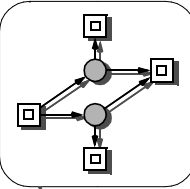
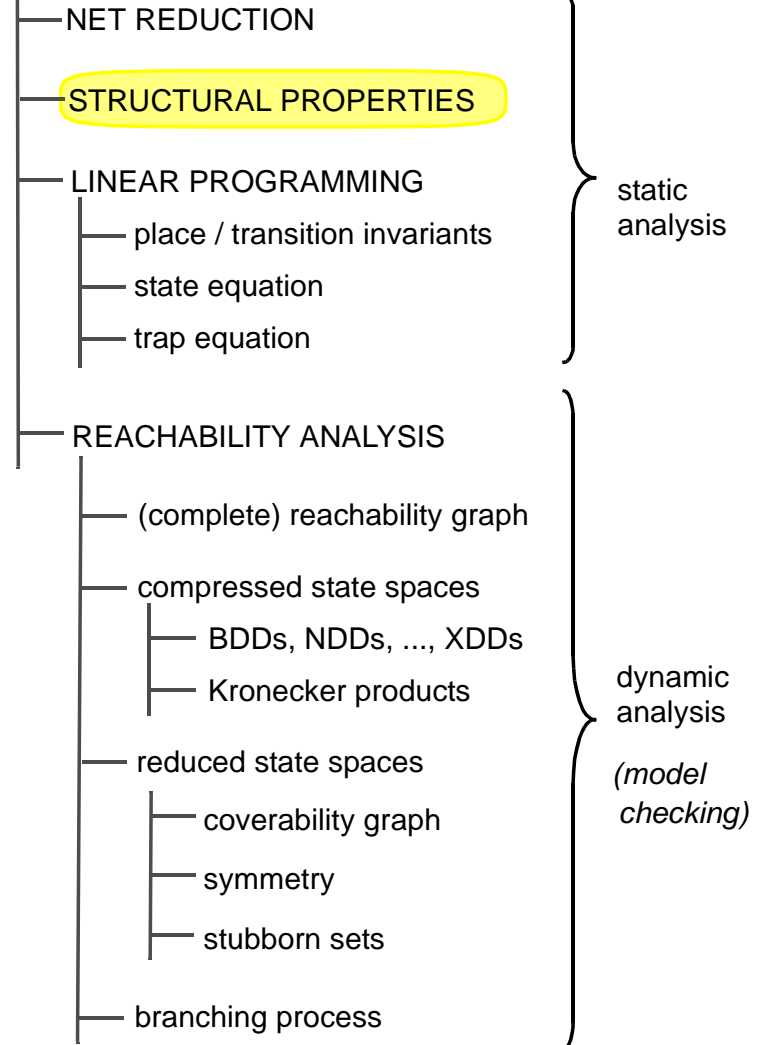
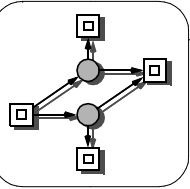


# STRUCTURAL PETRI NET ANALYSIS



## QUALITATIVE ANALYSIS METHODS, OVERVIEW





## QUALITATIVE PROPERTIES



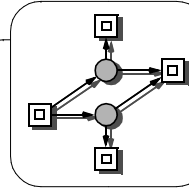
### behavioural properties

- general semantic properties
  - boundedness*
  - liveness*
  - reversibility*
- special semantic properties
  - safety properties*
  - progress properties*



### structural properties

- especially valuable:
  - local(ly decidable) structural properties;
- certain combinations of structural properties
  - allow conclusions to
  - behavioural properties;



## BEHAVIOURAL NET PROPERTIES

### MARKABILITY of places

- markable (*place liveness*)
- k-bounded (*safe*)

### LIVENESS of transitions

- zero times firing ( *$m_0$ -dead*)
- finite times firing (*dead, non-live*)
- infinite times (probably) firing (*live*)
- infinite times (definitely) firing (*livelock free*)

### REACHABILITY of states

- dead states
- reproducibility
- reversibility ( *$m_0$  - home state*)

