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Petri Net Representations of Bayesian Networks

Institut für Softwaretechnik
Universität Koblenz-Landau

Arbeitsgruppe
Petri-Netze

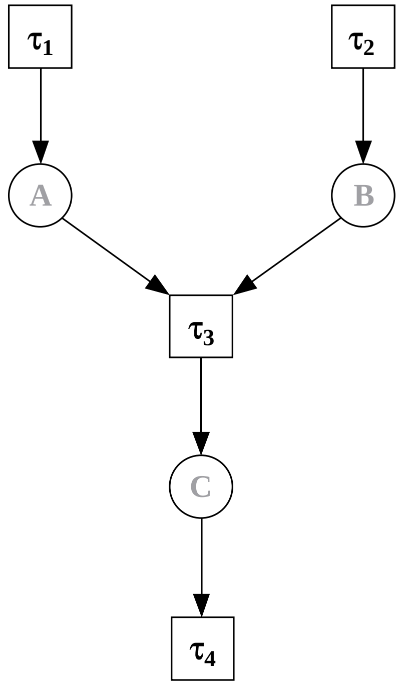
$$\alpha = A \wedge B \wedge (\neg A \vee \neg B \vee C) \wedge \neg C$$

$$\tau_1 = A$$

$$\tau_2 = B$$

$$\tau_3 = \neg A \vee \neg B \vee C$$

$$\tau_4 = \neg C$$

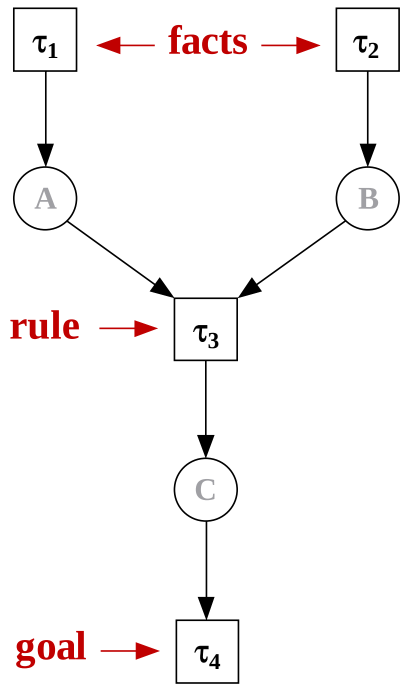


canonical net representation of α

Probabilistic Horn Abduction

$\alpha = A \wedge B \wedge (\neg A \vee \neg B \vee C) \wedge \neg C$

$\tau_1 = A$	fact
$\tau_2 = B$	fact
$\tau_3 = \neg A \vee \neg B \vee C$	rule
$\tau_4 = \neg C$	goal



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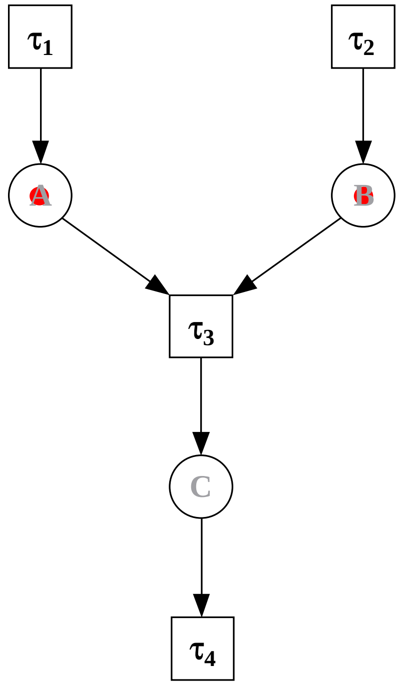

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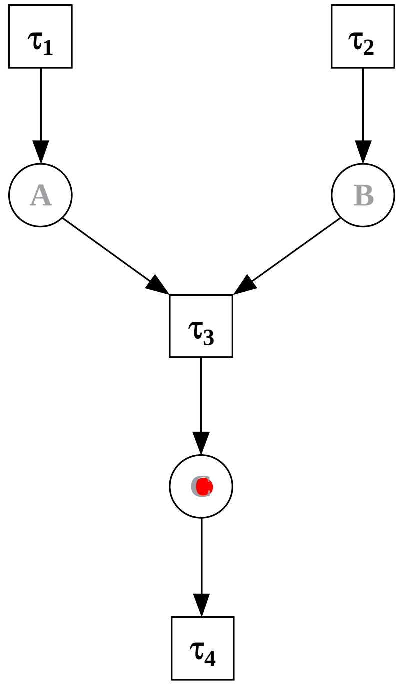

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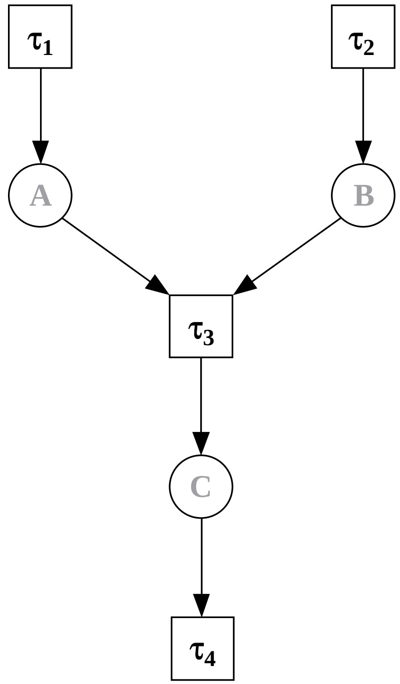

canonical net representation of α

Probabilistic Horn Abduction

$$\alpha = A \wedge B \wedge (\neg A \vee \neg B \vee C) \wedge \neg C$$

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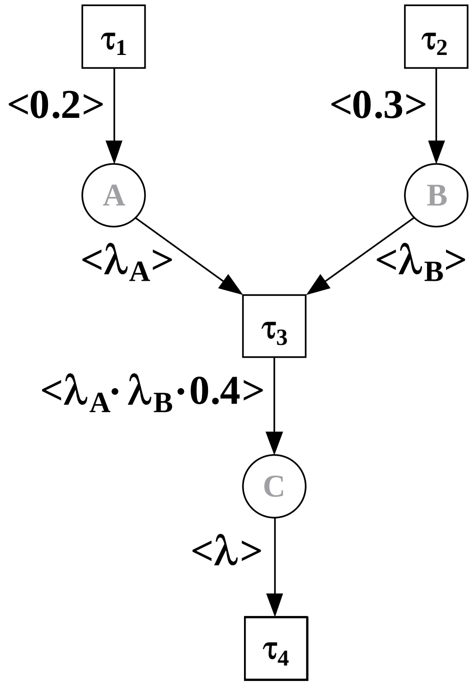
$\tau_3 = \neg A \vee \neg B \vee C$	**0.4**
$\tau_4 = \neg C$	**1.0**



probability propagation net

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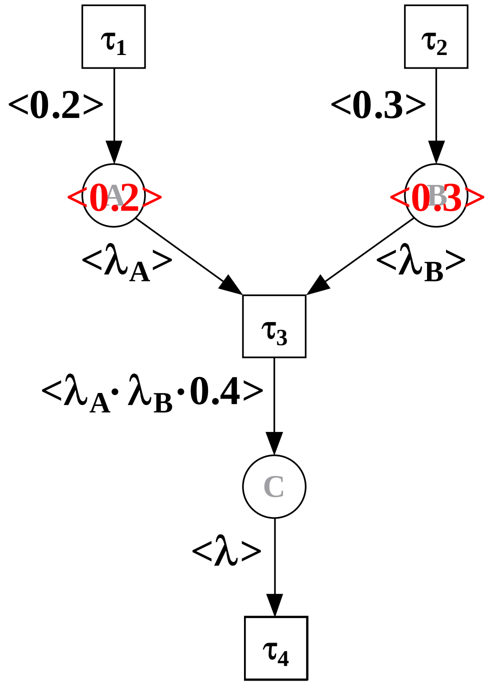

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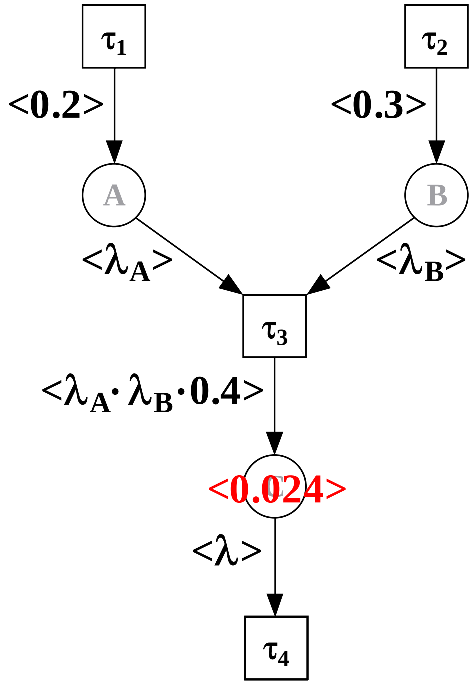

probability propagation net

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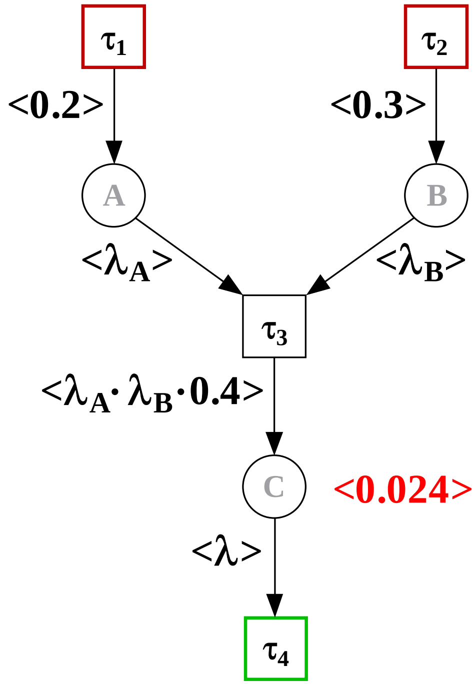
$\tau_4 = \neg C$	**1.0**

$A \wedge B = \tau_1 \wedge \tau_2$ is an explanation of $C = \neg\tau_4$

the probability of the explanation $A \wedge B$ is

$$P(A \wedge B \wedge (\neg A \vee \neg B \vee C)) = P(\tau_1 \wedge \tau_2 \wedge \neg\tau_4) = 0.024$$



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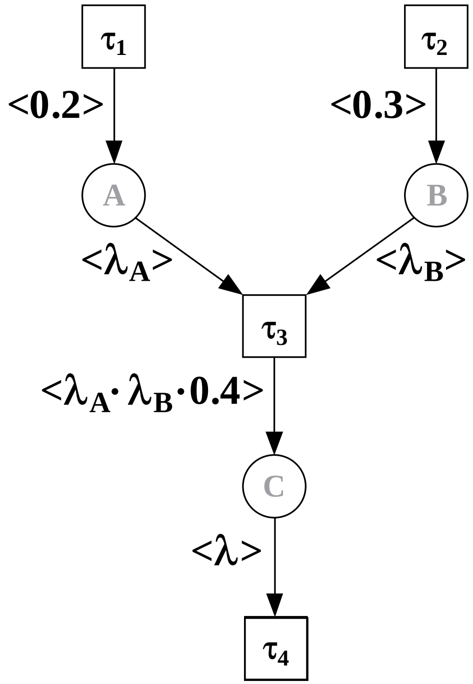

probability propagation net

$$\alpha = A \wedge (\neg A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee \neg C \vee D) \wedge \neg D$$

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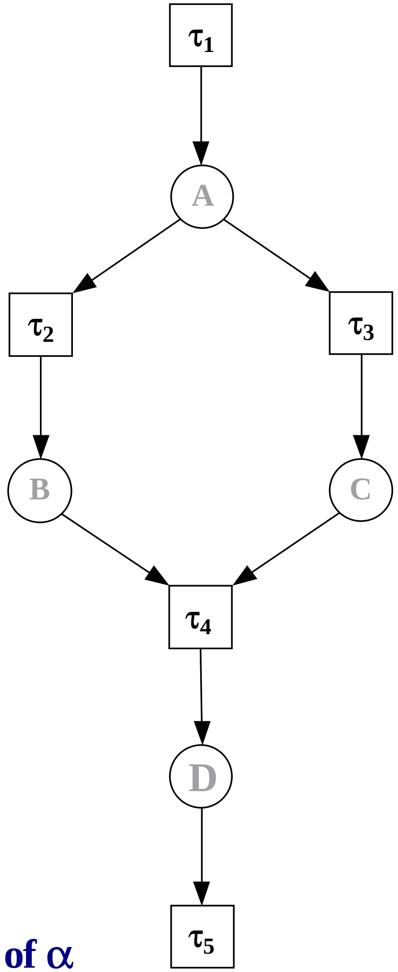

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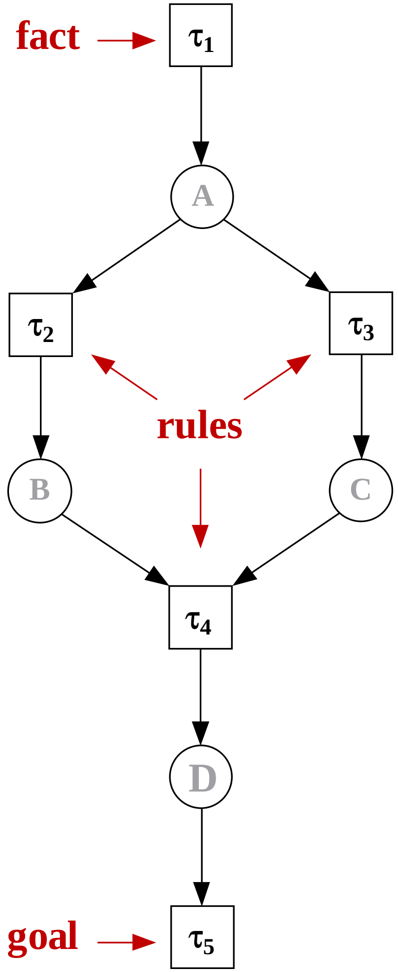
$\tau_3 = \neg A \vee C$	**rule**
$\tau_4 = \neg B \vee \neg C \vee D$	**rule**
$\tau_5 = \neg D$	**goal**



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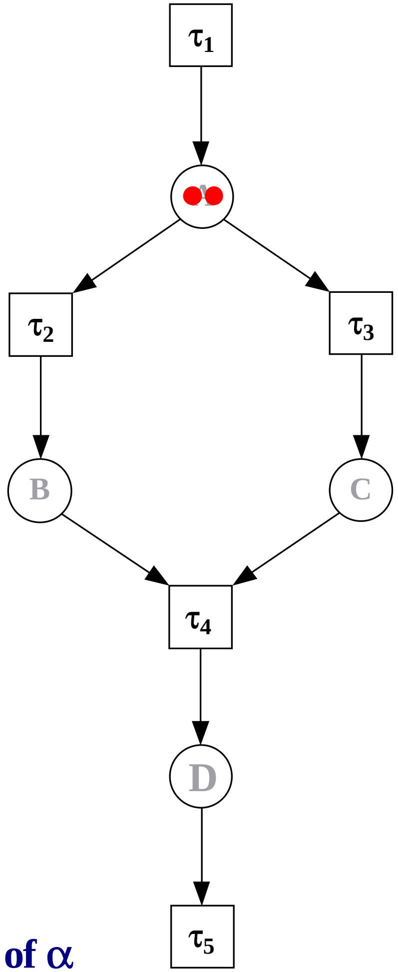

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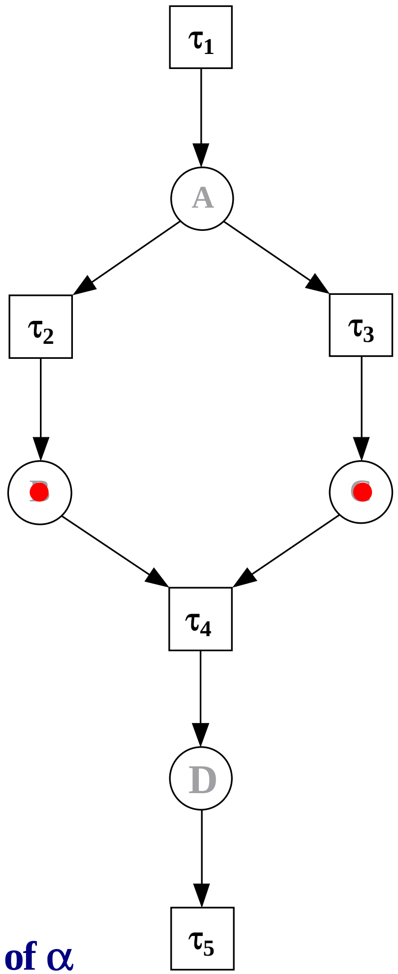

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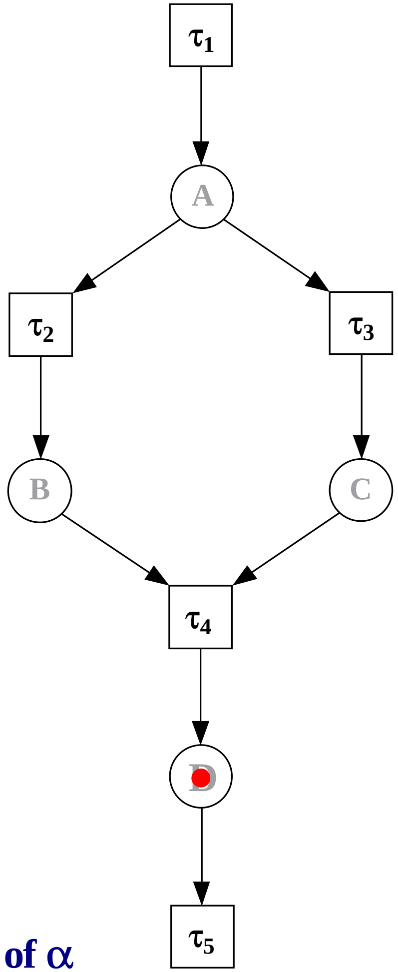

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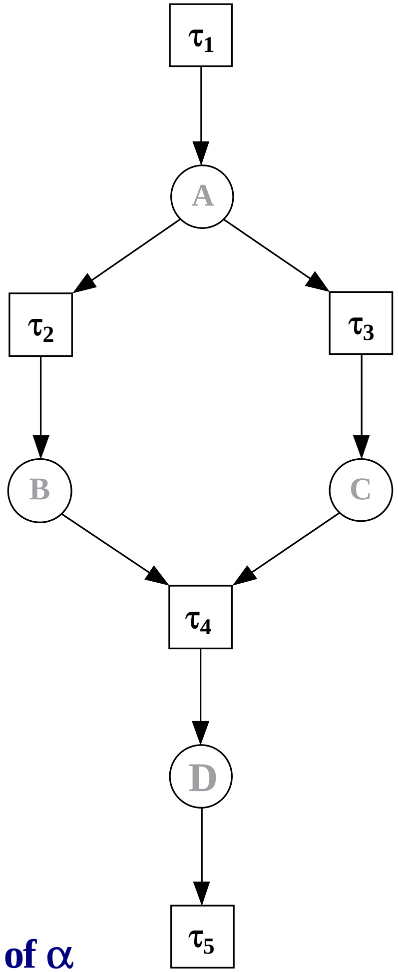

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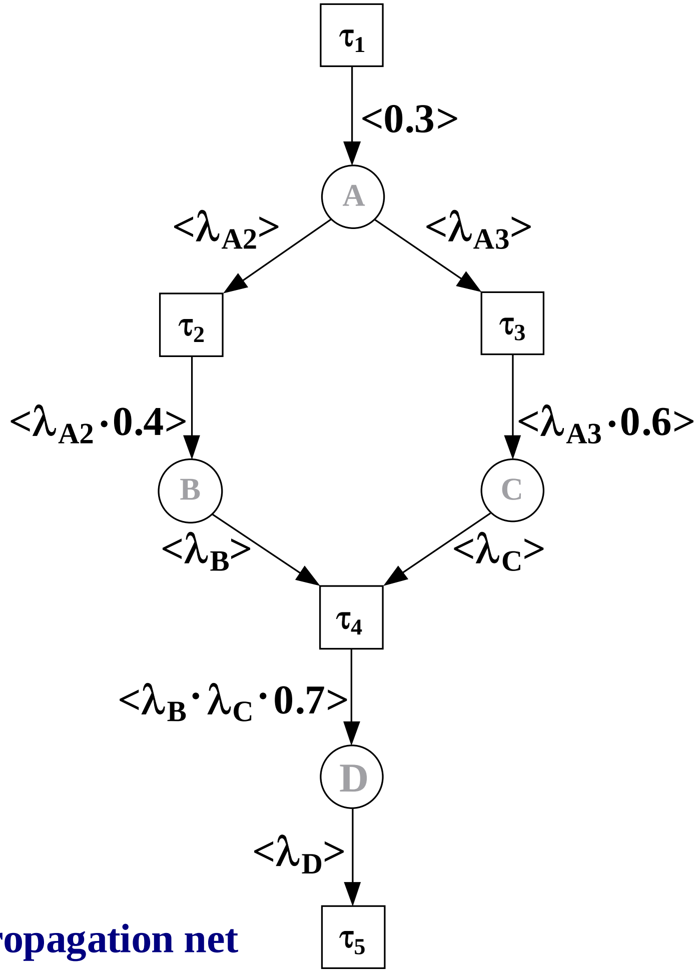

probability propagation net

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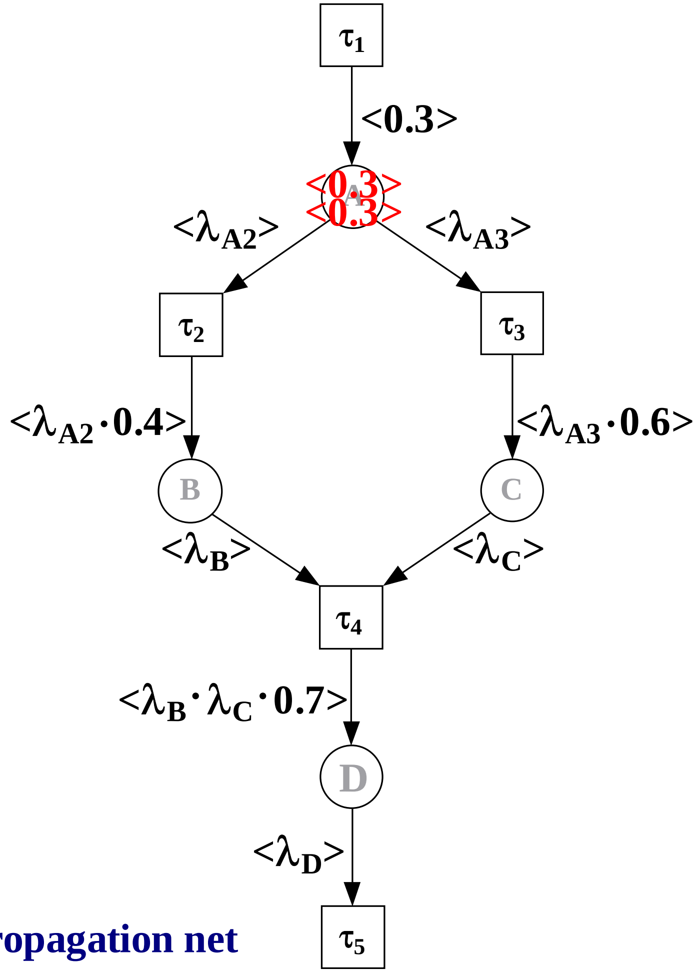

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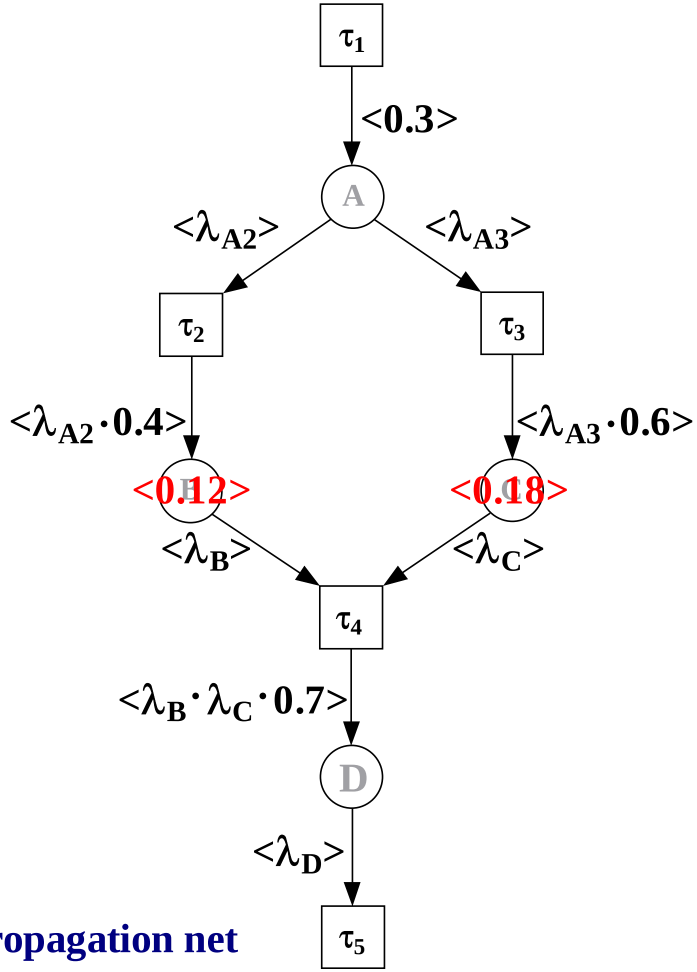

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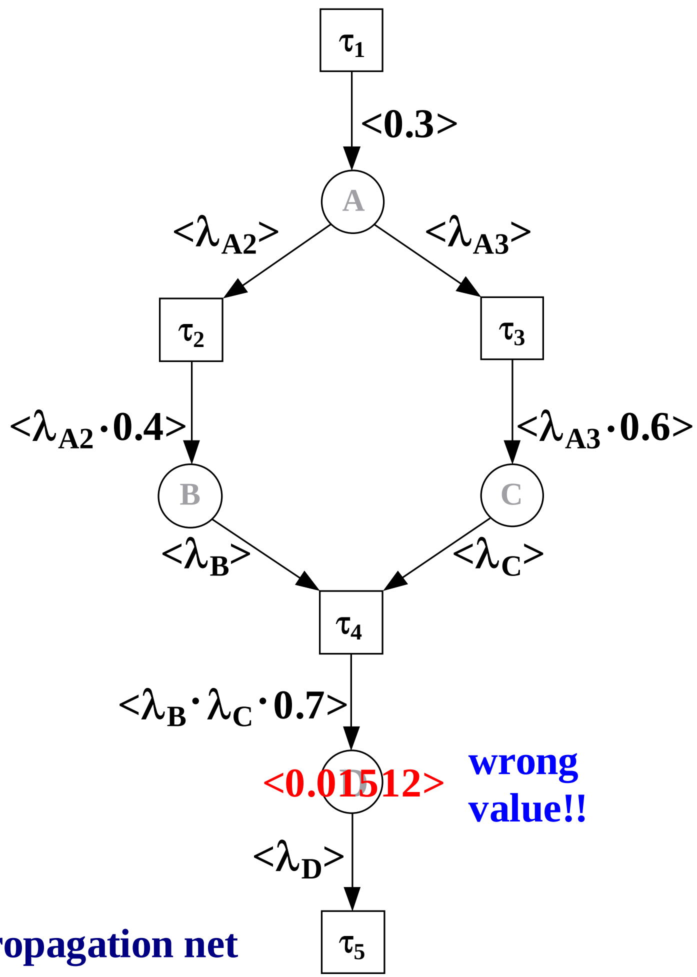

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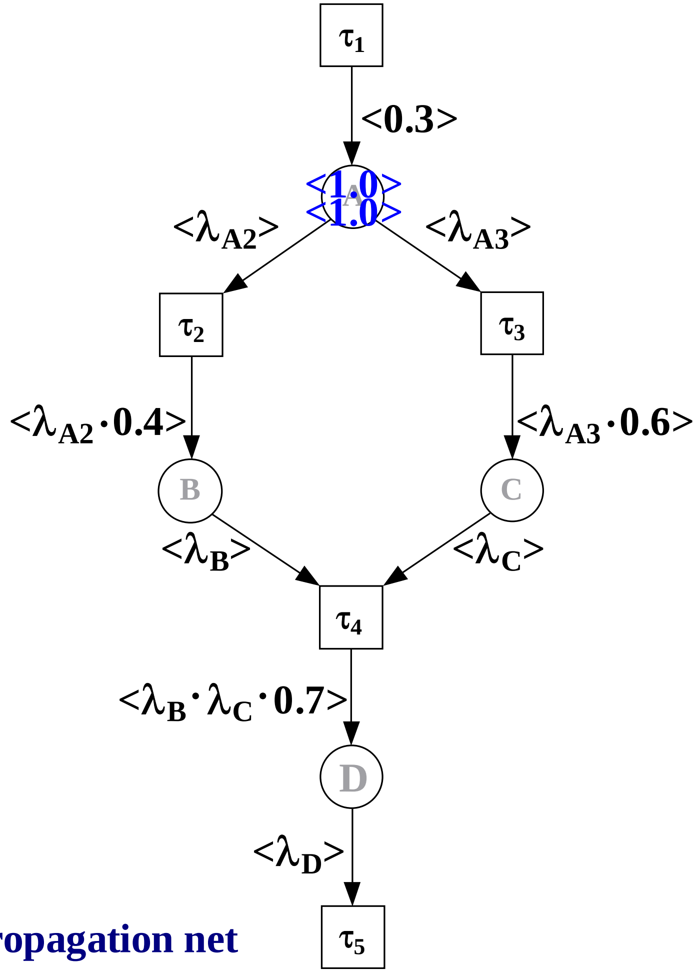

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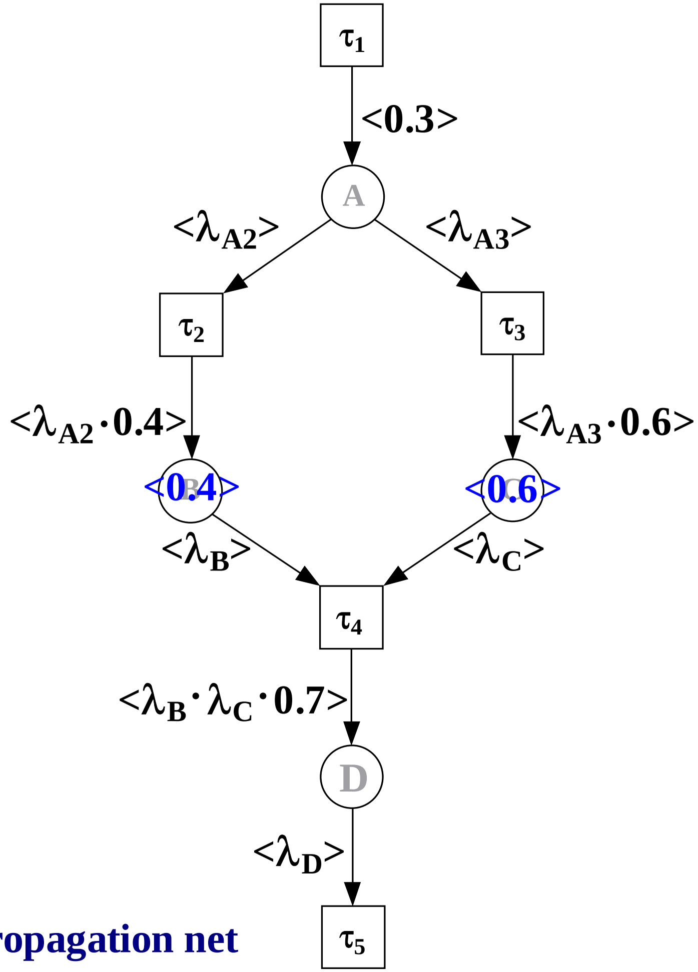

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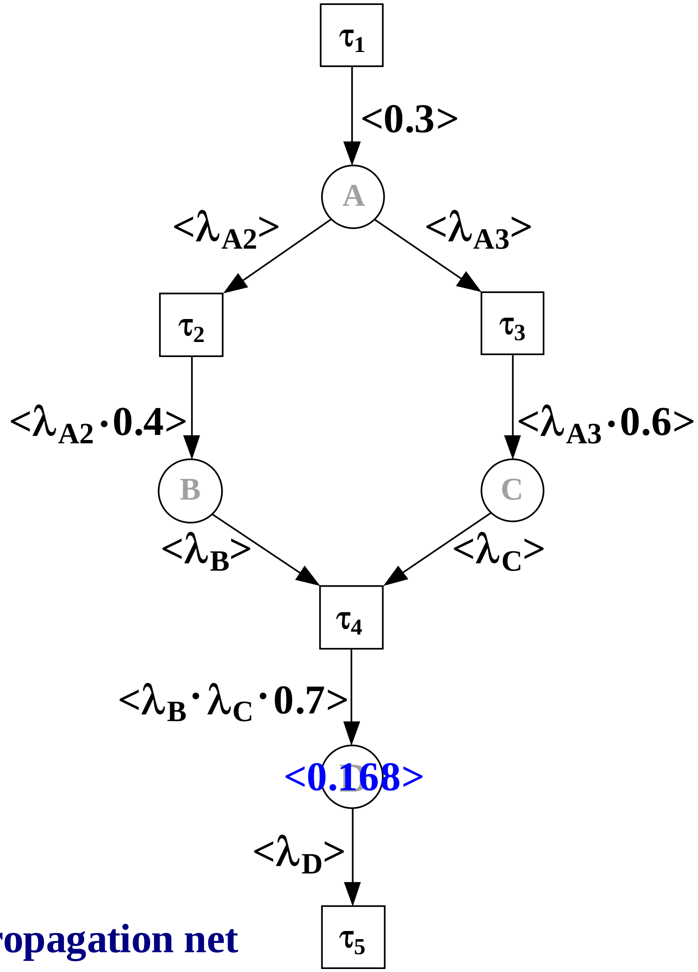

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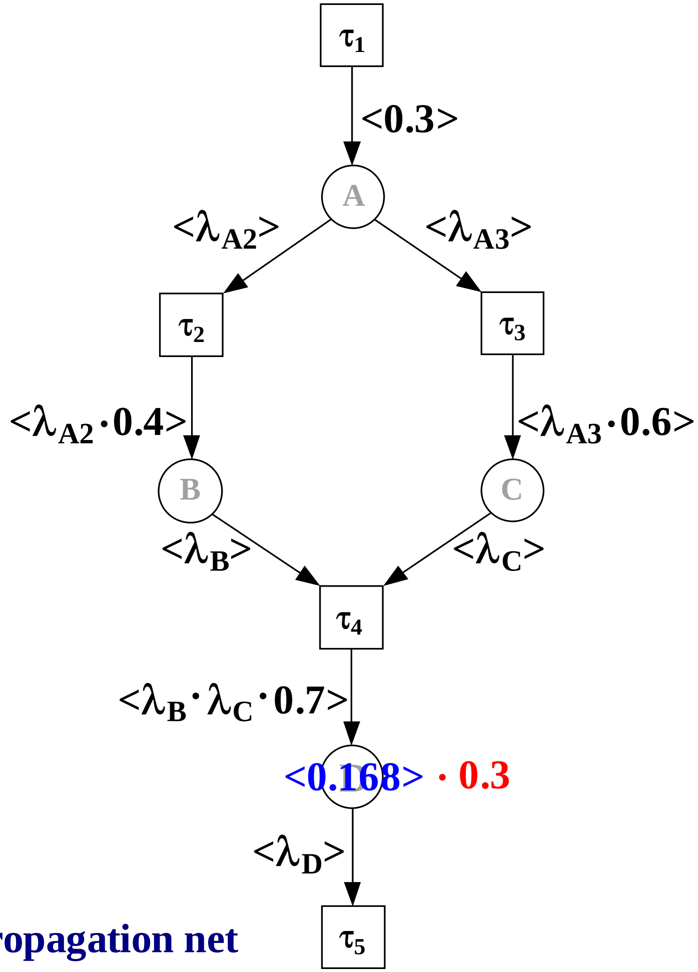

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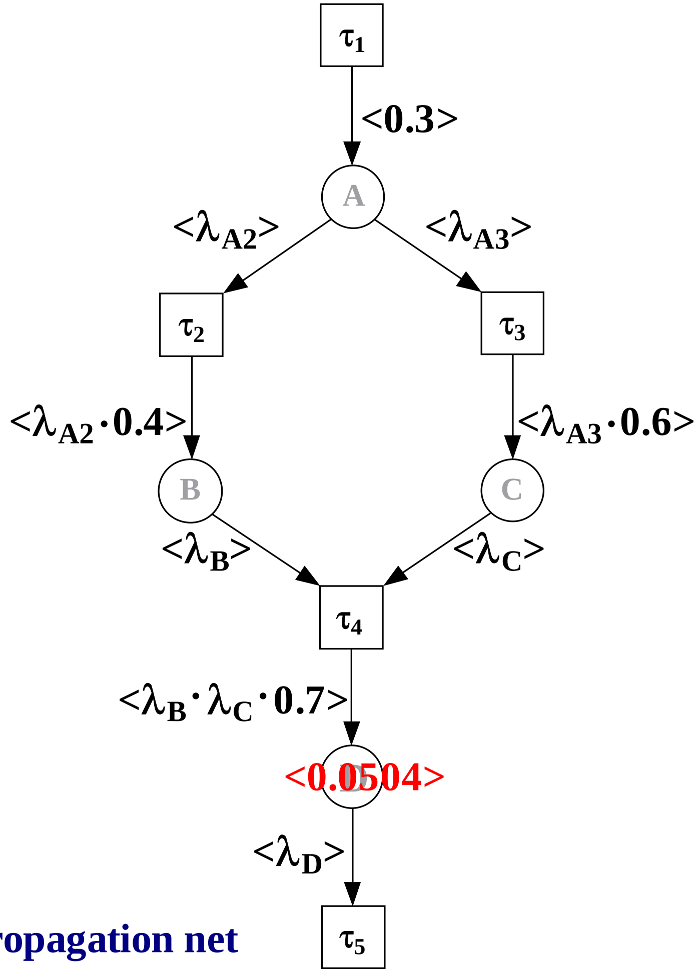

probability propagation net

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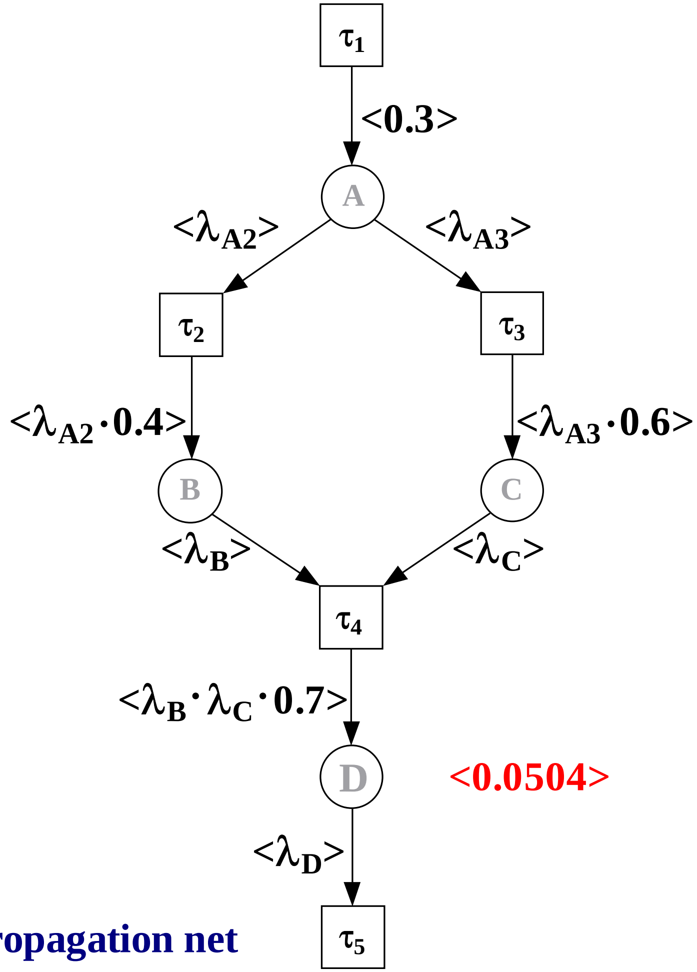
$\tau_5 = \neg D$	**1.0**

$A = \tau_1$ is an explanation of $D = \neg\tau_5$

the probability of the explanation A is

$P(A \wedge (\neg A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee \neg C \vee D)) =$

$P(\tau_1 \wedge \tau_2 \wedge \tau_3 \wedge \tau_4) = 0.0504$



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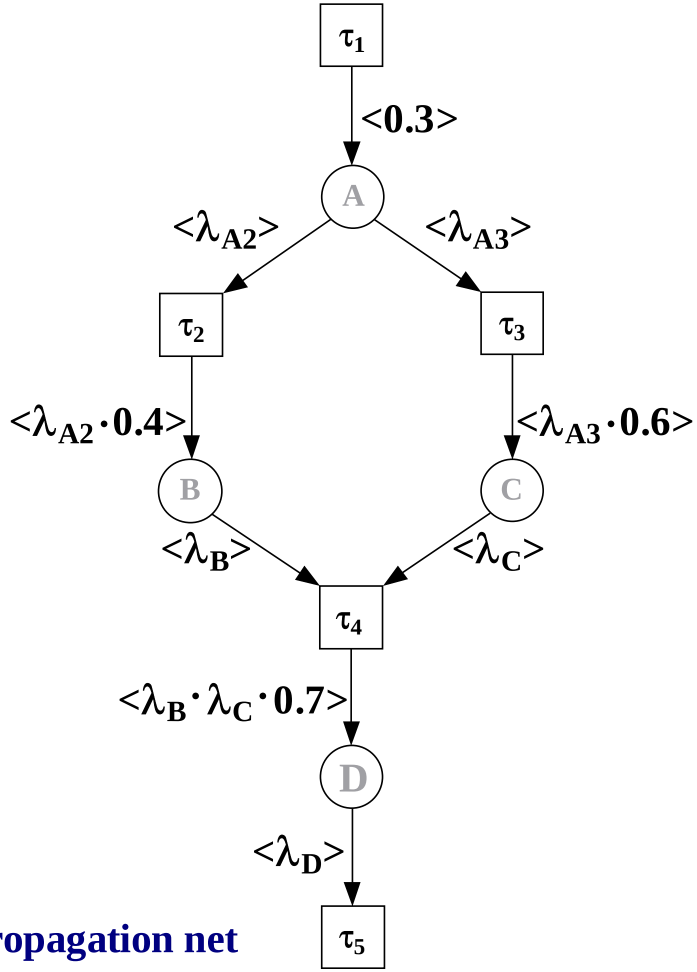


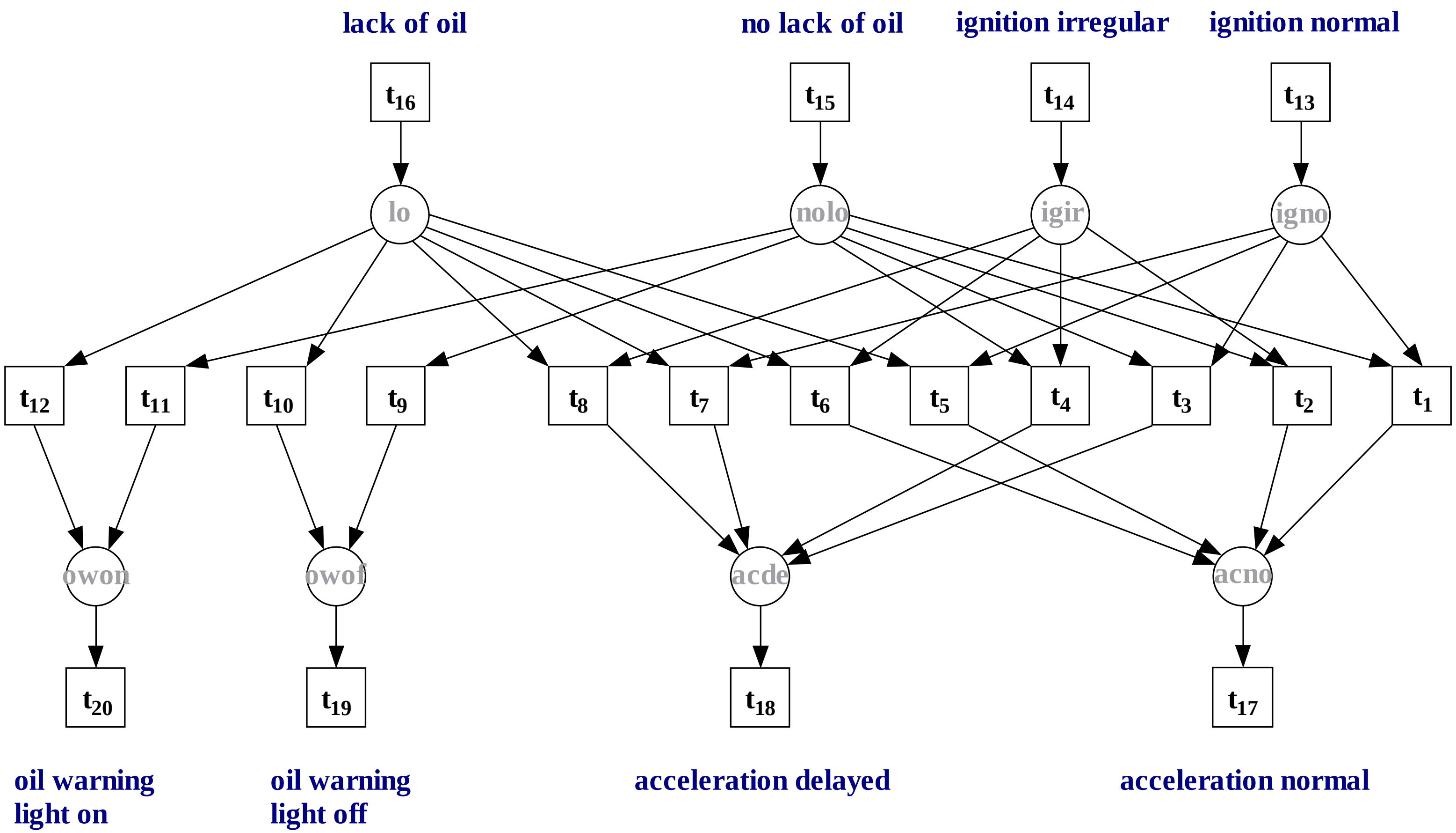
probability propagation net

Car Example

lack of oil	: 0.4	lack of oil	implies	oil warning light on	: 1.0
no lack of oil	: 0.6	no lack of oil	implies	oil warning light on	: 0.0
ignition normal	: 0.9	lack of oil	implies	oil warning light off	: 0.0
ignition irregular	: 0.1	no lack of oil	implies	oil warning light off	: 1.0

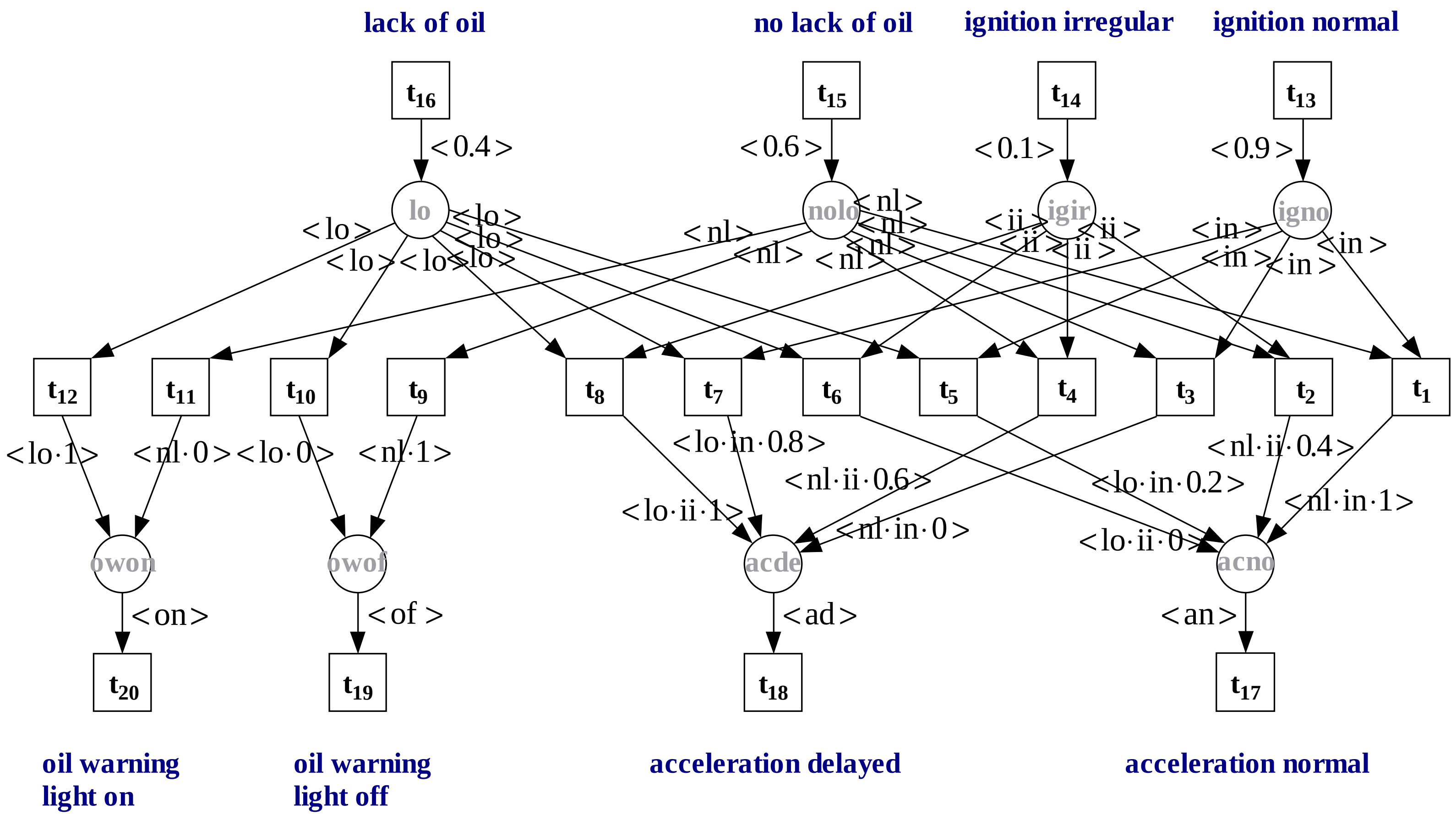
lack of oil	and	ignition irregular	implies	acceleration normal	: 0.0
lack of oil	and	ignition normal	implies	acceleration normal	: 0.2
lack of oil	and	ignition irregular	implies	acceleration delayed	: 1.0
lack of oil	and	ignition normal	implies	acceleration delayed	: 0.8
no lack of oil	and	ignition irregular	implies	acceleration normal	: 0.4
no lack of oil	and	ignition normal	implies	acceleration normal	: 1.0
no lack of oil	and	ignition irregular	implies	acceleration delayed	: 0.6
no lack of oil	and	ignition normal	implies	acceleration delayed	: 0.0

Car Example



canonical net representation

Car Example



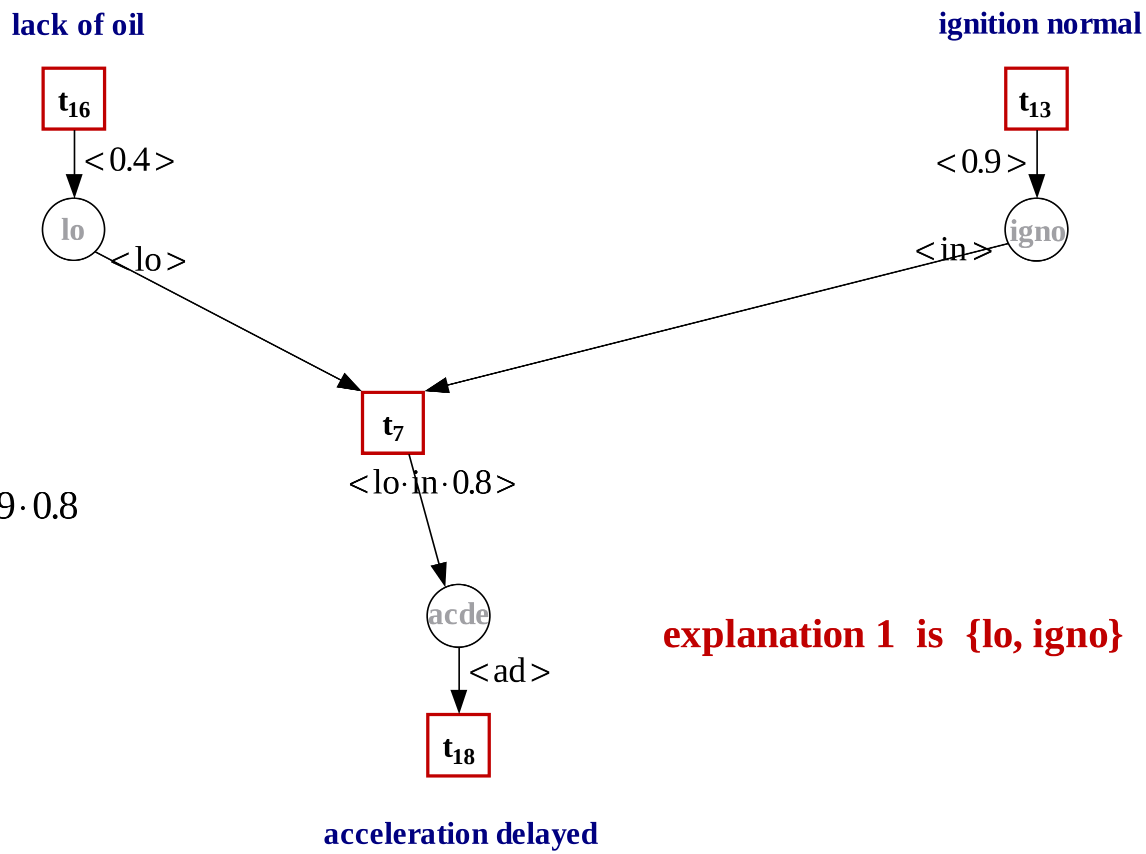
probability propagation net

Problem: **What is the probability of "acceleration delayed" (acde, t_{18}) ?**
What are the corresponding explanations?

