Fast Mutex; N – Number of processes competing to access the critical section.



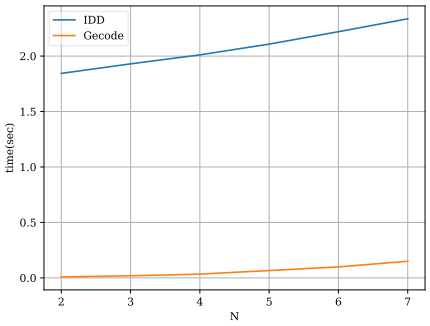
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
2	195	375	2106
3	384	1048	6032
4	665	2390	13865
5	1056	4746	27594
6	1575	8533	49616
7	2240	14240	82736
8	3069	22428	130167

Fig. 7: Neo Election; N – number of network nodes participating in the election.



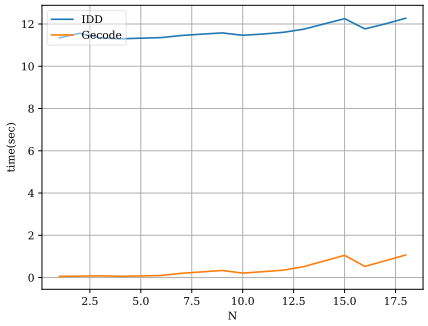
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
1	208	1024	6080
2	208	1024	6080
5	208	1024	6080
10	208	1024	6080
20	208	1024	6080
50	208	1024	6080

Fig. 8: PermAdmissibility; N – Multiplier for the marking of places.



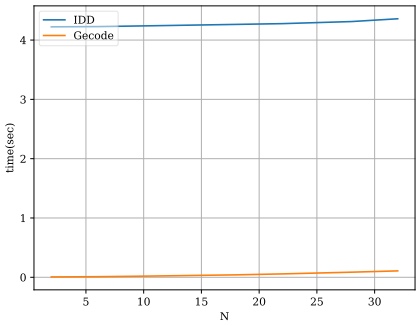
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
2	108	138	432
3	256	356	1112
4	500	730	2280
5	864	1302	4068
6	1372	2114	6608
7	2048	3208	10032

Fig. 9: Peterson; N – number of processes .



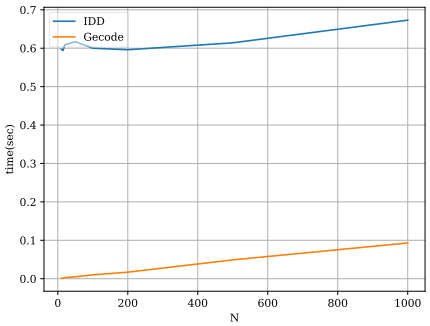
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
1	476	920	4794
2	614	1242	6468
3	752	1572	8182
4	536	1064	5418
5	690	1434	7300
6	844	1812	9222
7	554	2998	25218
8	712	4012	33696
9	870	5034	42214
10	618	3190	26034
11	792	4268	34784
12	966	5354	43574
13	454	6994	69280
14	632	10500	103962
15	810	14014	138684
16	506	7154	69952
17	700	10740	104970
18	894	14334	140028

Fig. 10: PolyORBLF; N – sequential model version number .



N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
2	86	56	223
6	270	116	659
10	550	176	1287
18	1398	296	3119
22	1966	356	4323
28	2998	446	6489
32	3806	506	8173

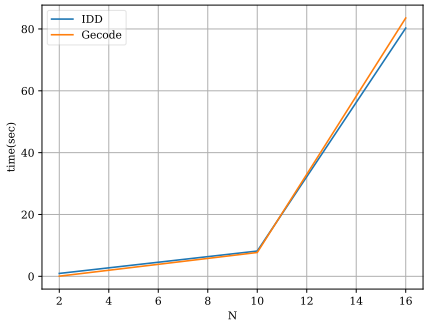
Fig. 11: Quasi certification protocol; N – Size of the Leaf Sets.