- bad states (*facts*)
- user-specified states

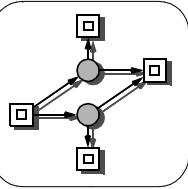
### NET INVARIANTS

- transition invariants
- place invariants

— temporal relationship of logic formulae

general semantic properties

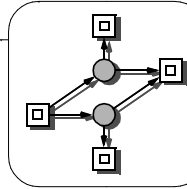
special semantic properties



## PETRI NET PROPERTIES, OVERVIEW / CHARLIE

### 1. SIMPLE STRUCTURAL PROPERTIES

- PUR pure (*no side conditions*)
- ORD ordinary (*1-multiplicity of all arcs*)
- HOM homogeneous (*all output arcs of a given place have the same multiplicity*)
- NBM non-blocking multiplicity (*for each place applies: MIN multiplicity of input arcs  $\geq$  MAX multiplicity of output arcs*)
- CSV conservative (*any firing preserves token amount*)
- SCF static conflict free
- Ft0 every transition has a pre-place
- tF0 every transition has a post-place
- FP0 every place has a pre-transition
- pF0 every place has a post-transition
- CON connected
- SC strongly connected
- MG marked graph (*synchronization graph*)
- SM state machine
- FC free choice net
- EFC extended free choice net
- ES extended simple net



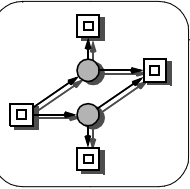
## MORE EXPENSIVE STRUCTURAL PROPERTIES

### 2. STRUCTURAL PROPERTIES

- RKTH rank theorem
- STP siphon trap property
- SMC state machine coverable (*covered with SM components*)
- SMD state machine decomposable (*covered with SCSM components*)
- SMA state machine allocatable
- CPI covered with place invariants
- CTI covered with transition invariants
- SCTI strongly covered by transition invariants
- SB structurally bounded

### 3. BEHAVIOURAL PROPERTIES

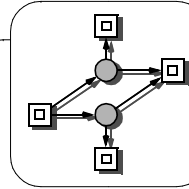
- k-B k-bounded
- DCF dynamically conflict free
- DSt dead states (*a state where no transition is enabled*)
- DTr dead transitions (*at the initial state*)
- LIV live
- REV reversible (*the initial state  $m_0$  can be reached again from all reachable states: home state*)



### CONCLUSIONS (1) STRUCTURAL -> BEHAVIOURAL PROPERTIES

<p><b>Ft0</b></p> <p>t live p unbounded</p>	<p><b>tF0</b></p> <p>t not live OR p unbounded</p>
<p><b>Fp0</b></p> <p>t not live p bounded</p>	<p><b>pF0</b></p> <p>t not live OR p unbounded</p>

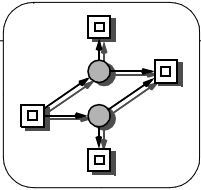
*input nodes allow net reduction:  
conclude properties -> delete nodes -> conclude properties ->*



### CONCLUSIONS (2) STRUCTURAL -> BEHAVIOURAL PROPERTIES

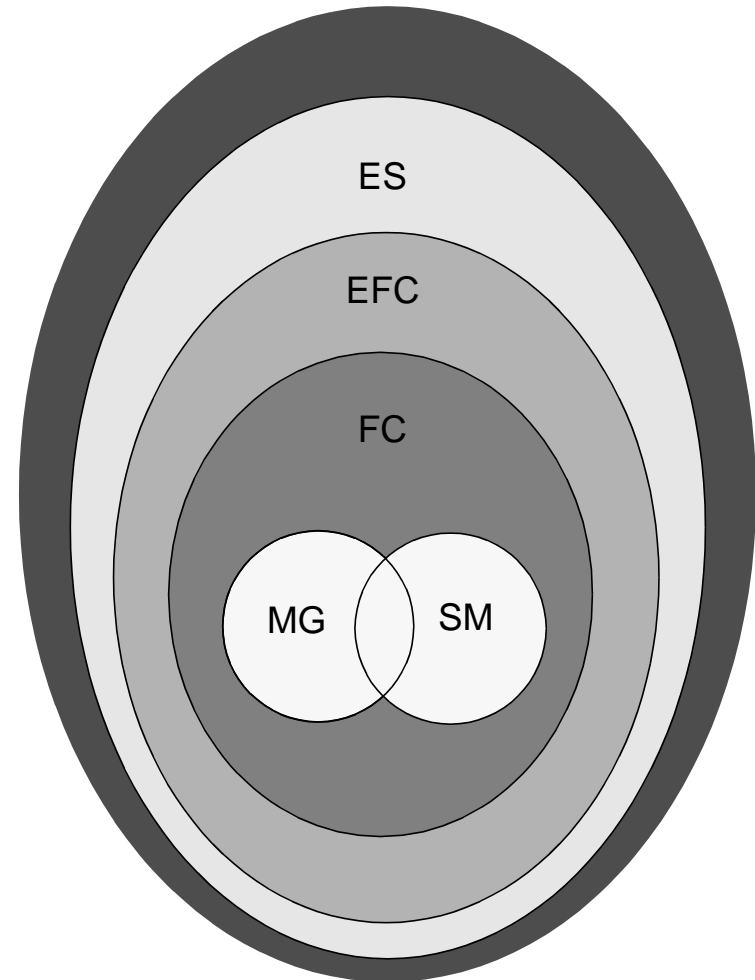
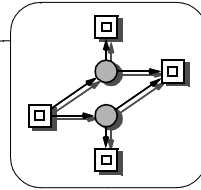
- CSV ⇒ BND
- CPI ⇒ BND
- covered by Sub-P-invariants (yC ≤ 0) ⇔ (structural) BND
- SC ⇔ LIVE & BND  
( not SC ⇒ not LIVE or not BND )
- CTI ⇔ LIVE & BND  
( not CTI ⇒ not LIVE or not BND )  
( not CTI & BND ⇒ not LIVE )