There are 4 t-invariants passing through t_{18}:

Car Example

t-invariant 1



$P_1(acde) = 0.4 \cdot 0.9 \cdot 0.8$

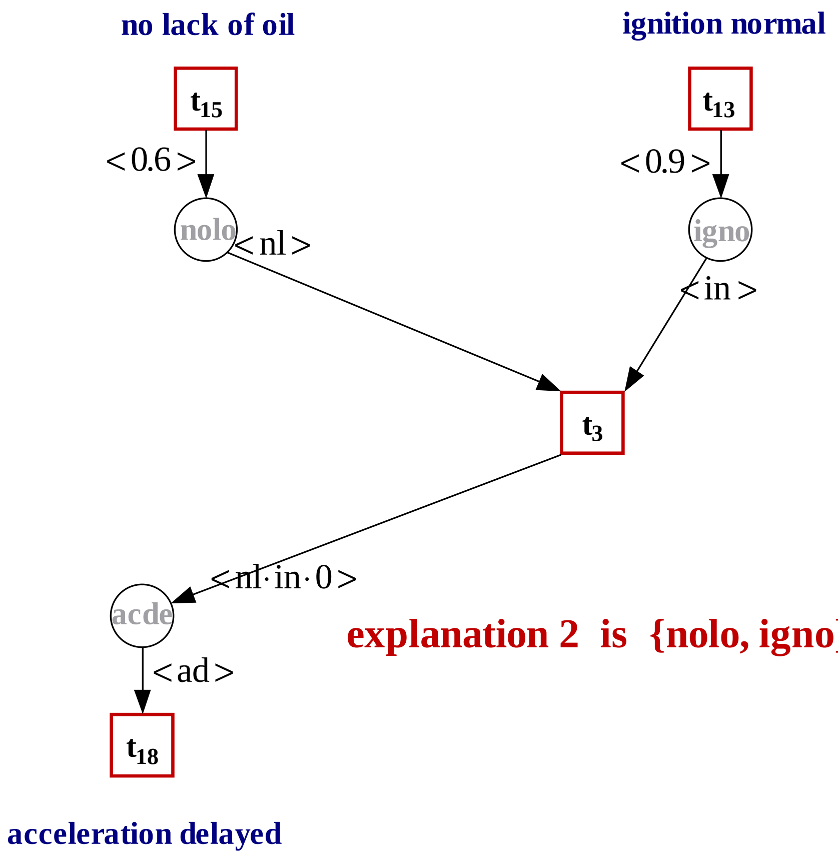
$= 0.288$

explanation 1 is {lo, igno}

Car Example

t-invariant 2

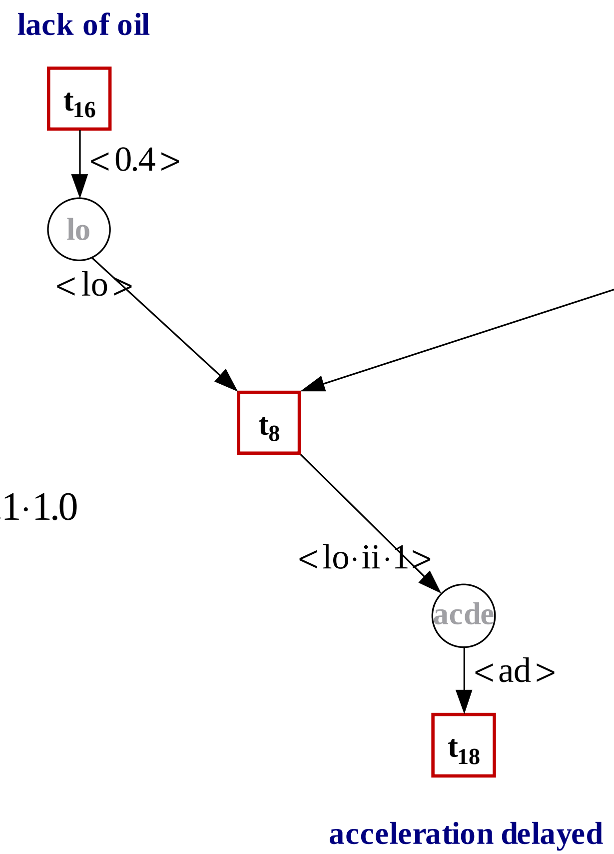
$$P_2(\text{acde}) = 0.6 \cdot 0.9 \cdot 0.0$$
$$= \mathbf{0.0}$$



explanation 2 is {nolo, igno}

Car Example

t-invariant 3



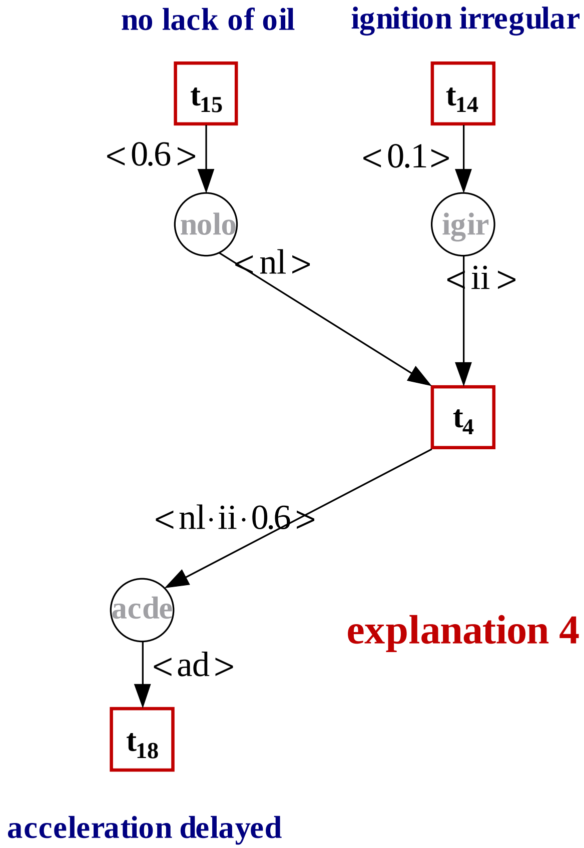
$P_3(acde) = 0.4 \cdot 0.1 \cdot 1.0$

$= \mathbf{0.04}$

explanation 3 is {lo, igir}

Car Example

t-invariant 4



$P_4(acde) = 0.6 \cdot 0.1 \cdot 0.6$

$= \mathbf{0.036}$

explanation 4 is {nolo, igir}

Problem: What is the probability of "acceleration delayed" (acde, t_{18}) ?

What are the corresponding explanations?

The explanations and their probabilities are:

{lo, igno}	0.288
{nolo, igno}	0.0
{lo, igir}	0.04
{nolo, igir}	0.036

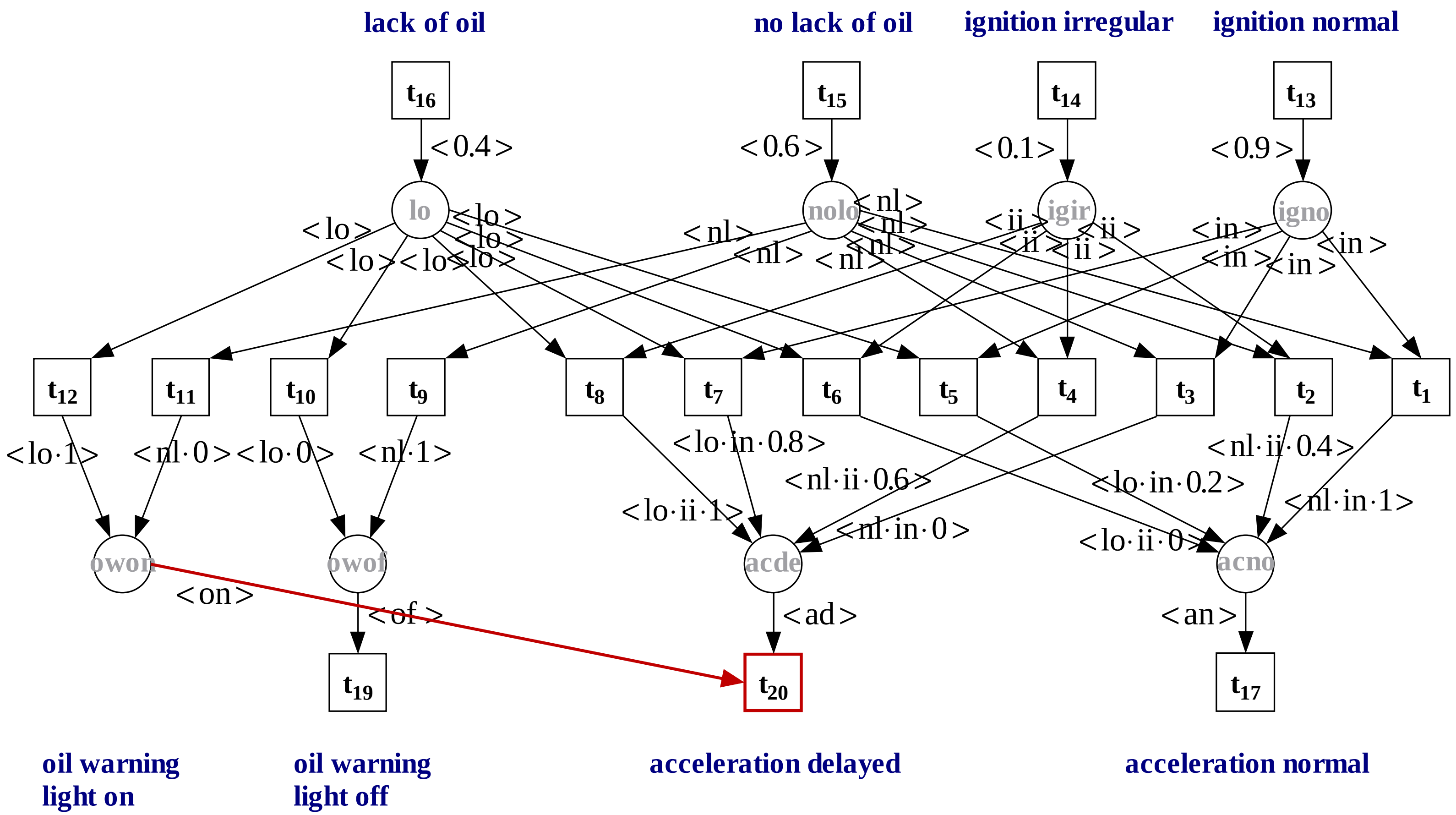
0.364 is the probability of "acceleration delayed".

Problem: **What is the probability of "acceleration delayed" and "oil warning light on"?**

What are the corresponding explanations?

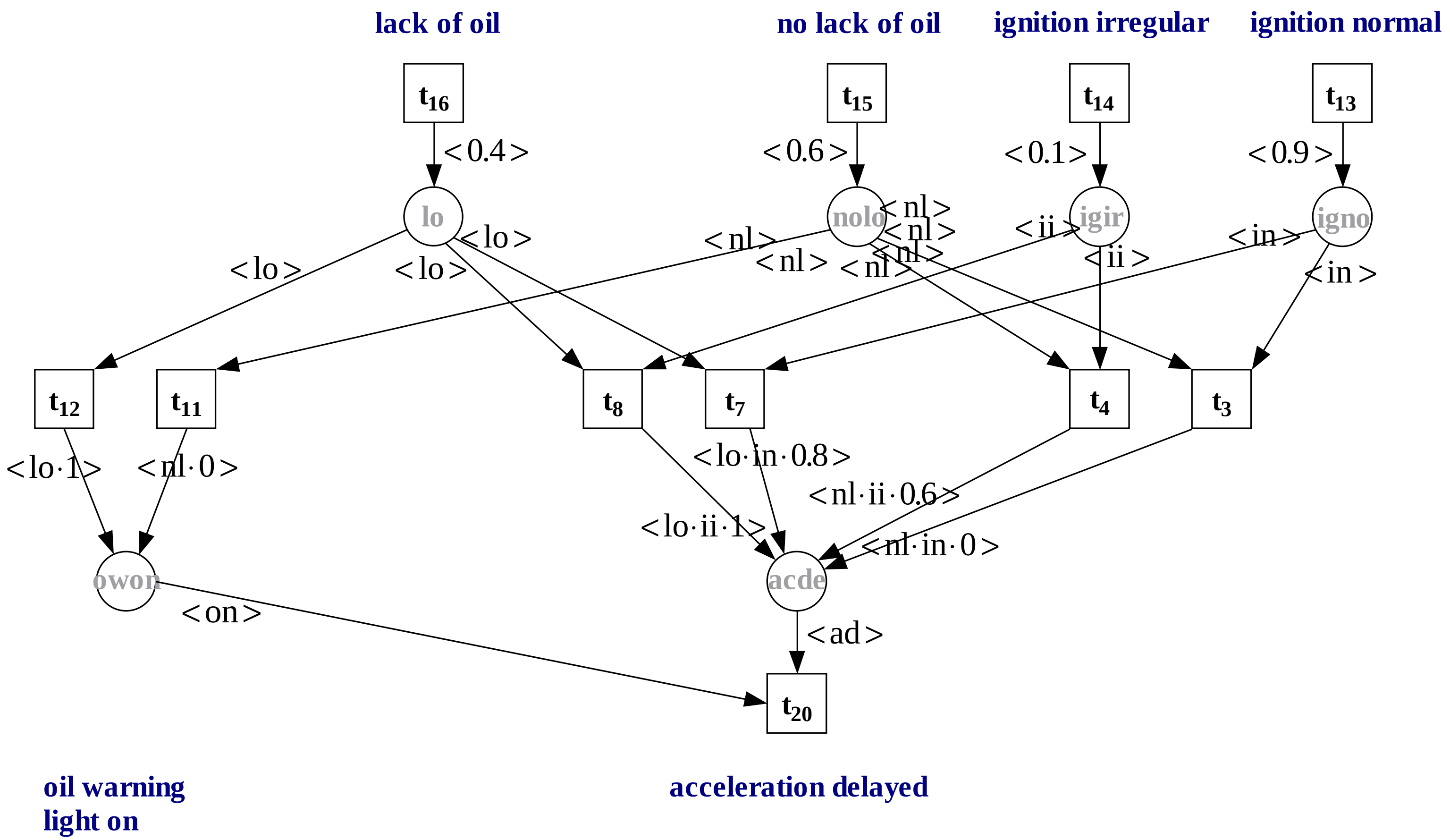
The net has to be slightly modified.

Car Example



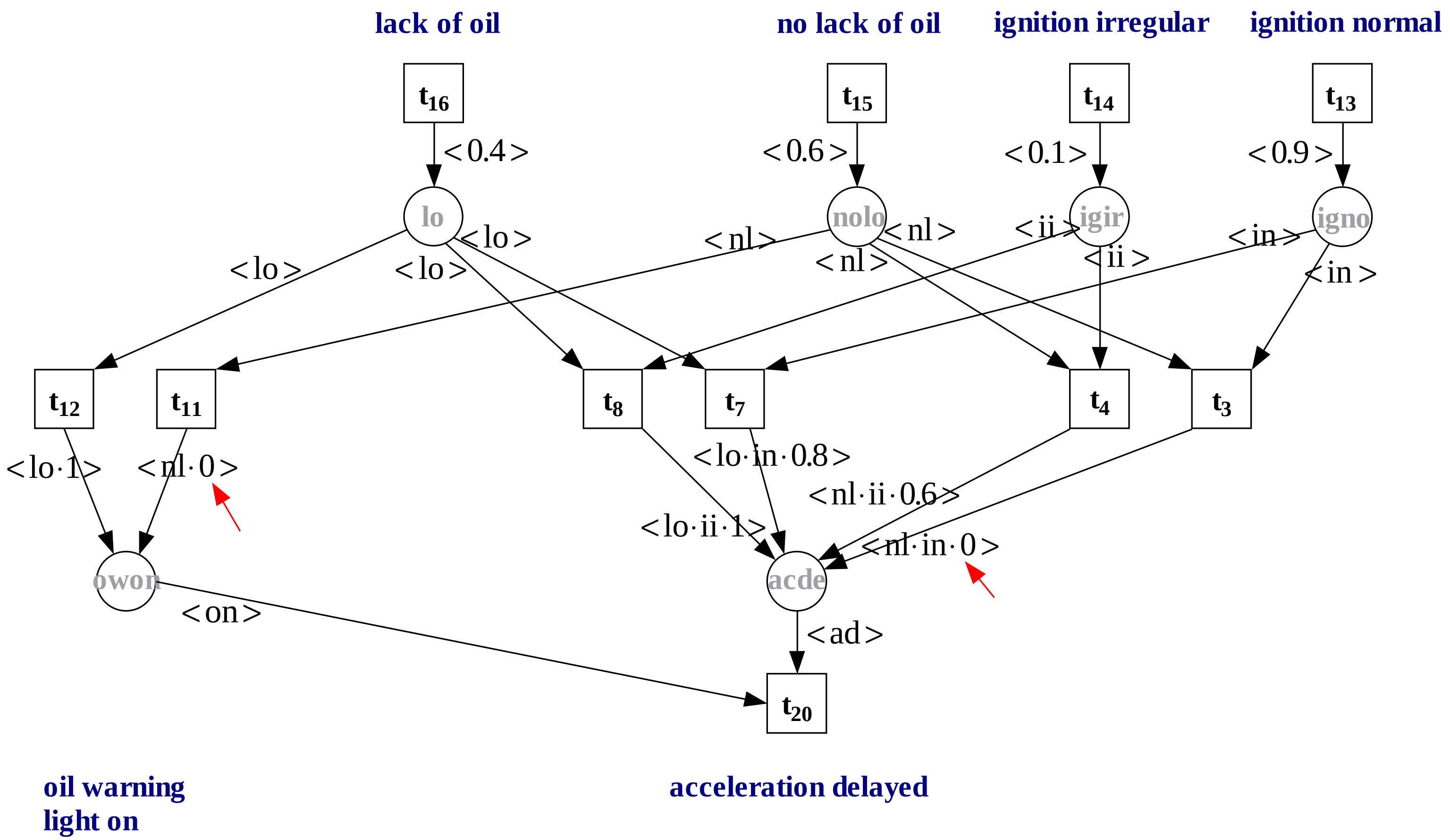
probability propagation net

Car Example

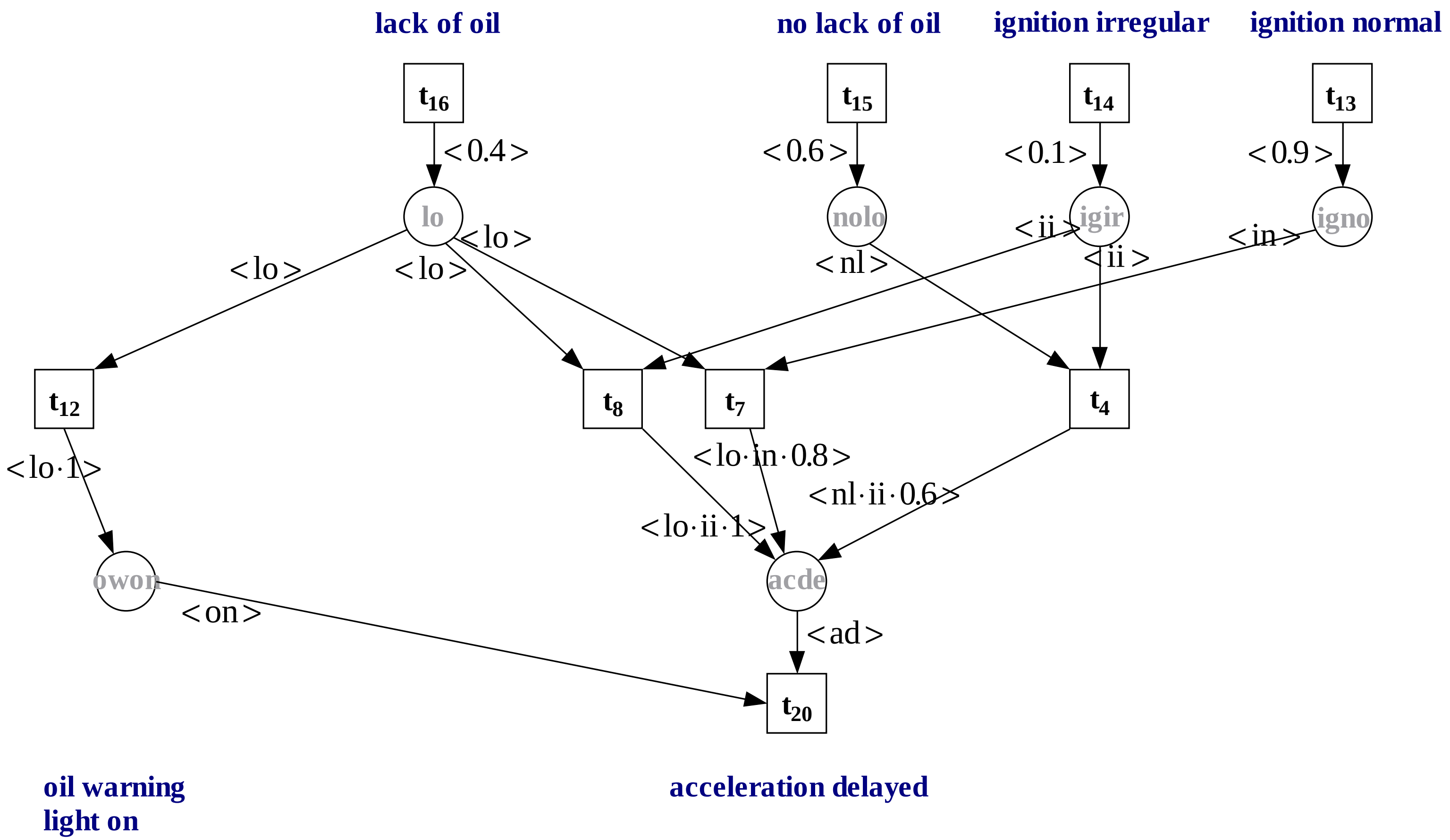


probability propagation net

Car Example

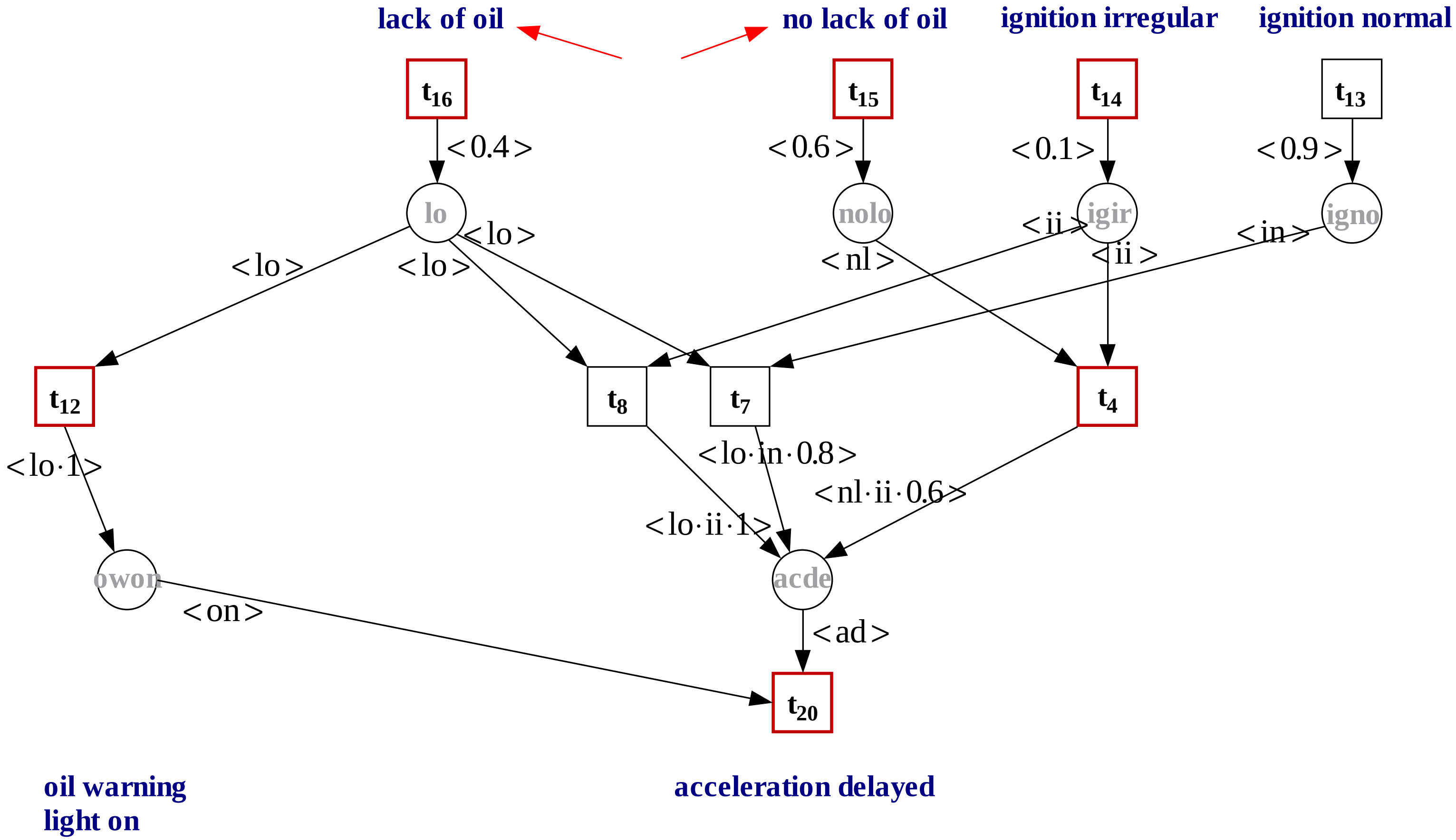


probability propagation net



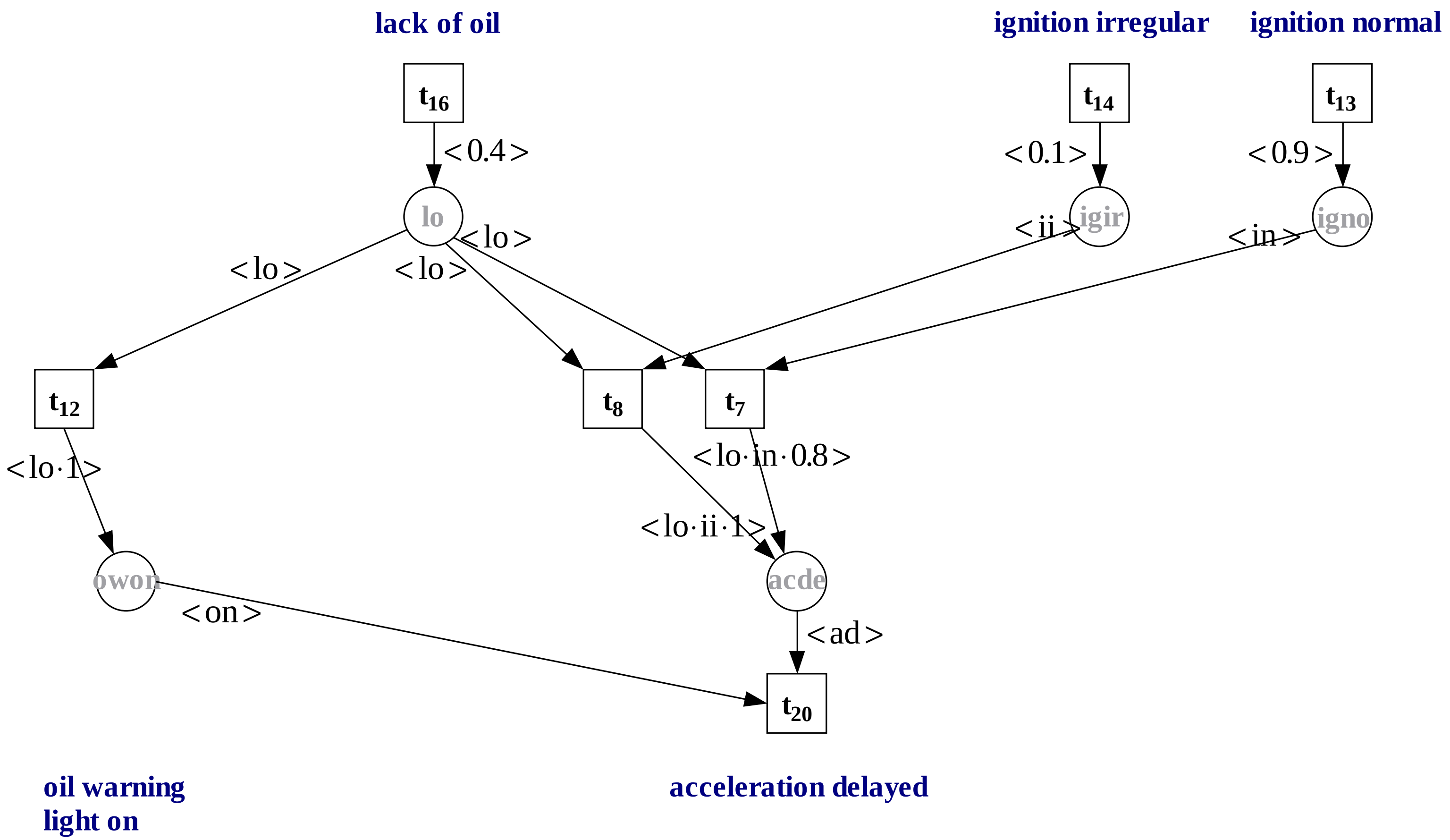
probability propagation net

Car Example



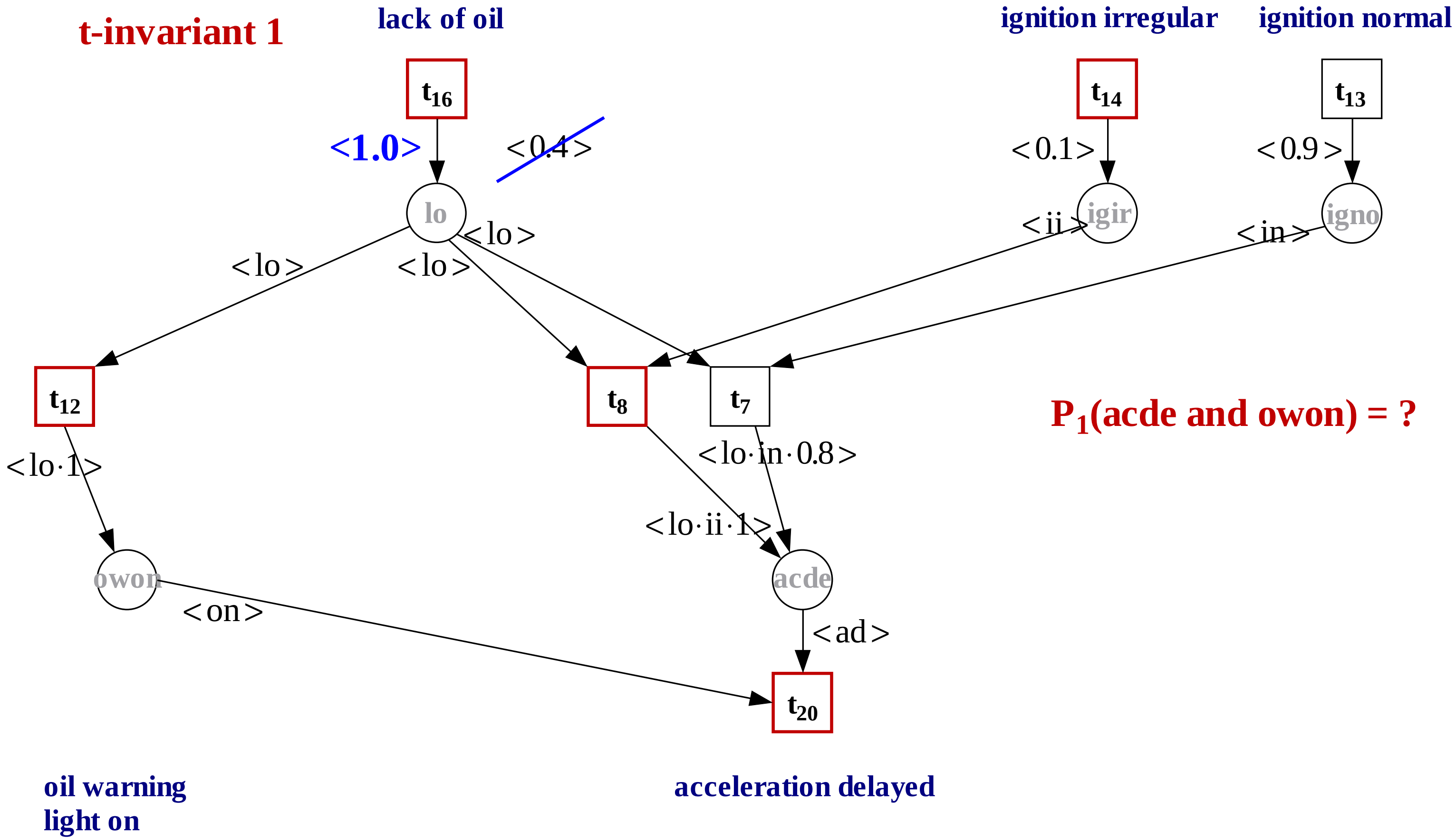
probability propagation net

Car Example



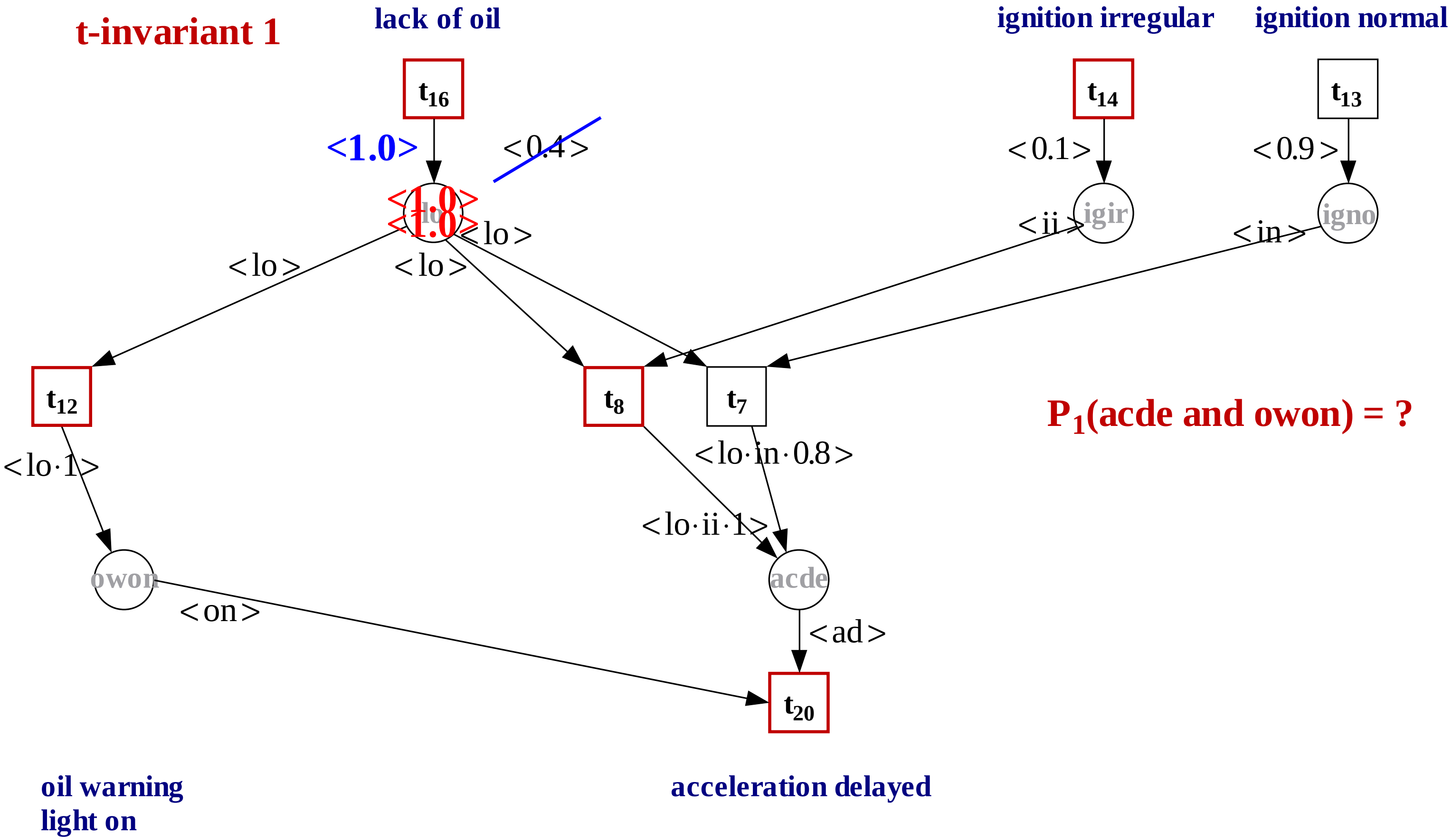
the relevant probability propagation net

Car Example


t-invariant 1
lack of oil
ignition irregular
ignition normal
t16
t14
t13
<1.0>
<0.4>
<0.1>
<0.9>
lo
igir
igno
<lo>
<lo>
<lo>
<ii>
<in>
t12
t8
t7
<lo·1>
<lo·in·0.8>
<lo·ii·1>
owon
acde
<on>
<ad>
t20
P1(acde and owon) = ?
oil warning light on
acceleration delayed

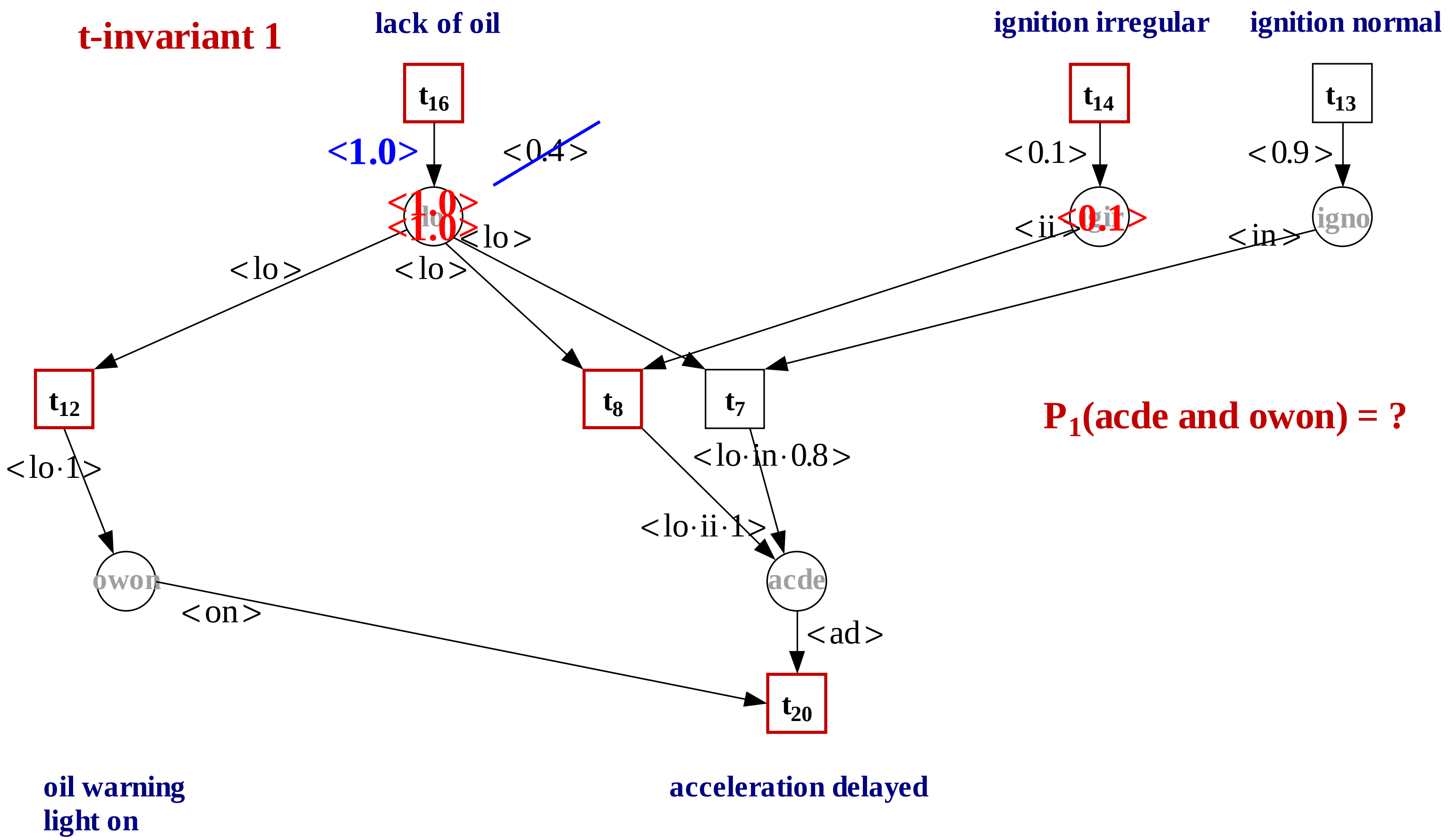
Car Example

t-invariant 1

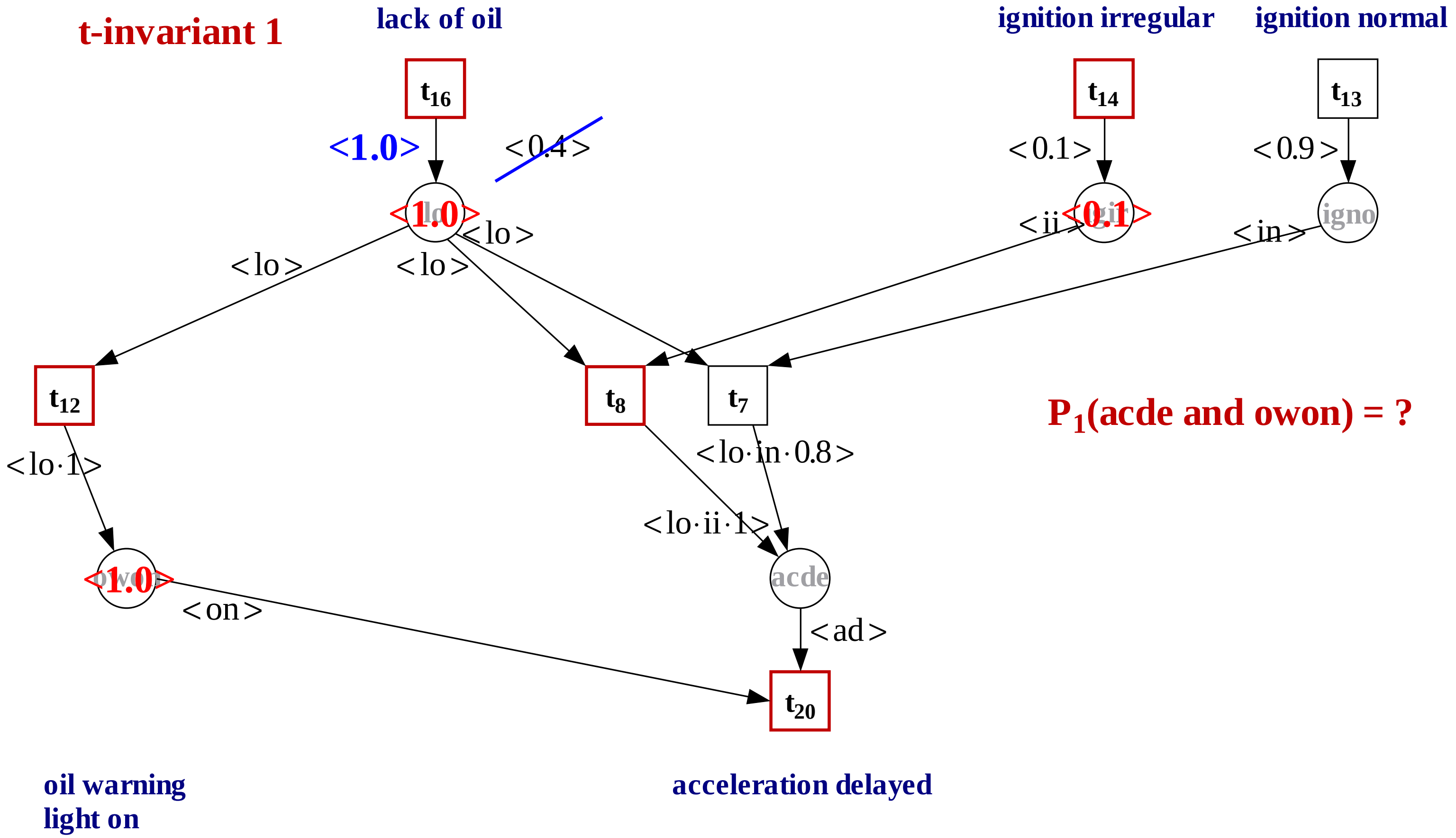


P_1(acde and owon) = ?

Car Example


t-invariant 1
lack of oil
ignition irregular
ignition normal
t16
t14
t13
<1.0>
<0.4>
<0.1>
<0.9>
<1.8>
lo
<lo>
<lo>
<lo>
gir
<ii>
<0.1>
igno
<in>
t12
t8
t7
<lo·1>
<lo·in·0.8>
<lo·ii·1>
owon
acde
<on>
<ad>
t20
oil warning
light on
acceleration delayed
P1(acde and owon) = ?

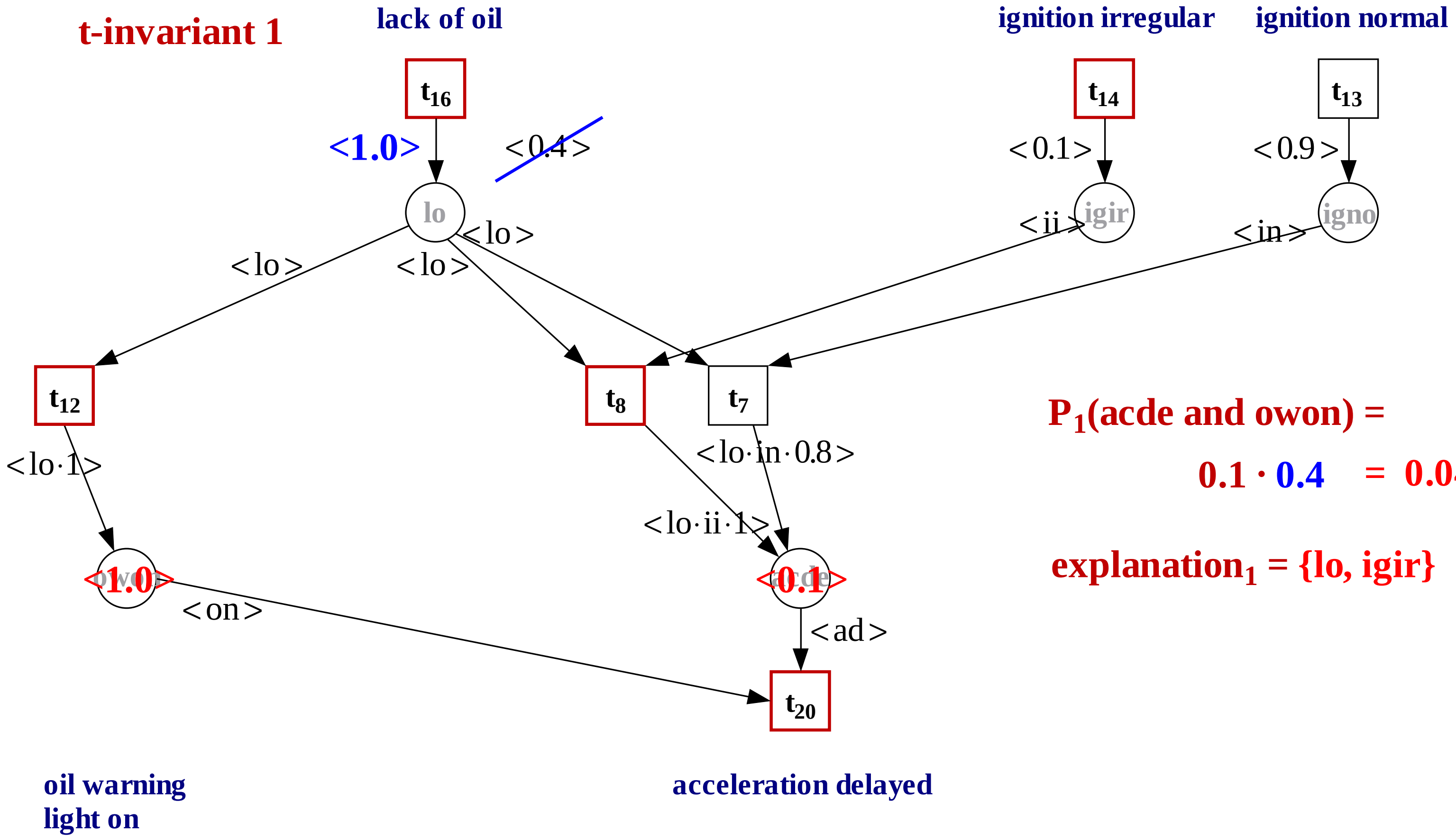
Car Example



P_1(acde and owon) = ?