N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
10	31	21	51
15	46	31	76
20	61	41	101
50	151	101	251
100	301	201	501
200	601	401	1001
500	1501	1001	2501
1000	3001	2001	5001

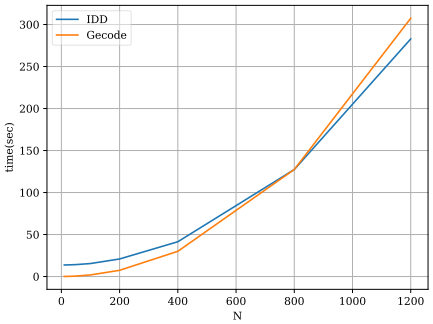
Fig. 12: Referendum; N – the maximum number of voters .

2 MCC models – Exponential run time increase



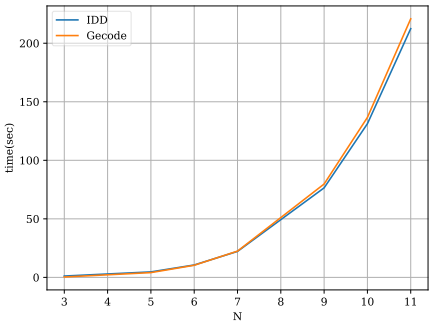
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
2	24	72	536
10	120	111160	1086520
16	192	1118560	11035840

Fig. 13: Drink Vending Machine; N – Number of products.



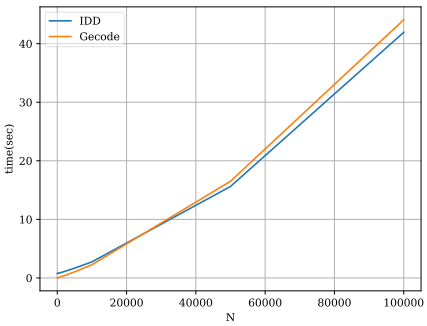
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
10	1475	1234	3799
20	3271	2753	8446
50	12194	10560	32238
100	40605	36871	112728
200	143908	134279	411469
400	537708	508489	1558729
800	2075308	1976909	6061249
1200	4612908	4405329	13507769

Fig. 14: Family Reunion; N – number of legal residents.



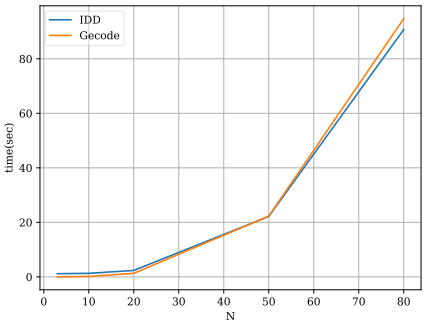
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
3	33	4791	46284
5	75	56105	551960
6	102	136662	1349364
7	133	291067	2880724
9	207	1003437	9959616
10	250	1688410	16773820
11	297	2705087	26893724

Fig. 15: Global Resource Allocation; N – Cardinality of process and resources.



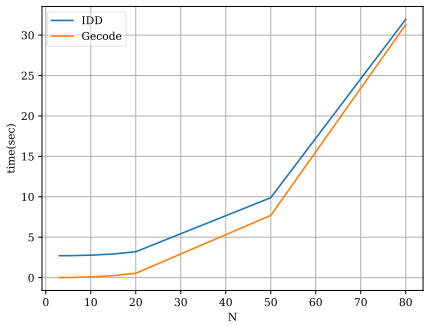
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
5	25	25	80
10	50	50	160
20	100	100	320
50	250	250	800
100	500	500	1600
200	1000	1000	3200
500	2500	2500	8000
1000	5000	5000	16000
2000	10000	10000	32000
5000	25000	25000	80000
10000	50000	50000	160000
50000	250000	250000	800000
100000	500000	500000	1600000

Fig. 16: philosophers; N – number of Philosophers.



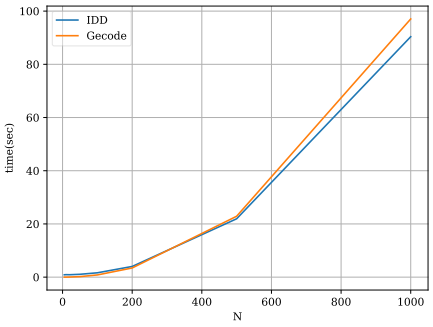
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
3	30	84	591
10	170	2310	18420
20	540	17220	141640
50	2850	257550	2160100
80	6960	1043280	8793760

Fig. 17: PhiloDyn; N – number of Philosophers.