### NET CLASSES, OVERVIEW

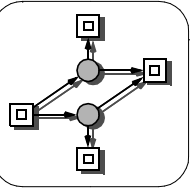


	allowed	not allowed
State Machines		
Marked Graphs		
FC nets		
EFC nets		
ES nets		

### RELATIONSHIP OF NET CLASSES

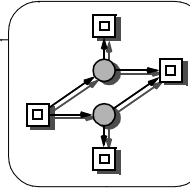
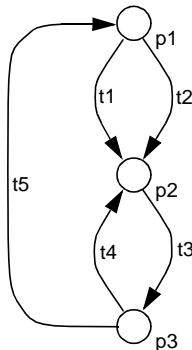
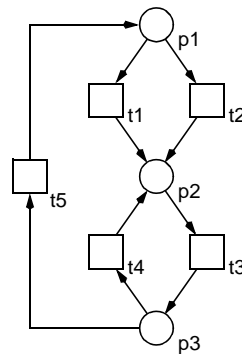


MG - synchronization graph, T-nets, SM - S-nets



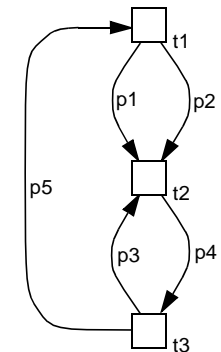
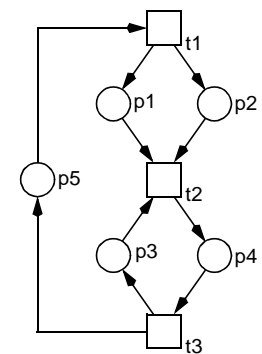
### NET CLASS: STATE MACHINE

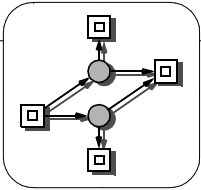
- ❑ no forward/backward branching of transitions  
-> finite (state) automaton
- ❑ no concurrency, but conflicts
- ❑ any static conflict = dynamic conflict
- ❑ no production/consumption of tokens  
-> conservative (CSV)  
-> bounded (BND)
- ❑ prototype of a bounded pn
- ❑ SM  $\Rightarrow$  [ SC & 'at least one token'  
 $\Leftrightarrow$  LIVE & BND & REV ]
- ❑ SM  $\Rightarrow$  [ SC & 'exactly one token'  
 $\Leftrightarrow$  LIVE & SAFE & REV ]
- ❑ SM and MG  
-> duality