Car Example

t-invariant 1

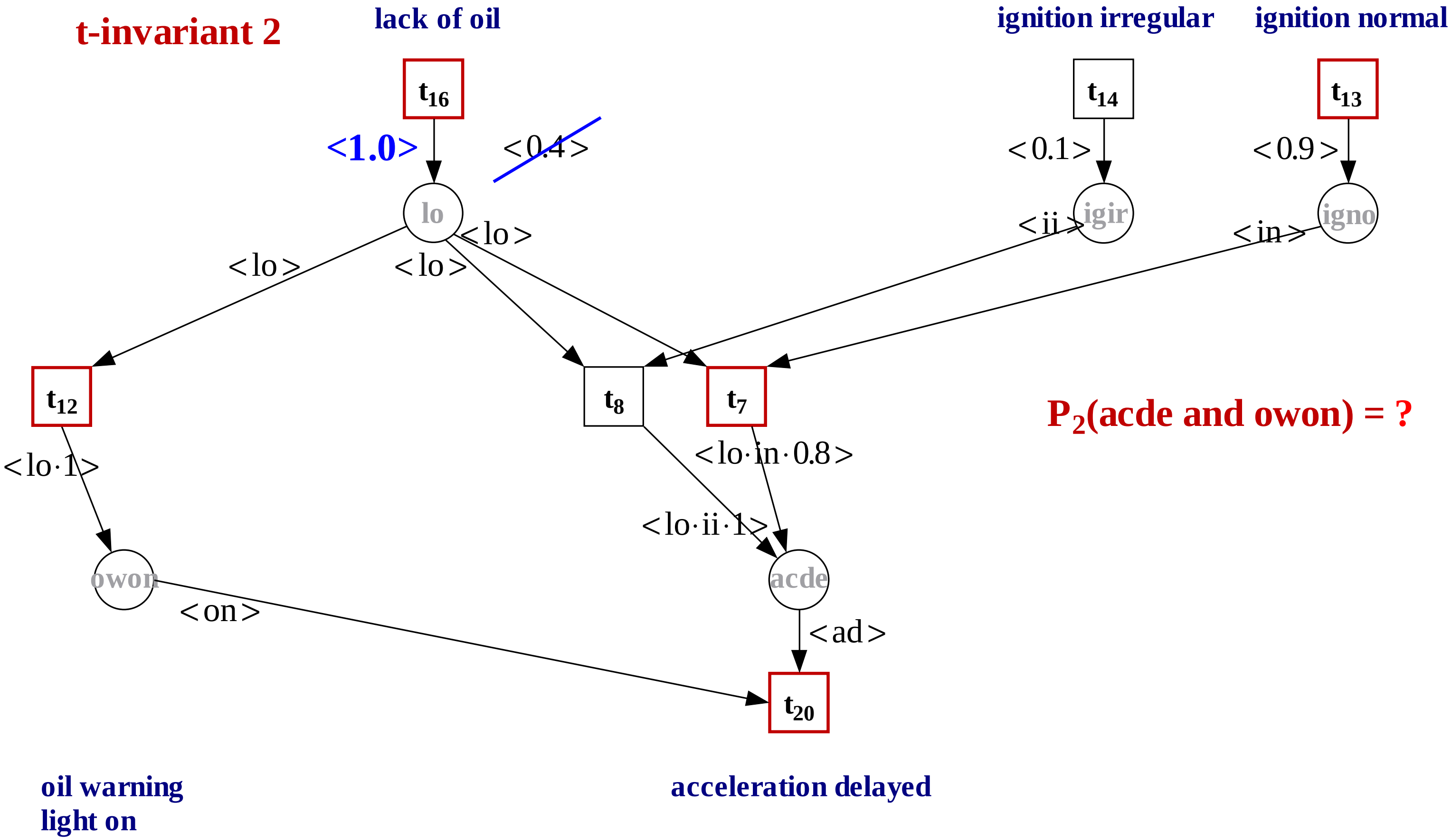


P_1(acde and owon) =

0.1 · 0.4 = 0.04

explanation$_1$ = {lo, igir}

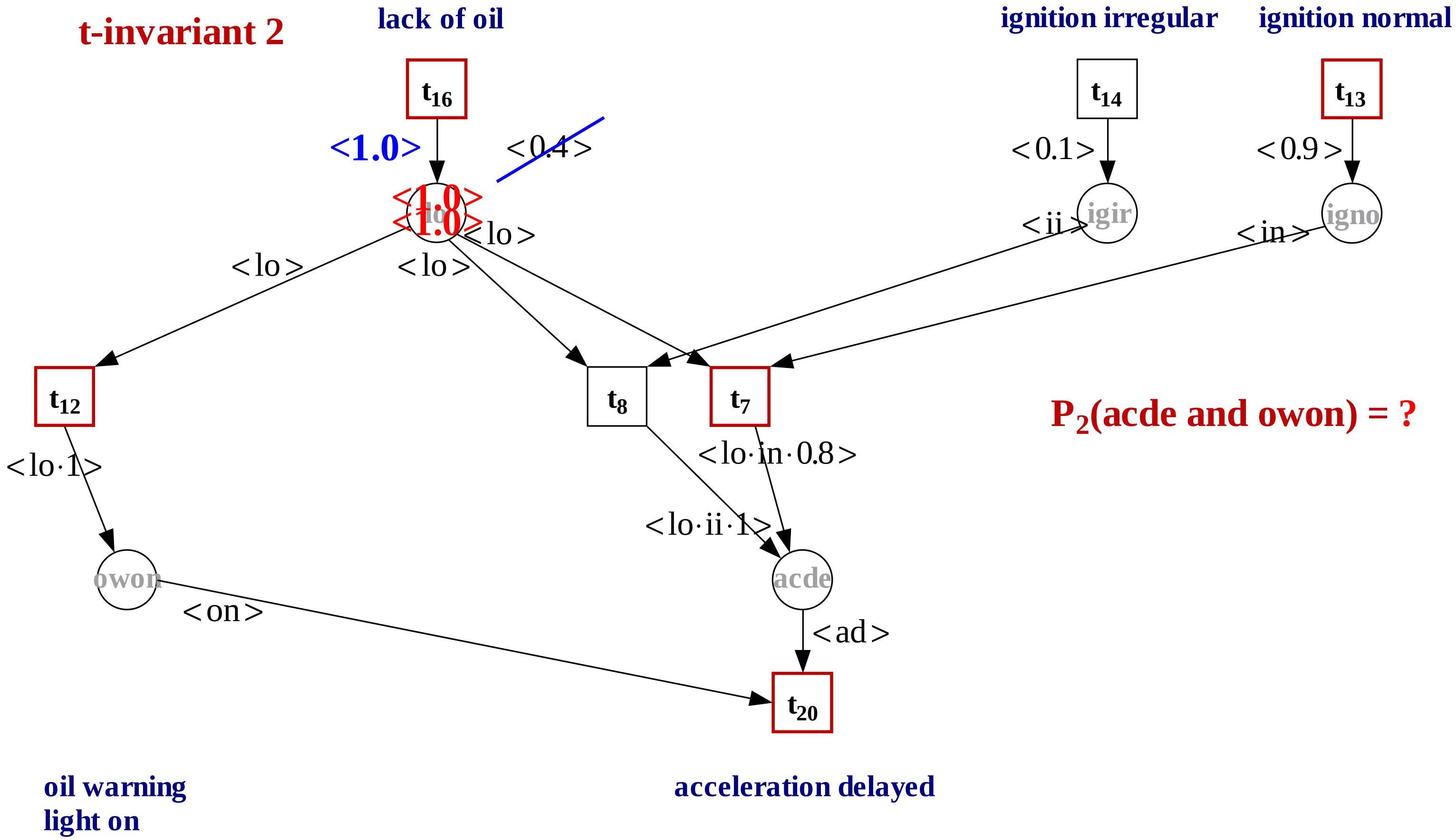
Car Example



P_2(acde and owon) = ?

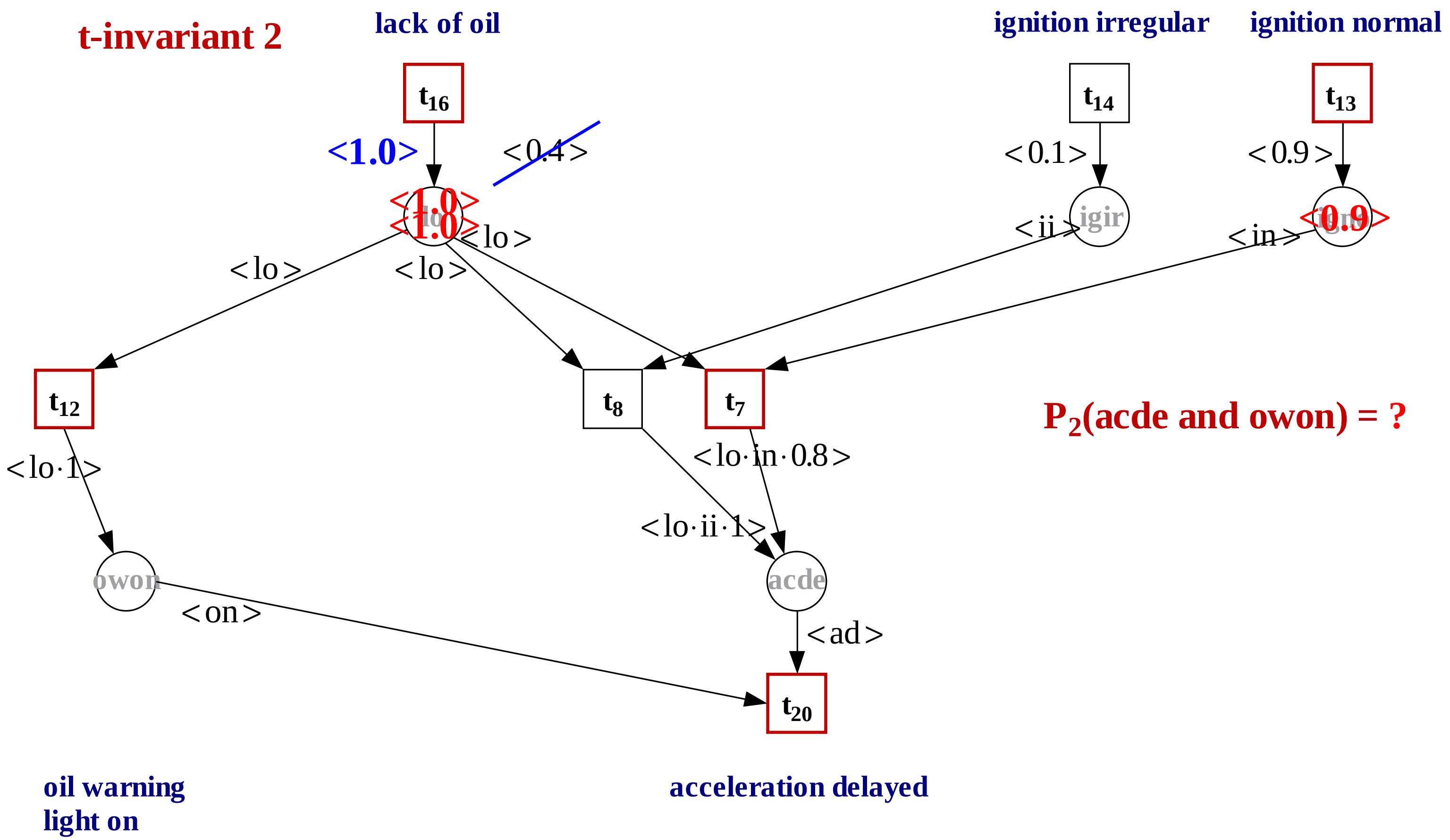
Car Example

t-invariant 2

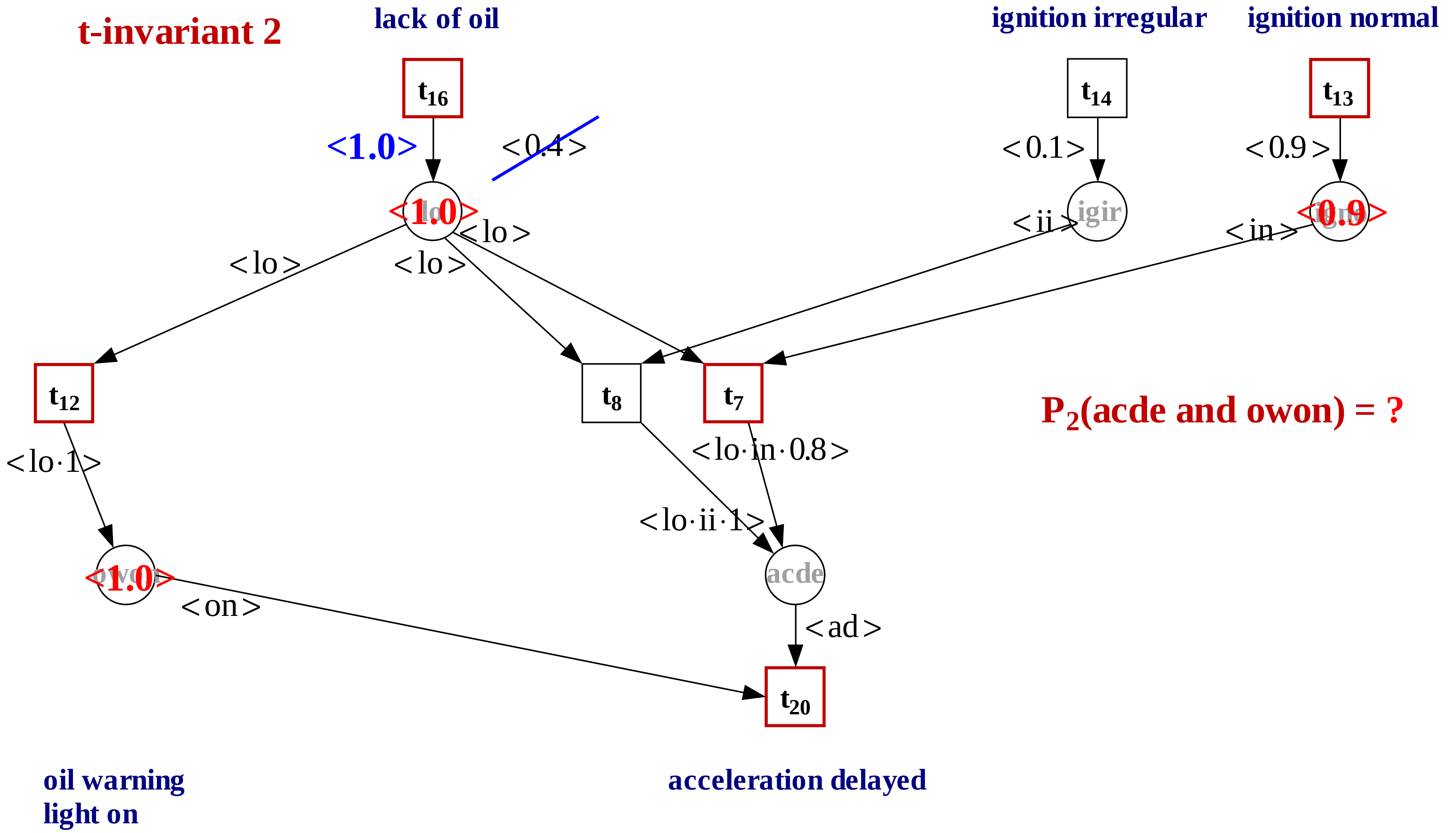


P_2(acde and owon) = ?

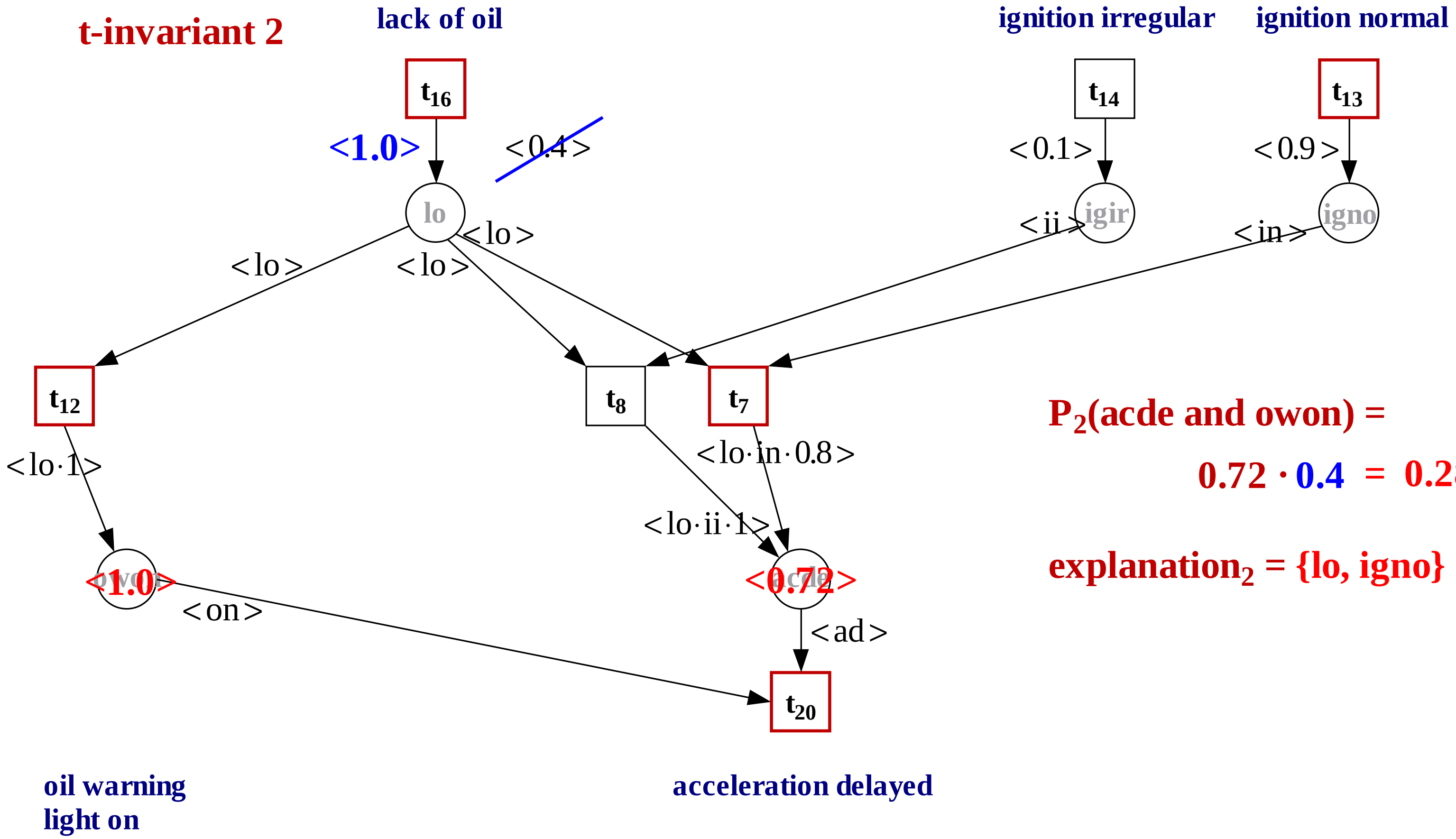
Car Example


t-invariant 2
lack of oil
ignition irregular
ignition normal
t16
t14
t13
<1.0>
<0.4>
<0.1>
<0.9>
<1.8>
lo
igir
ign
<0.9>
<lo>
<ii>
<in>
t12
t8
t7
<lo·1>
<lo·in·0.8>
<lo·ii·1>
owon
acde
<on>
<ad>
t20
oil warning light on
acceleration delayed
P2(acde and owon) = ?

Car Example


t-invariant 2
lack of oil
ignition irregular
ignition normal
t16
t14
t13
<1.0>
<0.4>
<0.1>
<0.9>
lo
<1.0>
igir
<ii>
ignn
<0.9>
<in>
<lo>
<lo>
<lo>
t12
t8
t7
P2(acde and owon) = ?
<lo·1>
<lo·in·0.8>
<lo·ii·1>
owon
<1.0>
<on>
acde
<ad>
t20
oil warning light on
acceleration delayed

t-invariant 2



P_2(acde and owon) =

$0.72 \cdot 0.4 = 0.288$

explanation$_2$ = {lo, igno}

Car Example

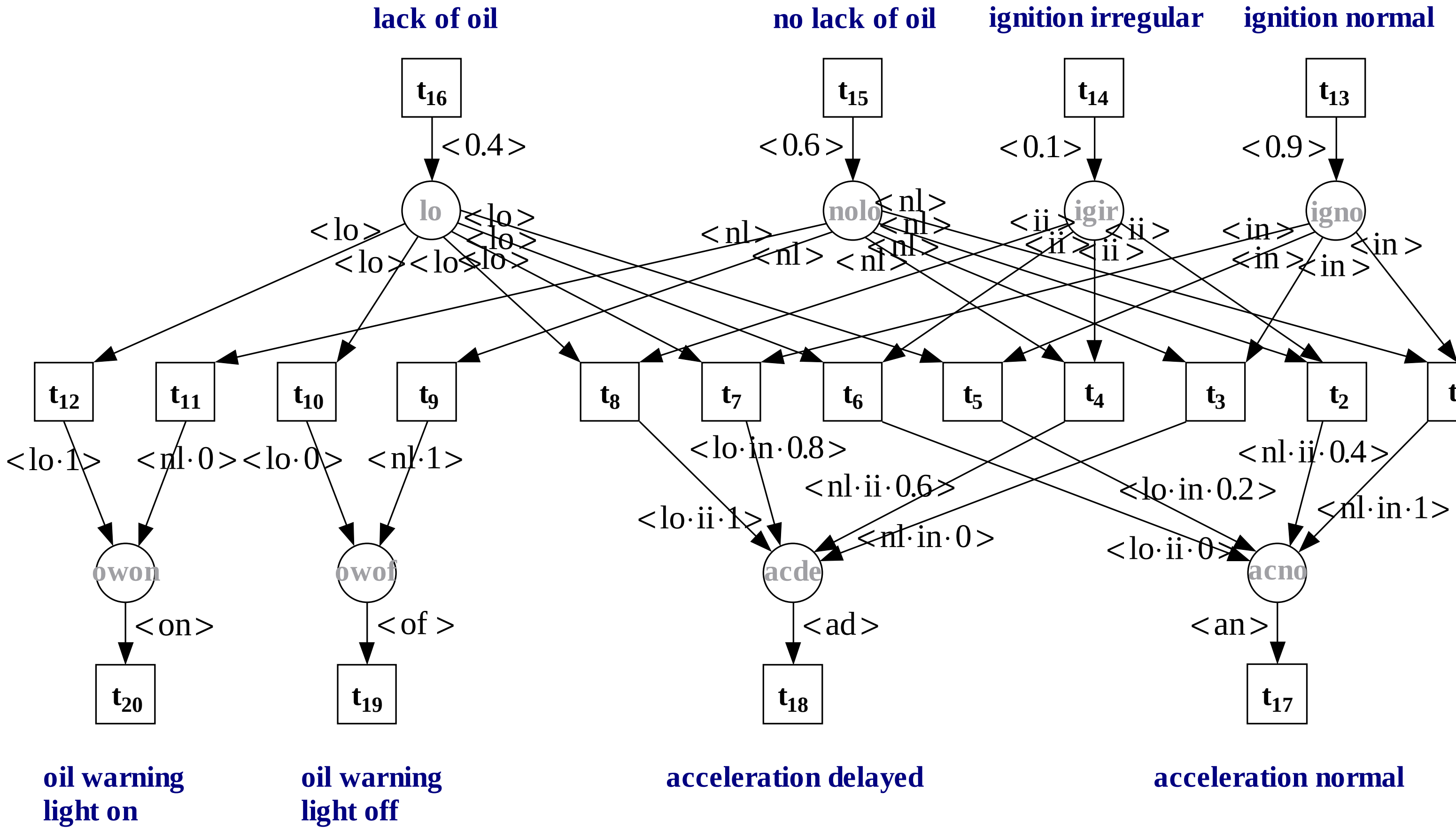
Problem: What is the probability of "acceleration delayed" and "oil warning light on"?

What are the corresponding explanations?

The explanations and their probabilities are:

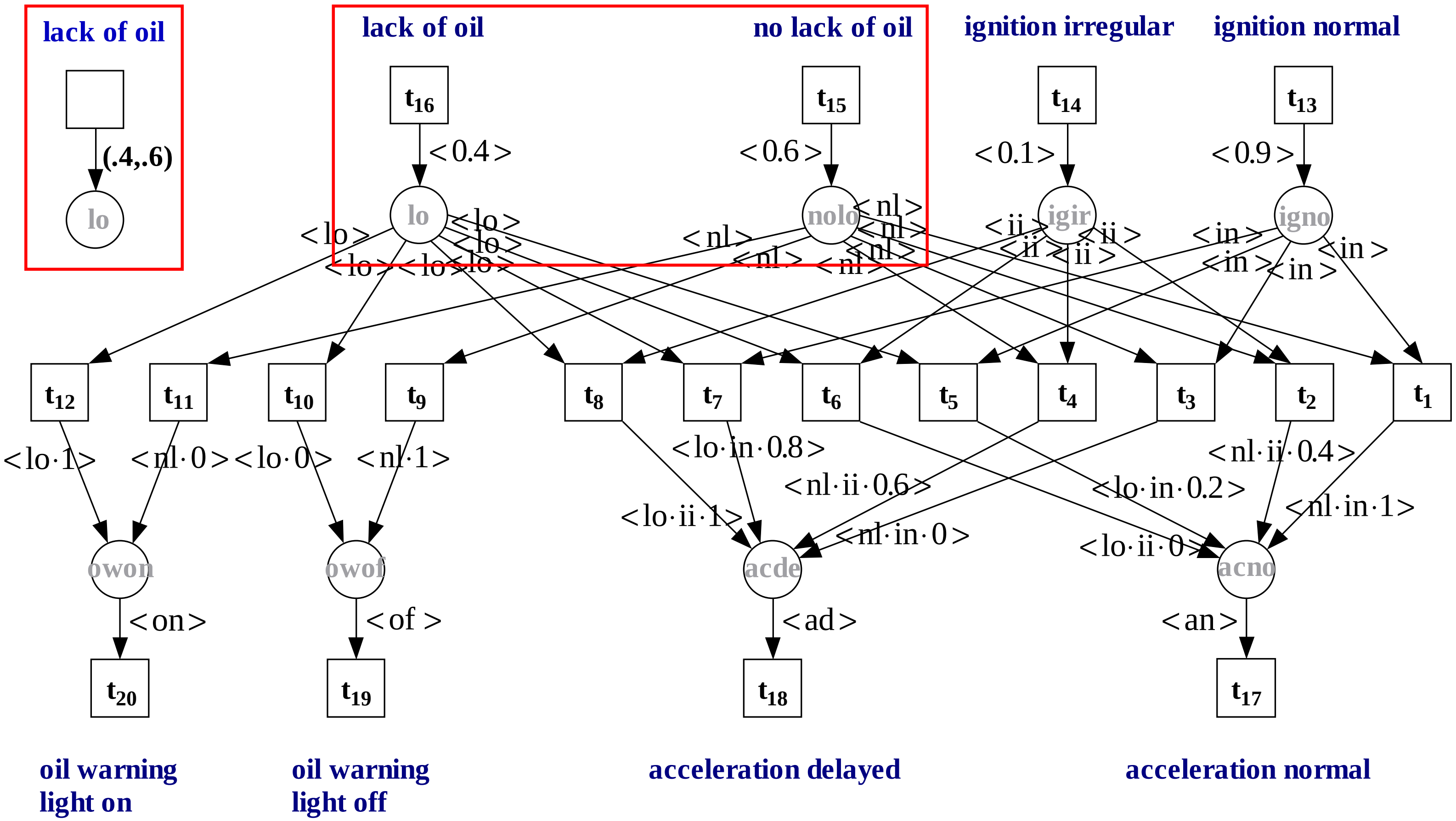
{lo, igir}	0.04
{lo, igno}	0.288
	0.328 is the probability of "acceleration delayed" and "oil warning light on".

Folding Probability Propagation Nets



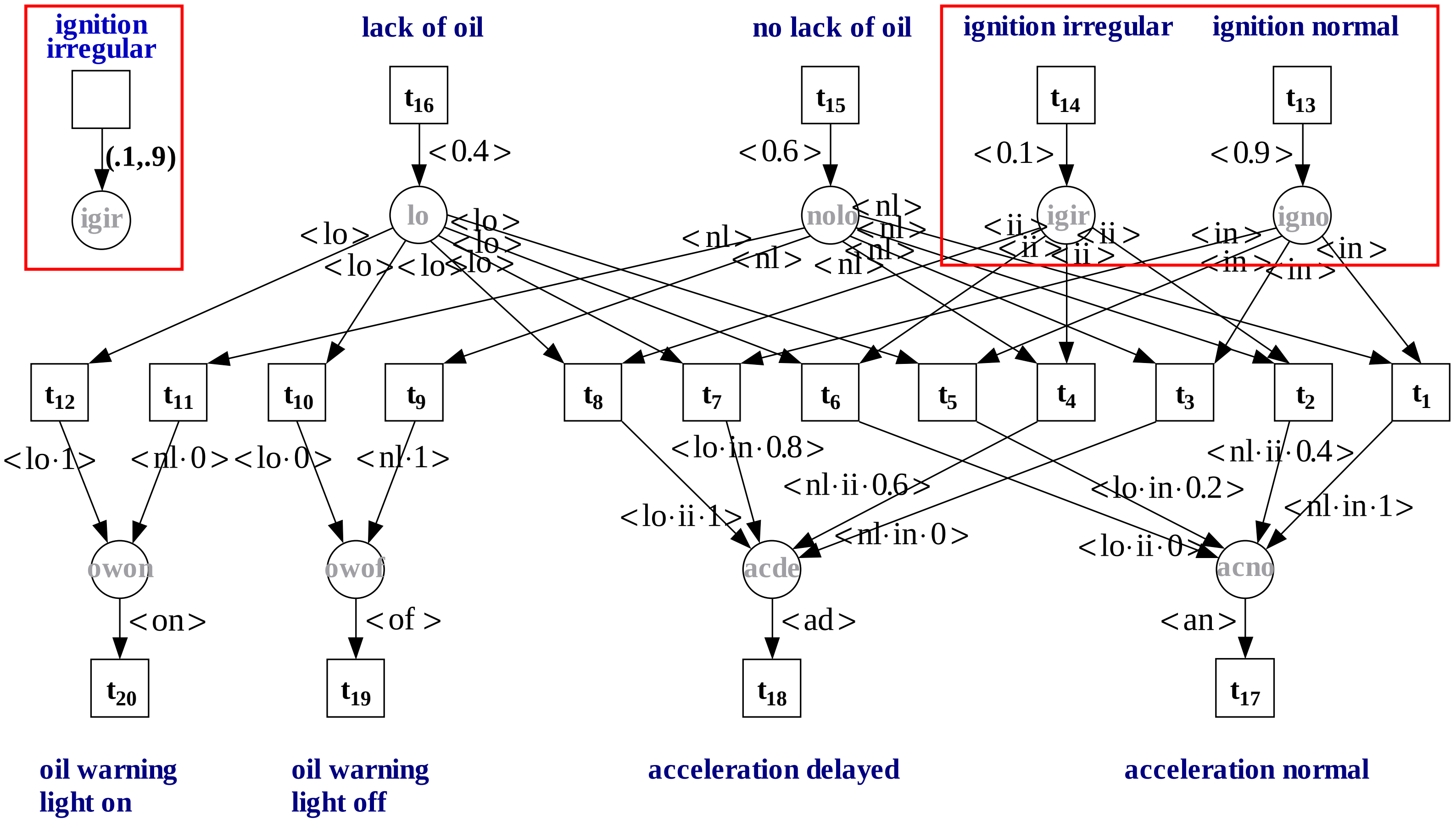
probability propagation net

Folding Probability Propagation Nets



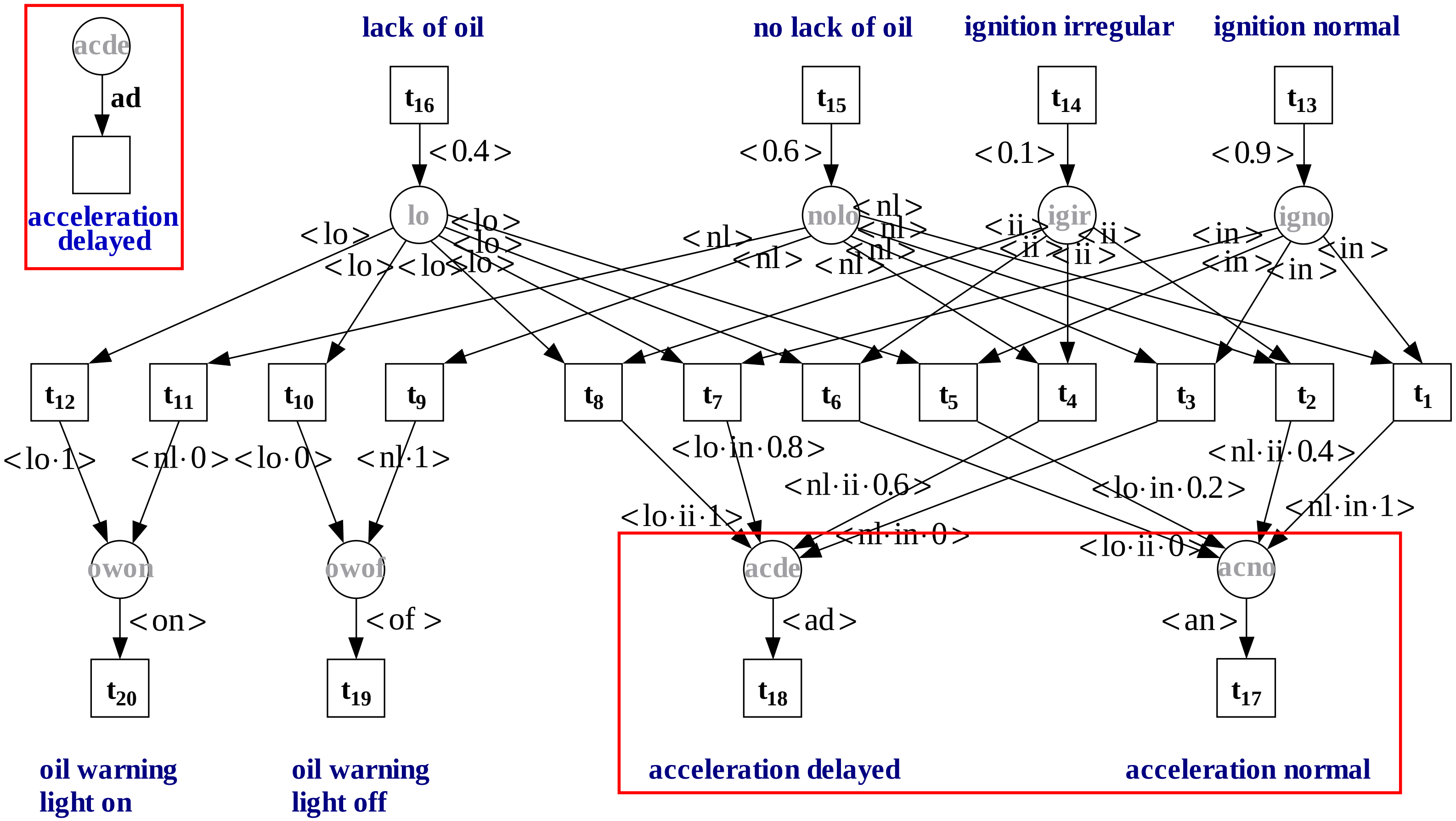
probability propagation net

Folding Probability Propagation Nets



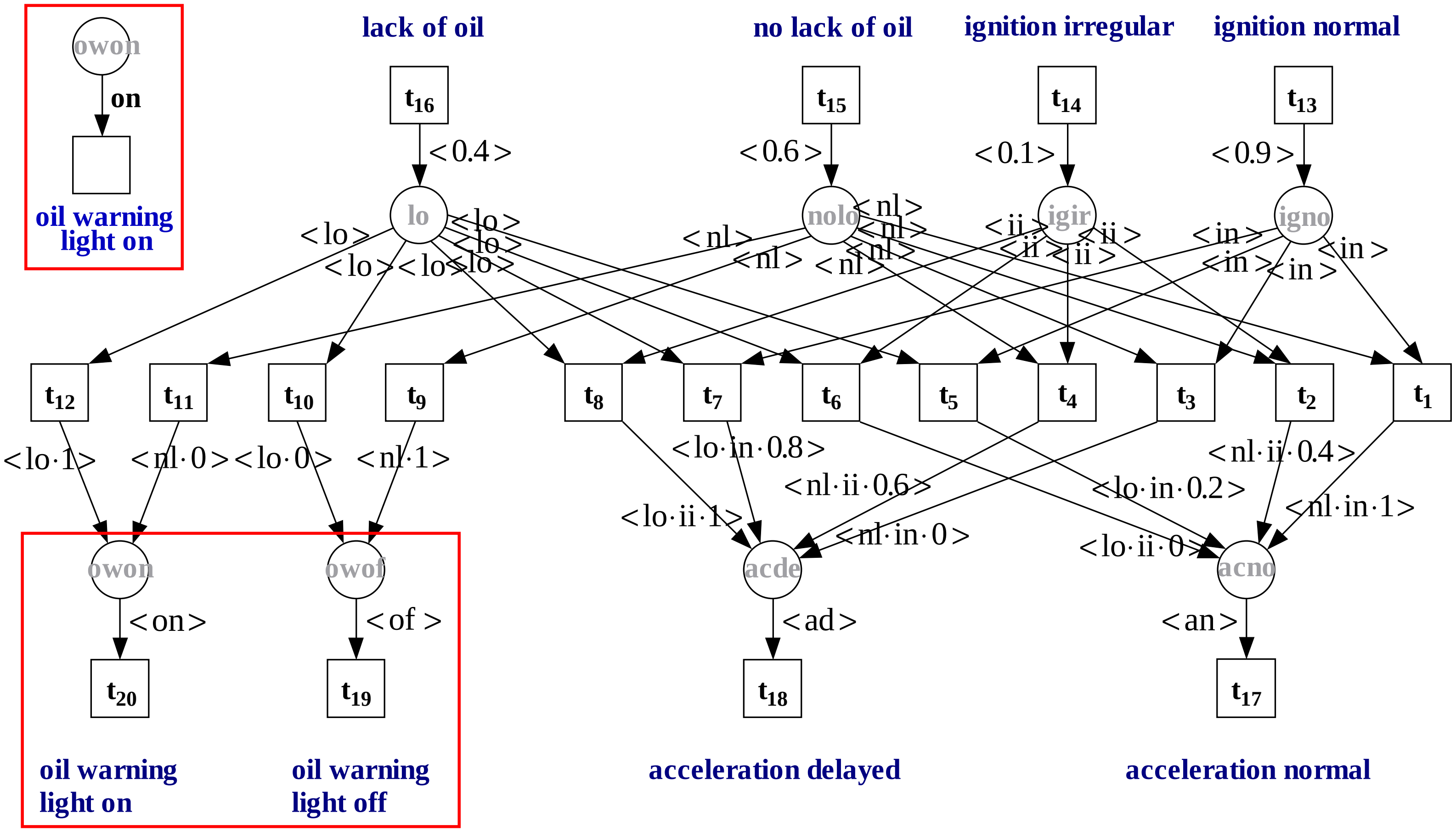
probability propagation net

Folding Probability Propagation Nets



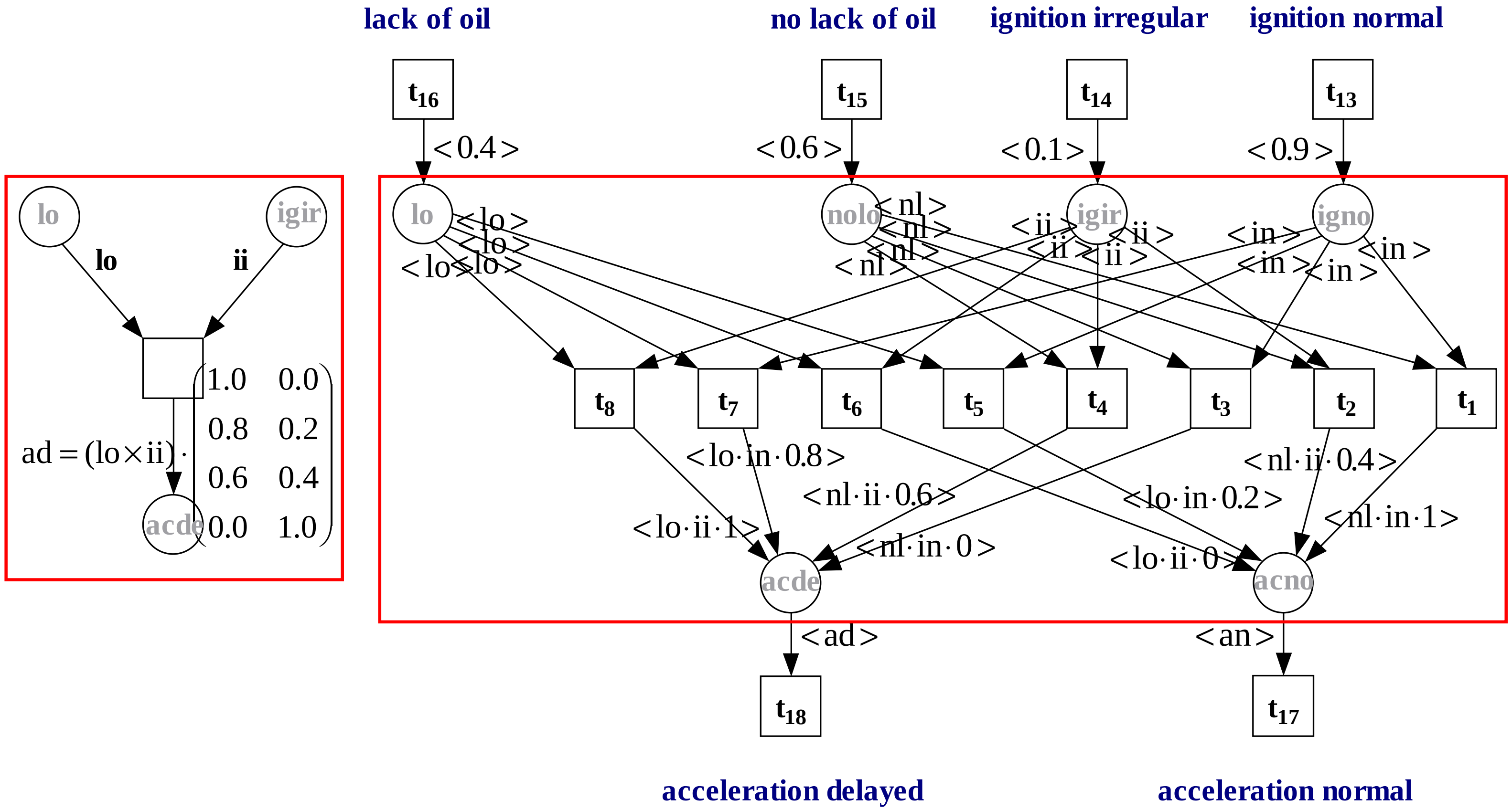
probability propagation net

Folding Probability Propagation Nets



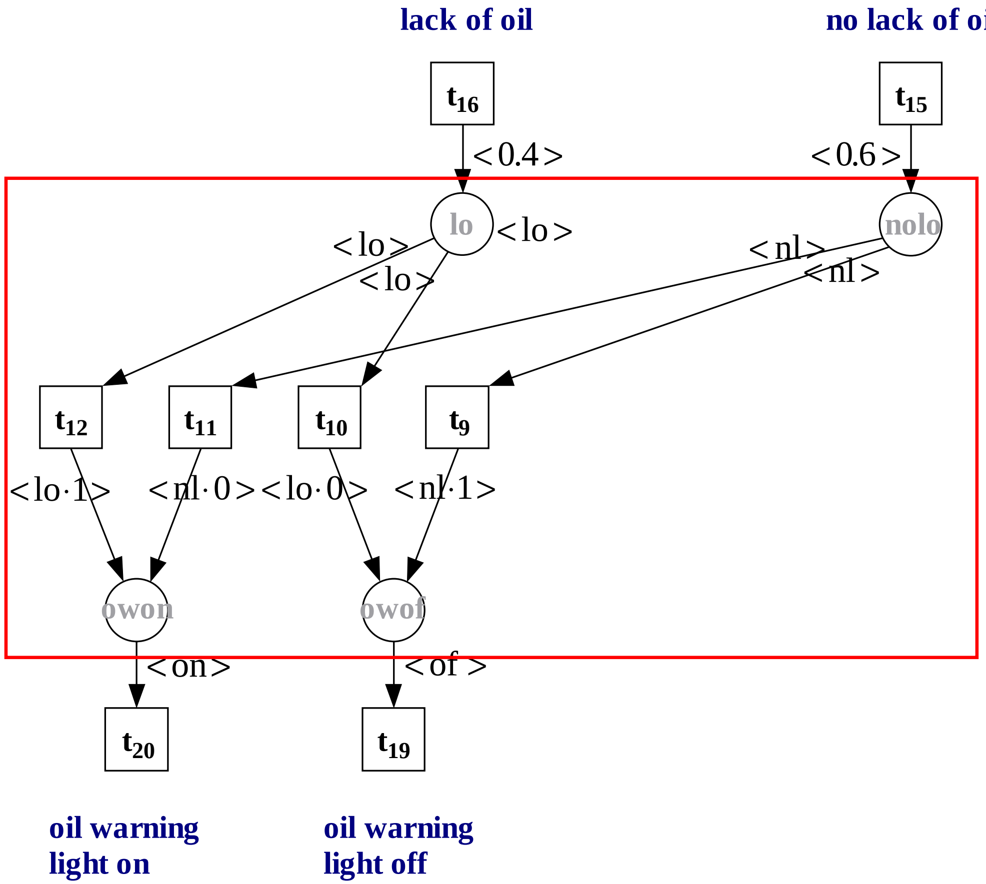
probability propagation net

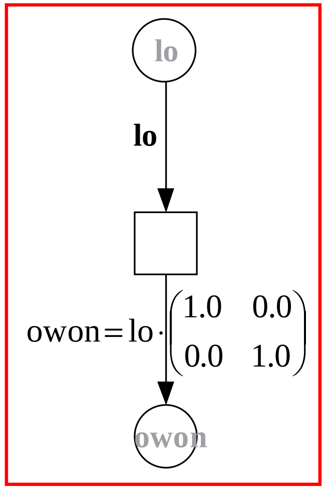
Folding Probability Propagation Nets



probability propagation net

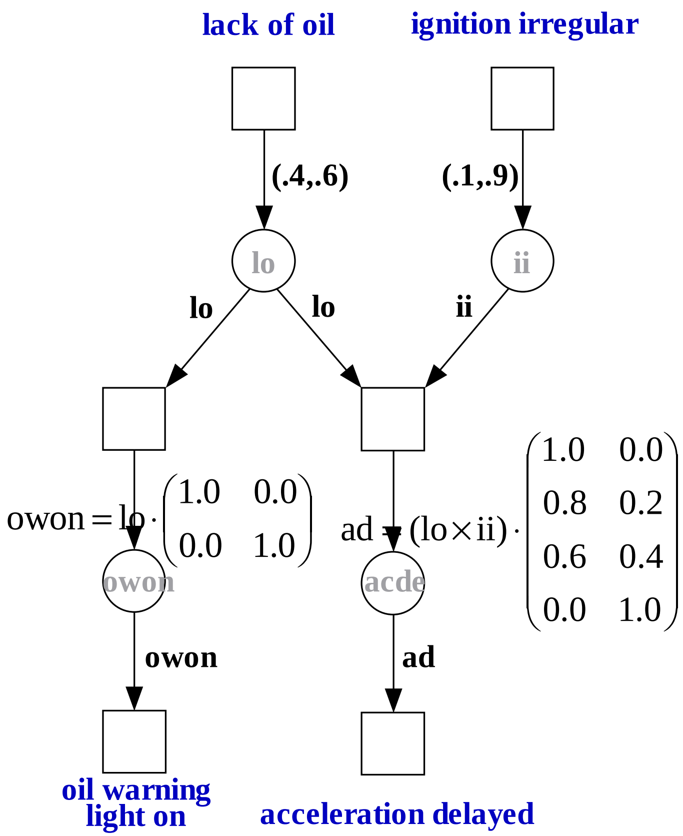
Folding Probability Propagation Nets





probability propagation net

Folding Probability Propagation Nets



$$owon = lo \cdot \begin{pmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix}$$

$$ad = (lo \times ii) \cdot \begin{pmatrix} 1.0 & 0.0 \\ 0.8 & 0.2 \\ 0.6 & 0.4 \\ 0.0 & 1.0 \end{pmatrix}$$

Bayesian belief network as Petri net

Burglar Alarm

BA-01

a_1 = Mr. Holmes' burglar alarm sounds

a_0 = Mr. Holmes' burglar alarm does not sound

b_1 = Mr. Holmes' residence is burglarized

b_0 = Mr. Holmes' residence is not burglarized

c_1 = there is an earthquake

c_0 = there is no earthquake

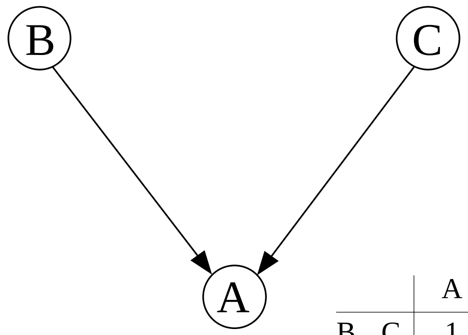
A Bayesian network

B	1	0
	0.01	0.99

= P(B)