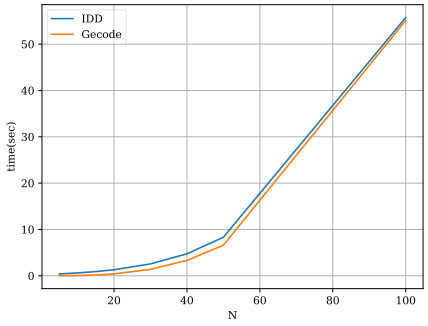
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
3	60	97	568
6	150	463	3022
10	326	1651	11474
15	636	4801	34684
20	1046	10501	77544
50	5606	140251	1083354
80	13766	550801	4308564

Fig. 18: Safe Bus; N – number of interlocutors connected to the bus.



N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
5	46	60	220
10	141	220	840
20	481	840	3280
50	2701	5100	20200
100	10401	20200	80400
200	40801	80400	320800
500	252001	501000	2002000
1000	1004001	2002000	8004000

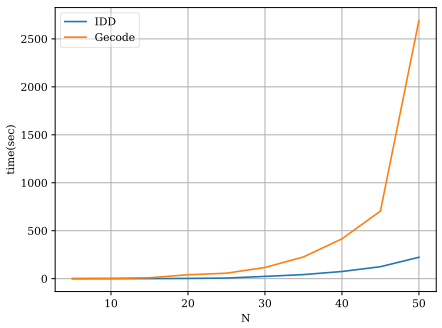
Fig. 19: Shared Memory; N – number of processors.



N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
5	36	156	624
10	121	1111	4444
15	256	3616	14464
20	441	8421	33684
30	961	27931	111724
40	1681	65641	262564
50	2601	127551	510204
100	10201	1010101	4040404

Fig. 20: Token Ring; N – Number of processes.

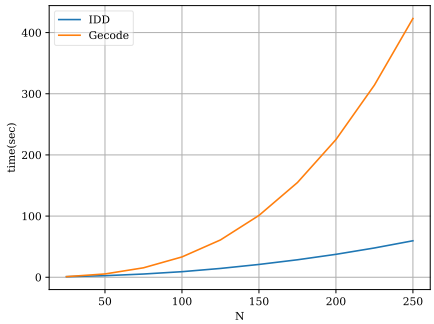
3 Test cases from our own Collection



N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
5	125	600	1200
10	1000	5400	10 800
15	3375	18 900	37 800
20	8000	72 998	145 996
25	15 625	90 000	180 000
30	27 000	156 600	313 200
35	42 875	249 900	499 800
40	64 000	374 400	748 800
45	91 125	534 600	1 069 200
50	125 000	735 000	1 470 000

Fig. 21: 3D Diffusion with six neighbours; N – grid size of a 3D cube.

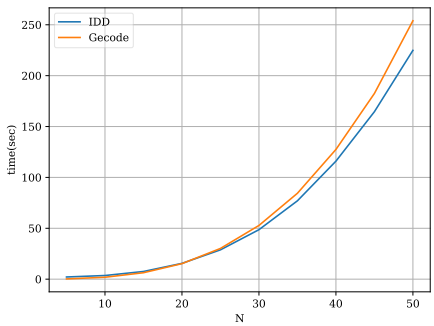
Diffusion is a basic mechanism underlying many biological processes and is thus crucial for many case studies undertaken by help of our unifying framework, building on scaleable $\mathcal{SPN}^{\mathcal{C}}$, $\mathcal{CPN}^{\mathcal{C}}$ or $\mathcal{HPN}^{\mathcal{C}}$, as introduced in [1]. Here, we consider a basic $\mathcal{CPN}^{\mathcal{C}}$ consisting of one place and one transition, and each grid element in the 3D cube has six neighbours. The scaling factor is the grid size, i.e., the edge length of the 3D cube.



N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
25	1250	11 908	23 191
50	5000	48 808	95 116
75	11 250	110 708	215 791
100	20 000	197 608	385 216
125	31 250	309 508	603 391
150	45 000	446 408	870 316
175	61 250	608 308	1 185 991
200	80 000	795 208	1 550 416
225	101 250	1 007 108	1 963 591
250	125 000	1 244 008	2 425 516

Fig. 22: Brusselator; N – grid size of a 2D square.