### NET CLASS: MARKED GRAPH

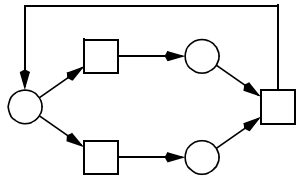
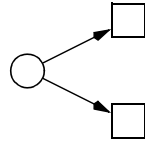
- ❑ no forward/backward branching of places  
-> precedence graph
- ❑ no conflicts, but concurrency  
-> DCF (persistent)
- ❑ prototype of a persistent pn
- ❑ number of tokens on each circle is invariant  
-> P-invariant
- ❑ elementary circle  
-> no node in the circle appears twice
- ❑ MG  $\Rightarrow$  [ SC & 'each elementary circle contains a token'  
 $\Leftrightarrow$  LIVE & BND & REV ]
- ❑ MG  $\Rightarrow$  [ SC & 'each elementary circle contains exactly one token'  
 $\Leftrightarrow$  LIVE & SAFE & REV ]



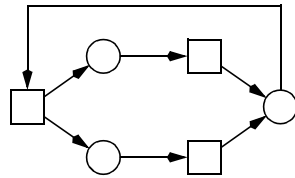


**NET CLASS:  
FREE-CHOICE NET**

- ❑ every shared place is the only pre-place of its post-transitions
- ❑ free choice for any conflict resolution
- ❑ conflict & concurrency
- ❑ sc not sufficient any more

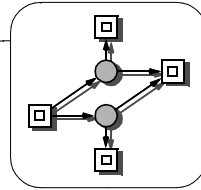
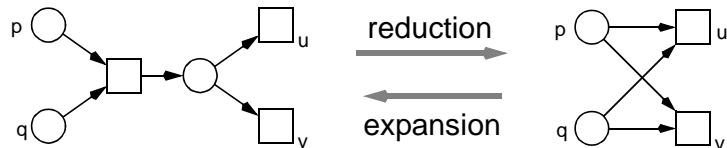


no live marking



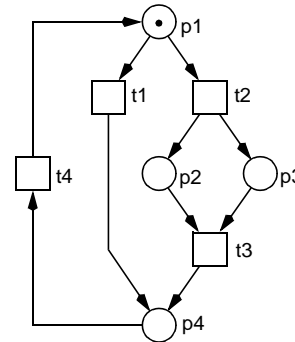
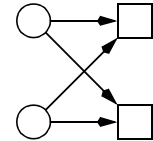
no bounded marking  
(except empty marking)

- ❑ all theorems for FCN hold also for EFCN
- > transformation FCN <-> EFCN

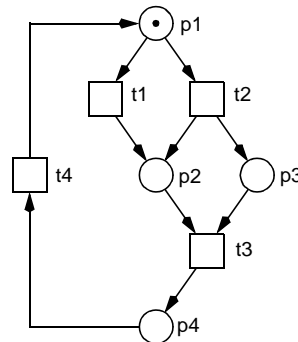


**NET CLASS:  
EXTENDED FREE-CHOICE NET**

- ❑ post-transitions of shared places have the same pre-places
- ❑ free choice for any conflict resolution
- ❑ EFC  $\Rightarrow$  [ STP ( & HOM & NBM )  $\Leftrightarrow$  LIVE ]

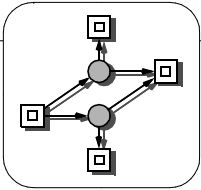


STP



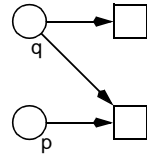
not STP

- ❑ EFC ( & HOM & NBM )  $\Rightarrow$  monotonicity of liveness

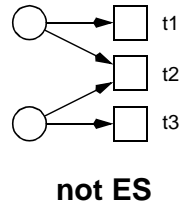
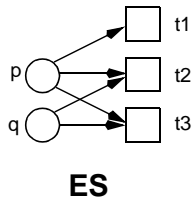


### NET CLASS: EXTENDED SIMPLE NET

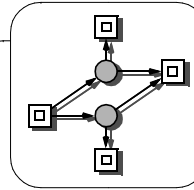
- IF two places  $p, q$  share post-transitions  
THEN post-transitions of  $q$  are also  
post-transitions of  $p$ ,  
or vice versa  
ENDIF  
-> one of the two places may have  
other post-transitions



- transitive conflict relation #  
->  $t1 \# t2$  and  $t2 \# t3 \rightarrow t1 \# t3$