C	1	0
	0.001	0.999

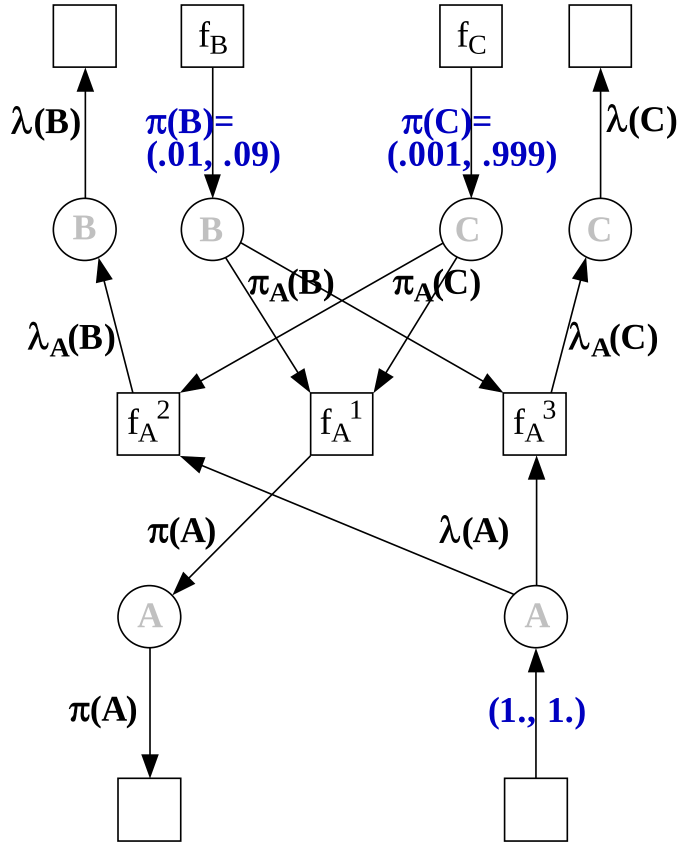
= P(C)

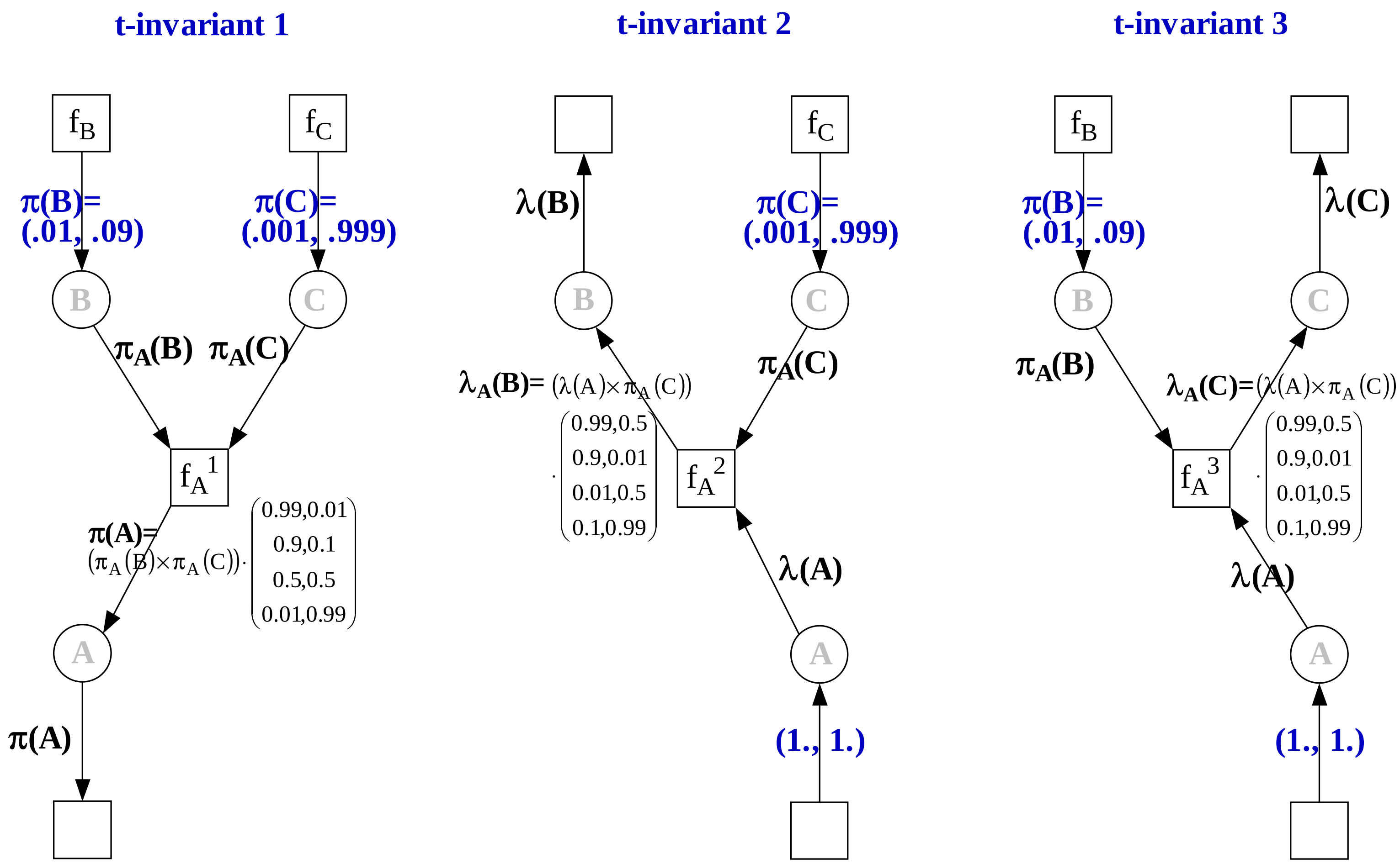


P(A|BC) =

		A	
B	C	1	0
1	1	0.99	0.01
1	0	0.9	0.1
0	1	0.5	0.5
0	0	0.01	0.99

A Petri net representation of the Bayesian network





t-invariant 1
t-invariant 2
t-invariant 3
fB
fC
π(B)= (.01, .09)
π(C)= (.001, .999)
B
C
πA(B)
πA(C)
fA1
π(A)= (πA(B)×πA(C))·
0.99,0.01
0.9,0.1
0.5,0.5
0.01,0.99
A
π(A)
λ(B)
λA(B)= (λ(A)×πA(C))·
0.99,0.5
0.9,0.01
0.01,0.5
0.1,0.99
fA2
λ(A)
(1., 1.)
λ(C)
λA(C)=(λ(A)×πA(C))·
fA3

$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$

$\pi(C) =$ **(.001,.999)**

$\lambda(C) =$

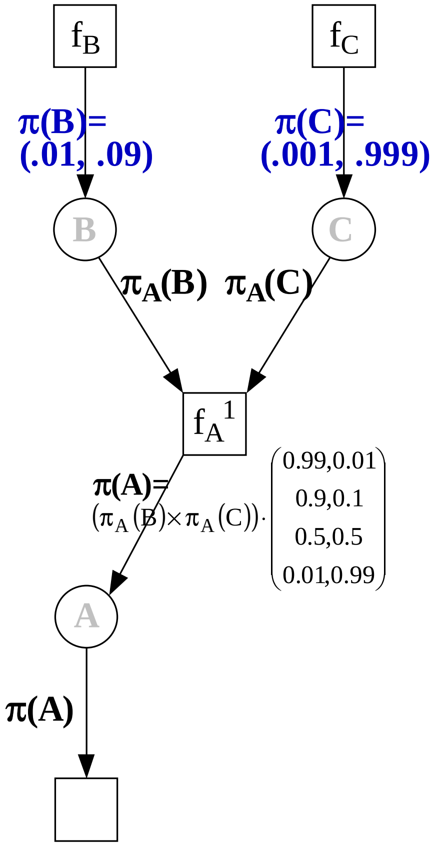
$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$

$\pi(A) =$

$\lambda(A) =$ **(1,1)**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$

t-invariant 1



Burglar Alarm

$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$

$\pi(C) =$ **(.001,.999)**

$\lambda(C) =$

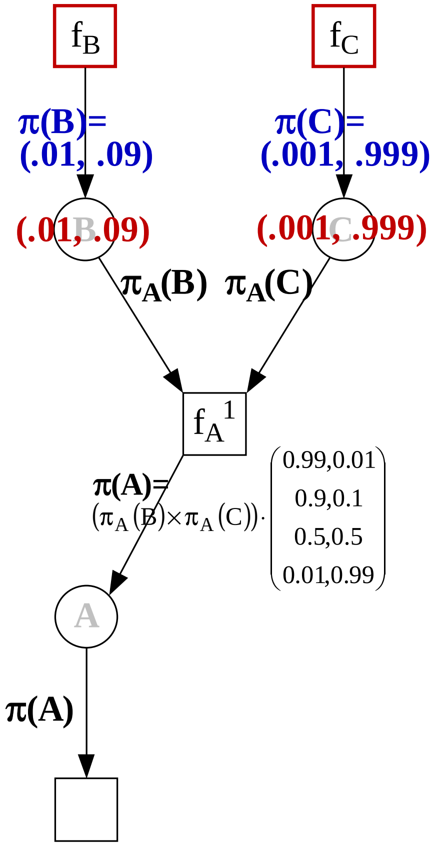
$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$

$\pi(A) =$

$\lambda(A) =$ **(1,1)**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$

t-invariant 1



$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$

$\pi(C) =$ **(.001,.999)**

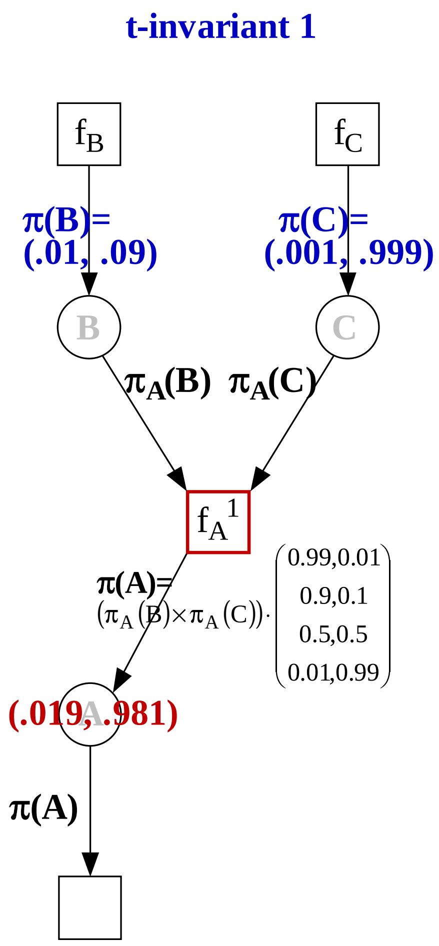
$\lambda(C) =$

$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$

$\pi(A) =$ **(.019, .981)**

$\lambda(A) =$ **(1,1)**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ **(.019, .981)**



$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$

$\pi(C) =$ **(.001,.999)**

$\lambda(C) =$

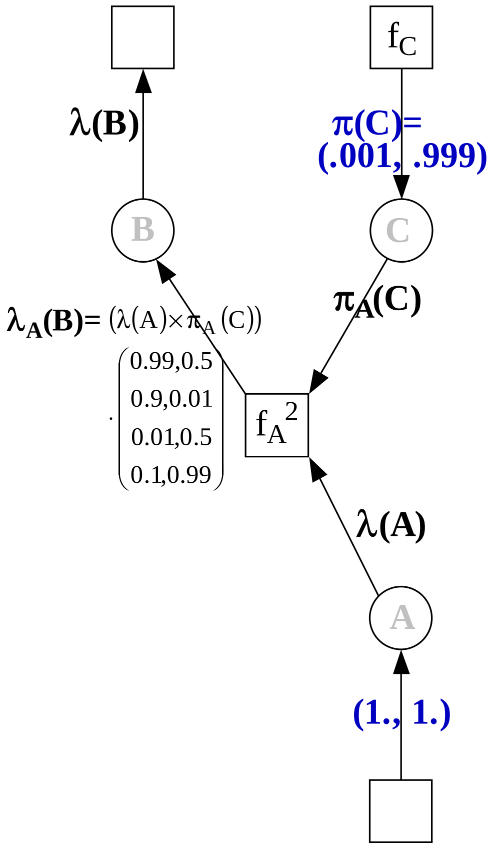
$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$

$\pi(A) =$ **(.019, .981)**

$\lambda(A) =$ **(1,1)**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ **(.019, .981)**

t-invariant 2



$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$

$\pi(C) =$ **(.001,.999)**

$\lambda(C) =$

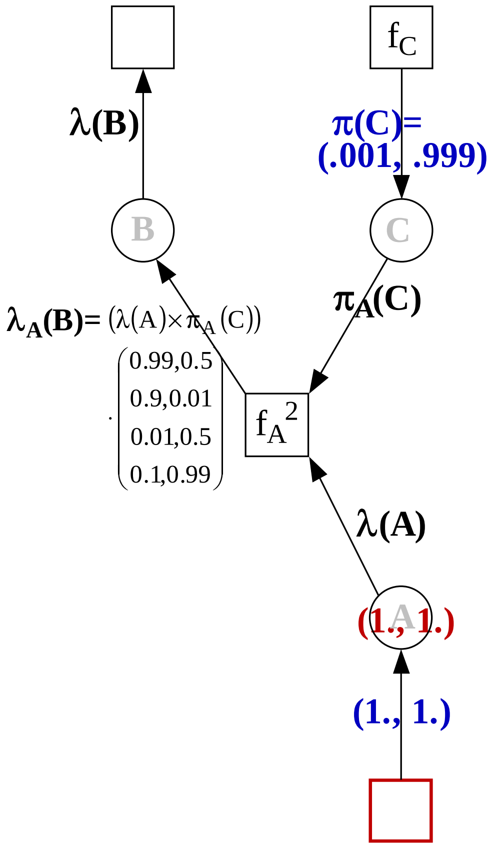
$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$

$\pi(A) =$ **(.019, .981)**

$\lambda(A) =$ **(1,1)**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ **(.019, .981)**

t-invariant 2



$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$

$\pi(C) =$ **(.001,.999)**

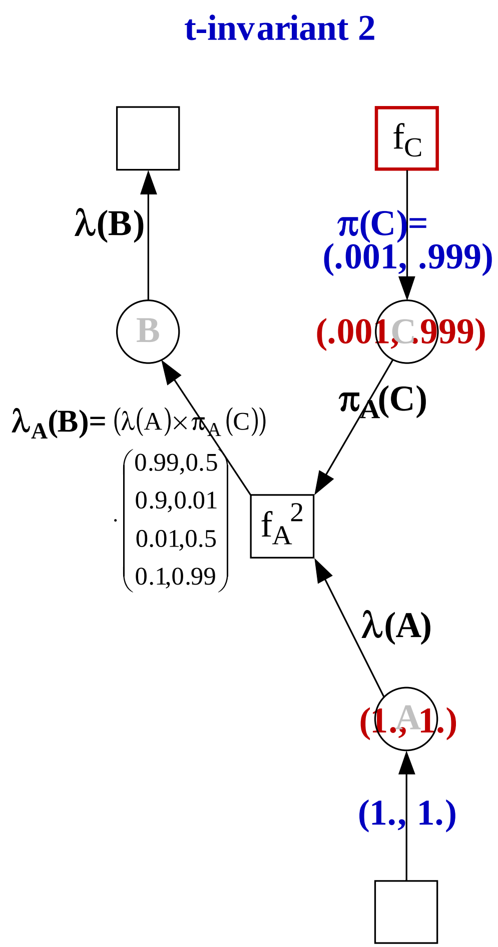
$\lambda(C) =$

$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$

$\pi(A) =$ **(.019, .981)**

$\lambda(A) =$ **(1,1)**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ **(.019, .981)**



$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$ **(1., 1.)**

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$ **(.01,.99)**

$\pi(C) =$ **(.001,.999)**

$\lambda(C) =$

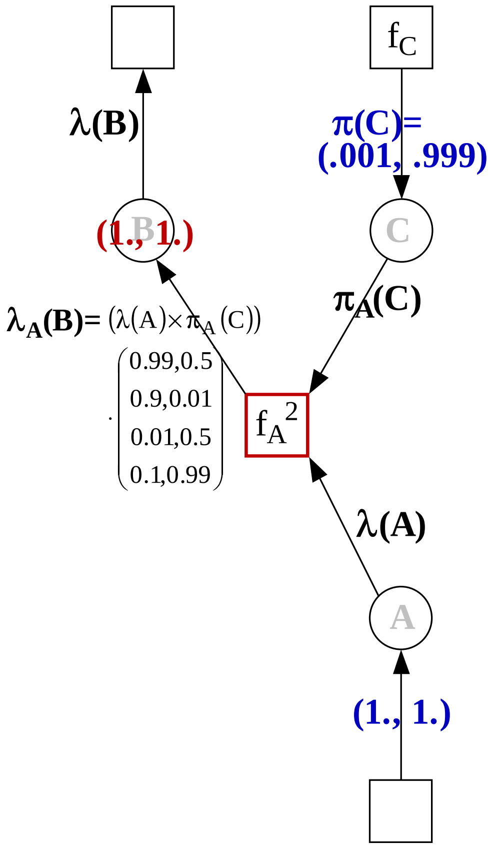
$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$

$\pi(A) =$ **(.019, .981)**

$\lambda(A) =$ **(1,1)**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ **(.019, .981)**

t-invariant 2



$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$ **(1., 1.)**

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$ **(.01,.99)**

$\pi(C) =$ **(.001,.999)**

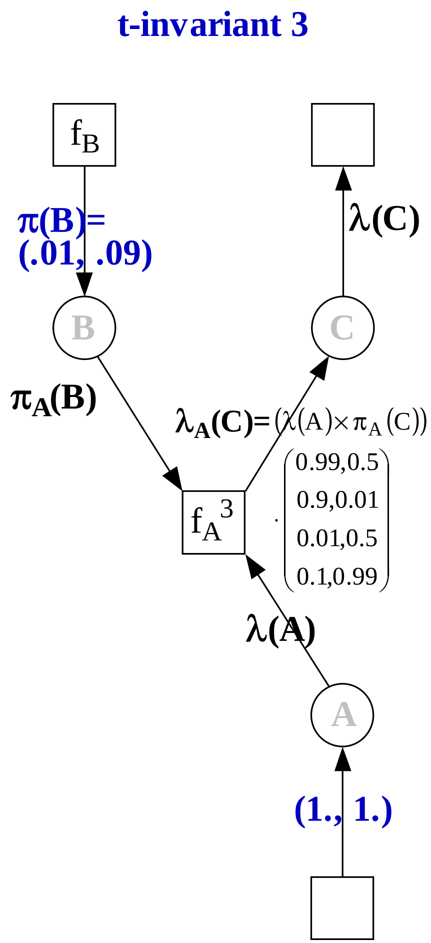
$\lambda(C) =$

$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$

$\pi(A) =$ **(.019, .981)**

$\lambda(A) =$ **(1,1)**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ **(.019, .981)**



$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$ **(1., 1.)**

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$ **(.01,.99)**

$\pi(C) =$ **(.001,.999)**

$\lambda(C) =$

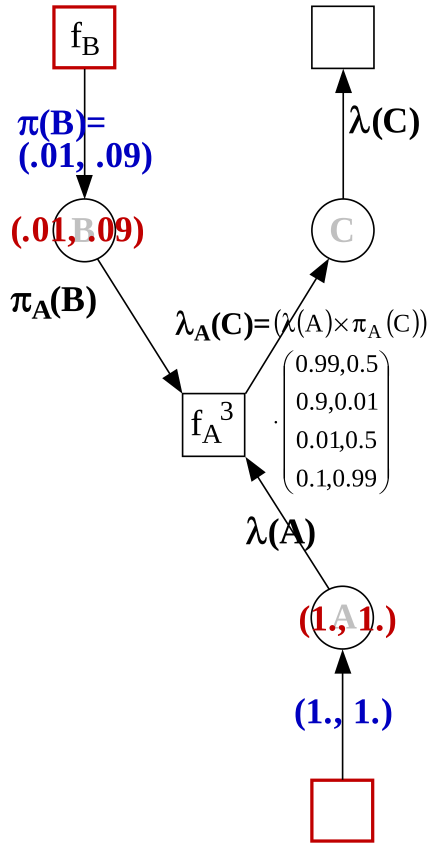
$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$

$\pi(A) =$ **(.019, .981)**

$\lambda(A) =$ **(1,1)**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ **(.019, .981)**

t-invariant 3



$$\pi(B) = (.01,.99)$$

$$\lambda(B) = (1., 1.)$$

$$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) = (.01,.99)$$

$$\pi(C) = (.001,.999)$$

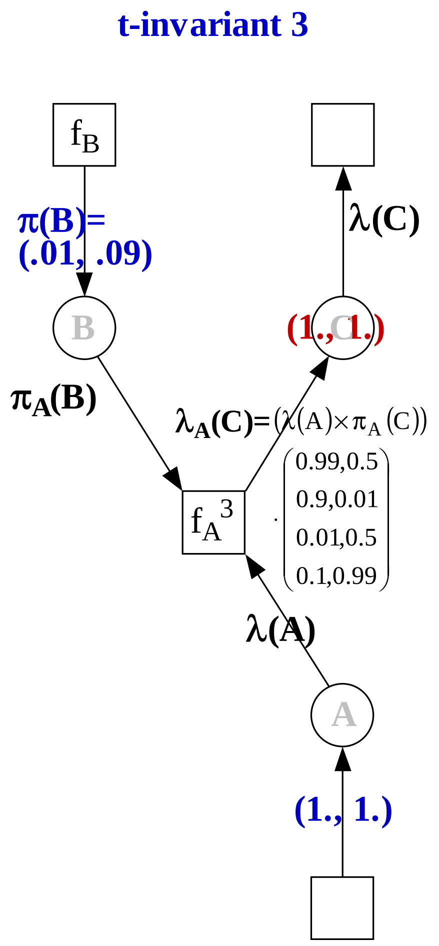
$$\lambda(C) = (1., 1.)$$

$$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) = (.001,.999)$$

$$\pi(A) = (.019, .981)$$

$$\lambda(A) = (1,1)$$

$$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) = (.019, .981)$$



a_1 = Mr. Holmes' burglar alarm sounds

b_1 = Mr. Holmes' residence is burglarized

b_2 = Mr. Holmes' residence is not burglarized

c_1 = there is an earthquake

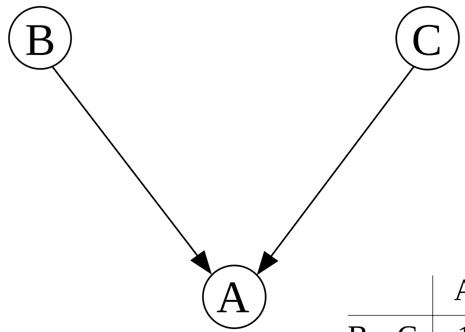
c_2 = there is no earthquake

B	1	0
	.01	.99

= P(B)

C	1	0
	.001	.999

= P(C)



New evidence !!

P(A|BC) =

		A	
B	C	1	0
1	1	.99	.01
1	0	.9	.1
0	1	.5	.5
0	0	.01	.99

$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$

$\pi(C) =$ **(.001,.999)**

$\lambda(C) =$

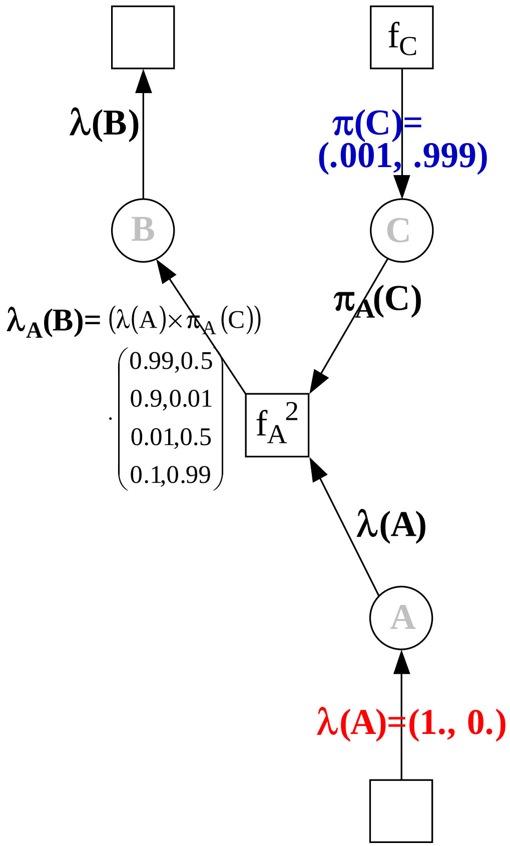
$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$

$\pi(A) =$

$\lambda(A) =$ **(1., 0.)** **new**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ **(1., 0.)** **evidence**

t-invariant 2



Burglar Alarm

$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$

$\pi(C) =$ **(.001,.999)**

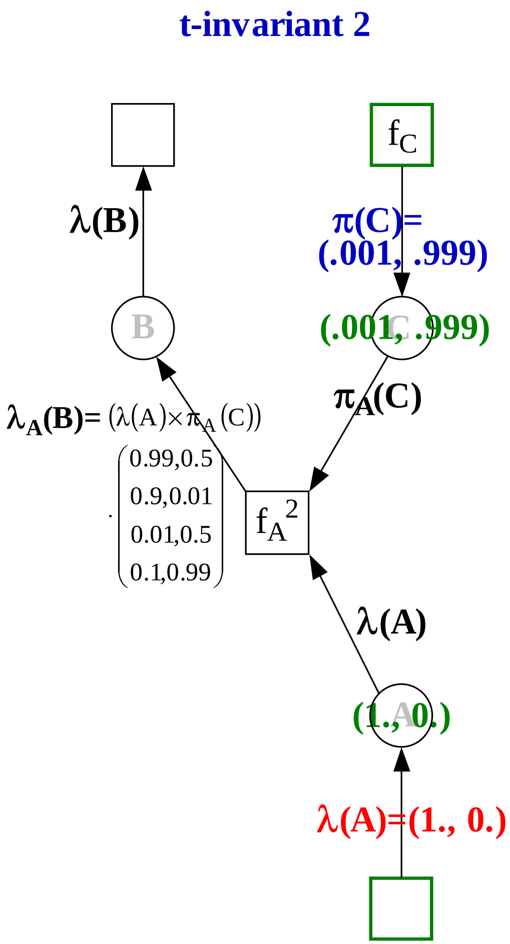
$\lambda(C) =$

$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$

$\pi(A) =$

$\lambda(A) =$ **(1., 0.)** **new**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ **(1., 0.)** **evidence**



Burglar Alarm

$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$ **(.9,.01)**

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$ **(.476,.524)**

$\pi(C) =$ **(.001,.999)**

$\lambda(C) =$

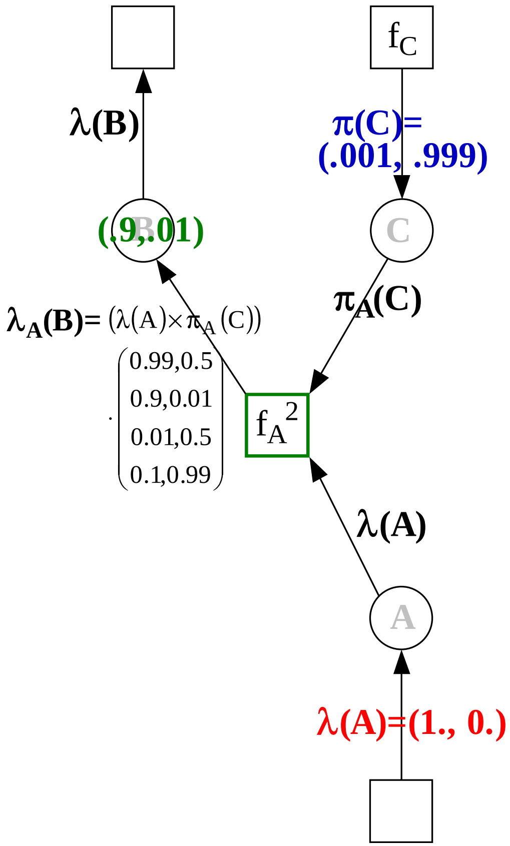
$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$

$\pi(A) =$

$\lambda(A) =$ **(1., 0.)** **new**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ **(1., 0.)** **evidence**

t-invariant 2



$\pi(B) =$ (.01,.99)

$\lambda(B) =$ (.9,.01)

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$ (.476,.524)

$\pi(C) =$ (.001,.999)

$\lambda(C) =$

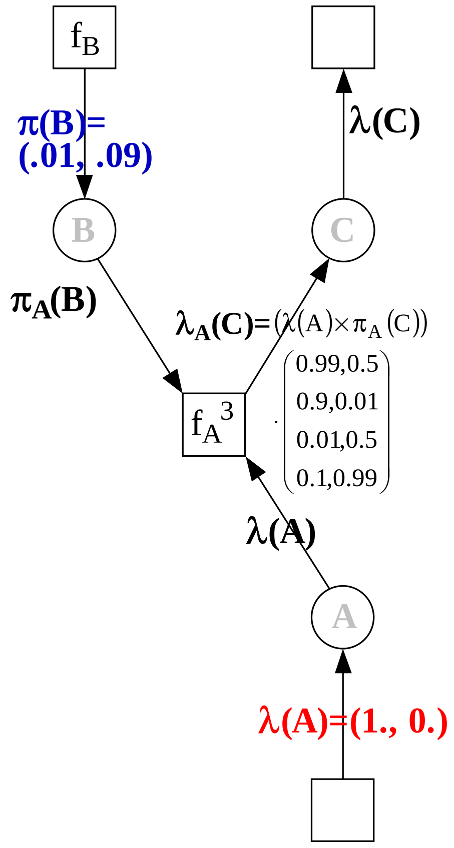
$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$

$\pi(A) =$

$\lambda(A) =$ (1., 0.) new

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ (1., 0.) evidence

t-invariant 3



$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$ **(.9,.01)**

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$ **(.476,.524)**

$\pi(C) =$ **(.001,.999)**

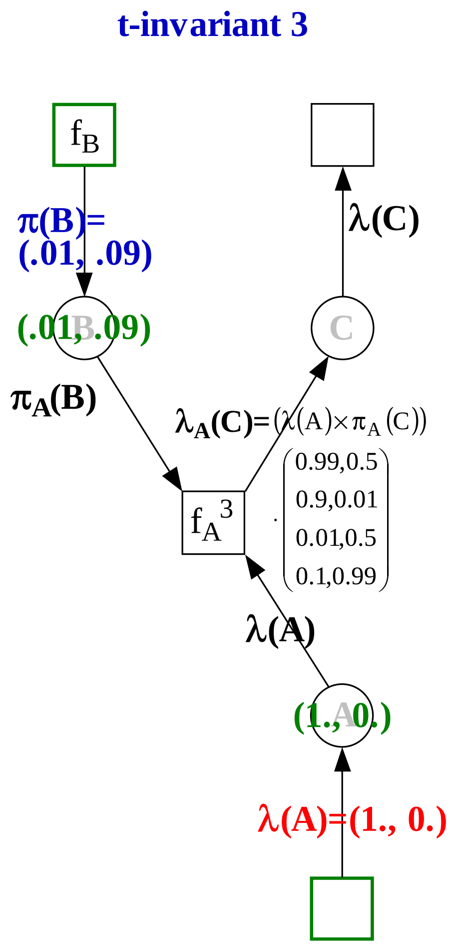
$\lambda(C) =$

$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$

$\pi(A) =$

$\lambda(A) =$ **(1., 0.)** **new**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ **(1., 0.)** **evidence**



$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$ **(.9,.01)**

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$ **(.476,.524)**

$\pi(C) =$ **(.001,.999)**

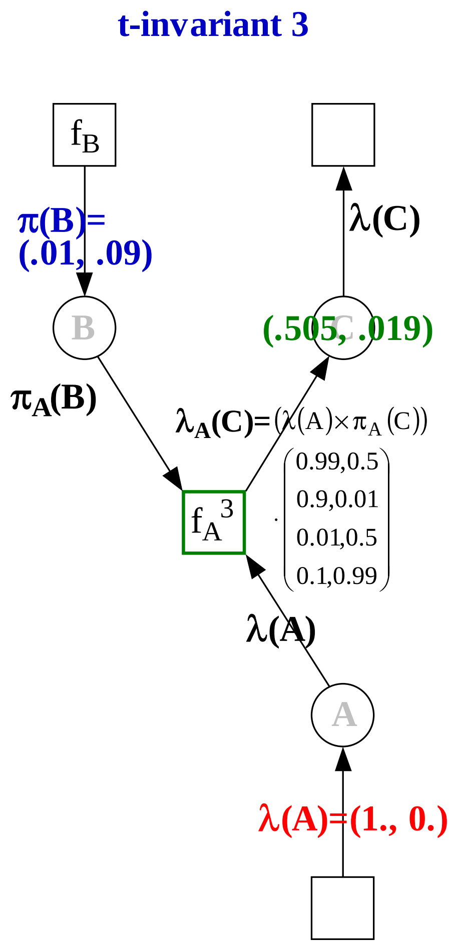
$\lambda(C) =$ **(.505, .019)**

$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$ **(.026, .974)**

$\pi(A) =$

$\lambda(A) =$ **(1., 0.)** **new**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ **(1., 0.)** **evidence**



$\pi(B) =$	**(.01,.99)**	**(.01,.99)**
$\lambda(B) =$	**(.9,.01)**	**(1., 1.)**
$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$	**(.476,.524)**	**(.01,.99)**
$\pi(C) =$	**(.001,.999)**	**(.001,.999)**
$\lambda(C) =$	**(.505, .019)**	**(1., 1.)**
$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$	**(.026, .974)**	**(.001,.999)**
$\pi(A) =$		**(.019, .981)**
$\lambda(A) =$	**(1., 0.)** **new**	**(1.,1.)**
$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$	**(1., 0.)** **evidence**	**(.019, .981)**

Burglar Alarm

a_1 = Mr. Holmes' burglar alarm sounds

b_1 = Mr. Holmes' residence is burglarized

b_2 = Mr. Holmes' residence is not burglarized

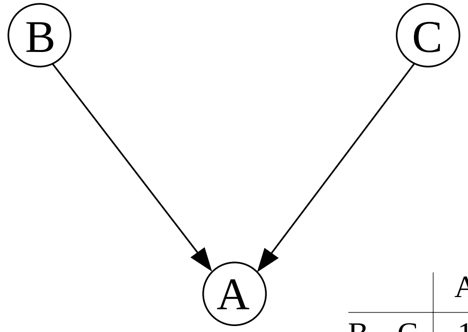
c_1 = there is an earthquake

B	1	0
	.01	.99

= P(B)

C	1	0
	.001	.999

= P(C)



More new evidence !!

$P(A|BC)$ =

		A	
B	C	1	0
1	1	.99	.01
1	0	.9	.1
0	1	.5	.5
0	0	.01	.99

$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$

$\pi(C) =$ **(1., 0.)** **new**

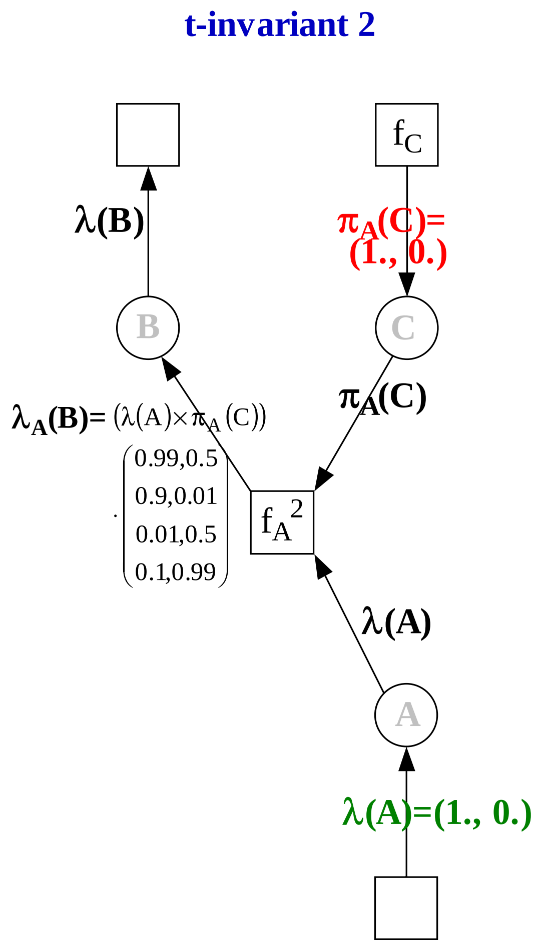
$\lambda(C) =$

$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$ **(1., 0.)** **evidence**

$\pi(A) =$

$\lambda(A) =$ **(1., 0.)**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ **(1., 0.)**



$\pi(B) =$ **(.01,.99)**

$\lambda(B) =$

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$

$\pi(C) =$ **(1., 0.)** **new**

$\lambda(C) =$

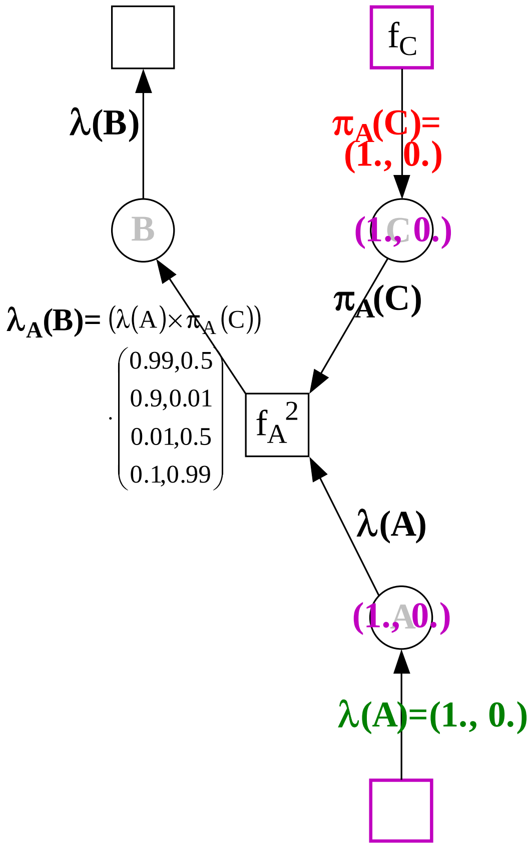
$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$ **(1., 0.)** **evidence**

$\pi(A) =$

$\lambda(A) =$ **(1., 0.)**

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ **(1., 0.)**

t-invariant 2



$\pi(B) =$ (.01,.99)

$\lambda(B) =$ (.99, .5)

$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$ (.02, .98)

$\pi(C) =$ (1., 0.) new

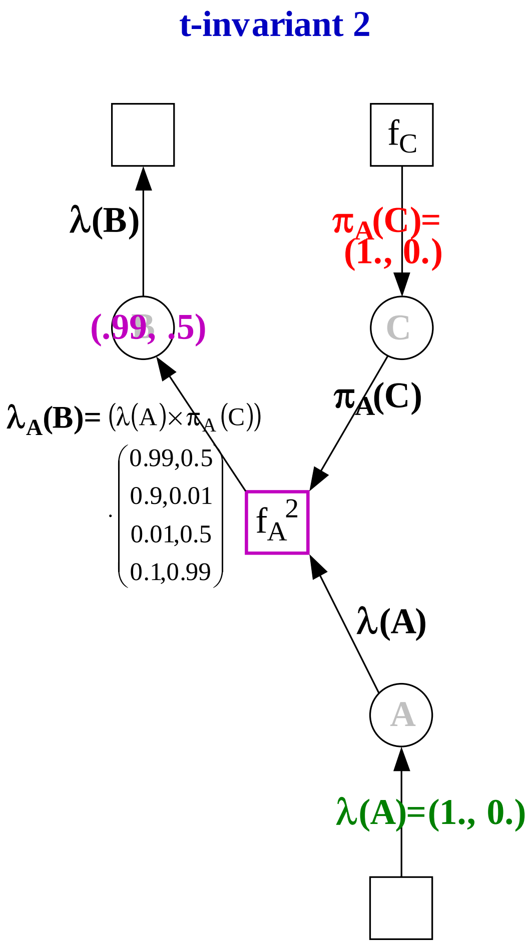
$\lambda(C) =$

$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$ (1., 0.) evidence

$\pi(A) =$

$\lambda(A) =$ (1., 0.)

$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$ (1., 0.)



Burglar Alarm

$\pi(B) =$	(.01,.99)	(.01,.99)	(.01,.99)
$\lambda(B) =$	(.99, .5)	(.9,.01)	(1., 1.)
$P(B) = \alpha \cdot \lambda(B) \cdot \pi(B) =$	(.02, .98)	(.476,.524)	(.01,.99)
$\pi(C) =$	(1., 0.) "new"	(.001,.999)	(.001,.999)
$\lambda(C) =$		(.505, .019)	(1., 1.)
$P(C) = \alpha \cdot \lambda(C) \cdot \pi(C) =$	(1., 0.) evidence	(.026, .974)	(.001,.999)
$\pi(A) =$			(.019, .981)
$\lambda(A) =$	(1., 0.)	(1., 0.) "new"	(1.,1.)
$P(A) = \alpha \cdot \lambda(A) \cdot \pi(A) =$	(1., 0.)	(1., 0.) evidence	(.019, .981)

Wet Grass

h_1 = Mr. Holmes' grass is wet

h_0 = Mr. Holmes' grass is not wet

r_1 = rain is the cause for Mr. Holmes' grass being wet

r_0 = rain is not the cause for Mr. Holmes' grass being wet

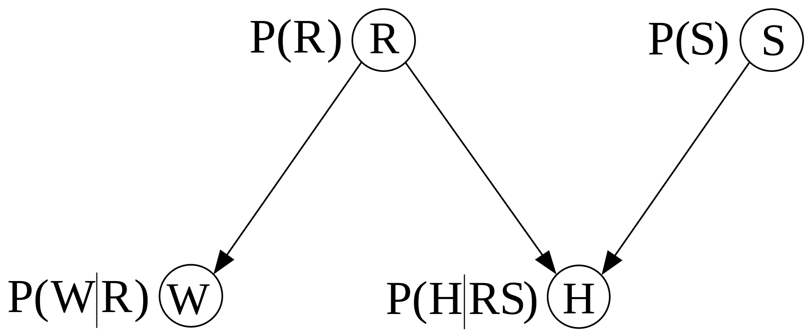
s_1 = sprinkler is the cause for Mr. Holmes' grass being wet

s_0 = sprinkler is not the cause for Mr. Holmes' grass being wet

w_1 = Dr. Watson's grass is wet

w_0 = Dr. Watson's grass is not wet

A Bayesian network

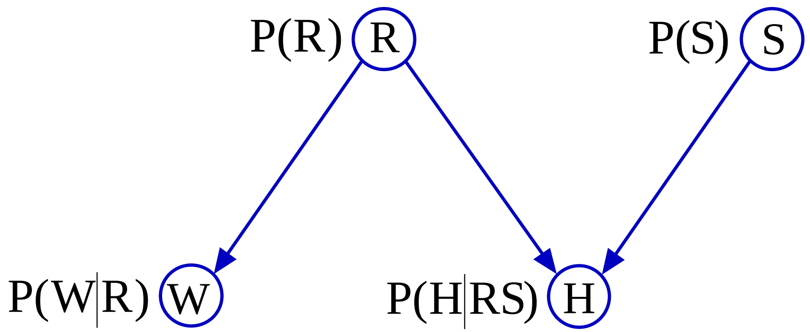


$f_R = P(R) =$

R	1	0
	0.2	0.8

$f_S = P(S) =$

S	1	0
	0.1	0.9



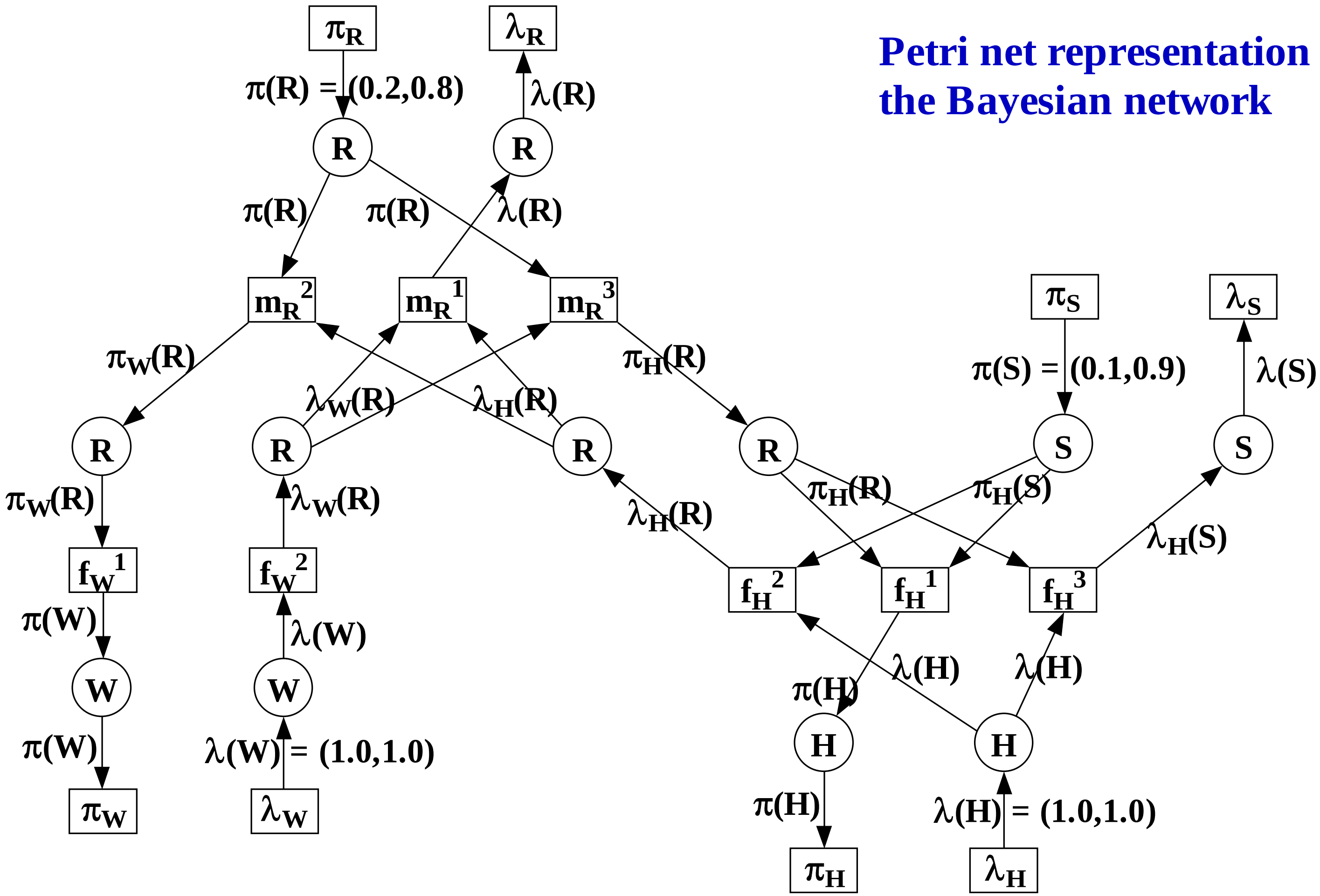
$f_W = P(W|R) =$

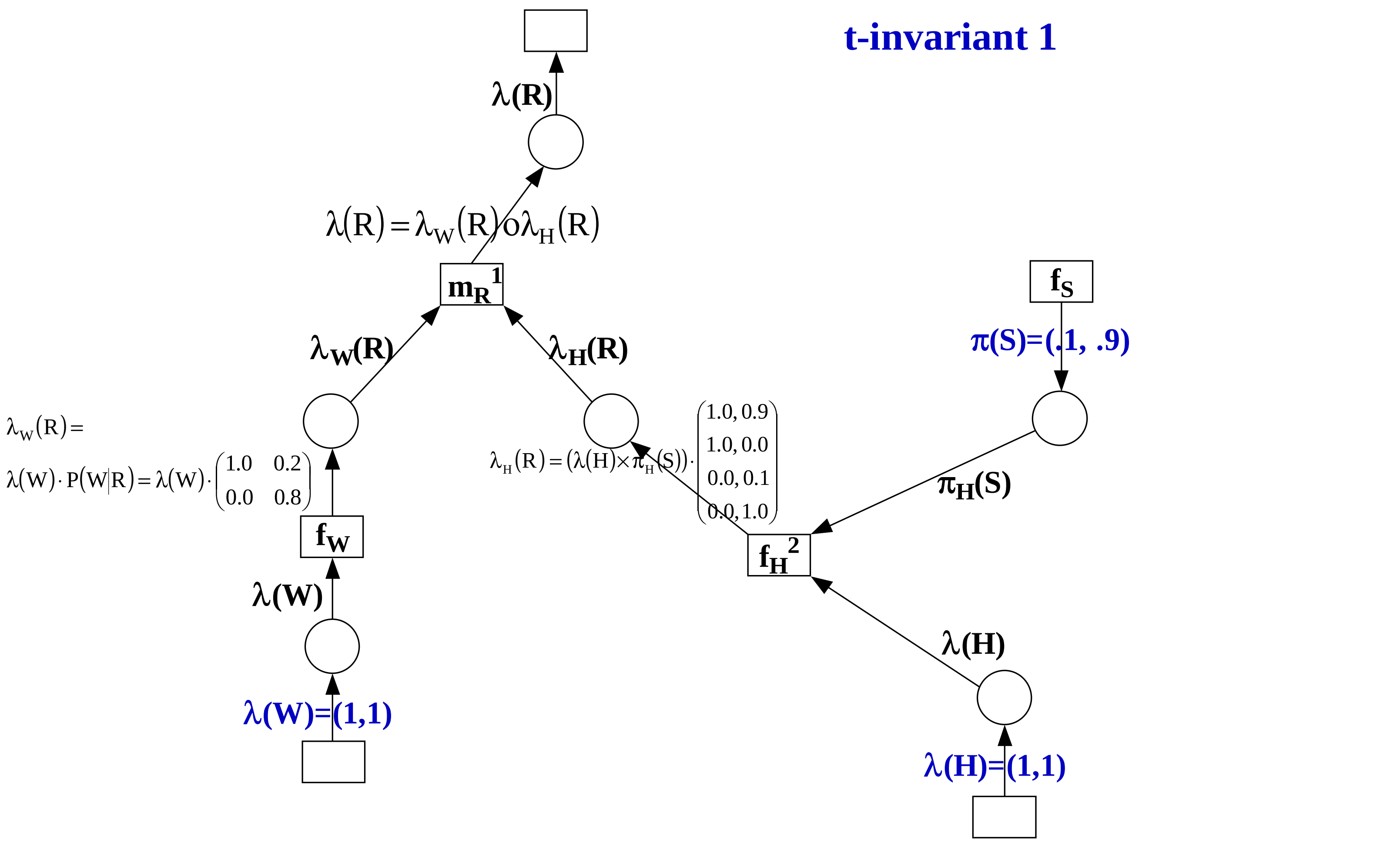
	R	
W	1	0
1	1.0	0.2
0	0.0	0.8

$f_H = P(H|RS) =$

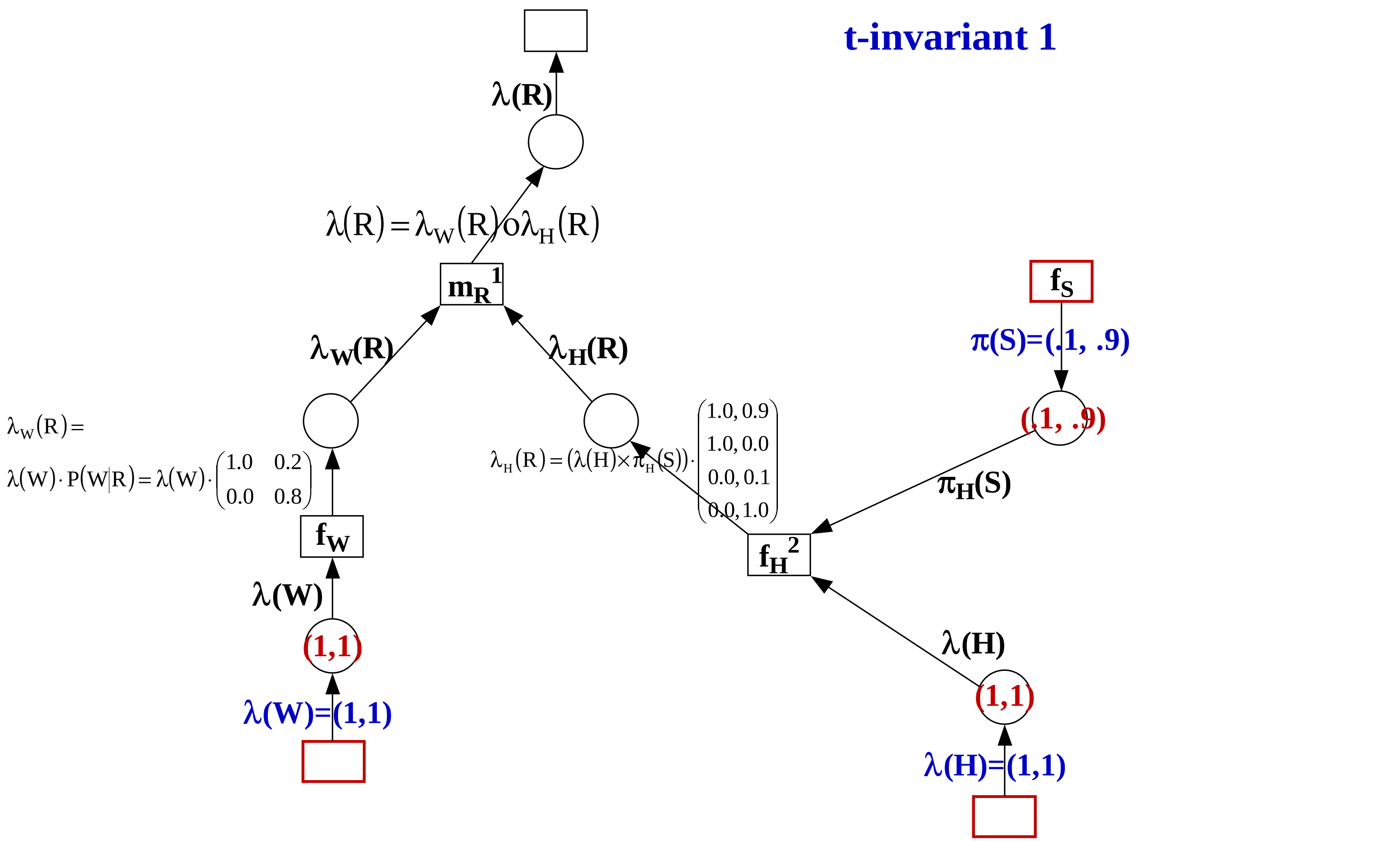
		H	
R	S	1	0
1	1	1.0	0.0
1	0	1.0	0.0
0	1	0.9	0.1
0	0	0.0	1.0