The Brusselator [6] is a popular reaction diffusion system, typically modelled as partial differential systems (PDE). Using this case study, we have shown in [4] how to systematically encode PDEs as $\mathcal{CPN}^{\mathcal{C}}$, which then can be equally read as $\mathcal{SPN}^{\mathcal{C}}$ or $\mathcal{HPN}^{\mathcal{C}}$. The coloured Petri net modelling the Brusselator involves diffusion in 2D with four neighbours and yields exactly the same results as obtained in [6] using PDEs. $\mathcal{CPN}^{\mathcal{C}}$ size: 2 places, 6 transitions, and 11 arcs. The nodes can either be read as discrete or continuous nodes. However, this does not have any effect on the unfolding efficiency. The scaling factor is the edge length of the 2D grid.

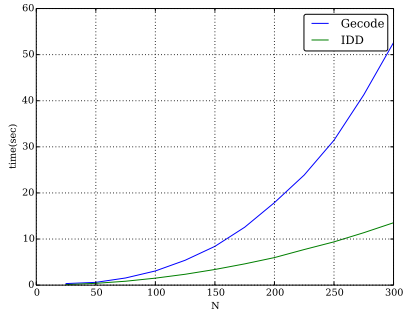


N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
5	225	5071	20 030
10	745	38 036	151 350
15	1565	125 901	501 970
20	2685	295 666	1 179 890
25	4105	574 331	2 293 110
30	5825	988 896	3 949 630
35	7845	1 566 361	6 257 450
40	10 165	2 333 726	9 324 570
45	12 785	3 317 991	13 258 990
50	15 705	4 546 156	18 168 710

Fig. 23: Botanical garden; N – number of concurrent processes (turnstiles).

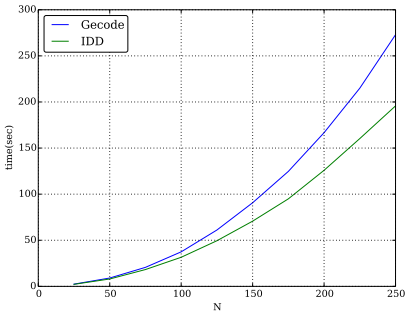
A $\mathcal{PN}^{\mathcal{C}}$ illustrating for teaching purposes the need for mutually exclusive synchronisation of processes concurrently writing to a shared global variable. This model is inspired by the ornamental garden problem [5]: people enter a botanical garden through either of two turnstiles. The management wishes to know how many visitors are in the garden at any time. The $\mathcal{PN}^{\mathcal{C}}$ is scaleable by the number of turnstiles, i.e., concurrent processes, and the number of people using each turnstile.

The following experiments were done by Martin Schwarick.



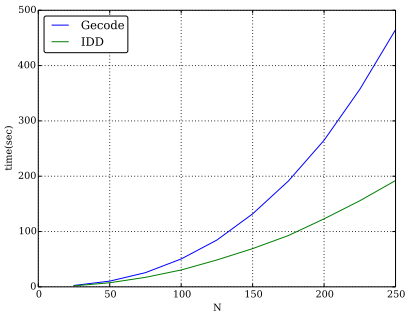
N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
25	625	2400	4800
50	2500	9800	19600
75	5625	22200	44400
100	10000	39600	79200
125	15625	62000	124000
150	22500	89400	178800
175	30625	121800	243600
200	40000	159200	318400
225	50625	201600	403200
250	62500	249000	498000
275	75625	301400	602800
300	90000	358800	717600

Fig. 24: Gradient 2D with four neighbours; N – grid size of a 2D square.



N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
25	825	37079	167658
50	2700	147404	667683
75	5825	331479	1501458
100	10200	589304	2668983
125	15825	920879	4170258
150	22700	1326204	6005283
175	30825	1805279	8174058
200	40200	2358104	10676583
225	50825	2984679	13512858
250	62700	3685004	16682883

Fig. 25: Dicty; N – grid size of a 2D square.



N	$\mid P \mid$	$\mid T \mid$	$\mid A \mid$
25	1877	21317	106582
50	7502	87617	438082
75	16877	198917	994582
100	30002	355217	1776082
125	46877	556517	2782582
150	67502	802817	4014082
175	91877	1094117	5470582
200	120002	1430417	7152082
225	151877	1811717	9058582
250	187502	2238017	11190082

Fig. 26: Phase variation; N – grid size of a 2D square.

References

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