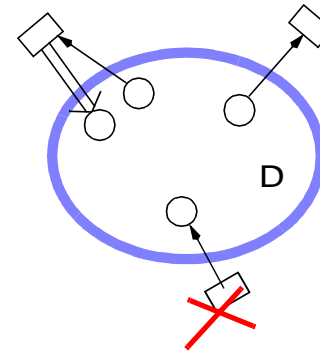


- ES & STP ( & HOM & NBM )  
⇒ LIVE
- ES & LIVE  
⇒ time-independently LIVE



### SIPHON-TRAP-PROPERTY (STP)

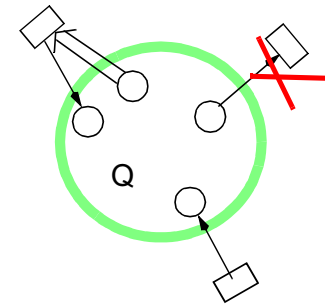
**Siphon D**  
 $FD \subseteq DF$



any transition  
putting token into the set  
also takes token from it

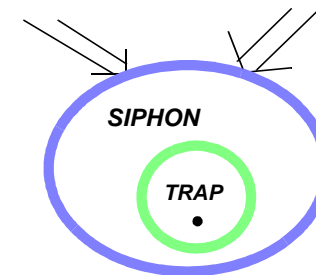
an empty siphon will  
never again carry a token

**Trap Q**  
 $QF \subseteq FQ$



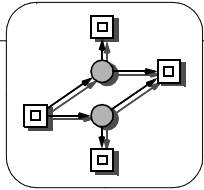
any transition  
taking tokens from the set  
also puts token into it

a marked trap will  
never again be empty



STP: each siphon contains  
a (sufficiently) marked trap (at  $m_0$ )





### STP, EXAMPLES

□ example1

-> siphon1: {p1, p2}

$$F(p1, p2) = \{t1, t3\}$$

$$(p1, p2)F = \{t1, t2, t3\}$$

-> siphon2: {p1, p3}

$$F(p1, p3) = \{t2, t3\}$$

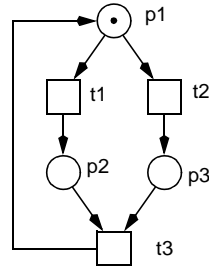
$$(p1, p3)F = \{t1, t2, t3\}$$

-> trap: {p1, p2, p3}

$$(p1, p2, p3)F = T$$

$$F(p1, p2, p3) = T$$

=> **not STP**



□ example2

-> siphon: {p1, p2, p3}

$$F(p1, p2, p3) = T$$

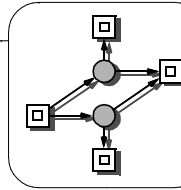
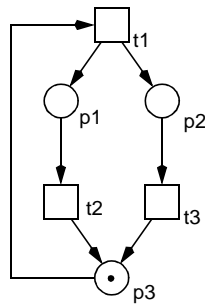
$$(p1, p2, p3)F = T$$

-> trap: {p1, p2, p3}

$$(p1, p2, p3)F = T$$

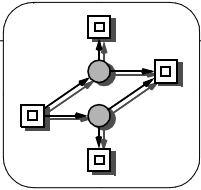
$$F(p1, p2, p3) = T$$

=> **STP**



### CONCLUSIONS (3) STRUCTURAL -> BEHAVIOURAL PROPERTIES:

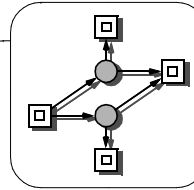
- SM  $\Rightarrow$  [ SC & 'at least one token'  
 $\Leftrightarrow$  LIVE & BND & REV ]
- SM  $\Rightarrow$  [ SC & 'exactly one token'  
 $\Leftrightarrow$  LIVE & SAFE & REV ]
- MG  $\Rightarrow$  [ SC & 'each elementary circle  
contains a token'  
 $\Leftrightarrow$  LIVE & BND & REV ]
- MG  $\Rightarrow$  [ SC & 'each elementary circle  
contains exactly one token'  
 $\Leftrightarrow$  LIVE & SAFE & REV ]
- EFC  $\Rightarrow$  [ STP ( & HOM & NBM )  $\Leftrightarrow$  LIVE ]
- ES & STP ( & HOM & NBM )  $\Rightarrow$  LIVE
- STP ( & HOM & NBM )  $\Rightarrow$  not DSt
- ORD & 'there are no siphons'  $\Rightarrow$  LIVE
- ORD & SC & SMA  $\Rightarrow$  structural LIVE