Petri net representation of the Bayesian network



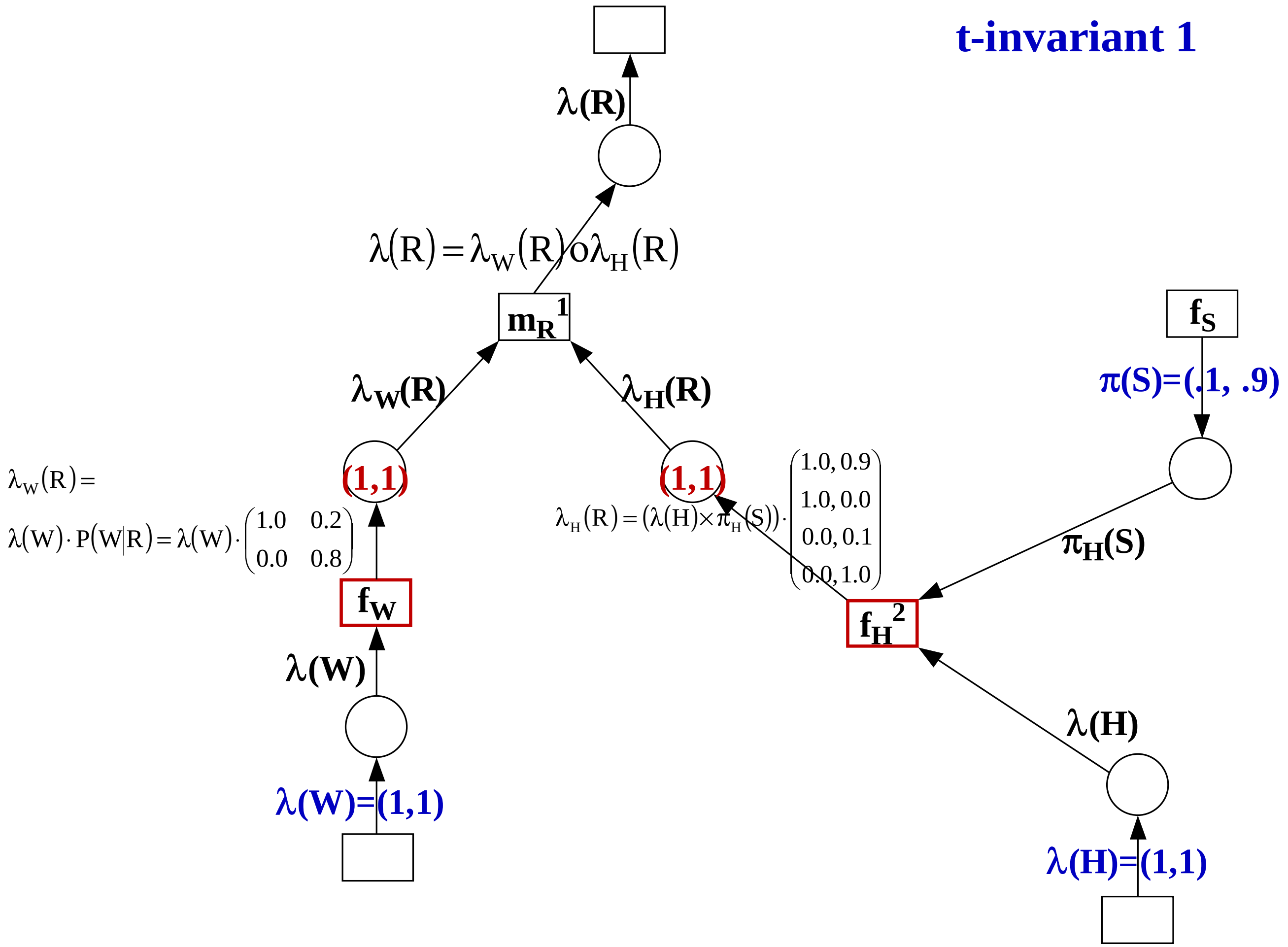


t-invariant 1
λ(R)
$\lambda(R) = \lambda_W(R) \circ \lambda_H(R)$
m_R^1
$\lambda_W(R)$
$\lambda_H(R)$
f_S
π(S)=(.1, .9)
$\lambda_W(R) =$
$\lambda(W) \cdot P(W|R) = \lambda(W) \cdot \begin{pmatrix} 1.0 & 0.2 \\ 0.0 & 0.8 \end{pmatrix}$
$\lambda_H(R) = (\lambda(H) \times \pi_H(S)) \cdot \begin{pmatrix} 1.0, 0.9 \\ 1.0, 0.0 \\ 0.0, 0.1 \\ 0.0, 1.0 \end{pmatrix}$
$\pi_H(S)$
f_W
f_H^2
λ(W)
λ(H)
λ(W)=(1,1)
λ(H)=(1,1)



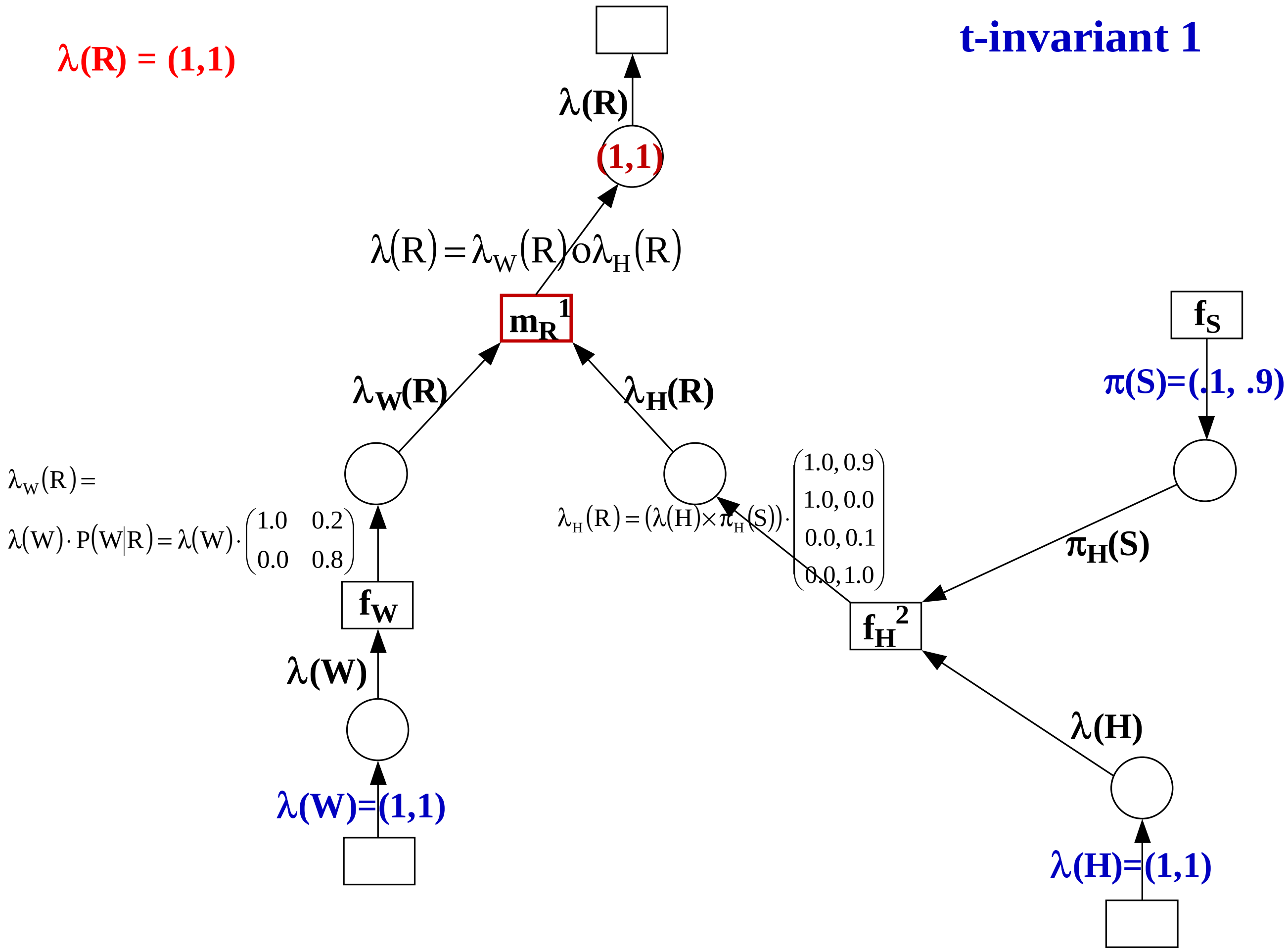
t-invariant 1
λ(R)
λ(R)=λ_W(R)oλ_H(R)
m_R^1
λ_W(R)
λ_H(R)
f_S
π(S)=(.1, .9)
(.1, .9)
λ_W(R)=
λ(W)·P(W|R)=λ(W)·(1.0 0.2; 0.0 0.8)
λ_H(R)=(λ(H)×π_H(S))·(1.0, 0.9; 1.0, 0.0; 0.0, 0.1; 0.0, 1.0)
π_H(S)
f_W
f_H^2
λ(W)
(1,1)
λ(H)
(1,1)
λ(W)=(1,1)
λ(H)=(1,1)

t-invariant 1

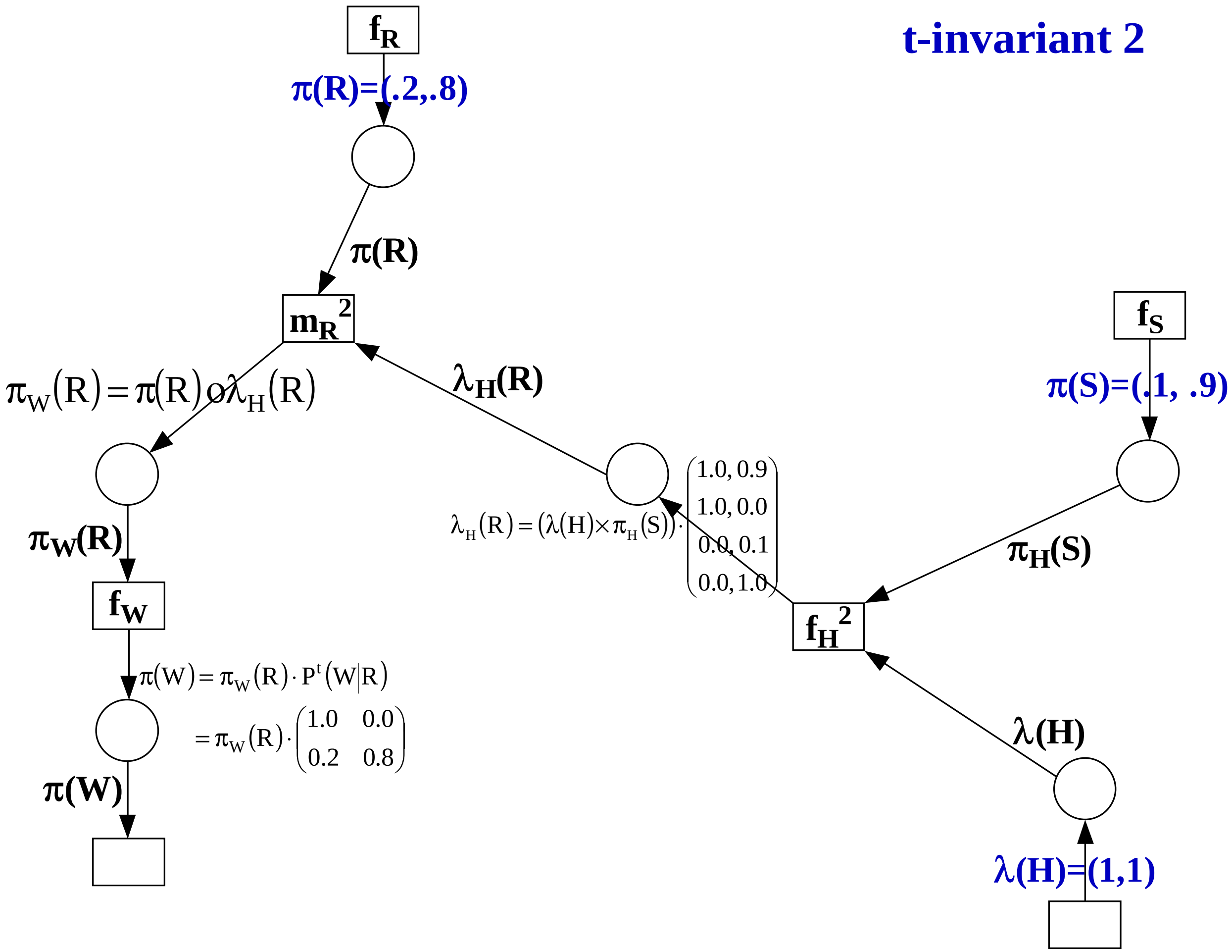


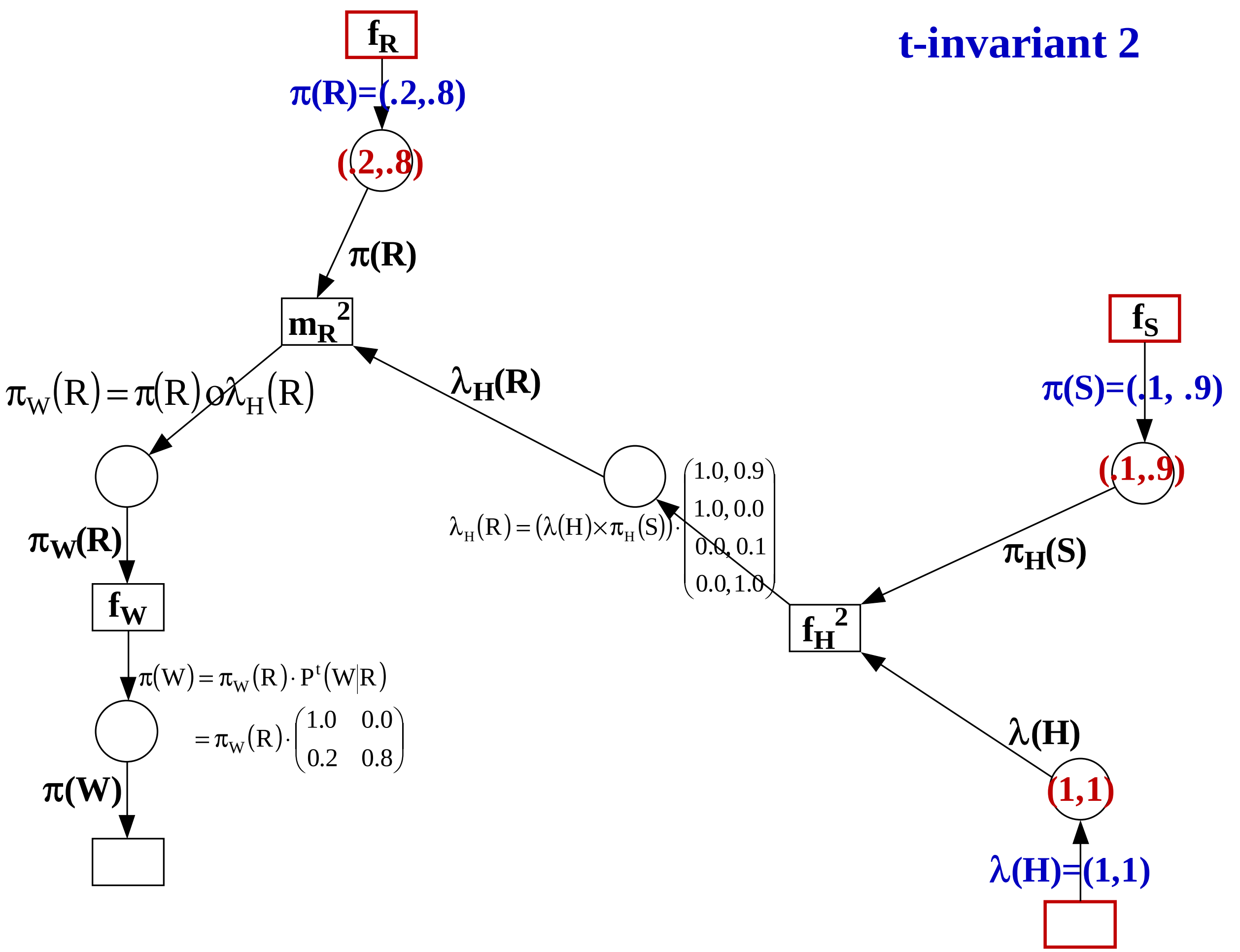
$\lambda(R) = (1,1)$

t-invariant 1

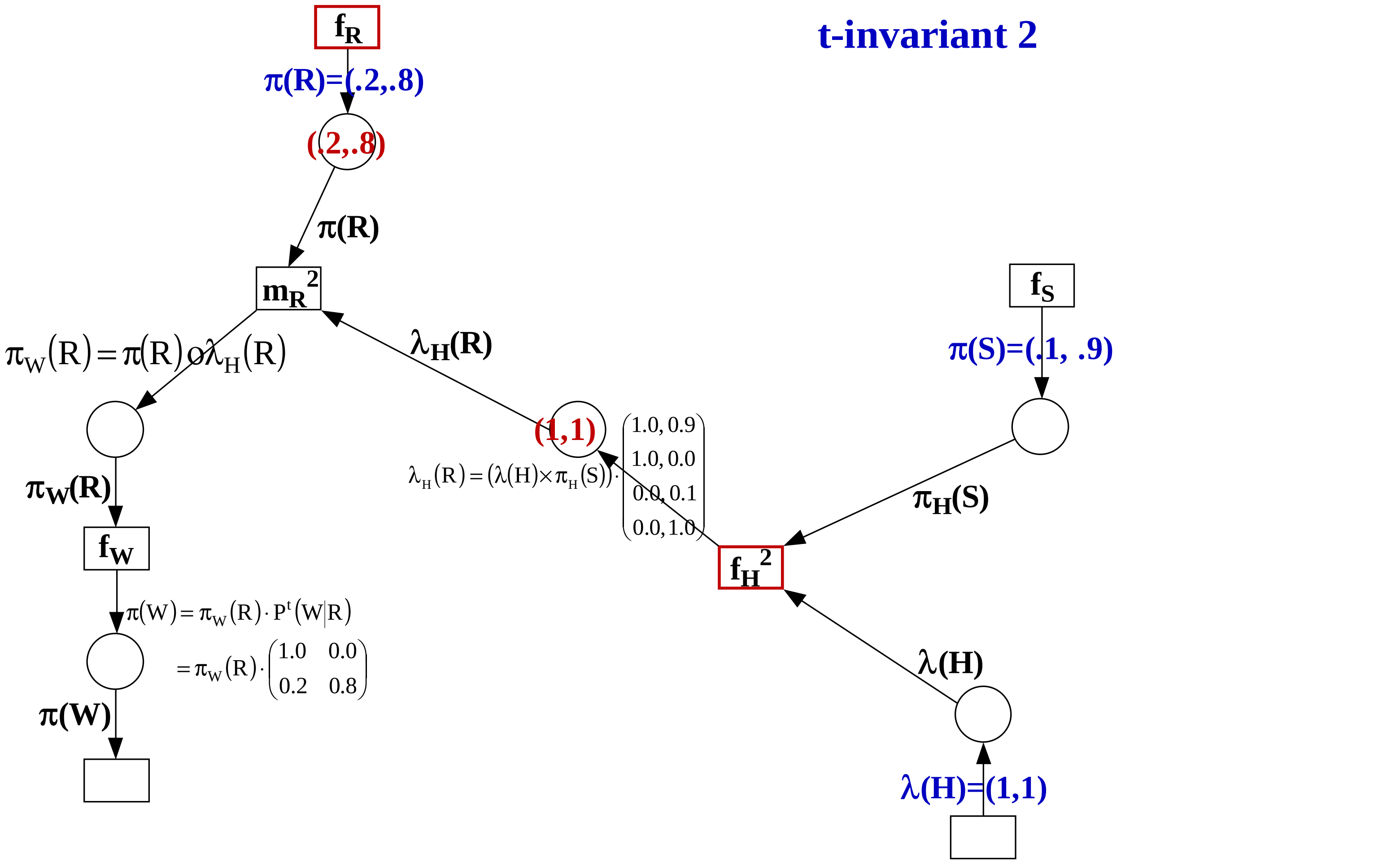


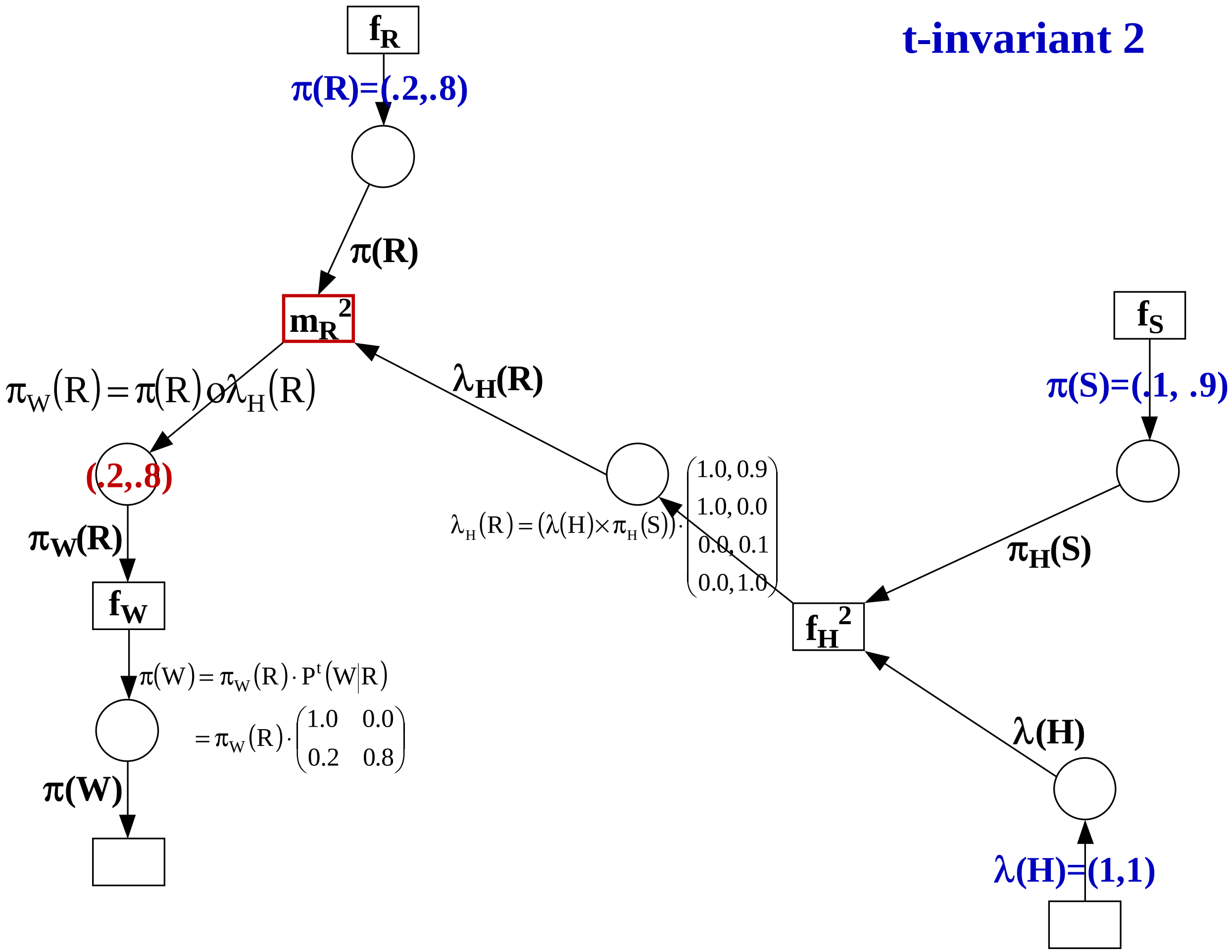
Wet Grass

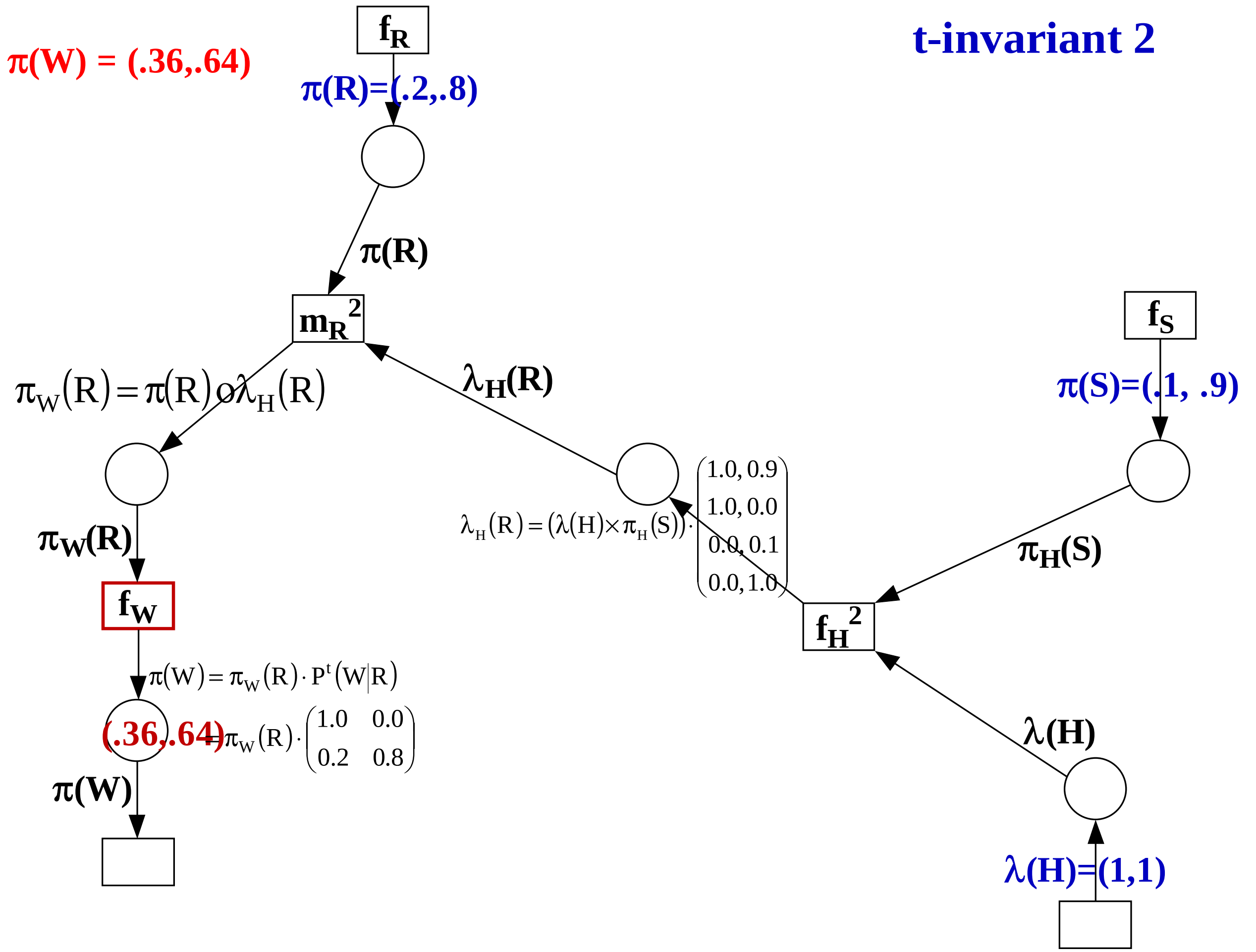

t-invariant 2
f_R
π(R)=(.2,.8)
π(R)
m_R^2
π_W(R) = π(R) o λ_H(R)
λ_H(R)
f_S
π(S)=(.1, .9)
λ_H(R) = (λ(H)×π_H(S))·
1.0, 0.9
1.0, 0.0
0.0, 0.1
0.0, 1.0
π_H(S)
f_H^2
π_W(R)
f_W
π(W) = π_W(R)·P^t(W|R)
= π_W(R)·
1.0 0.0
0.2 0.8
π(W)
λ(H)
λ(H)=(1,1)

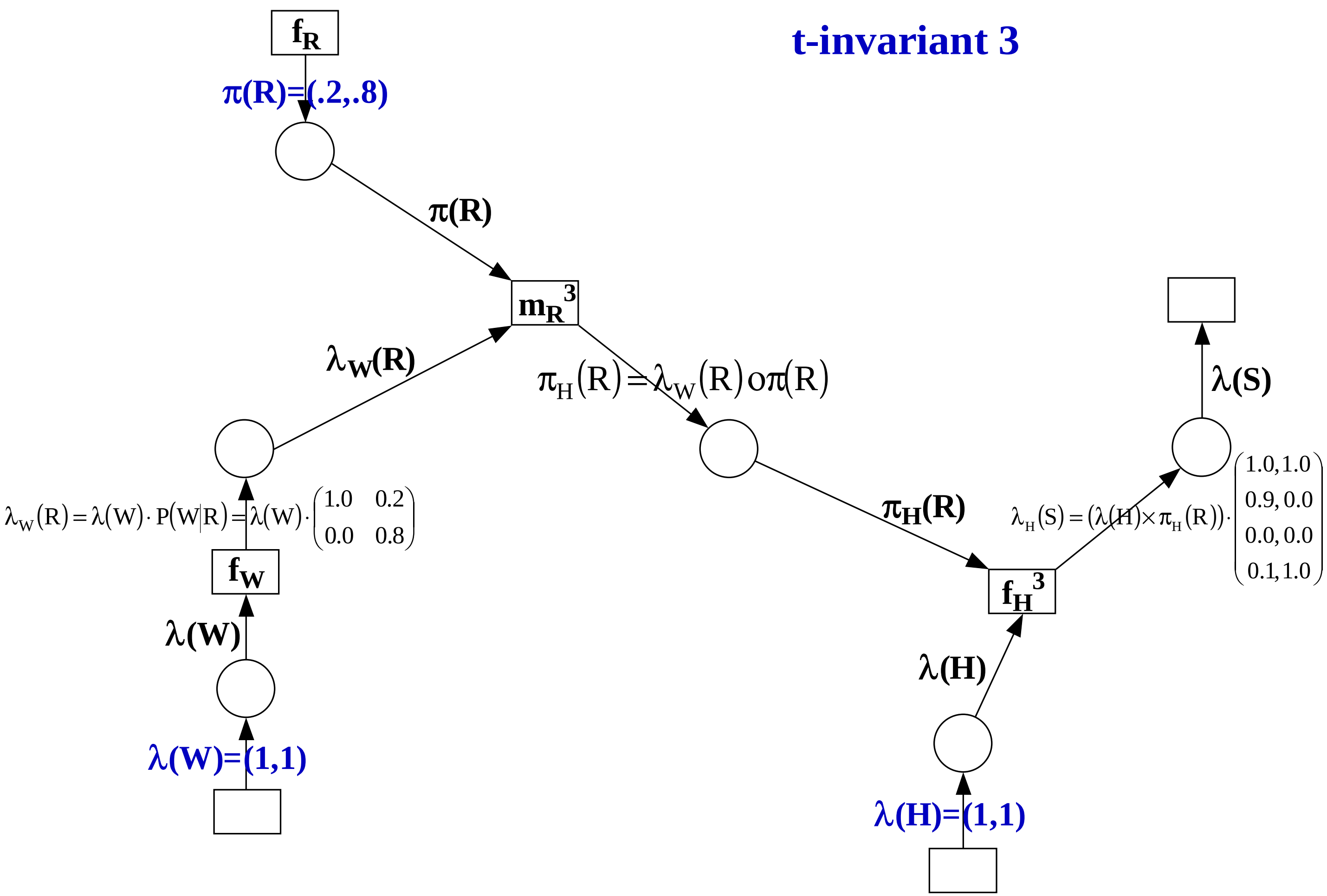


t-invariant 2
f_R
π(R)=(.2,.8)
(.2,.8)
π(R)
m_R 2
λ_H(R)
π_W(R) = π(R) o λ_H(R)
f_S
π(S)=(.1, .9)
(.1,.9)
π_H(S)
λ_H(R) = (λ(H)×π_H(S)) · (1.0, 0.9; 1.0, 0.0; 0.0, 0.1; 0.0, 1.0)
f_H 2
λ(H)
(1,1)
λ(H)=(1,1)
π_W(R)
f_W
π(W) = π_W(R) · P^t(W|R) = π_W(R) · (1.0 0.0; 0.2 0.8)
π(W)

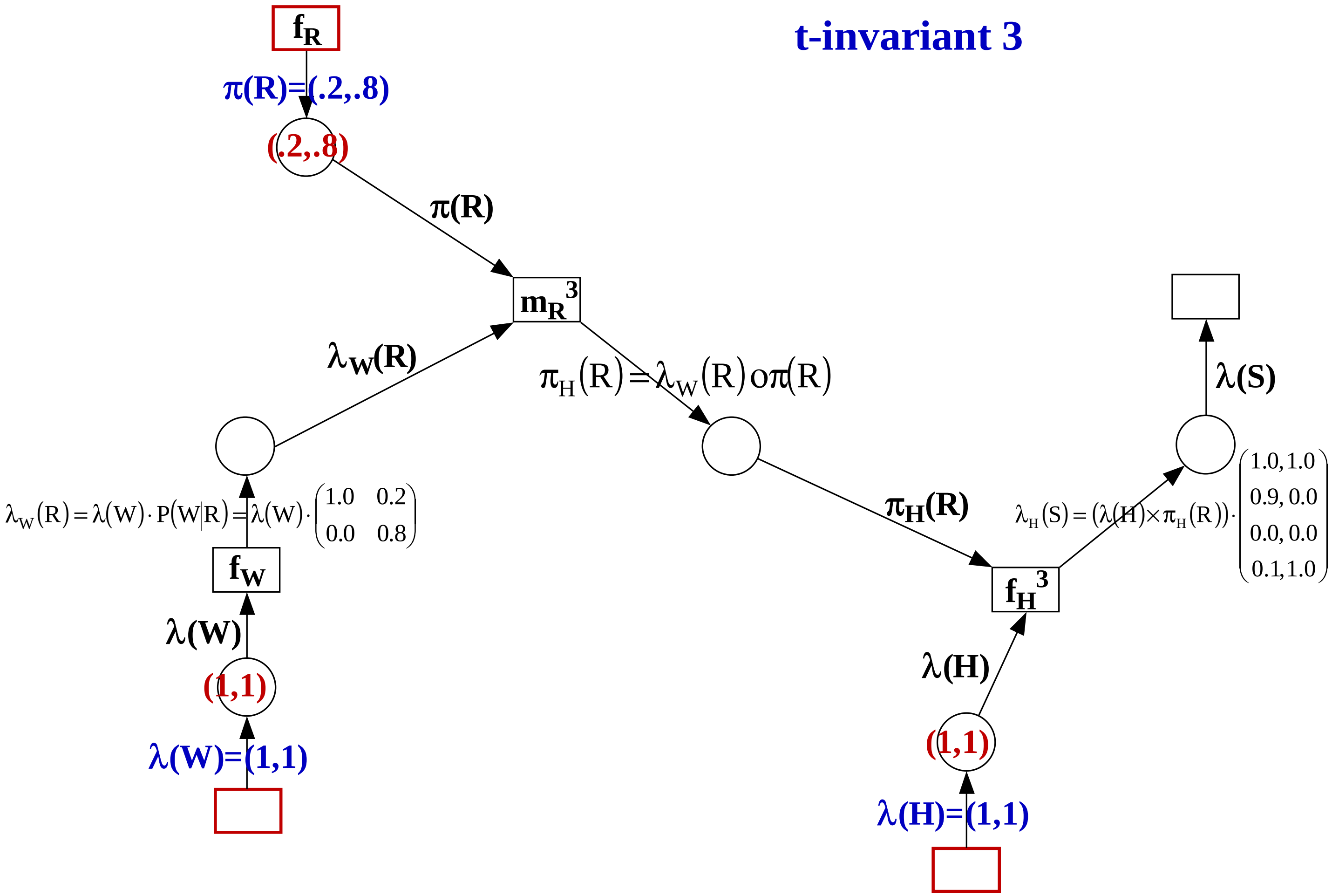

t-invariant 2
f_R
π(R)=(.2,.8)
(.2,.8)
π(R)
m_R^2
λ_H(R)
π_W(R)=π(R)oλ_H(R)
π_W(R)
f_W
π(W)=π_W(R)·P^t(W|R)
=π_W(R)·(1.0 0.0; 0.2 0.8)
π(W)
(1,1)
λ_H(R)=(λ(H)×π_H(S))·(1.0,0.9; 1.0,0.0; 0.0,0.1; 0.0,1.0)
f_H^2
f_S
π(S)=(.1, .9)
π_H(S)
λ(H)
λ(H)=(1,1)

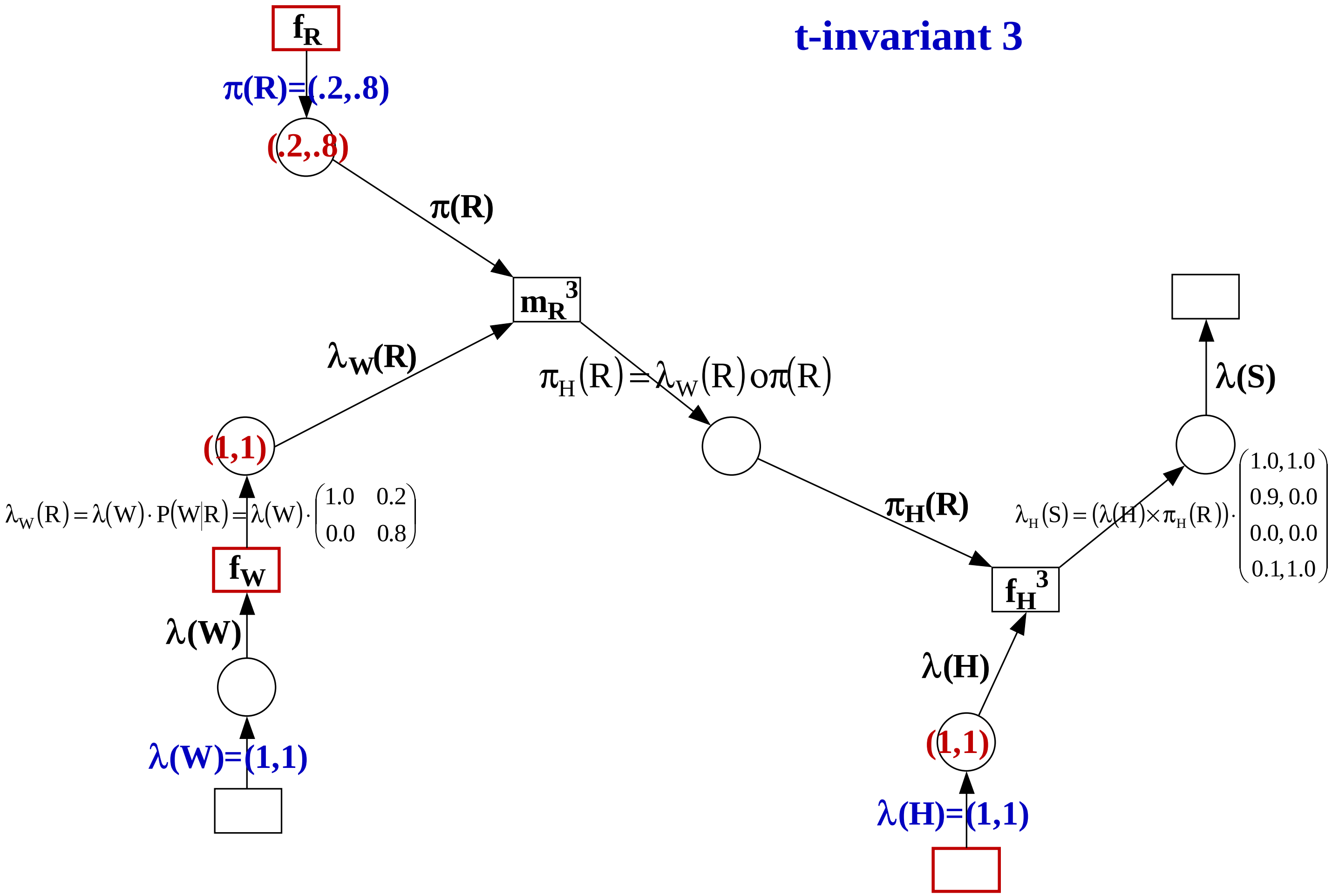

t-invariant 2
f_R
π(R)=(.2,.8)
π(R)
m_R^2
λ_H(R)
π_W(R)=π(R)∘λ_H(R)
(.2,.8)
π_W(R)
f_W
π(W)=π_W(R)·P^t(W|R)
=π_W(R)·(1.0 0.0; 0.2 0.8)
π(W)
λ_H(R)=(λ(H)×π_H(S))·(1.0, 0.9; 1.0, 0.0; 0.0, 0.1; 0.0, 1.0)
f_S
π(S)=(.1, .9)
π_H(S)
f_H^2
λ(H)
λ(H)=(1,1)

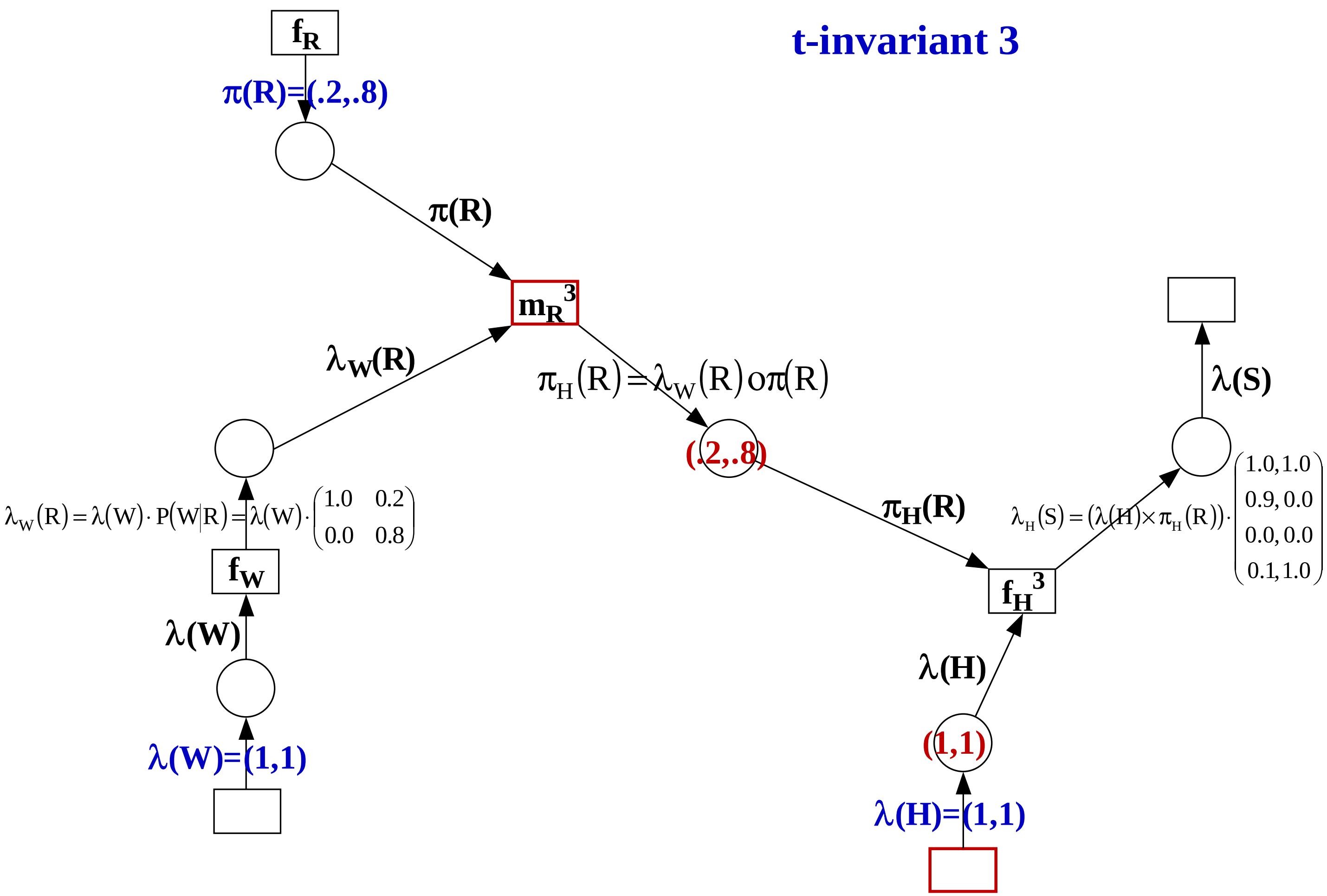

π(W) = (.36,.64)
f_R
π(R)=(.2,.8)
t-invariant 2
π(R)
m_R^2
λ_H(R)
f_S
π(S)=(.1, .9)
π_W(R)=π(R)oλ_H(R)
λ_H(R)=(λ(H)×π_H(S))·
1.0, 0.9
1.0, 0.0
0.0, 0.1
0.0, 1.0
π_W(R)
π_H(S)
f_W
f_H^2
π(W)=π_W(R)·P^t(W|R)
(.36,.64)
=π_W(R)·
1.0 0.0
0.2 0.8
λ(H)
π(W)
λ(H)=(1,1)


t-invariant 3
f_R
π(R)=(.2,.8)
π(R)
m_R^3
λ_W(R)
π_H(R)=λ_W(R)oπ(R)
λ(S)
λ_W(R)=λ(W)·P(W|R)=λ(W)·(1.0 0.2; 0.0 0.8)
f_W
π_H(R)
λ_H(S)=(λ(H)×π_H(R))·(1.0,1.0; 0.9,0.0; 0.0,0.0; 0.1,1.0)
f_H^3
λ(W)
λ(H)
λ(W)=(1,1)
λ(H)=(1,1)

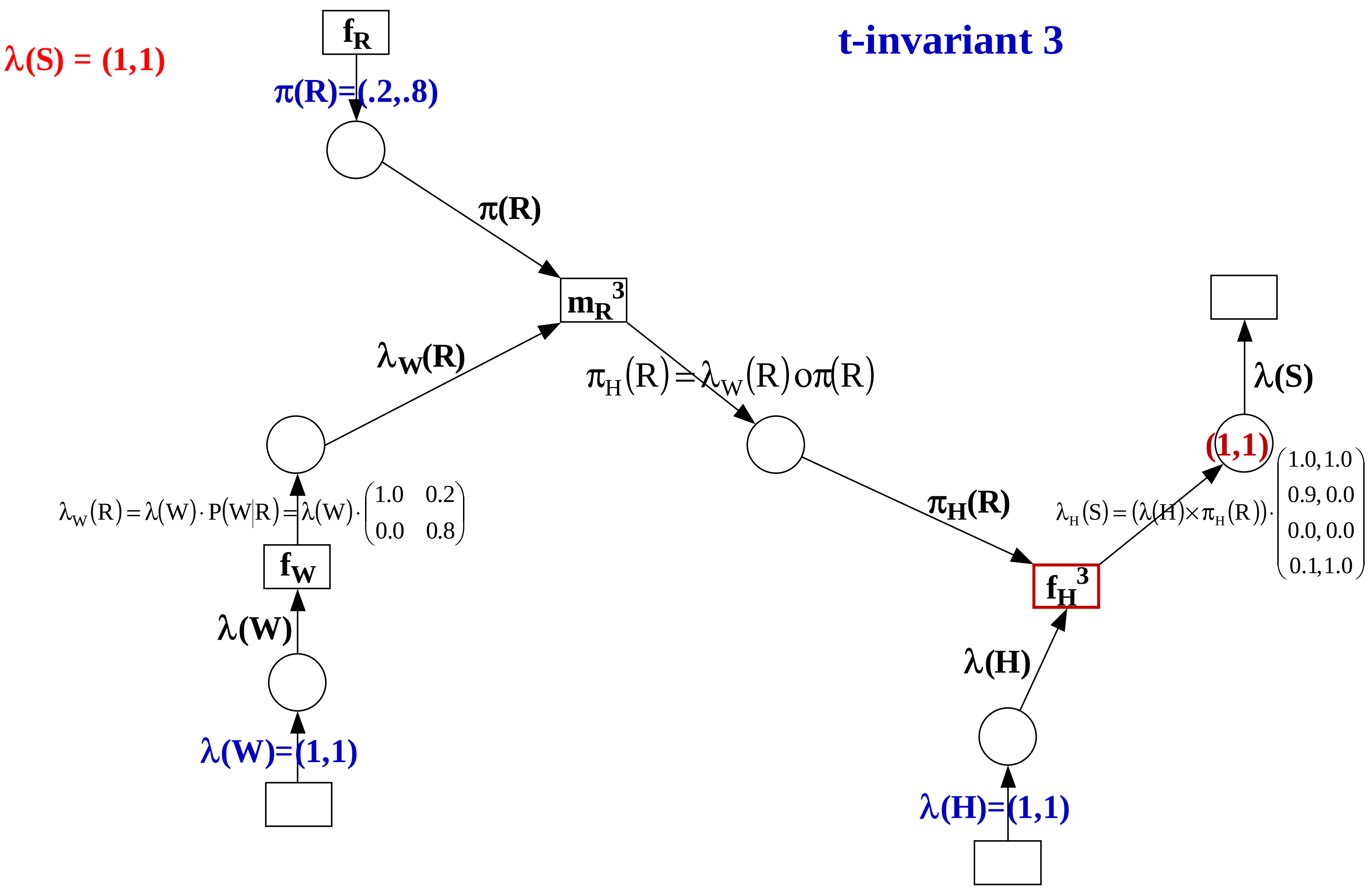
t-invariant 3

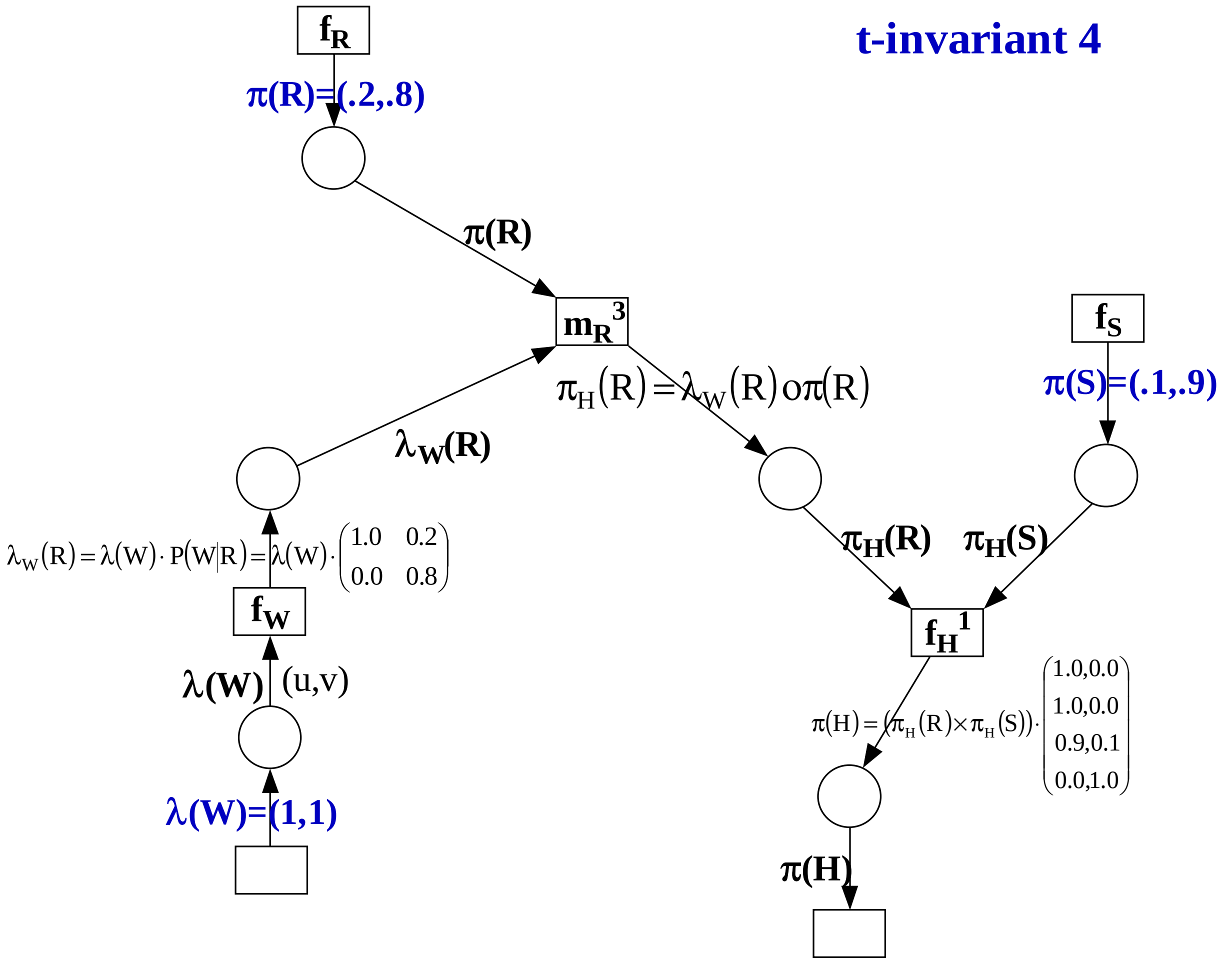


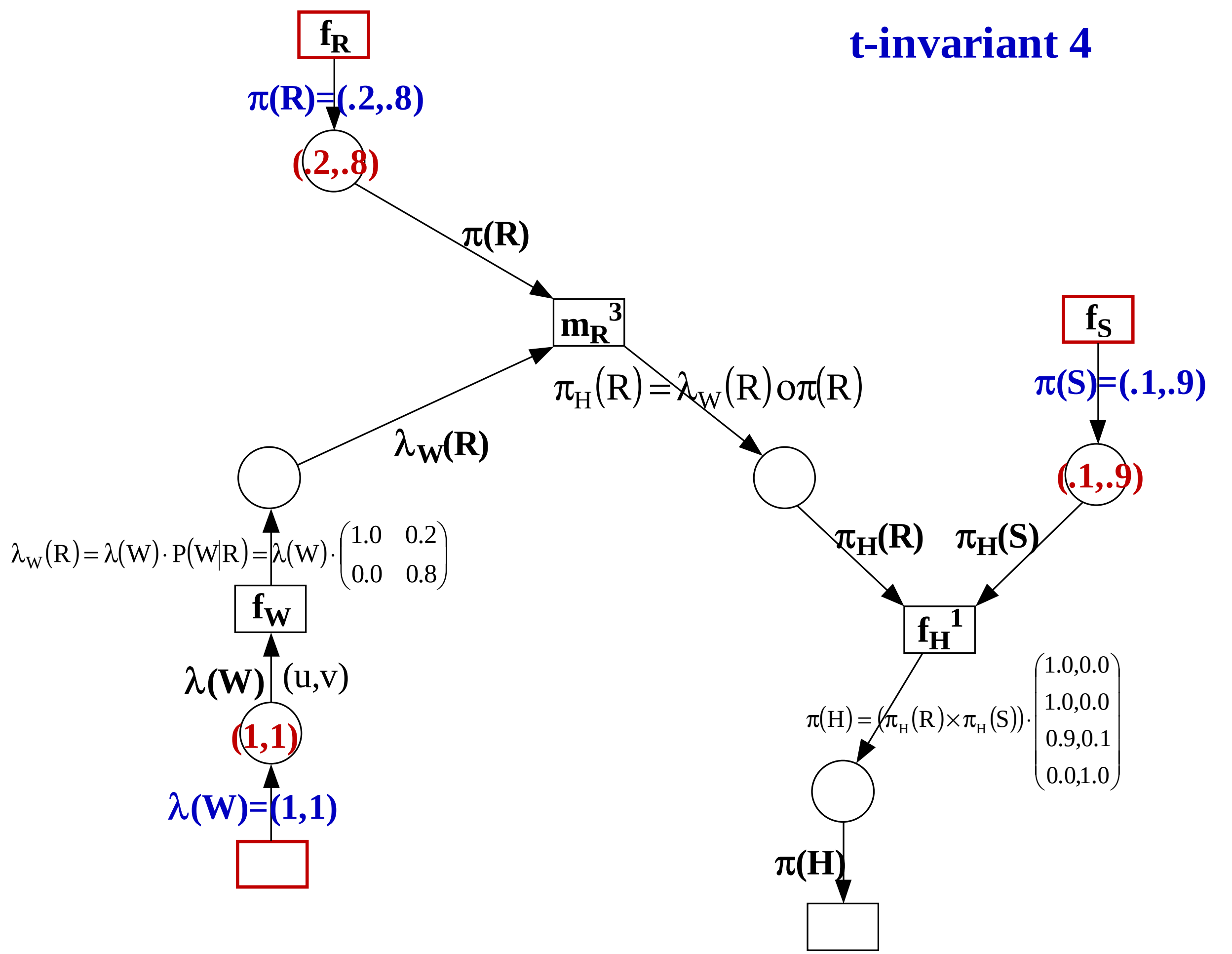

t-invariant 3
f_R
$\pi(R)=(.2,.8)$
(.2,.8)
$\pi(R)$
m_R^3
$\lambda_W(R)$
$\pi_H(R)=\lambda_W(R) \circ \pi(R)$
(1,1)
$\lambda_W(R)=\lambda(W)\cdot P(W|R)=\lambda(W)\cdot\begin{pmatrix}1.0 & 0.2\\ 0.0 & 0.8\end{pmatrix}$
f_W
$\lambda(W)$
$\lambda(W)=(1,1)$
$\pi_H(R)$
$\lambda(S)$
$\lambda_H(S)=(\lambda(H)\times\pi_H(R))\cdot\begin{pmatrix}1.0, 1.0\\ 0.9, 0.0\\ 0.0, 0.0\\ 0.1, 1.0\end{pmatrix}$
f_H^3
$\lambda(H)$
(1,1)
$\lambda(H)=(1,1)$



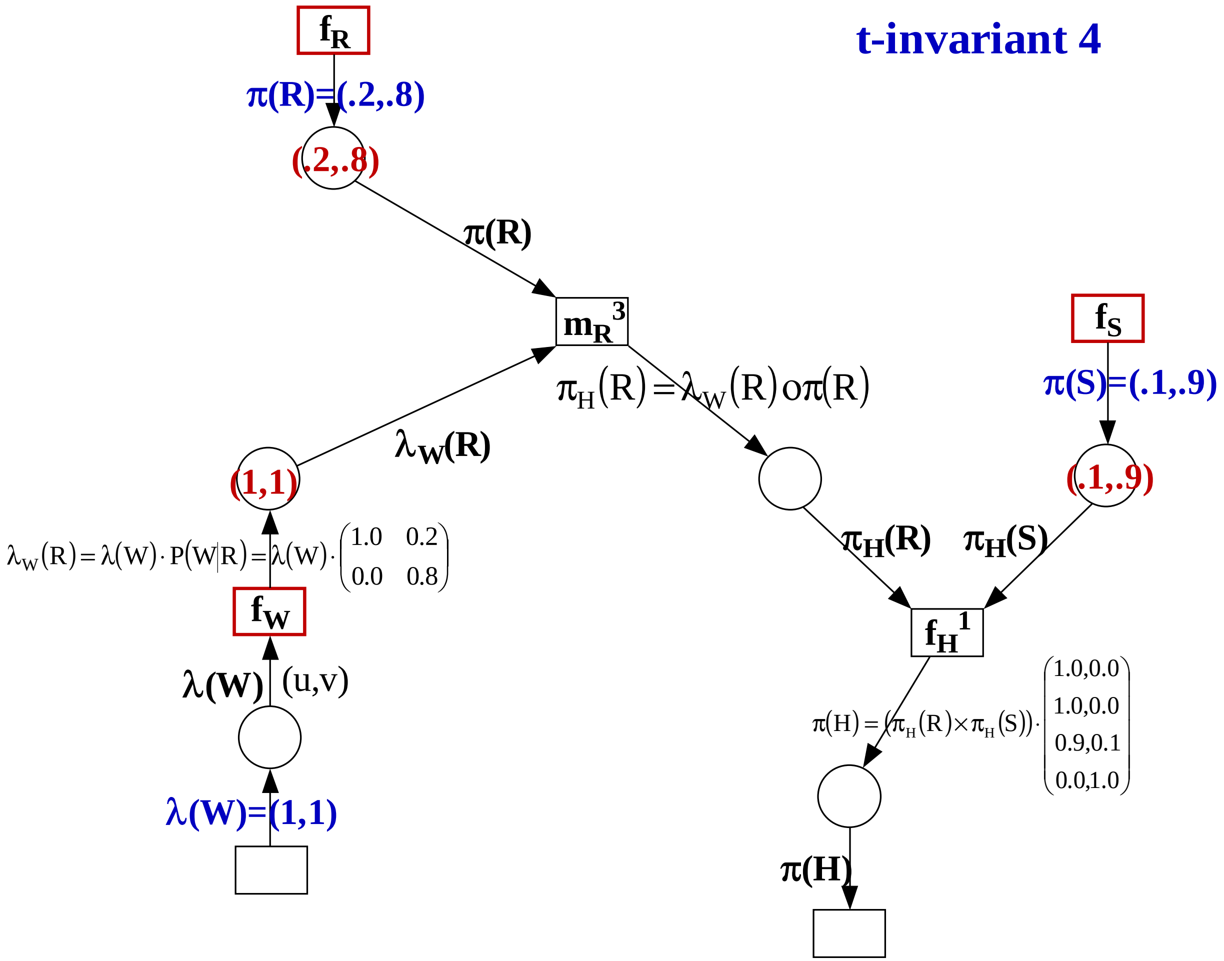
t-invariant 3
f_R
$\pi(R)=(.2,.8)$
$\pi(R)$
m_R^3
$\lambda_W(R)$
$\pi_H(R)=\lambda_W(R)\circ\pi(R)$
(.2,.8)
$\lambda(S)$
$\lambda_W(R)=\lambda(W)\cdot P(W|R)=\lambda(W)\cdot\begin{pmatrix}1.0 & 0.2\\ 0.0 & 0.8\end{pmatrix}$
f_W
$\pi_H(R)$
$\lambda_H(S)=(\lambda(H)\times\pi_H(R))\cdot\begin{pmatrix}1.0,1.0\\ 0.9,0.0\\ 0.0,0.0\\ 0.1,1.0\end{pmatrix}$
f_H^3
$\lambda(W)$
$\lambda(H)$
(1,1)
$\lambda(W)=(1,1)$
$\lambda(H)=(1,1)$

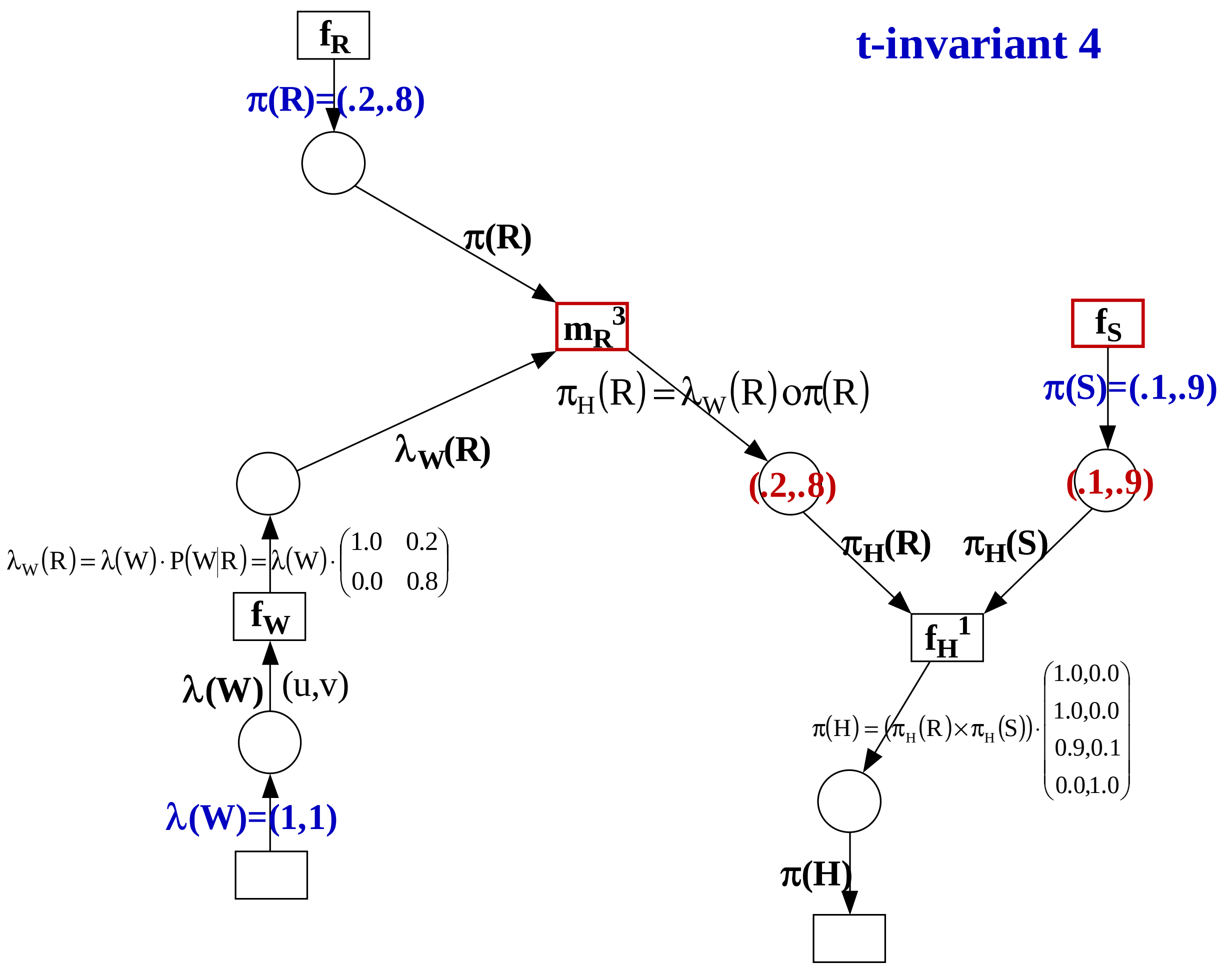

λ(S) = (1,1)
f_R
π(R)=(.2,.8)
t-invariant 3
π(R)
m_R^3
λ_W(R)
π_H(R)=λ_W(R)oπ(R)
λ(S)
(1,1)
λ_W(R)=λ(W)·P(W|R)=λ(W)·(1.0 0.2 / 0.0 0.8)
π_H(R)
λ_H(S)=(λ(H)×π_H(R))·(1.0,1.0 / 0.9,0.0 / 0.0,0.0 / 0.1,1.0)
f_W
f_H^3
λ(W)
λ(H)
λ(W)=(1,1)
λ(H)=(1,1)

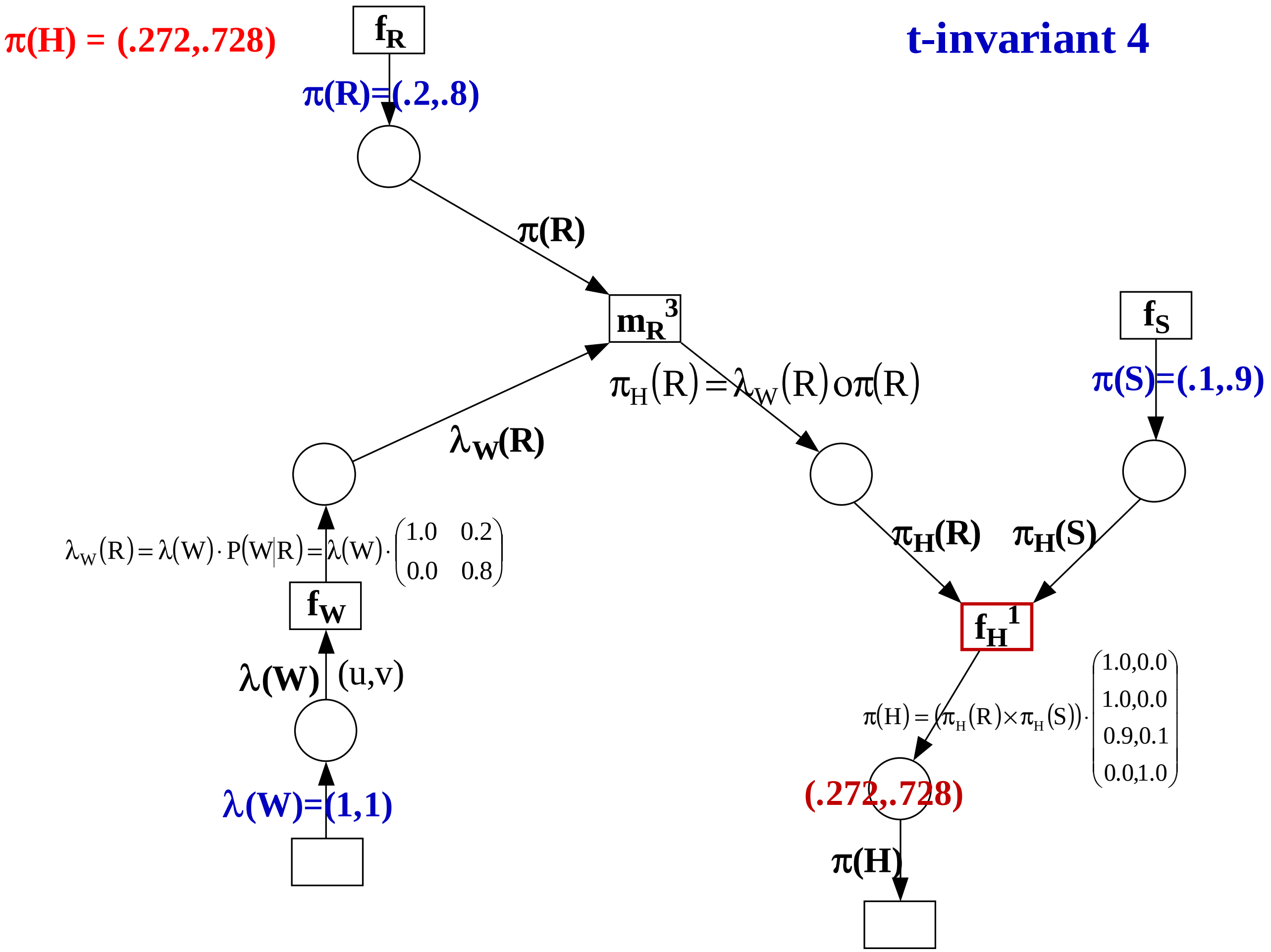

t-invariant 4
f_R
$\pi(R)=(.2,.8)$
$\pi(R)$
m_R^3
f_S
$\pi(S)=(.1,.9)$
$\pi_H(R)=\lambda_W(R)\circ\pi(R)$
$\lambda_W(R)$
$\pi_H(R)$
$\pi_H(S)$
$\lambda_W(R)=\lambda(W)\cdot P(W|R)=\lambda(W)\cdot\begin{pmatrix}1.0 & 0.2\\ 0.0 & 0.8\end{pmatrix}$
f_W
f_H^1
$\lambda(W)$
(u,v)
$\pi(H)=(\pi_H(R)\times\pi_H(S))\cdot\begin{pmatrix}1.0,0.0\\ 1.0,0.0\\ 0.9,0.1\\ 0.0,1.0\end{pmatrix}$
$\lambda(W)=(1,1)$
$\pi(H)$



t-invariant 4
fR
π(R)=(.2,.8)
(.2,.8)
π(R)
mR3
fS
π(S)=(.1,.9)
πH(R)=λW(R)oπ(R)
λW(R)
(.1,.9)
πH(R)
πH(S)
λW(R) = λ(W)·P(W|R) = λ(W)·(1.0 0.2; 0.0 0.8)
fW
fH1
λ(W)
(u,v)
π(H) = (πH(R)×πH(S))·(1.0,0.0; 1.0,0.0; 0.9,0.1; 0.0,1.0)
(1,1)
λ(W)=(1,1)
π(H)


t-invariant 4
f_R
π(R)=(.2,.8)
(.2,.8)
π(R)
m_R^3
f_S
π(S)=(.1,.9)
π_H(R)=λ_W(R)oπ(R)
λ_W(R)
(1,1)
(.1,.9)
π_H(R)
π_H(S)
λ_W(R)=λ(W)·P(W|R)=λ(W)·(1.0 0.2; 0.0 0.8)
f_W
f_H^1
λ(W)
(u,v)
π(H)=(π_H(R)×π_H(S))·(1.0,0.0; 1.0,0.0; 0.9,0.1; 0.0,1.0)
λ(W)=(1,1)
π(H)


t-invariant 4
f_R
π(R)=(.2,.8)
π(R)
m_R^3
f_S
π(S)=(.1,.9)
π_H(R)=λ_W(R)oπ(R)
λ_W(R)
(.2,.8)
(.1,.9)
π_H(R)
π_H(S)
λ_W(R)=λ(W)·P(W|R)=λ(W)·(1.0 0.2 / 0.0 0.8)
f_W
f_H^1
λ(W)
(u,v)
π(H)=(π_H(R)×π_H(S))·(1.0,0.0 / 1.0,0.0 / 0.9,0.1 / 0.0,1.0)
λ(W)=(1,1)
π(H)



π(H) = (.272,.728)
t-invariant 4
f_R
π(R)=(.2,.8)
π(R)
m_R^3
λ_W(R)
π_H(R)=λ_W(R)oπ(R)
f_S
π(S)=(.1,.9)
π_H(R)
π_H(S)
λ_W(R)=λ(W)·P(W|R)=λ(W)·(1.0 0.2; 0.0 0.8)
f_W
λ(W)
(u,v)
λ(W)=(1,1)
f_H^1
π(H)=(π_H(R)×π_H(S))·(1.0,0.0; 1.0,0.0; 0.9,0.1; 0.0,1.0)
(.272,.728)
π(H)

$\lambda(R)$ **= (1 ,1)**

$\pi(R)$ **= (.2,.8)**

$P(R) = \alpha\lambda(R)\pi(R)$ **= (.2,.8)**

$\lambda(S)$ **= (1 ,1)**

$\pi(S)$ **= (.1,.9)**

$P(S) = \alpha\lambda(S)\pi(S)$ **= (.1,.9)**

$\lambda(W)$ **= (1 ,1)**

$\pi(W)$ **= (.36,.64)**

$P(W) = \alpha\lambda(W)\pi(W)$ **= (.36,.64)**

$\lambda(H)$ **= (1 ,1)**

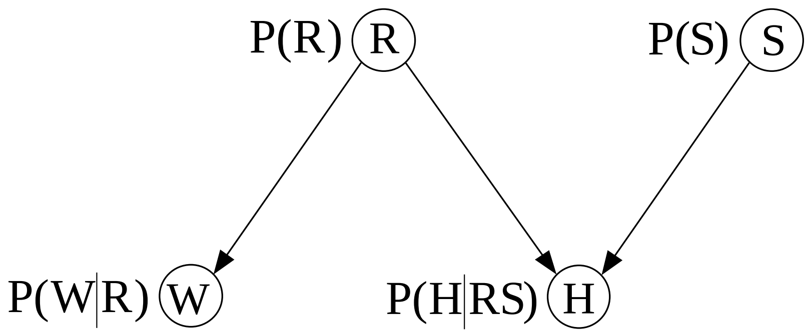
$\pi(H)$ **= (.272, 728)**

$P(H) = \alpha\lambda(H)\pi(H)$ **= (.272, 728)**

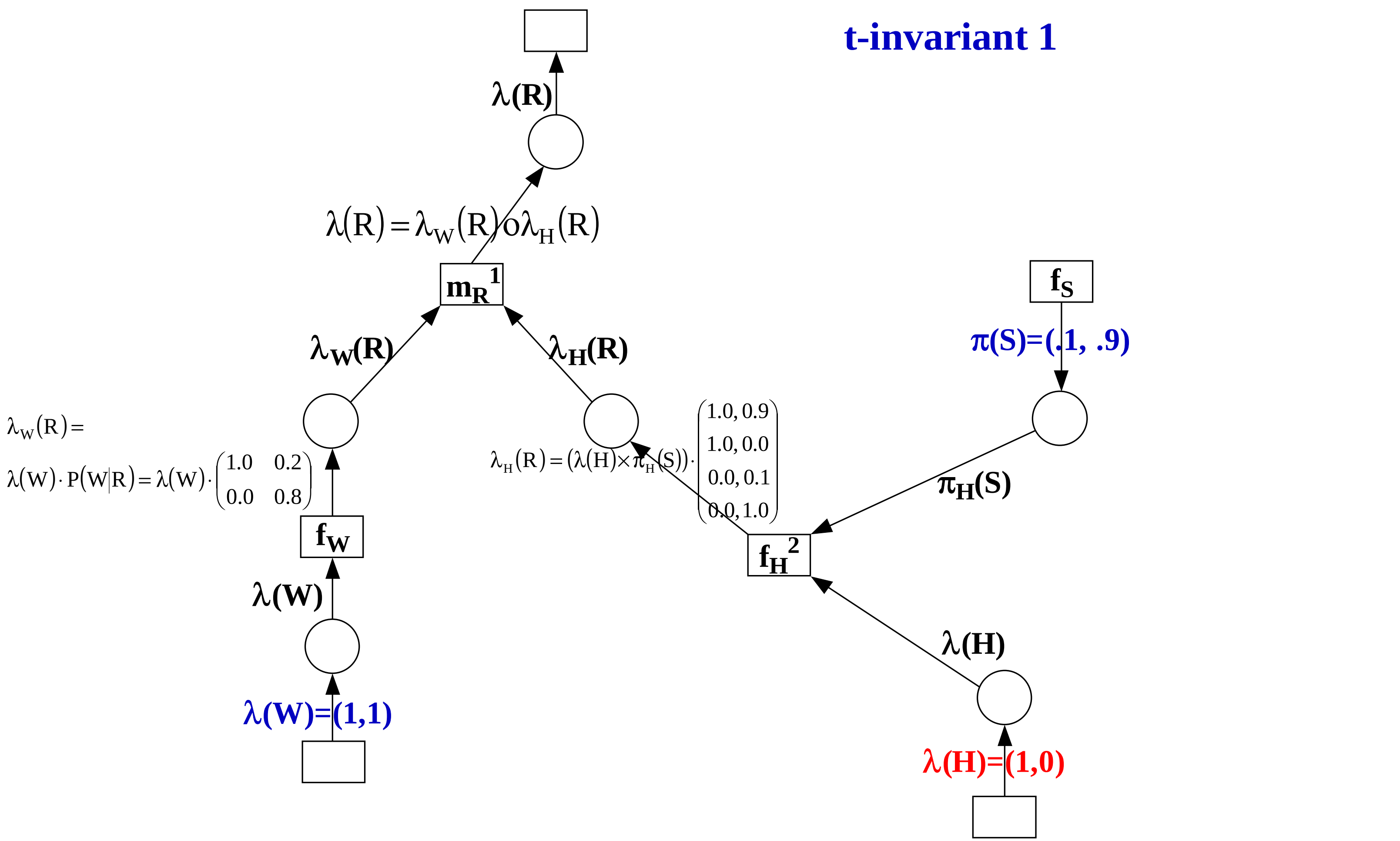
h_1 = Mr. Holmes' grass is wet

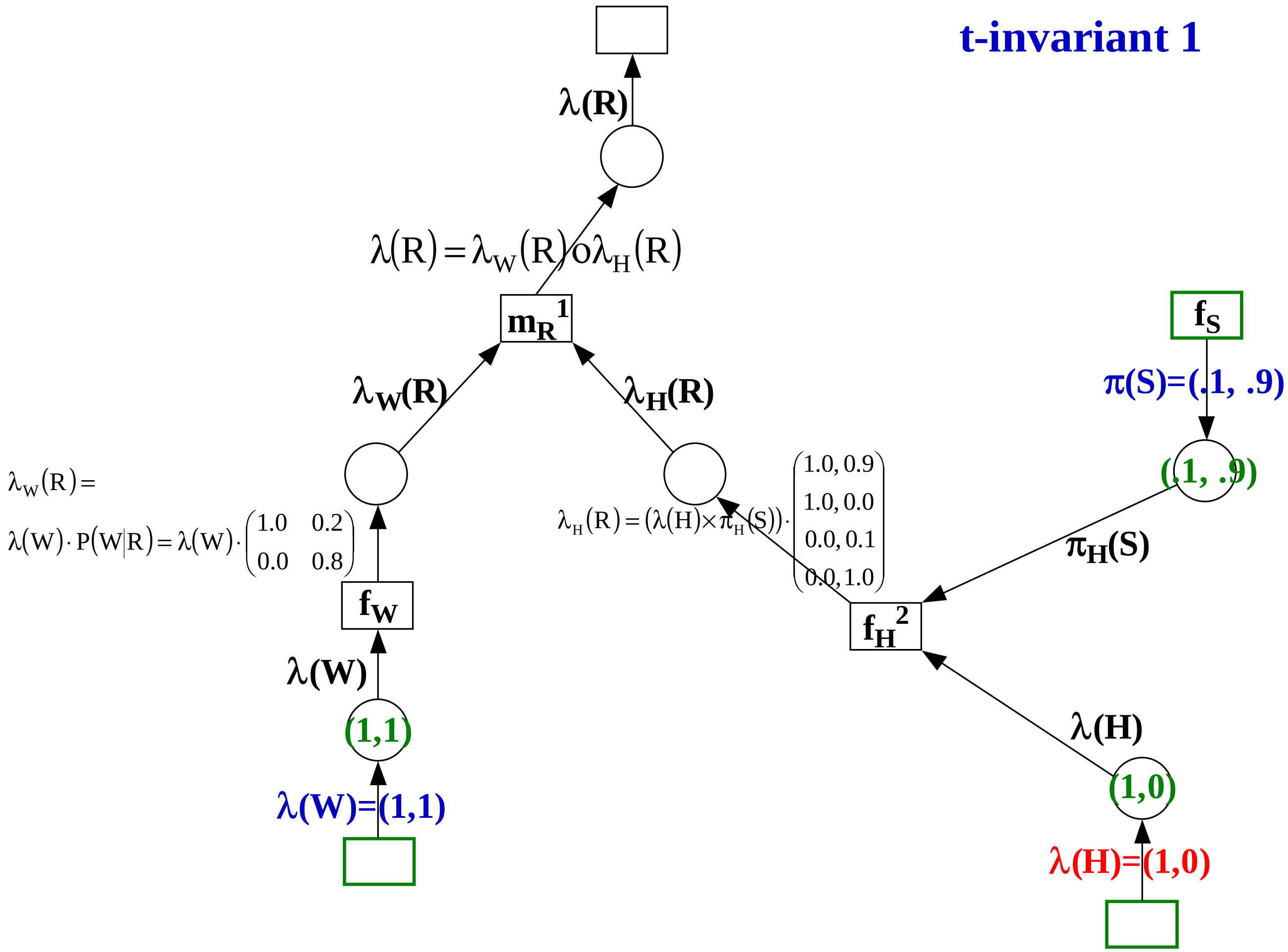
r_1 = rain is the cause for Mr. Holmes' grass being wet
r_0 = rain is not the cause for Mr. Holmes' grass being wet
s_1 = sprinkler is the cause for Mr. Holmes' grass being wet
s_0 = sprinkler is not the cause for Mr. Holmes' grass being wet
w_1 = Dr. Watson's grass is wet
w_0 = Dr. Watson's grass is not wet

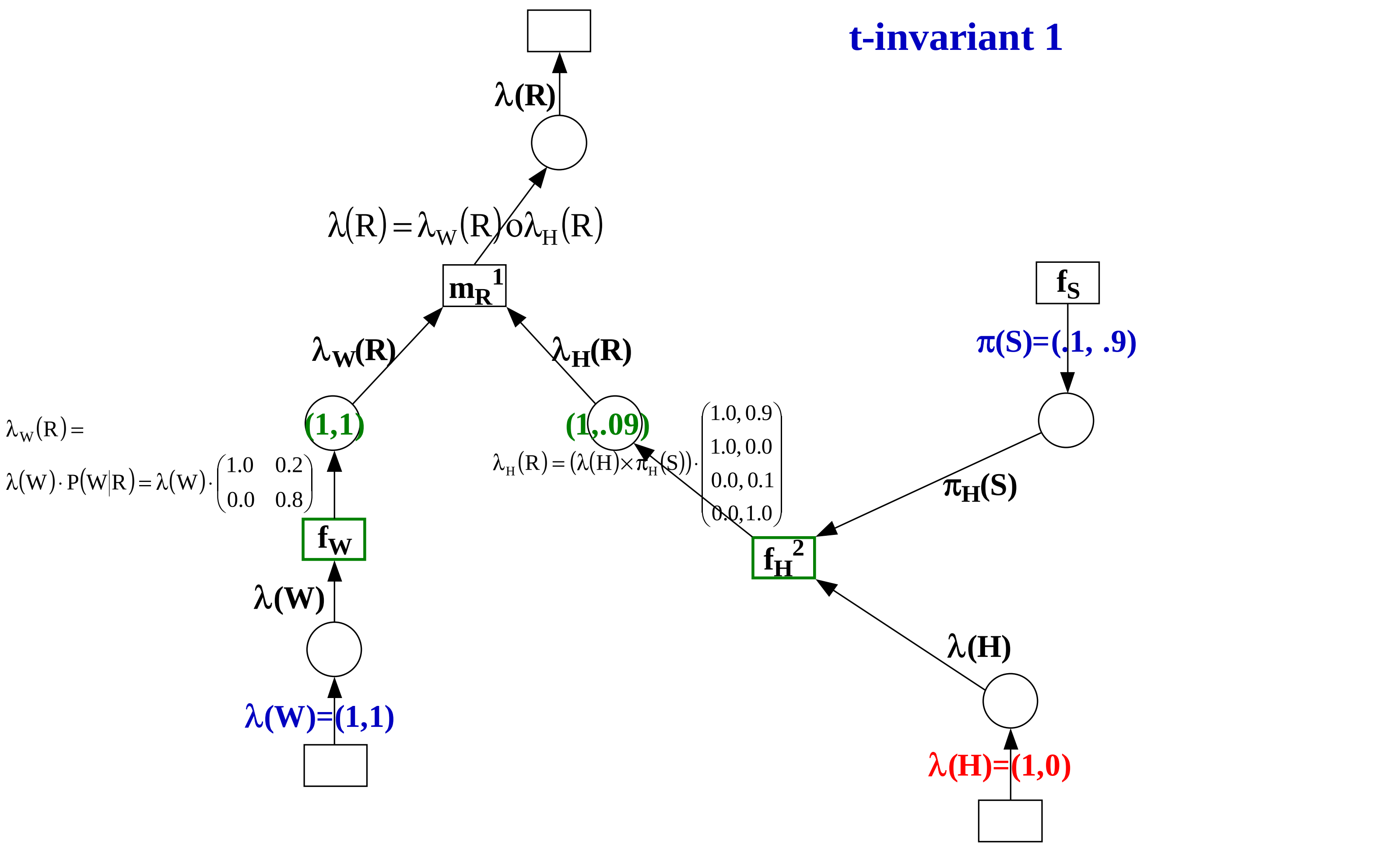
New evidence



$\lambda(R)$		**= (1 ,1)**
$\pi(R)$	**= (.2,.8)**	**= (.2,.8)**
$P(R) = \alpha\lambda(R)\pi(R)$		**= (.2,.8)**
$\lambda(S)$		**= (1 ,1)**
$\pi(S)$	**= (.1,.9)**	**= (.1,.9)**
$P(S) = \alpha\lambda(S)\pi(S)$		**= (.1,.9)**
$\lambda(W)$	**= (1 ,1)**	**= (1 ,1)**
$\pi(W)$		**= (.36,.64)**
$P(W) = \alpha\lambda(W)\pi(W)$		**= (.36,.64)**
$\lambda(H)$	**= (1 ,1)**	**= (1 ,1)**
$\pi(H)$	**= (1 ,0)** **new**	**= (.272, 728)**
$P(H) = \alpha\lambda(H)\pi(H)$	**= (1 ,0)** **evidence**	**= (.272, 728)**


t-invariant 1
λ(R)
λ(R)=λW(R)oλH(R)
mR1
λW(R)
λH(R)
fS
π(S)=(.1, .9)
λW(R)=
λ(W)·P(W|R)=λ(W)·(1.0 0.2 / 0.0 0.8)
λH(R)=(λ(H)×πH(S))·(1.0, 0.9 / 1.0, 0.0 / 0.0, 0.1 / 0.0, 1.0)
πH(S)
fW
fH2
λ(W)
λ(H)
λ(W)=(1,1)
λ(H)=(1,0)

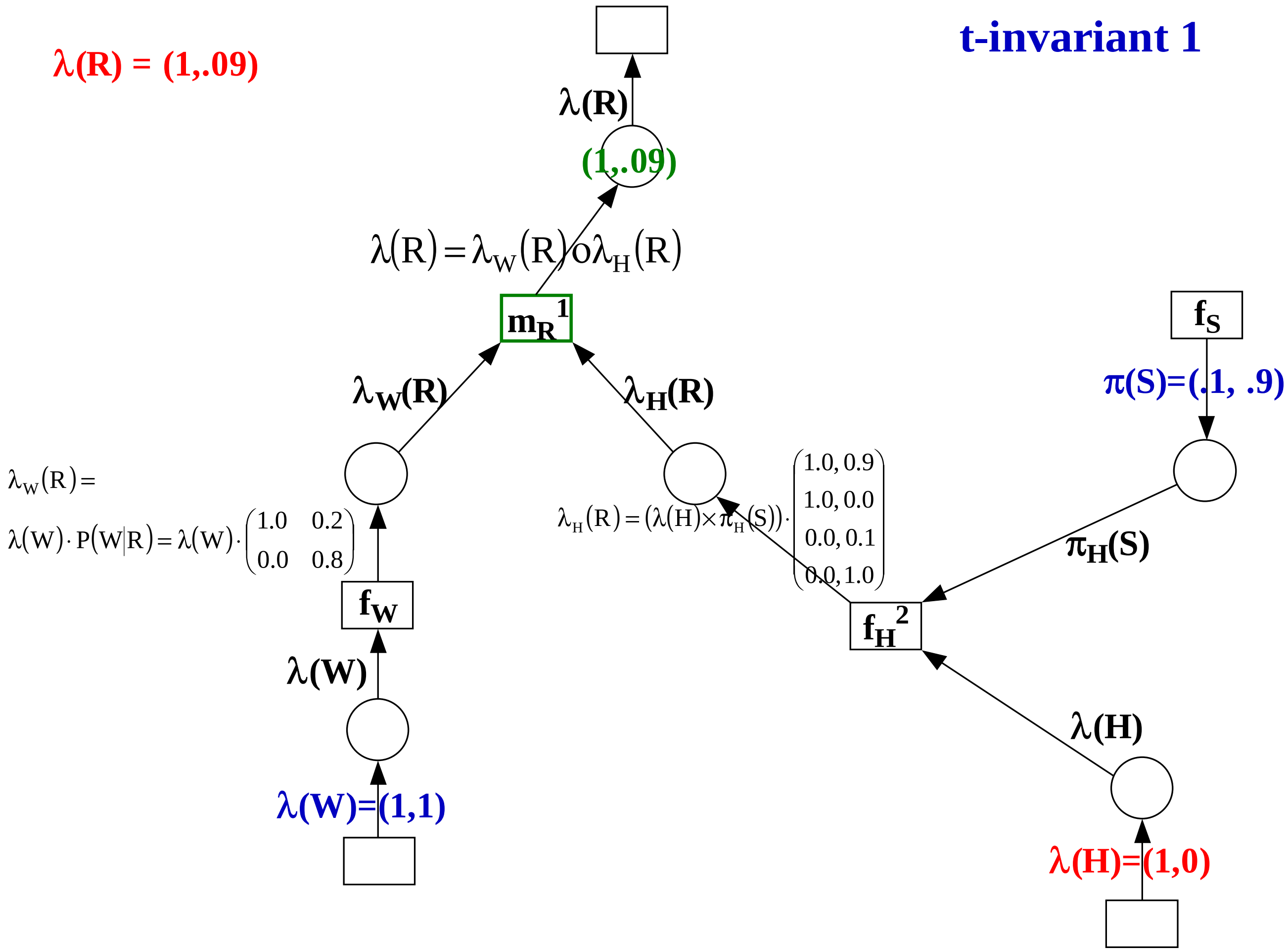

t-invariant 1
λ(R)
λ(R)=λ_W(R)oλ_H(R)
m_R 1
λ_W(R)
λ_H(R)
f_S
π(S)=(.1, .9)
(.1, .9)
λ_W(R)=
λ(W)·P(W|R)=λ(W)·(1.0 0.2; 0.0 0.8)
λ_H(R)=(λ(H)×π_H(S))·(1.0, 0.9; 1.0, 0.0; 0.0, 0.1; 0.0, 1.0)
π_H(S)
f_W
f_H 2
λ(W)
(1,1)
λ(H)
(1,0)
λ(W)=(1,1)
λ(H)=(1,0)


t-invariant 1
λ(R)
λ(R)=λW(R)oλH(R)
mR1
λW(R)
λH(R)
fS
π(S)=(.1, .9)
λW(R)=
λ(W)·P(W|R)=λ(W)·(1.0 0.2; 0.0 0.8)
(1,1)
(1,.09)
λH(R)=(λ(H)×πH(S))·(1.0, 0.9; 1.0, 0.0; 0.0, 0.1; 0.0, 1.0)
πH(S)
fW
fH2
λ(W)
λ(H)
λ(W)=(1,1)
λ(H)=(1,0)