## SOME NOTIONS

- ❑ **empty siphon (trap):**  
siphon (trap), not containing a token  
-> *A deadlocked Petri net has an empty siphon.*
- ❑ **minimal siphon (trap):**  
siphon (trap) not including a siphon (trap) as a proper subset;
- ❑ **bad siphon:**  
siphon, which does not include a trap;  
-> *If there is a bad siphon, the DTP does not hold.*
- ❑ **language L of Petri net N with initial marking m:**  
 $L(N(m))$  - set of all transition firing sequences;  
= all path through the RG  $(N(m))$ ;
- ❑ For any net N and two markings m and m', with  $m \leq m'$ , it holds

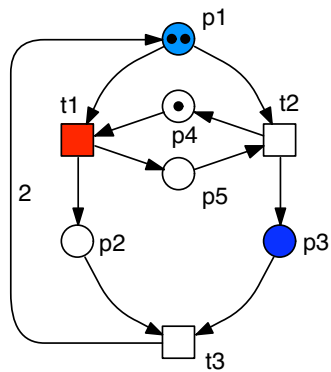
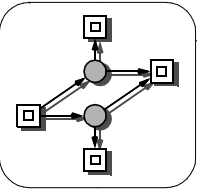
$$L(N(m)) \subseteq L(N(m')).$$



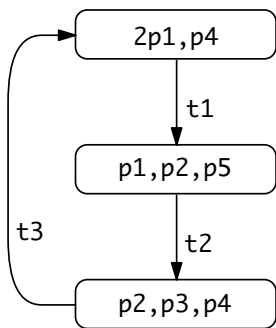
## SOME OBSERVATIONS

- ❑ A source place p establishes a bad siphon  $D = \{p\}$  on its own, and a sink place q a trap  $Q = \{q\}$ .
- ❑ If each transition has a pre-place, then  $PF = T$ .  
If each transition has a post-place, then  $FP = T$ .  
Thus, in a net without boundary transitions, the whole set of places is a siphon as well as a trap (however, not necessarily minimal ones).
- ❑ For a P-invariants x it holds  $F\text{supp}(x) = \text{supp}(x)F$ .  
Thus, the support of a P-invariant is siphon and trap as well (however, generally not vice versa).
- ❑ If R1 and R2 are siphons (traps), then the union of R1 and R2 is also a siphon (trap).
- ❑ A minimal siphon (trap) is a P-strongly-connected component, i.e., its places are strongly connected.
- ❑ If an ordinary N is monotonically live, then there are no bad siphons.
- ❑ If  $N(m_0)$  is monotonically live, then the DTP holds.

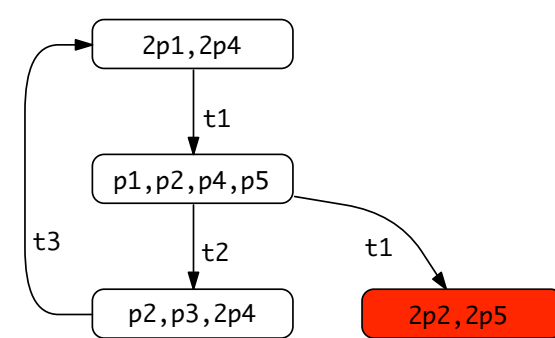
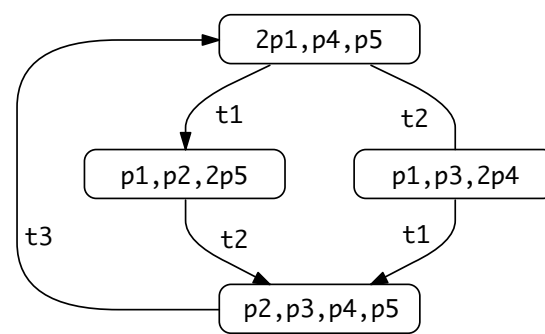
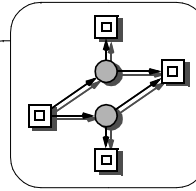
### BAD SIPHON 1

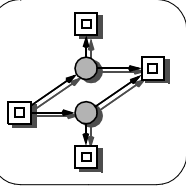


{p1, p3} - bad siphon  
t1- troublemaker



### BAD SIPHON 2





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