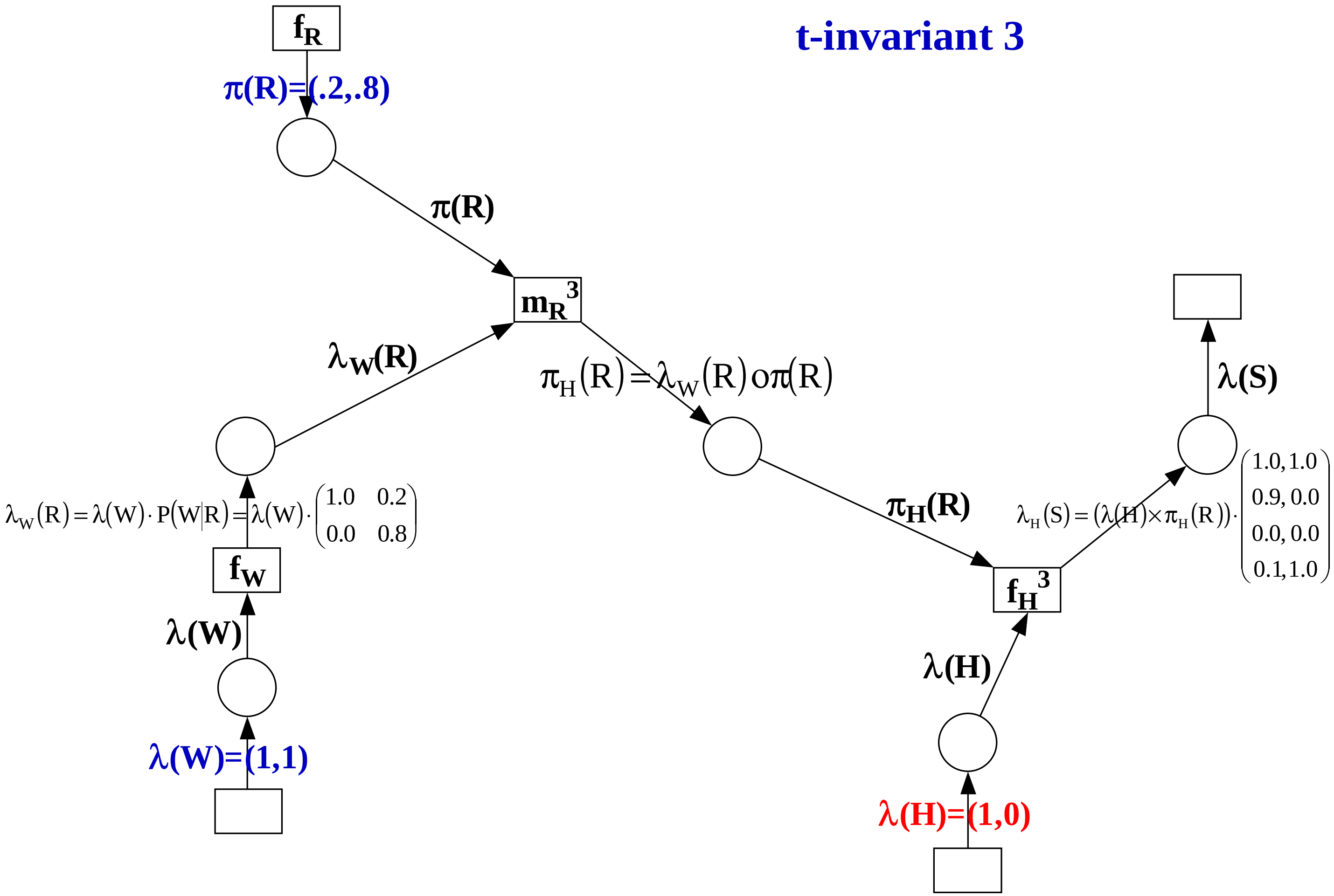
Wet Grass

t-invariant 1

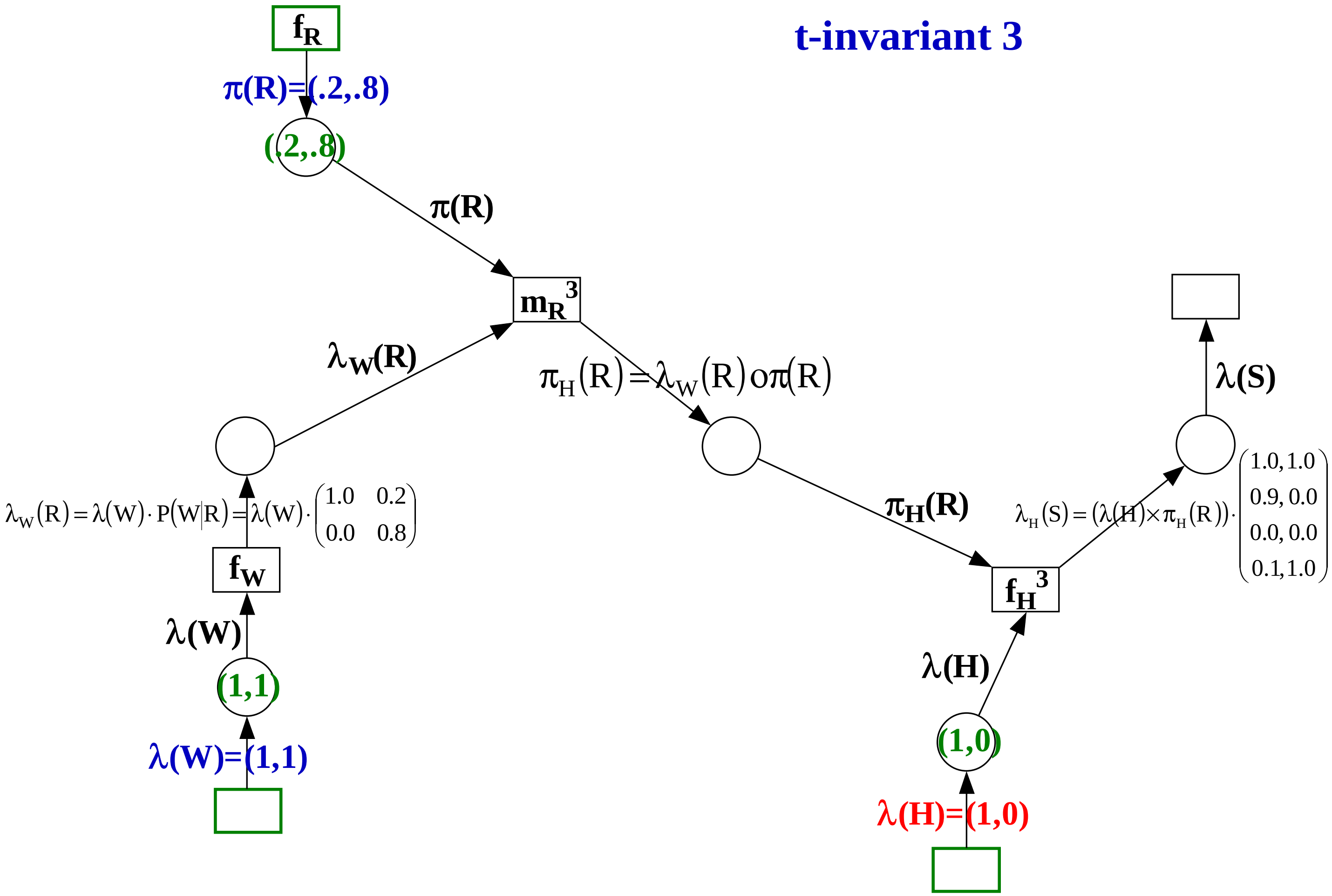
$\lambda(R) = (1,.09)$

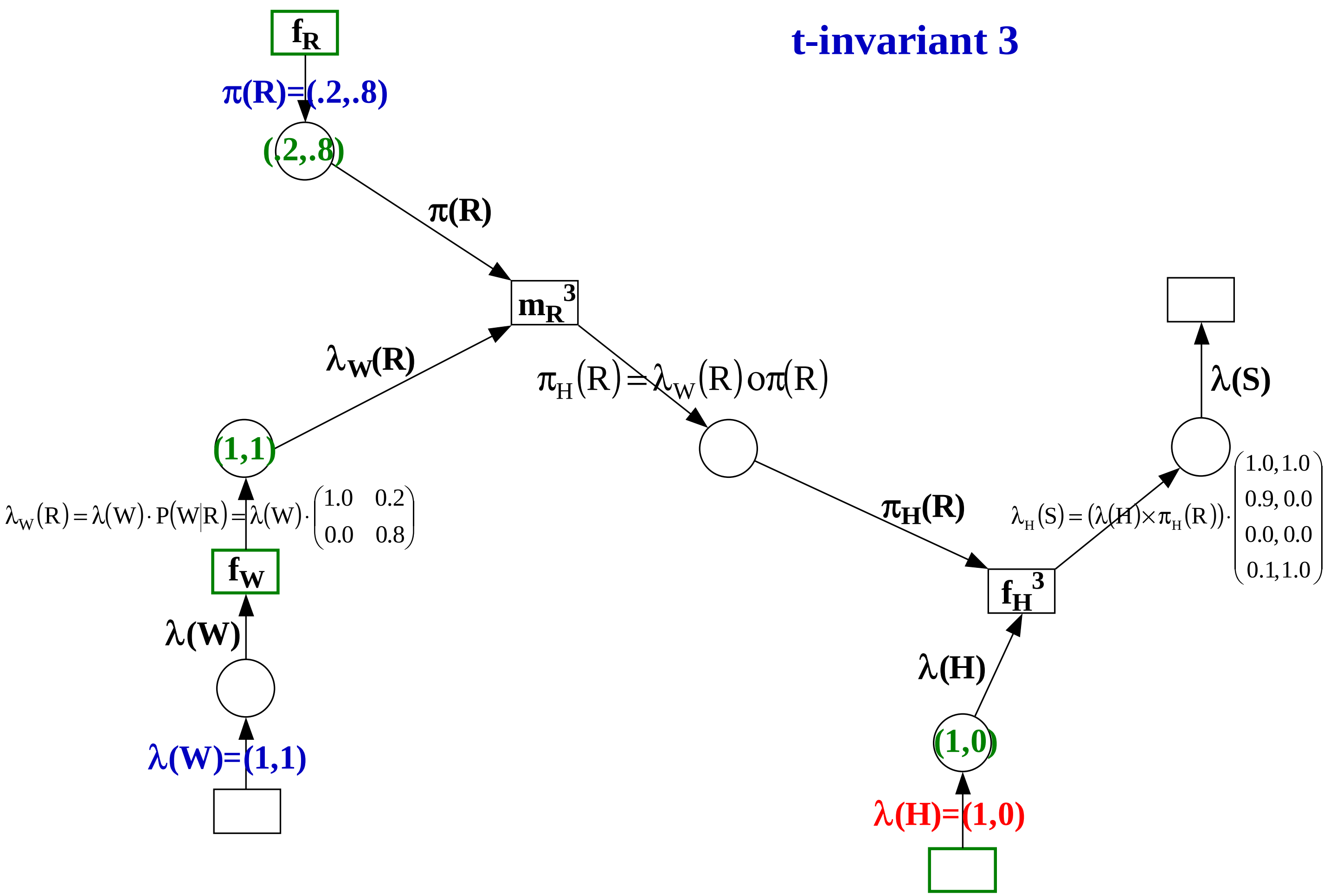


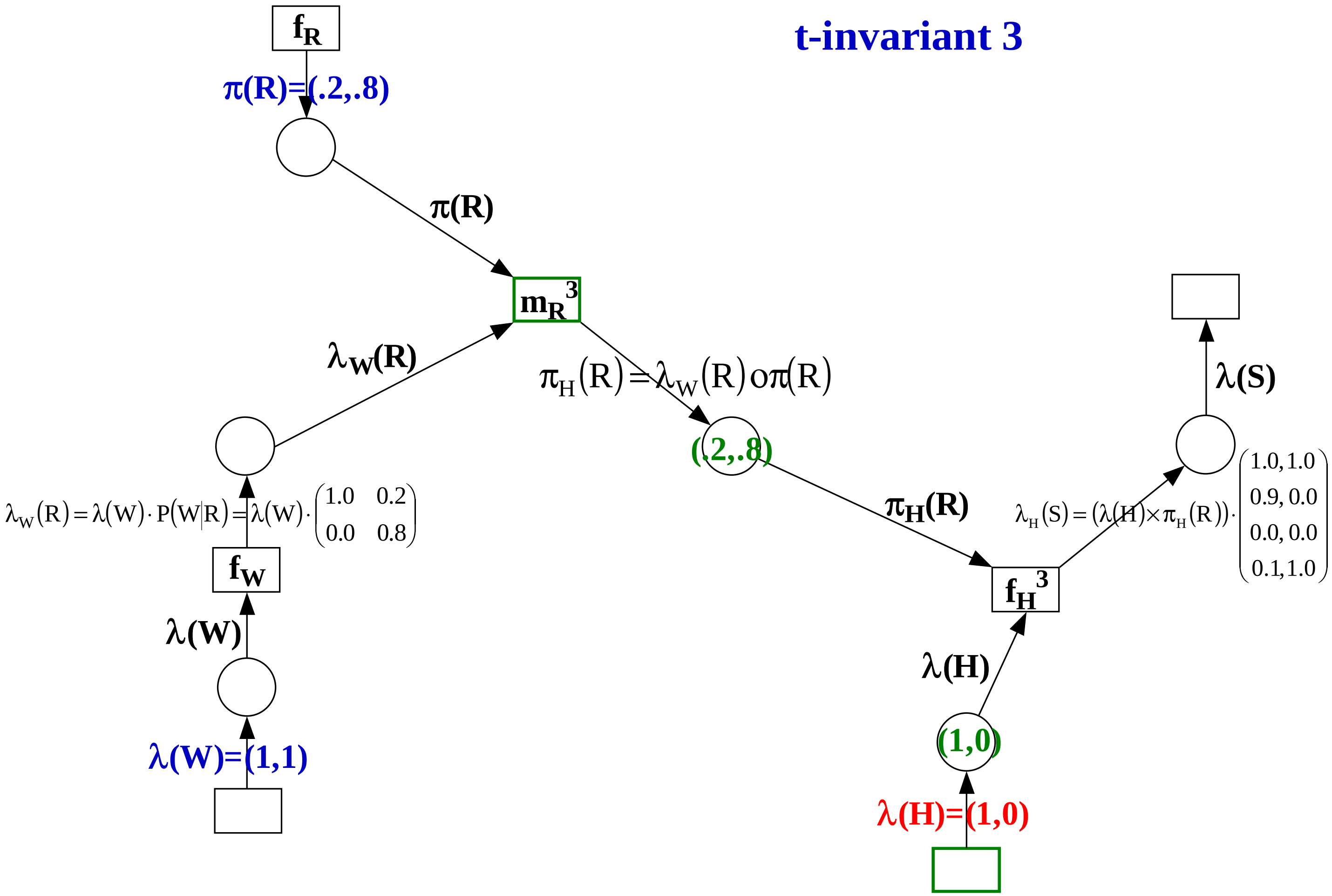
t-invariant 3



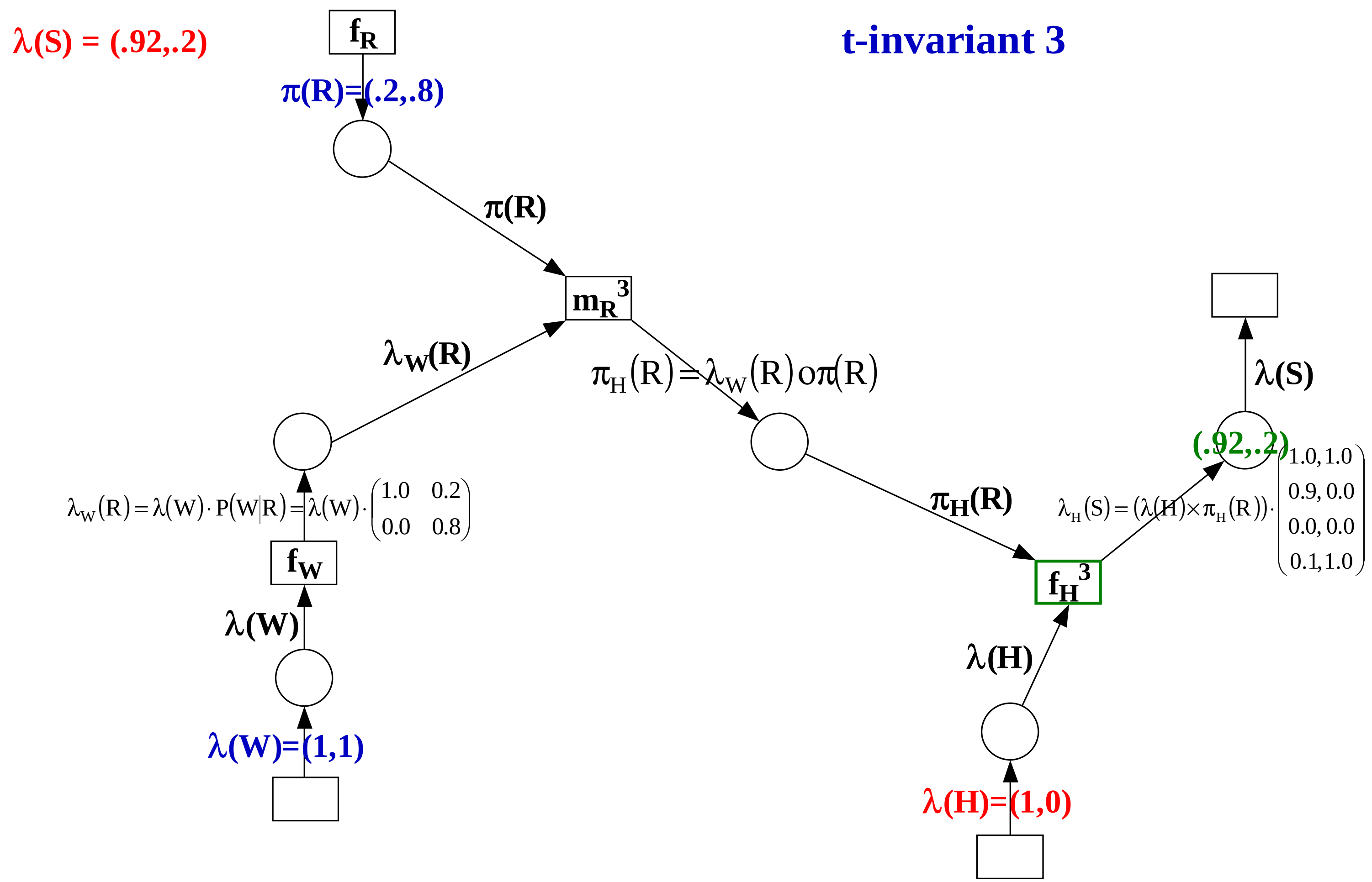
t-invariant 3




t-invariant 3
fR
π(R)=(.2,.8)
(.2,.8)
π(R)
mR3
λW(R)
πH(R)=λW(R)oπ(R)
(1,1)
λW(R) = λ(W)·P(W|R) = λ(W)·(1.0 0.2; 0.0 0.8)
fW
λ(W)
λ(W)=(1,1)
πH(R)
λH(S) = (λ(H)×πH(R))·(1.0, 1.0; 0.9, 0.0; 0.0, 0.0; 0.1, 1.0)
fH3
λ(S)
λ(H)
(1,0)
λ(H)=(1,0)


t-invariant 3
fR
π(R)=(.2,.8)
π(R)
mR 3
λW(R)
πH(R)=λW(R)oπ(R)
(.2,.8)
πH(R)
λ(S)
λW(R)=λ(W)·P(W|R)=λ(W)·(1.0 0.2 / 0.0 0.8)
fW
λ(W)
λ(W)=(1,1)
λH(S)=(λ(H)×πH(R))·(1.0,1.0 / 0.9,0.0 / 0.0,0.0 / 0.1,1.0)
fH 3
λ(H)
(1,0)
λ(H)=(1,0)

Wet Grass


λ(S) = (.92,.2)
t-invariant 3
f_R
π(R)=(.2,.8)
π(R)
m_R^3
λ_W(R)
π_H(R) = λ_W(R) o π(R)
λ(S)
(.92,.2)
λ_W(R) = λ(W) · P(W|R) = λ(W) · (1.0 0.2; 0.0 0.8)
f_W
λ(W)
λ(W)=(1,1)
π_H(R)
λ_H(S) = (λ(H) × π_H(R)) · (1.0, 1.0; 0.9, 0.0; 0.0, 0.0; 0.1, 1.0)
f_H^3
λ(H)
λ(H)=(1,0)

$\lambda(R)$	**= (1,.09)**	**= (1 ,1)**
$\pi(R)$	**= (.2,.8)**	**= (.2,.8)**
$P(R) = \alpha\lambda(R)\pi(R)$	**= (.735,.265)**	**= (.2,.8)**
$\lambda(S)$	**= (.92,.2)**	**= (1 ,1)**
$\pi(S)$	**= (.1,.9)**	**= (.1,.9)**
$P(S) = \alpha\lambda(S)\pi(S)$	**= (.338,.662)**	**= (.1,.9)**
$\lambda(W)$	**= (1 ,1)**	**= (1 ,1)**
$\pi(W)$		**= (.36,.64)**
$P(W) = \alpha\lambda(W)\pi(W)$		**= (.36,.64)**
$\lambda(H)$	**= (1 ,1)**	**= (1 ,1)**
$\pi(H)$	**= (1 ,0)** **new**	**= (.272, 728)**
$P(H) = \alpha\lambda(H)\pi(H)$	**= (1 ,0)** **evidence**	**= (.272, 728)**

h_1 = Mr. Holmes' grass is wet

r_1 = rain is the cause for Mr. Holmes' grass being wet

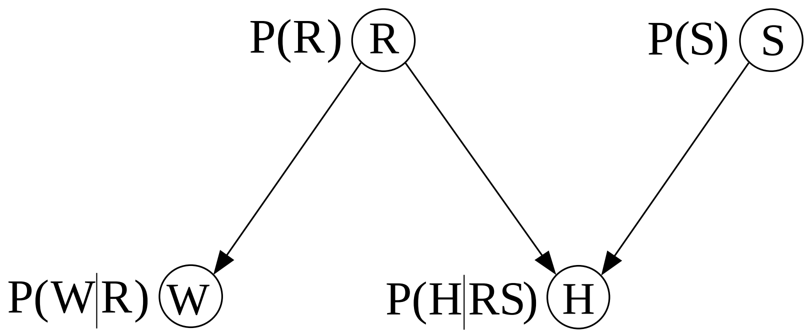
r_0 = rain is not the cause for Mr. Holmes' grass being wet

s_1 = sprinkler is the cause for Mr. Holmes' grass being wet

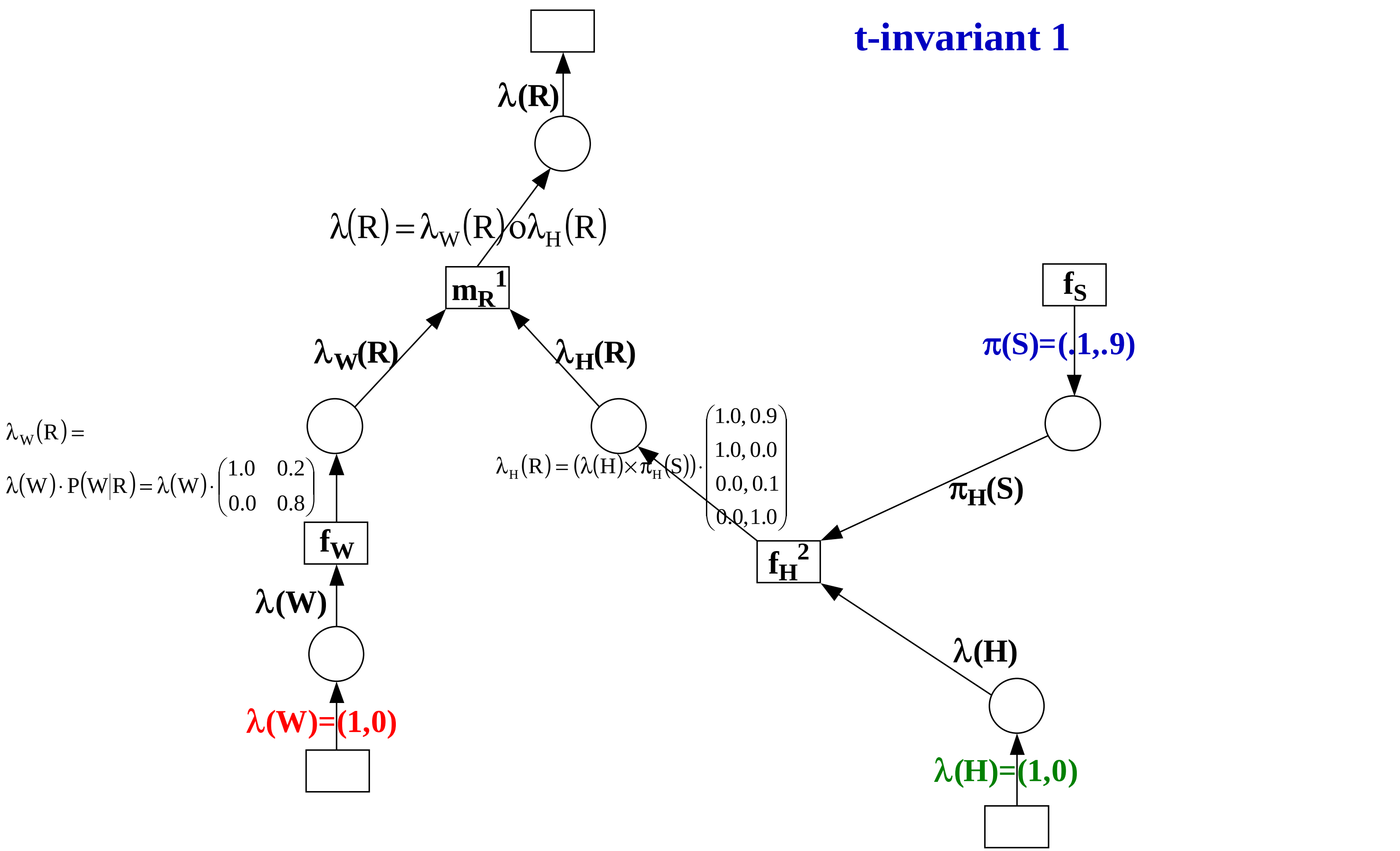
s_0 = sprinkler is not the cause for Mr. Holmes' grass being wet

w_1 = Dr. Watson's grass is wet

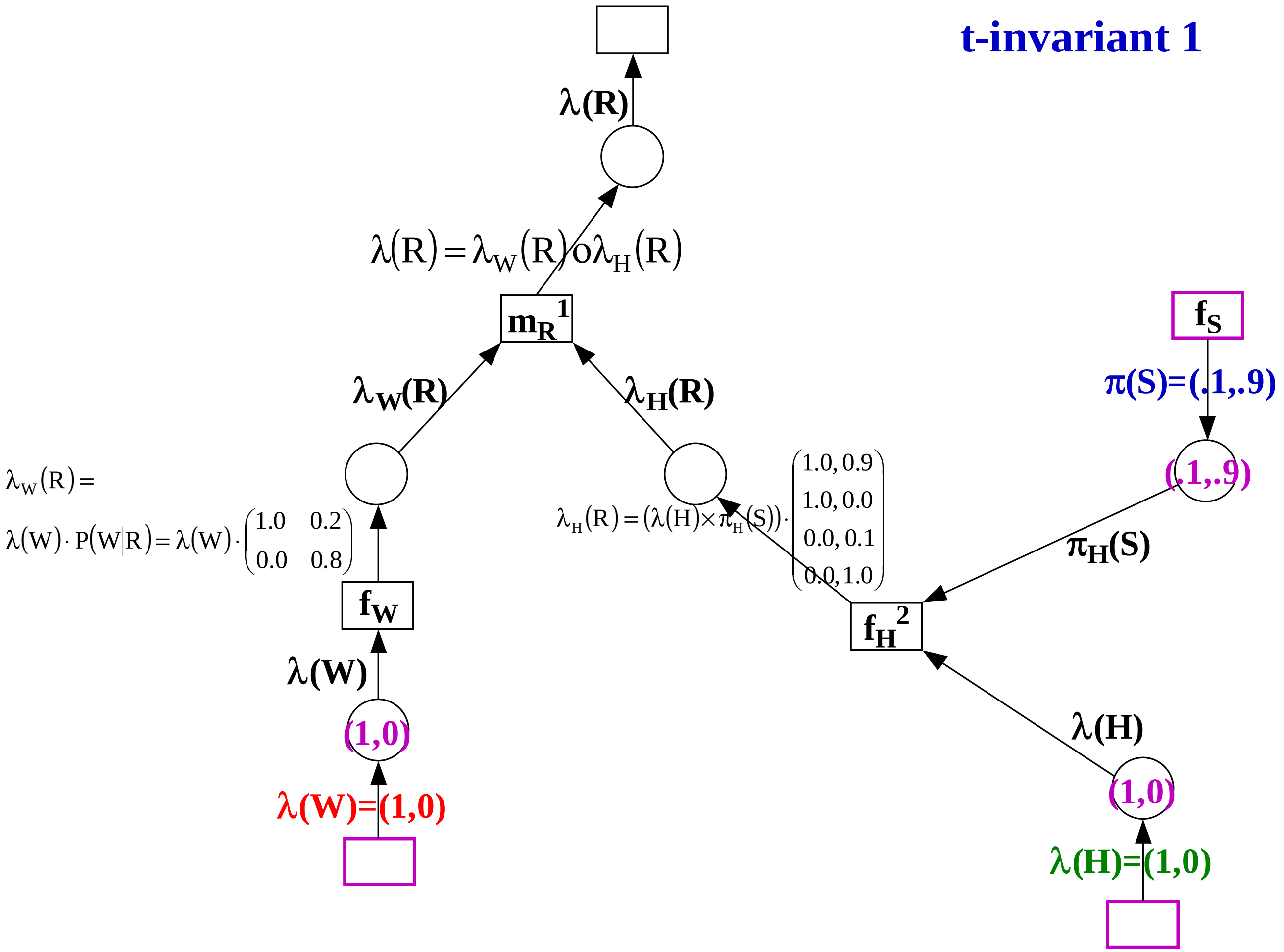
More new evidence

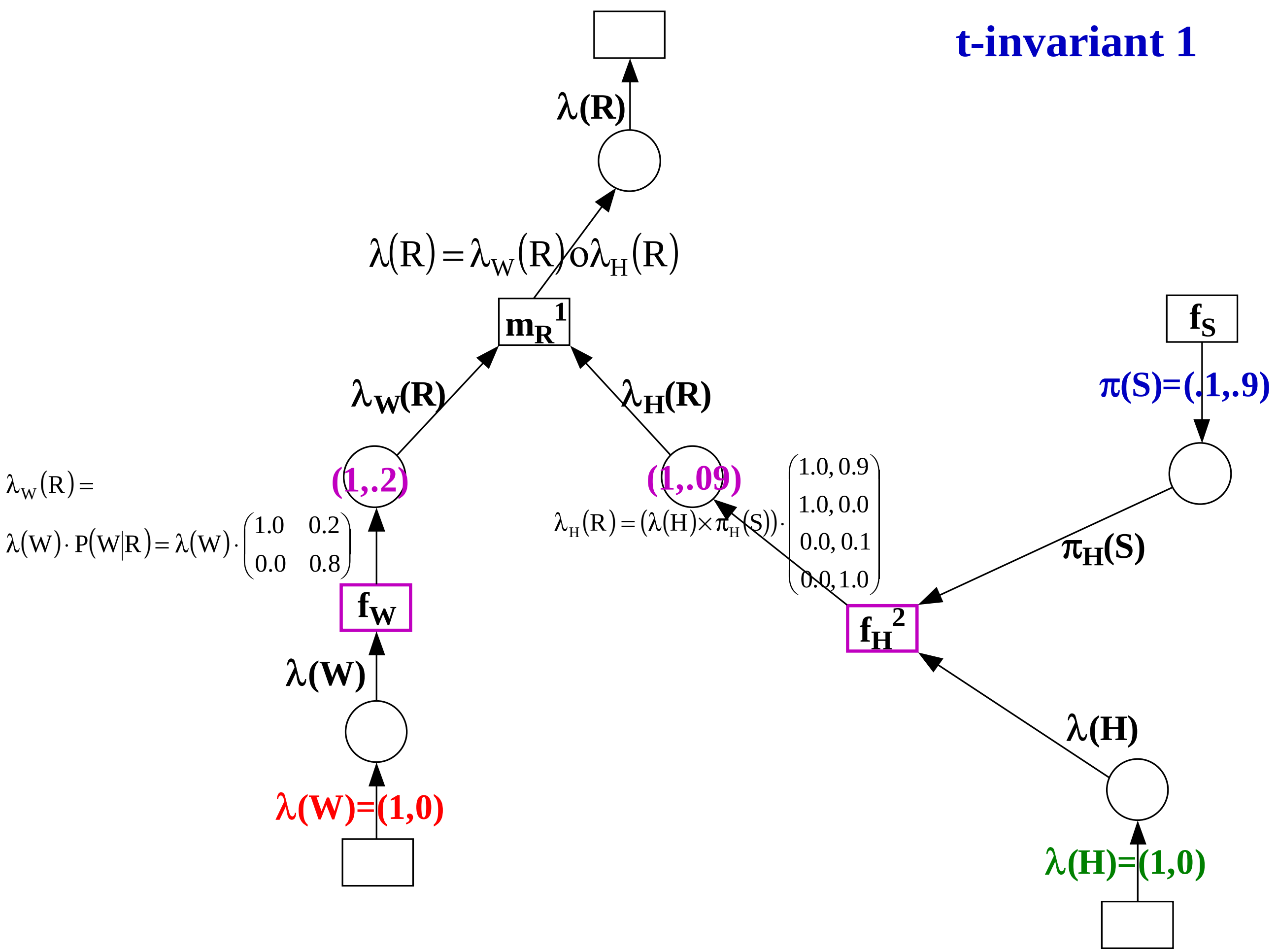


$\lambda(R)$			= (1,.09)	= (1 ,1)
$\pi(R)$	= (.2,.8)		= (.2,.8)	= (.2,.8)
$P(R) = \alpha\lambda(R)\pi(R)$			= (.735,.265)	= (.2,.8)
$\lambda(S)$			= (.92,.2)	= (1 ,1)
$\pi(S)$	= (.1,.9)		= (.1,.9)	= (.1,.9)
$P(S) = \alpha\lambda(S)\pi(S)$			= (.338,.662)	= (.1,.9)
$\lambda(W)$	= (1 ,1)		= (1 ,1)	= (1 ,1)
$\pi(W)$	= (1 ,0)	new		= (.36,.64)
$P(W) = \alpha\lambda(W)\pi(W)$	= (1 ,0)	evidence		= (.36,.64)
$\lambda(H)$	= (1 ,1)		= (1 ,1)	= (1 ,1)
$\pi(H)$	= (1 ,0)		= (1 ,0) "new"	= (.272, 728)
$P(H) = \alpha\lambda(H)\pi(H)$	= (1 ,0)		= (1 ,0) evid.	= (.272, 728)


t-invariant 1
λ(R)
λ(R)=λ_W(R)oλ_H(R)
m_R^1
λ_W(R)
λ_H(R)
f_S
π(S)=(.1,.9)
λ_W(R)=
λ(W)·P(W|R)=λ(W)·(1.0 0.2 / 0.0 0.8)
λ_H(R)=(λ(H)×π_H(S))·(1.0, 0.9 / 1.0, 0.0 / 0.0, 0.1 / 0.0, 1.0)
π_H(S)
f_W
f_H^2
λ(W)
λ(H)
λ(W)=(1,0)
λ(H)=(1,0)

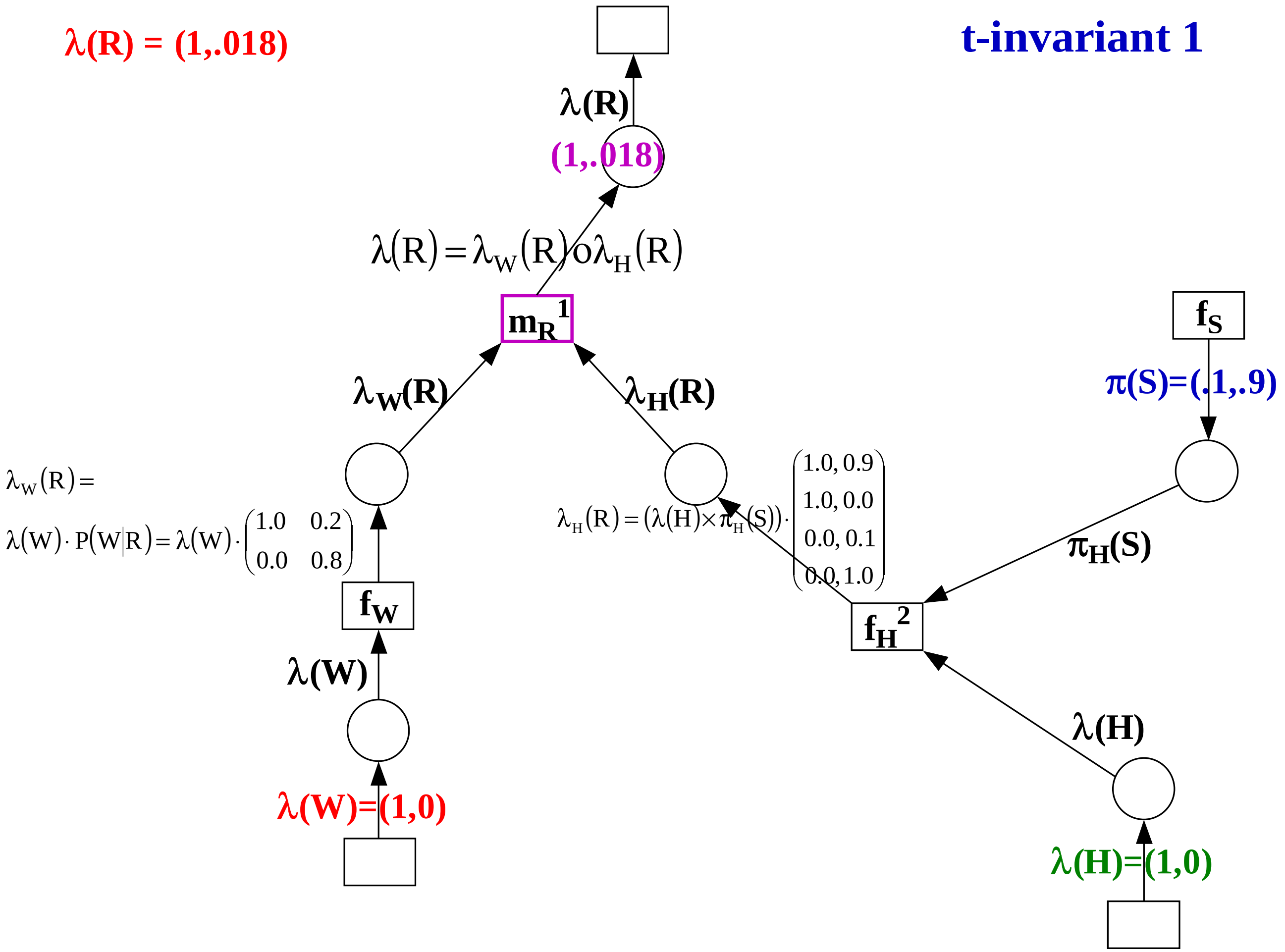
Wet Grass

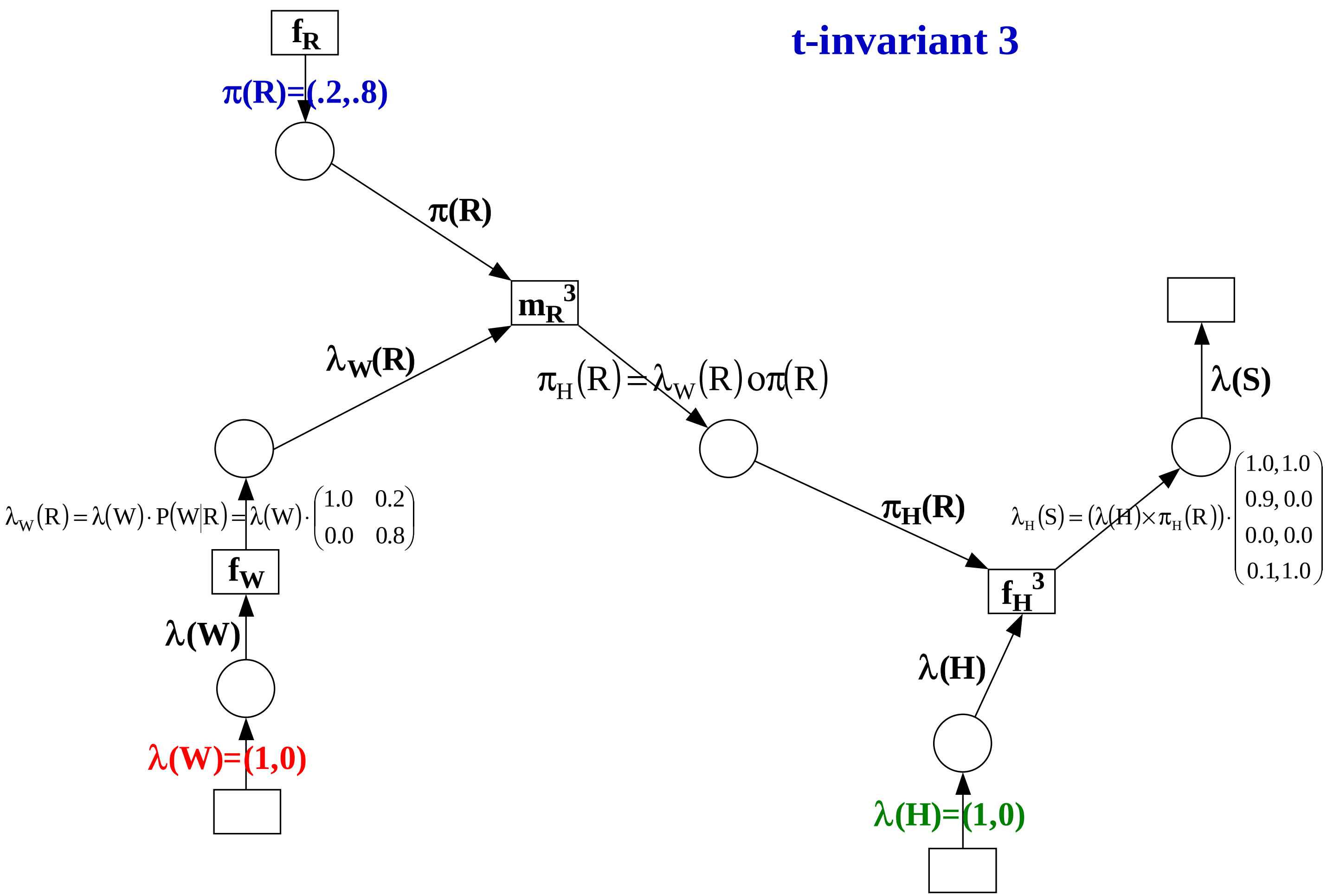

t-invariant 1
λ(R)
λ(R)=λW(R)oλH(R)
mR 1
λW(R)
λH(R)
fS
π(S)=(.1,.9)
(.1,.9)
λW(R)=
λ(W)·P(W|R)=λ(W)·
1.0 0.2
0.0 0.8
λH(R)=(λ(H)×πH(S))·
1.0, 0.9
1.0, 0.0
0.0, 0.1
0.0, 1.0
πH(S)
fW
fH 2
λ(W)
(1,0)
λ(H)
λ(W)=(1,0)
(1,0)
λ(H)=(1,0)


t-invariant 1
λ(R)
λ(R)=λ_W(R)oλ_H(R)
m_R 1
λ_W(R)
λ_H(R)
f_S
π(S)=(.1,.9)
λ_W(R)=
λ(W)·P(W|R)=λ(W)·(1.0 0.2 / 0.0 0.8)
(1,.2)
(1,.09)
λ_H(R)=(λ(H)×π_H(S))·(1.0, 0.9 / 1.0, 0.0 / 0.0, 0.1 / 0.0, 1.0)
π_H(S)
f_W
f_H 2
λ(W)
λ(H)
λ(W)=(1,0)
λ(H)=(1,0)

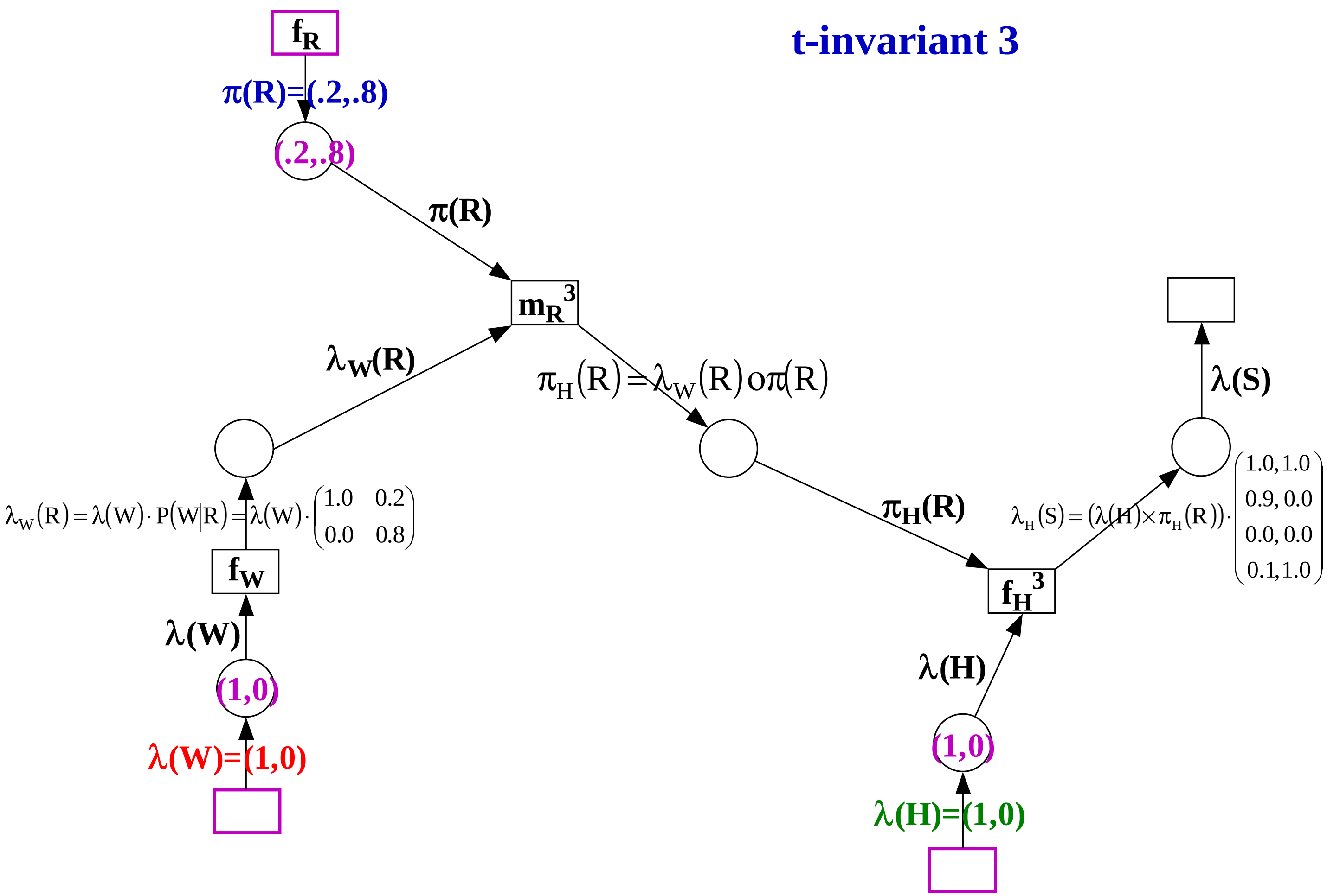
$\lambda(R)$ = (1,.018)

t-invariant 1

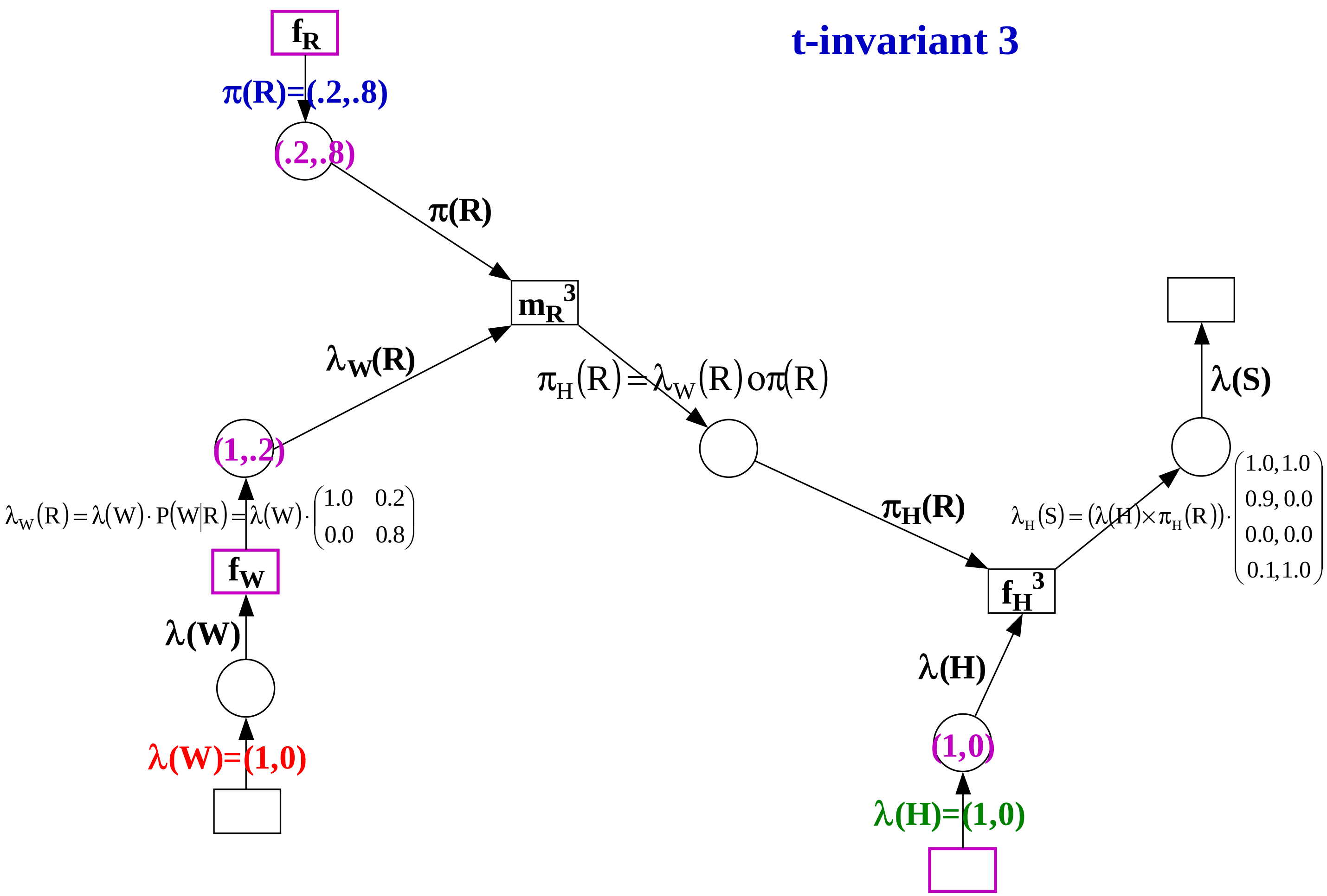




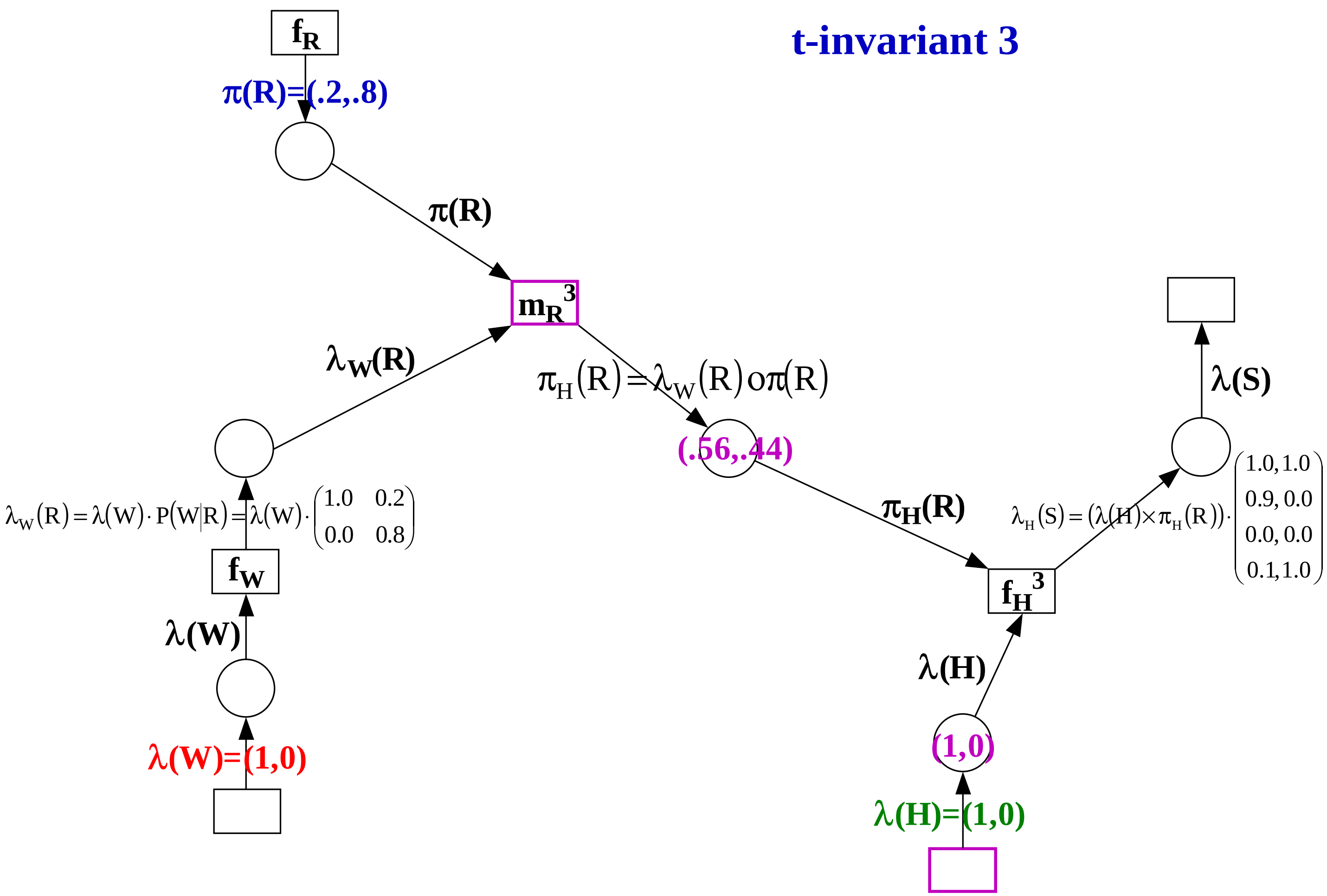
t-invariant 3
f_R
$\pi(R)=(.2,.8)$
$\pi(R)$
m_R^3
$\lambda_W(R)$
$\pi_H(R)=\lambda_W(R)\circ\pi(R)$
$\lambda(S)$
$\lambda_W(R)=\lambda(W)\cdot P(W|R)=\lambda(W)\cdot\begin{pmatrix}1.0 & 0.2\\ 0.0 & 0.8\end{pmatrix}$
$\pi_H(R)$
$\lambda_H(S)=(\lambda(H)\times\pi_H(R))\cdot\begin{pmatrix}1.0, 1.0\\ 0.9, 0.0\\ 0.0, 0.0\\ 0.1, 1.0\end{pmatrix}$
f_W
f_H^3
$\lambda(W)$
$\lambda(H)$
$\lambda(W)=(1,0)$
$\lambda(H)=(1,0)$

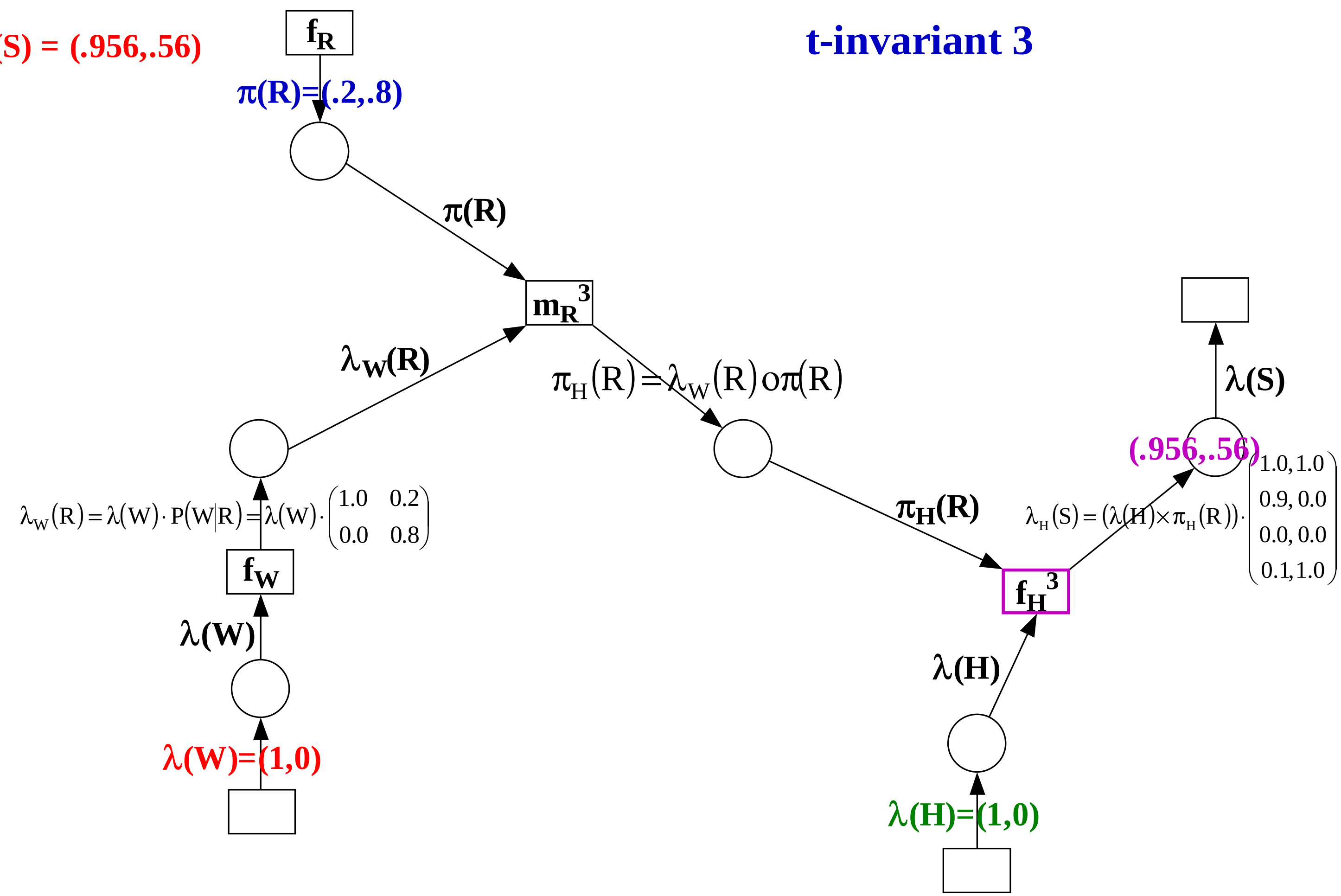


t-invariant 3
f_R
$\pi(R)=(.2,.8)$
(.2,.8)
$\pi(R)$
m_R^3
$\lambda_W(R)$
$\pi_H(R)=\lambda_W(R) o \pi(R)$
$\lambda(S)$
$\lambda_W(R)=\lambda(W)\cdot P(W|R)=\lambda(W)\cdot\begin{pmatrix}1.0 & 0.2\\0.0 & 0.8\end{pmatrix}$
f_W
$\pi_H(R)$
$\lambda_H(S)=(\lambda(H)\times\pi_H(R))\cdot\begin{pmatrix}1.0,1.0\\0.9,0.0\\0.0,0.0\\0.1,1.0\end{pmatrix}$
f_H^3
$\lambda(W)$
(1,0)
$\lambda(H)$
$\lambda(W)=(1,0)$
(1,0)
$\lambda(H)=(1,0)$



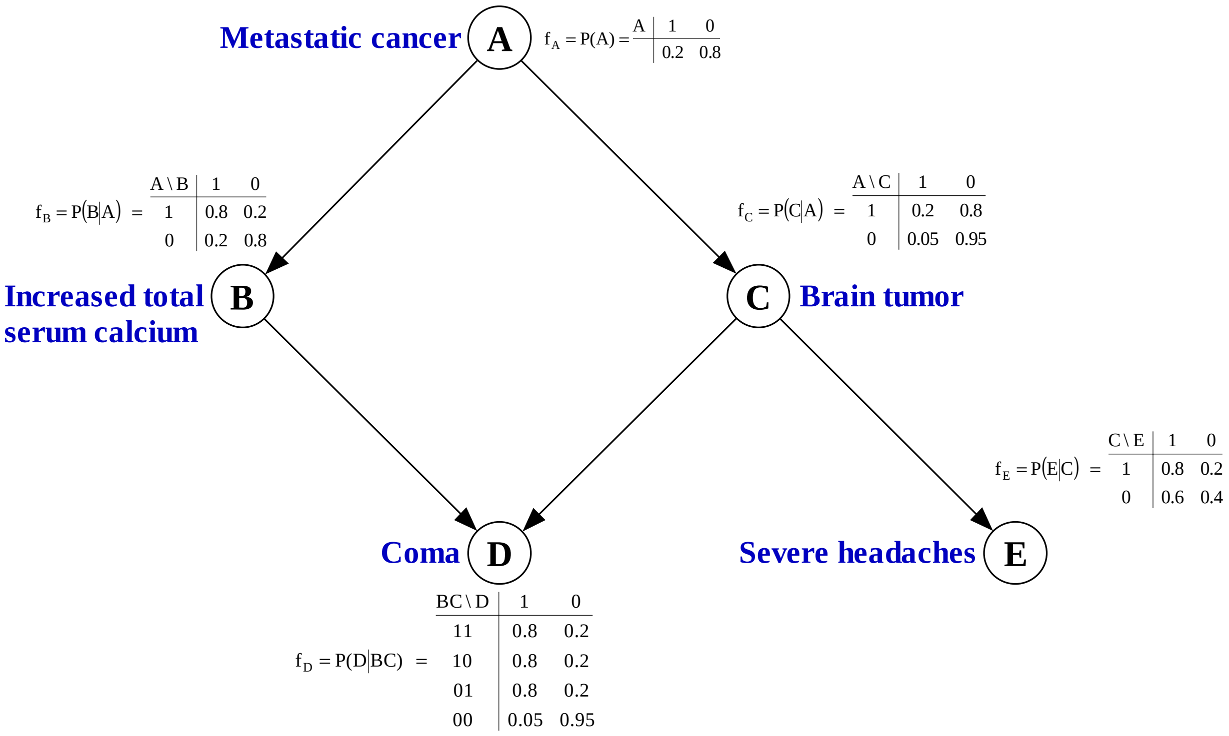
t-invariant 3
f_R
π(R)=(.2,.8)
(.2,.8)
π(R)
m_R 3
λ_W(R)
π_H(R)=λ_W(R)oπ(R)
(1,.2)
λ_W(R)=λ(W)·P(W|R)=λ(W)·(1.0 0.2; 0.0 0.8)
f_W
λ(W)
λ(W)=(1,0)
π_H(R)
λ_H(S)=(λ(H)×π_H(R))·(1.0,1.0; 0.9,0.0; 0.0,0.0; 0.1,1.0)
λ(S)
f_H 3
λ(H)
(1,0)
λ(H)=(1,0)


t-invariant 3
fR
π(R)=(.2,.8)
π(R)
mR 3
λW(R)
πH(R)=λW(R)oπ(R)
(.56,.44)
πH(R)
λ(S)
λH(S)=(λ(H)×πH(R))·
1.0, 1.0
0.9, 0.0
0.0, 0.0
0.1, 1.0
λW(R)=λ(W)·P(W|R)=λ(W)·
1.0 0.2
0.0 0.8
fW
λ(W)
λ(W)=(1,0)
fH 3
λ(H)
(1,0)
λ(H)=(1,0)

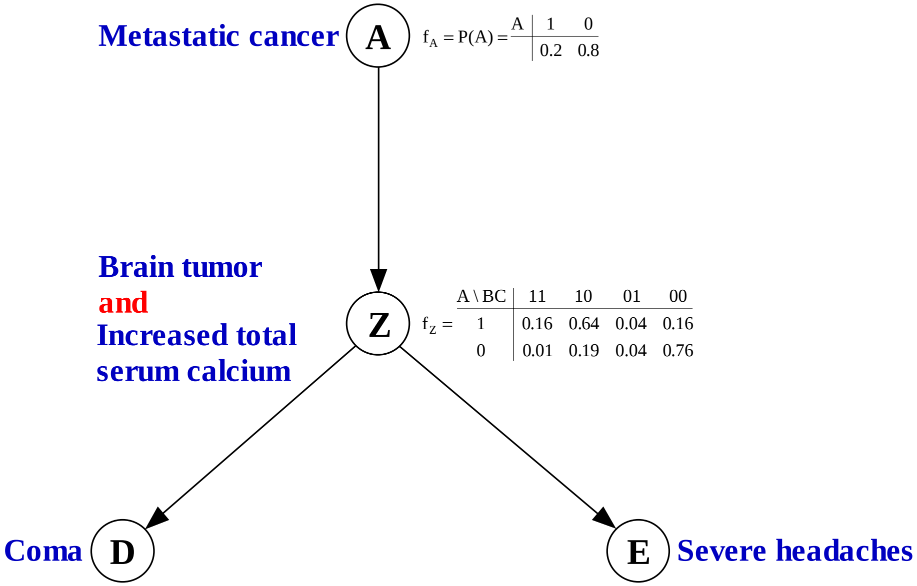

λ(S) = (.956,.56)
t-invariant 3
f_R
π(R)=(.2,.8)
π(R)
m_R^3
λ_W(R)
π_H(R)=λ_W(R)oπ(R)
λ(S)
(.956,.56)
λ_W(R) = λ(W)·P(W|R) = λ(W)·(1.0 0.2; 0.0 0.8)
f_W
λ(W)
λ(W)=(1,0)
π_H(R)
λ_H(S) = (λ(H)×π_H(R))·(1.0,1.0; 0.9,0.0; 0.0,0.0; 0.1,1.0)
f_H^3
λ(H)
λ(H)=(1,0)

λ(R)	= (1,.018)	= (1,.09)	= (1 ,1)
π(R)	= (.2,.8)	= (.2,.8)	= (.2,.8)
P(R) = αλ(R)π(R)	= (.93,.07)	= (.735,.265)	= (.2,.8)
λ(S)	= (.956,.56)	= (.92,.2)	= (1 ,1)
π(S)	= (.1,.9)	= (.1,.9)	= (.1,.9)
P(S) = αλ(S)π(S)	= (.16,.84)	= (.338,.662)	= (.1,.9)
λ(W)	= (1 ,1)	= (1 ,1)	= (1 ,1)
π(W)	= (1 ,0) "new"		= (.36,.64)
P(W) = αλ(W)π(W)	= (1 ,0) evid.		= (.36,.64)
λ(H)	= (1 ,1)	= (1 ,1)	= (1 ,1)
π(H)	= (1 ,0)	= (1 ,0) "new"	= (.272, 728)
P(H) = αλ(H)π(H)	= (1 ,0)	= (1 ,0) evid.	= (.272, 728)

Coping with Loops (Pearl)



Clustering (Pearl)



$f_A = P(A) =$

A	1	0
	0.2	0.8

$f_Z =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$$Z \coloneqq B \wedge C$$

$$P(Z|A) = P(BC|A) = P(B|A) \cdot P(C|A)$$

$$P(E|Z) = P(E|BC) = P(E|C)$$

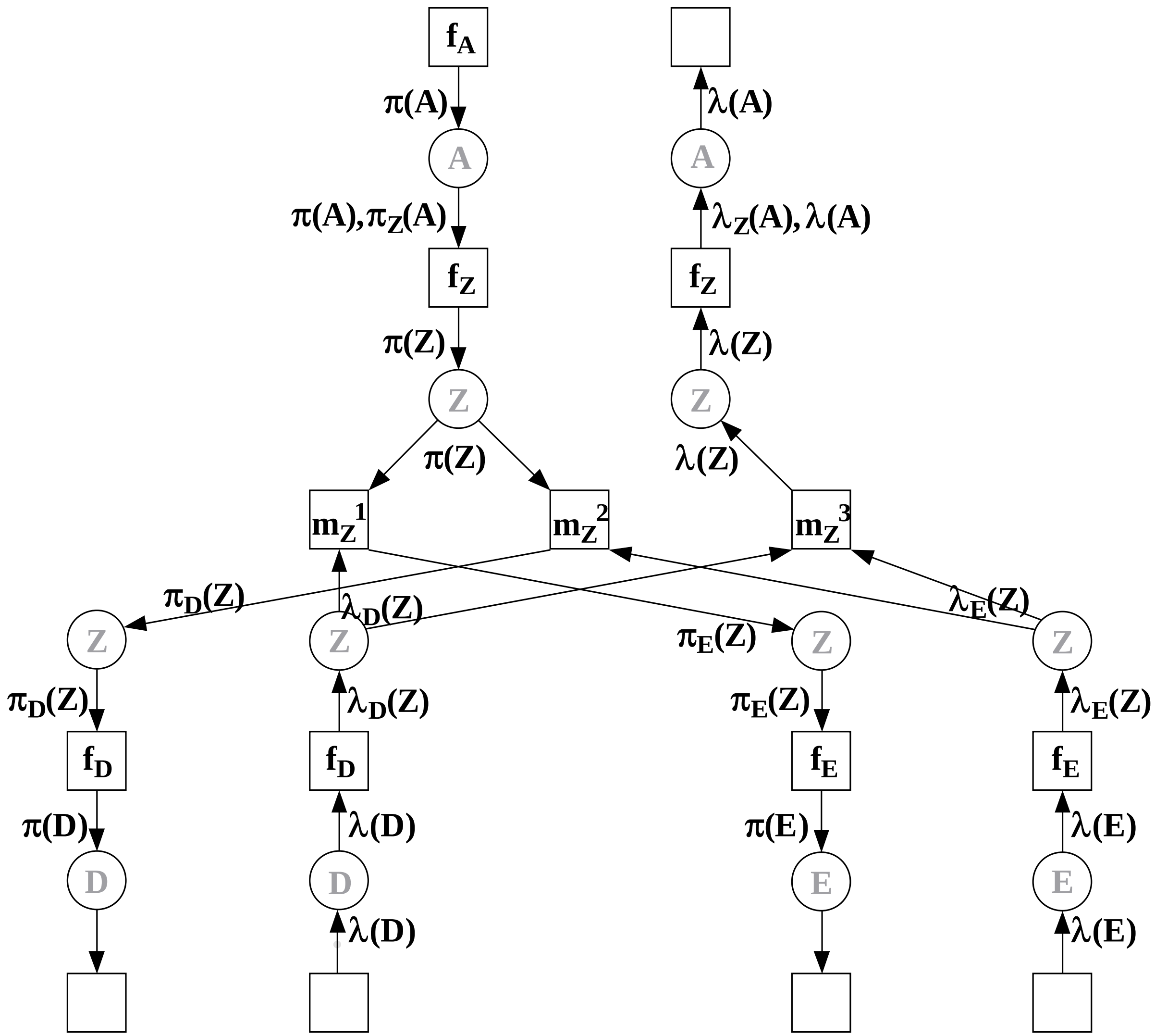
$f_D = P(D|BC) =$

BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

$f_E = P(E|BC) =$

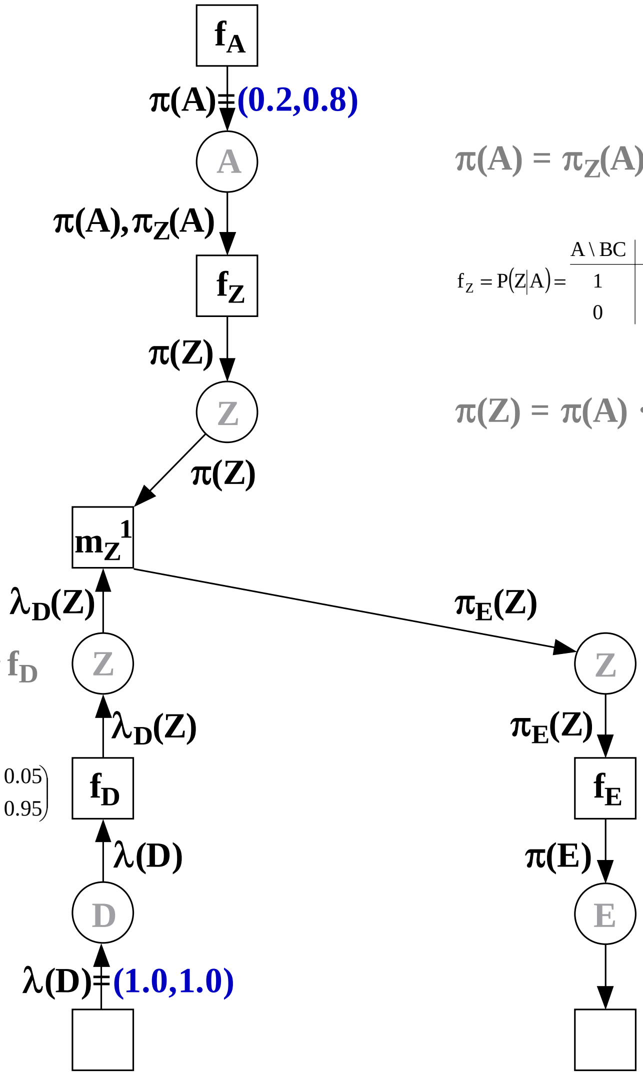
BC \ E	1	0	
11	0.8	0.2	
10	0.6	0.4	augmented
01	0.8	0.2	
00	0.6	0.4	

Clustering (Pearl)



Clustering (Pearl)

t-invariant 1



$$\pi(A) = \pi_Z(A) = f_A = (0.2,\ 0.8)$$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$$\pi(Z) = \pi(A) \cdot f_Z$$

$$\lambda_D(Z) = \lambda(D) \cdot f_D$$

$$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$$

$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

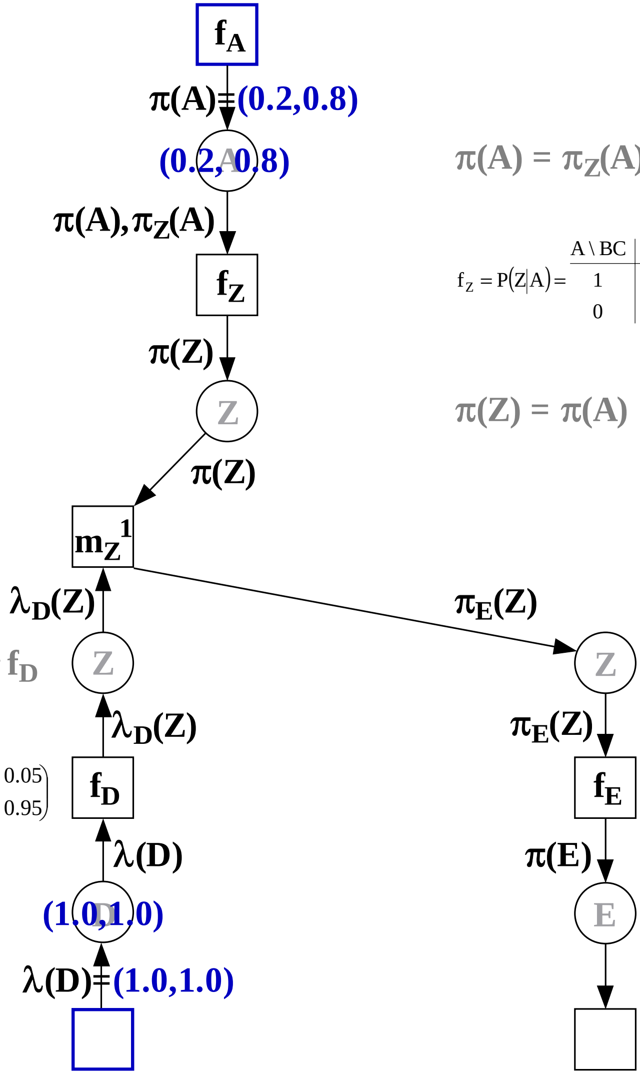
$f_E = P(E|BC) =$

BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$$\pi(E) = \pi_E(Z) \cdot f_E$$

Clustering (Pearl)

t-invariant 1



$\pi(A) = \pi_Z(A) = f_A = (0.2,\ 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_D(Z) = \lambda(D) \cdot f_D$

$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$

$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

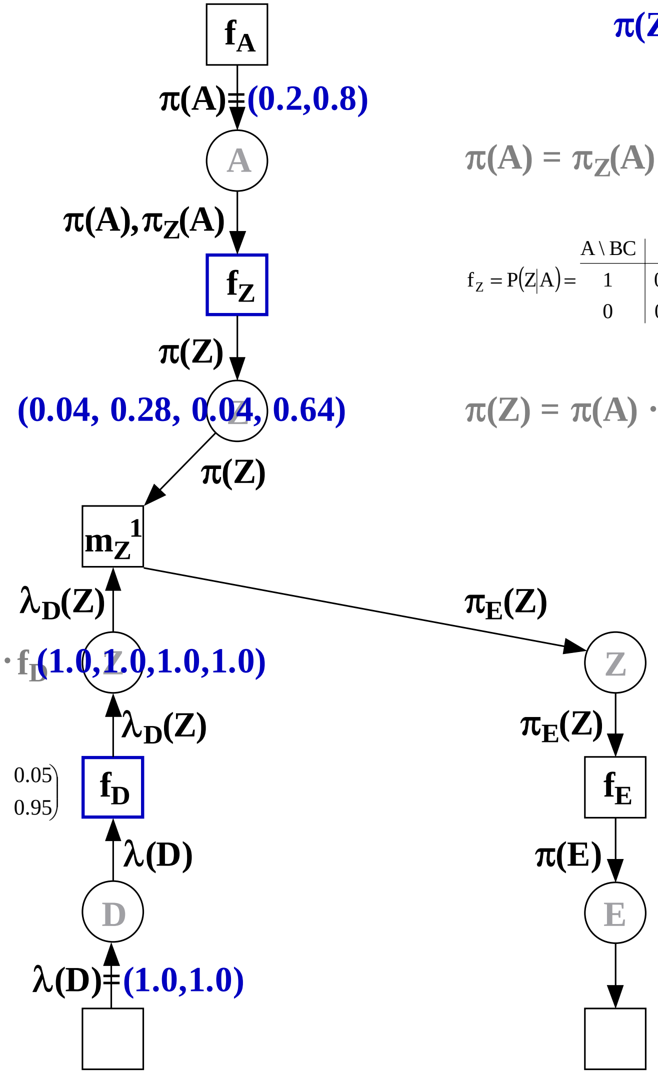
$f_E = P(E|BC) =$

BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$\pi(E) = \pi_E(Z) \cdot f_E$

t-invariant 1

$\pi(Z) = (0.04, 0.28, 0.04, 0.64)$



$\pi(A) = \pi_Z(A) = f_A = (0.2, 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_D(Z) = \lambda(D) \cdot f_D$

$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$

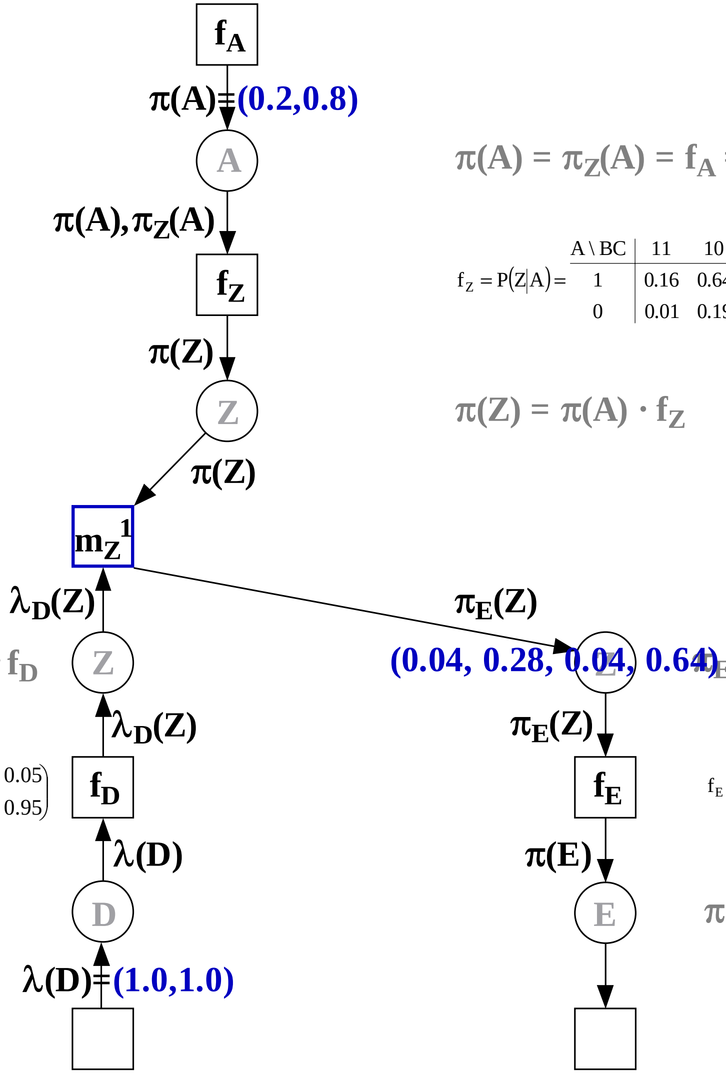
$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

$f_E = P(E|BC) =$

BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$\pi(E) = \pi_E(Z) \cdot f_E$

t-invariant 1



$\pi(A) = \pi_Z(A) = f_A = (0.2,\ 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_D(Z) = \lambda(D) \cdot f_D$

$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$

$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

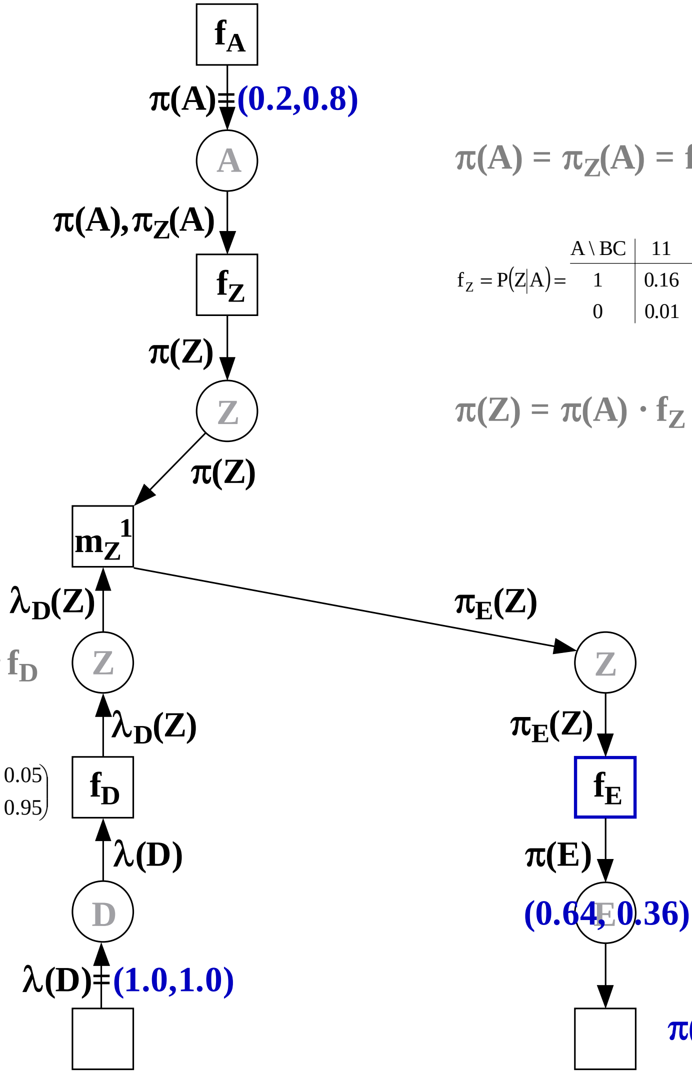
$f_E = P(E|BC) =$

BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$\pi(E) = \pi_E(Z) \cdot f_E$

Clustering (Pearl)

t-invariant 1



$\pi(A) = \pi_Z(A) = f_A = (0.2, 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_D(Z) = \lambda(D) \cdot f_D$

$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$

$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

$f_E = P(E|BC) =$

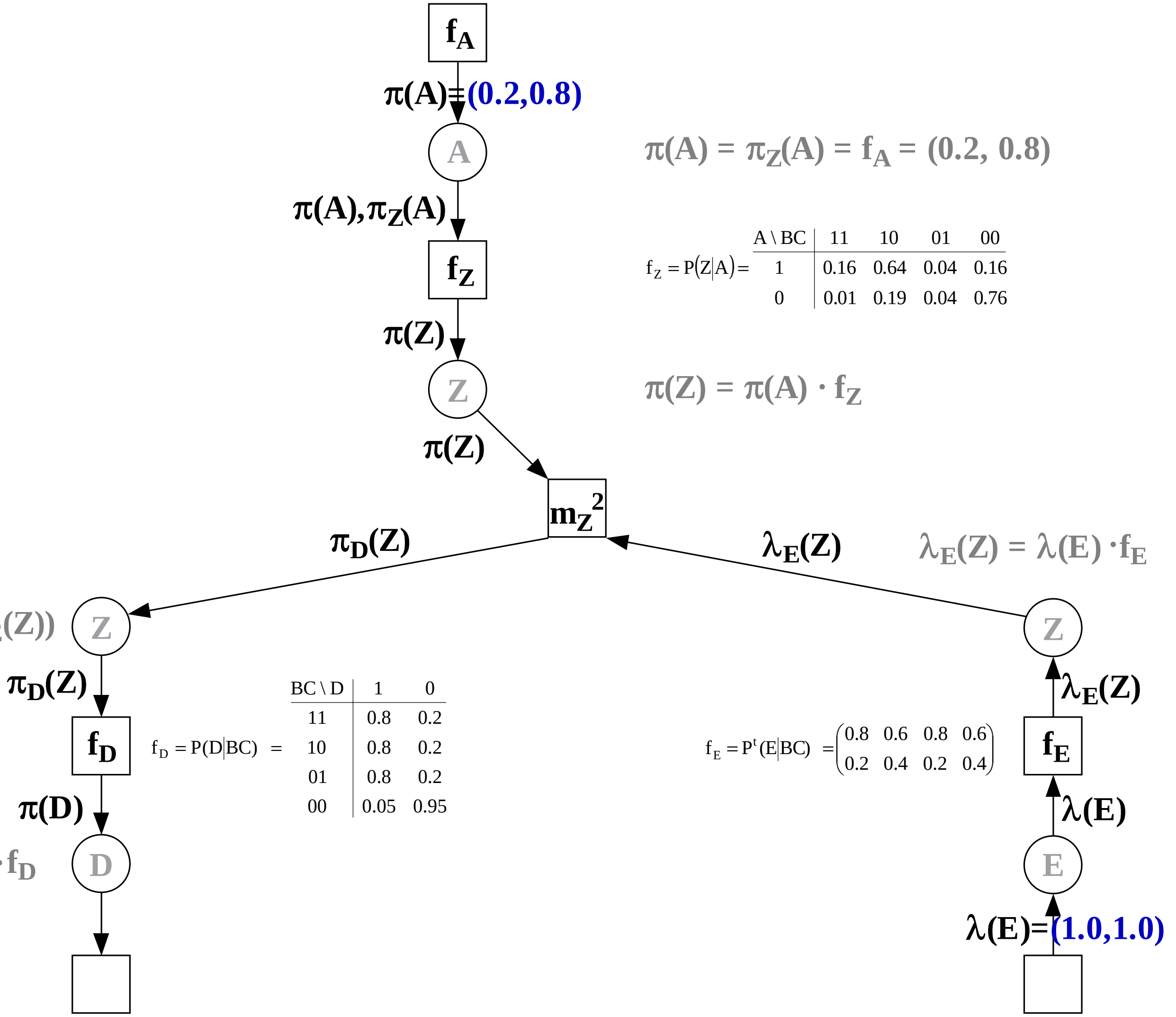
BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$\pi(E) = \pi_E(Z) \cdot f_E$

$\pi(E) = (0.64, 0.36)$

Clustering (Pearl)

t-invariant 2



$\pi(A) = \pi_Z(A) = f_A = (0.2,\ 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_E(Z) = \lambda(E) \cdot f_E$

$\pi_D(Z) = \alpha \cdot (\pi(Z) \circ \lambda_E(Z))$

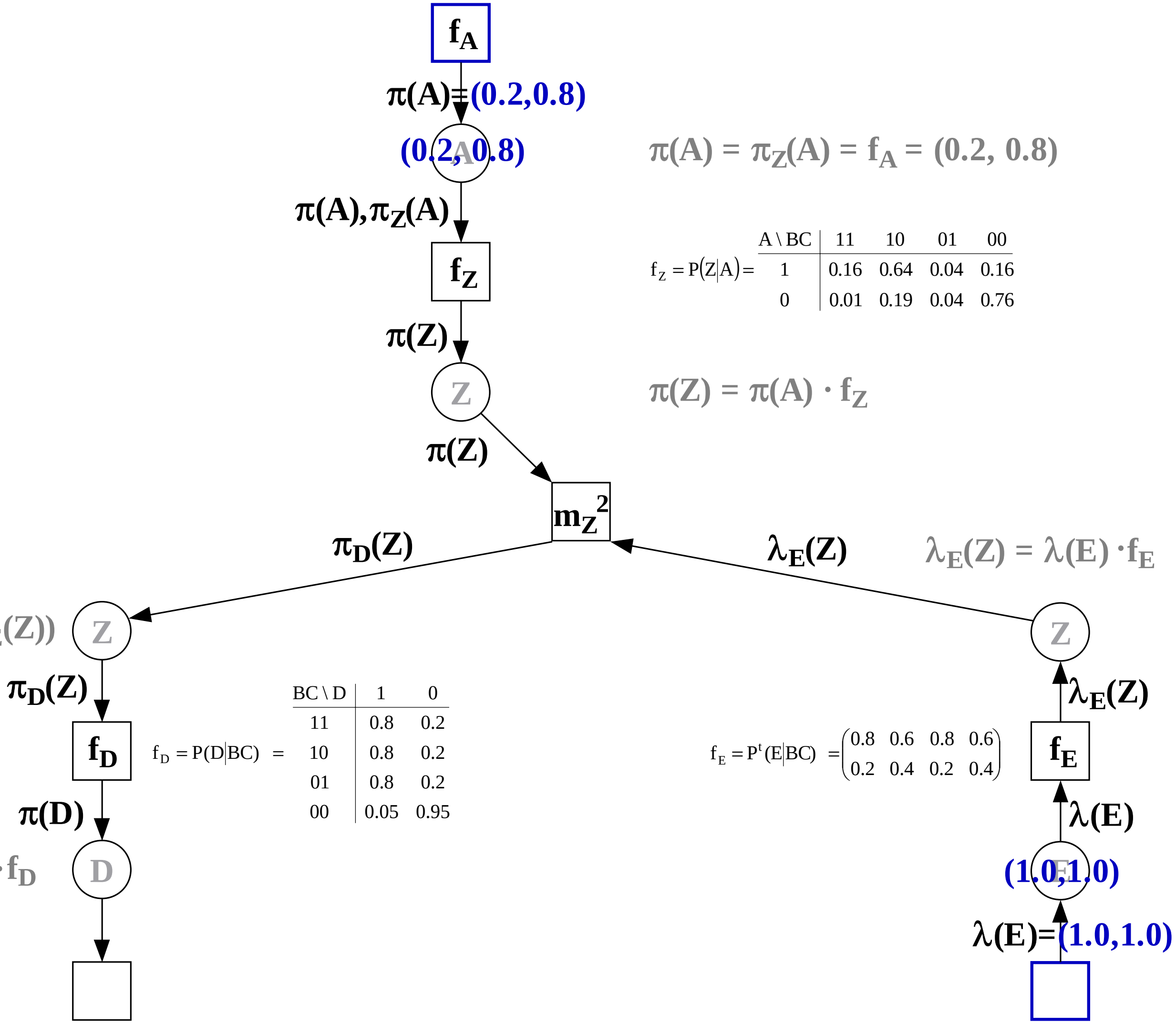
$f_D = P(D|BC) =$

BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

$f_E = P^t(E|BC) = \begin{pmatrix} 0.8 & 0.6 & 0.8 & 0.6 \\ 0.2 & 0.4 & 0.2 & 0.4 \end{pmatrix}$

$\pi(D) = \pi_D(Z) \cdot f_D$

t-invariant 2



$\pi(A) = \pi_Z(A) = f_A = (0.2,\ 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_E(Z) = \lambda(E) \cdot f_E$

$\pi_D(Z) = \alpha \cdot (\pi(Z) \circ \lambda_E(Z))$

$f_D = P(D|BC) =$

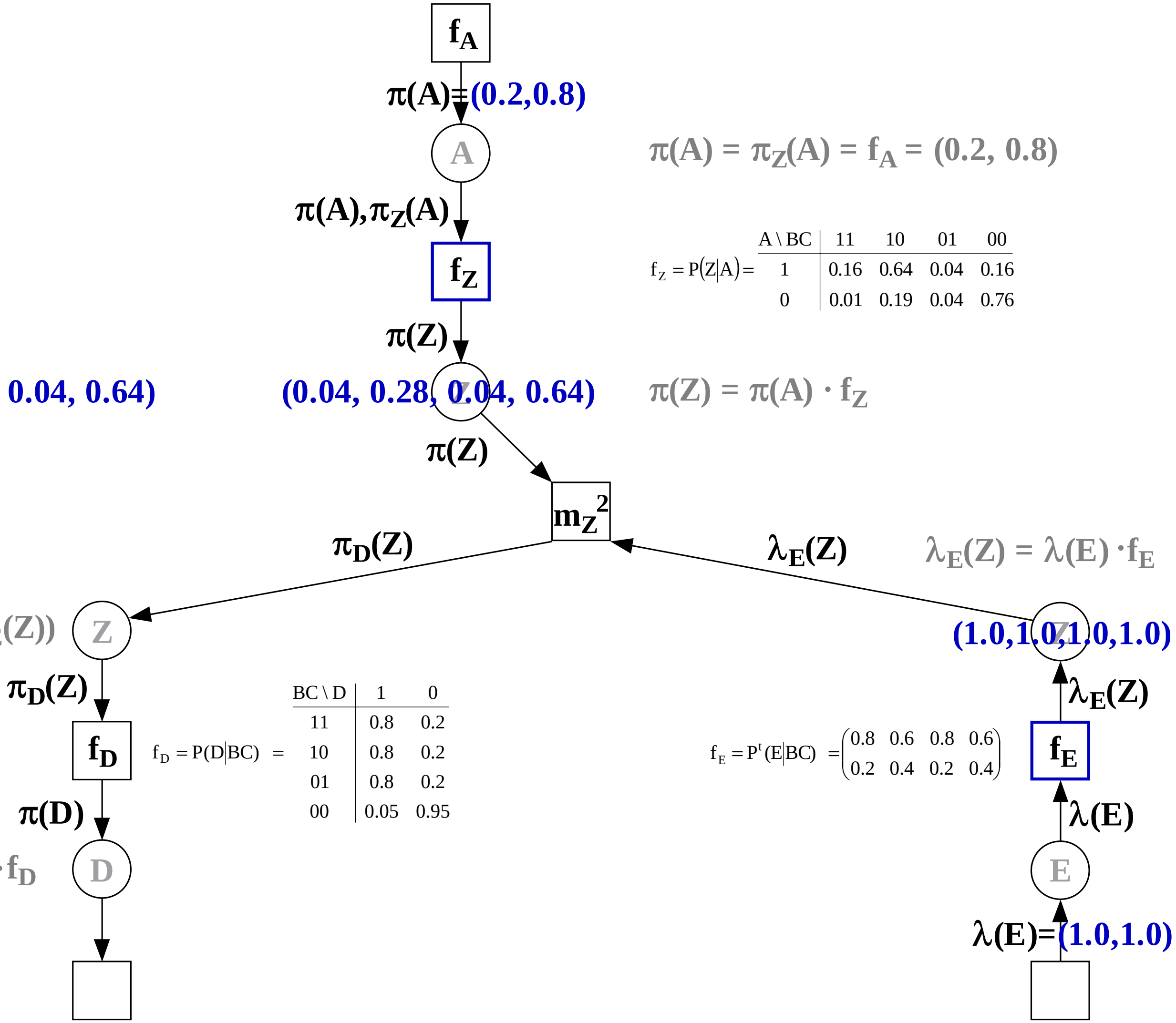
BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

$$f_E = P^t(E|BC) = \begin{pmatrix} 0.8 & 0.6 & 0.8 & 0.6 \\ 0.2 & 0.4 & 0.2 & 0.4 \end{pmatrix}$$

$\pi(D) = \pi_D(Z) \cdot f_D$

Clustering (Pearl)

t-invariant 2



$\pi(A) = \pi_Z(A) = f_A = (0.2,\ 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = (0.04,\ 0.28,\ 0.04,\ 0.64)$

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_E(Z) = \lambda(E) \cdot f_E$

$\pi_D(Z) = \alpha \cdot (\pi(Z) \circ \lambda_E(Z))$

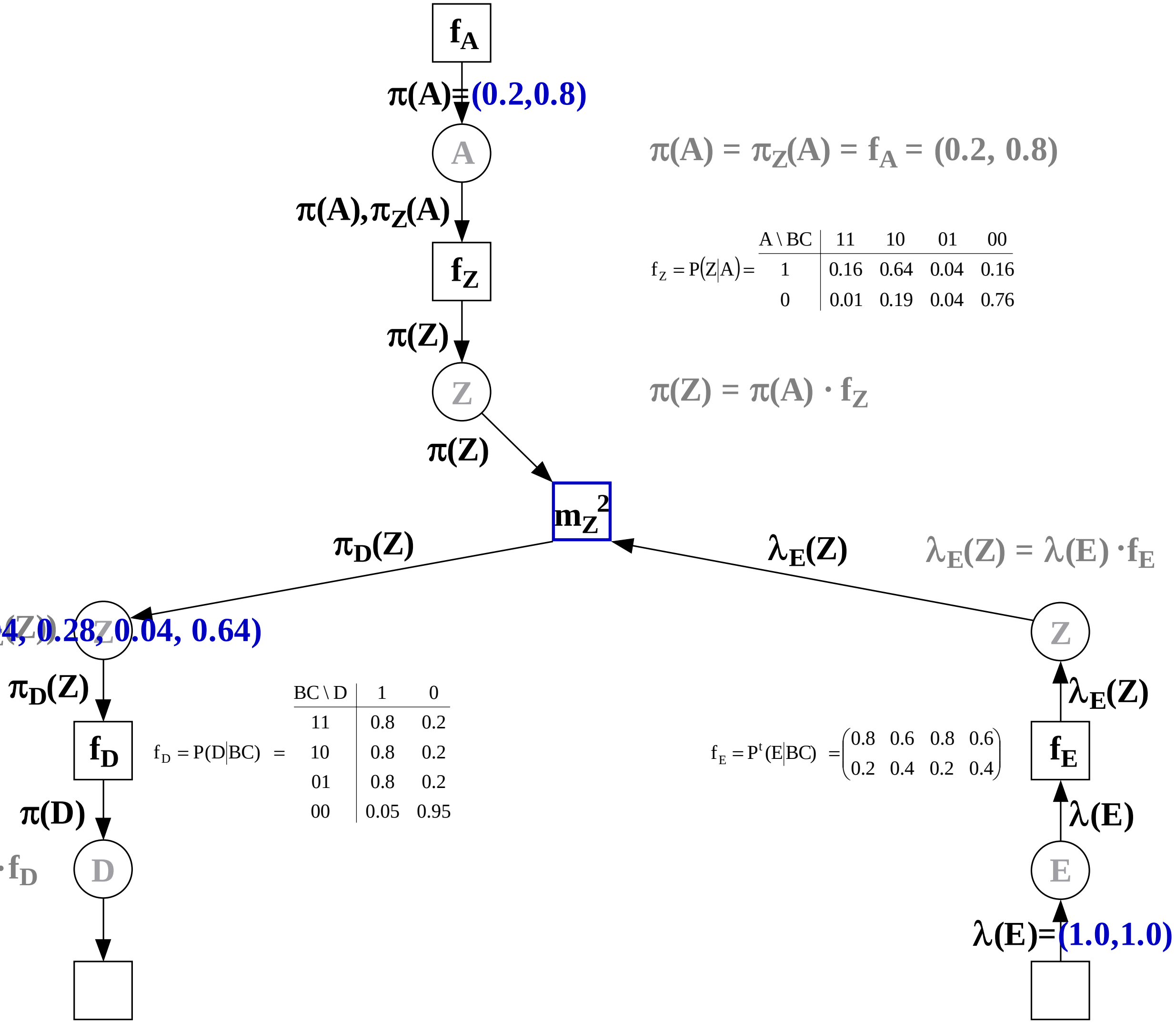
$f_D = P(D|BC) =$

BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

$f_E = P^t(E|BC) = \begin{pmatrix} 0.8 & 0.6 & 0.8 & 0.6 \\ 0.2 & 0.4 & 0.2 & 0.4 \end{pmatrix}$

$\pi(D) = \pi_D(Z) \cdot f_D$

t-invariant 2



$\pi(A) = \pi_Z(A) = f_A = (0.2,\ 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_E(Z) = \lambda(E) \cdot f_E$

$\pi_D(Z) = \alpha \cdot (\pi(Z) \cdot \lambda_E(Z))$

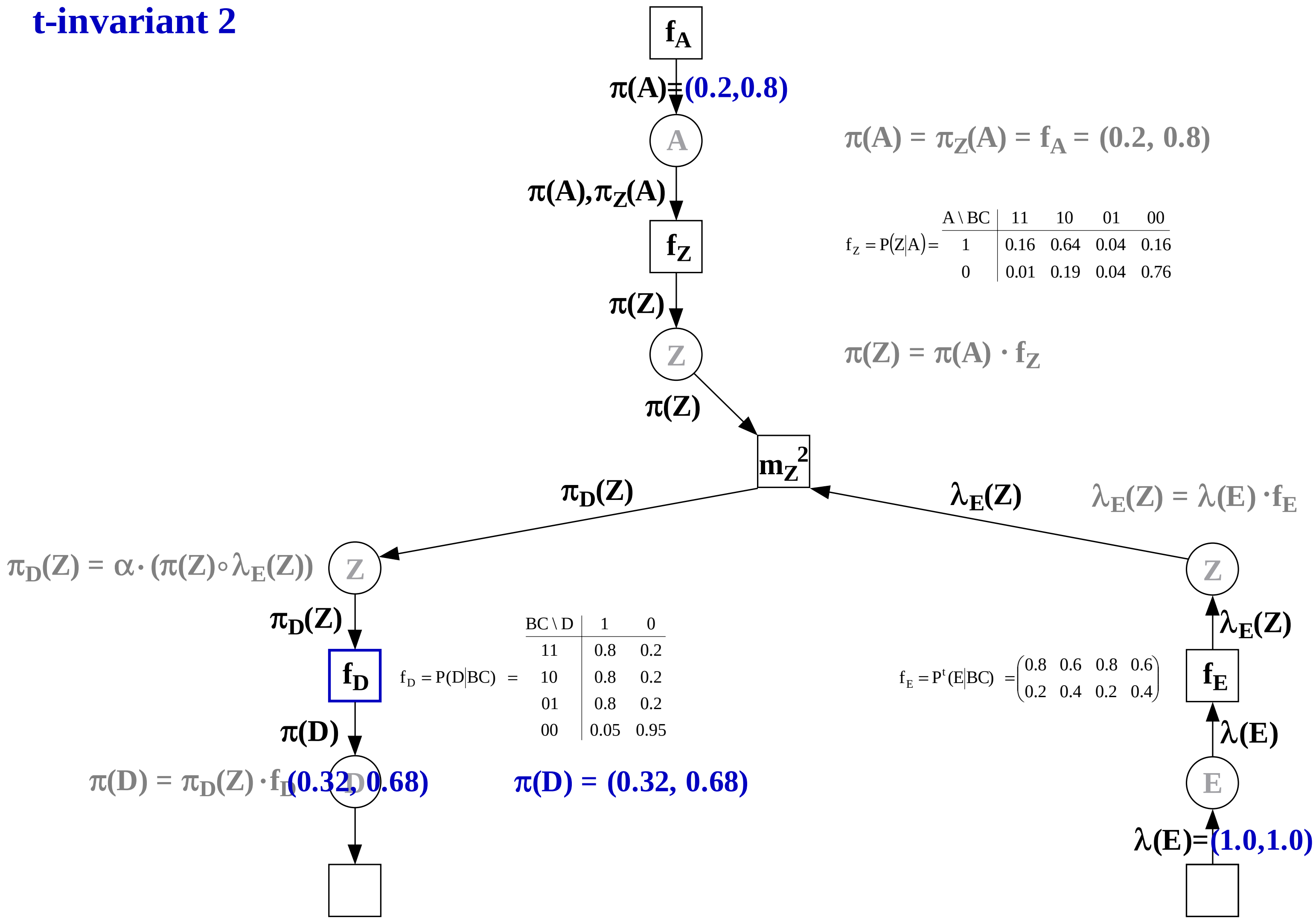
$f_D = P(D|BC) =$

BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

$$f_E = P^t(E|BC) = \begin{pmatrix} 0.8 & 0.6 & 0.8 & 0.6 \\ 0.2 & 0.4 & 0.2 & 0.4 \end{pmatrix}$$

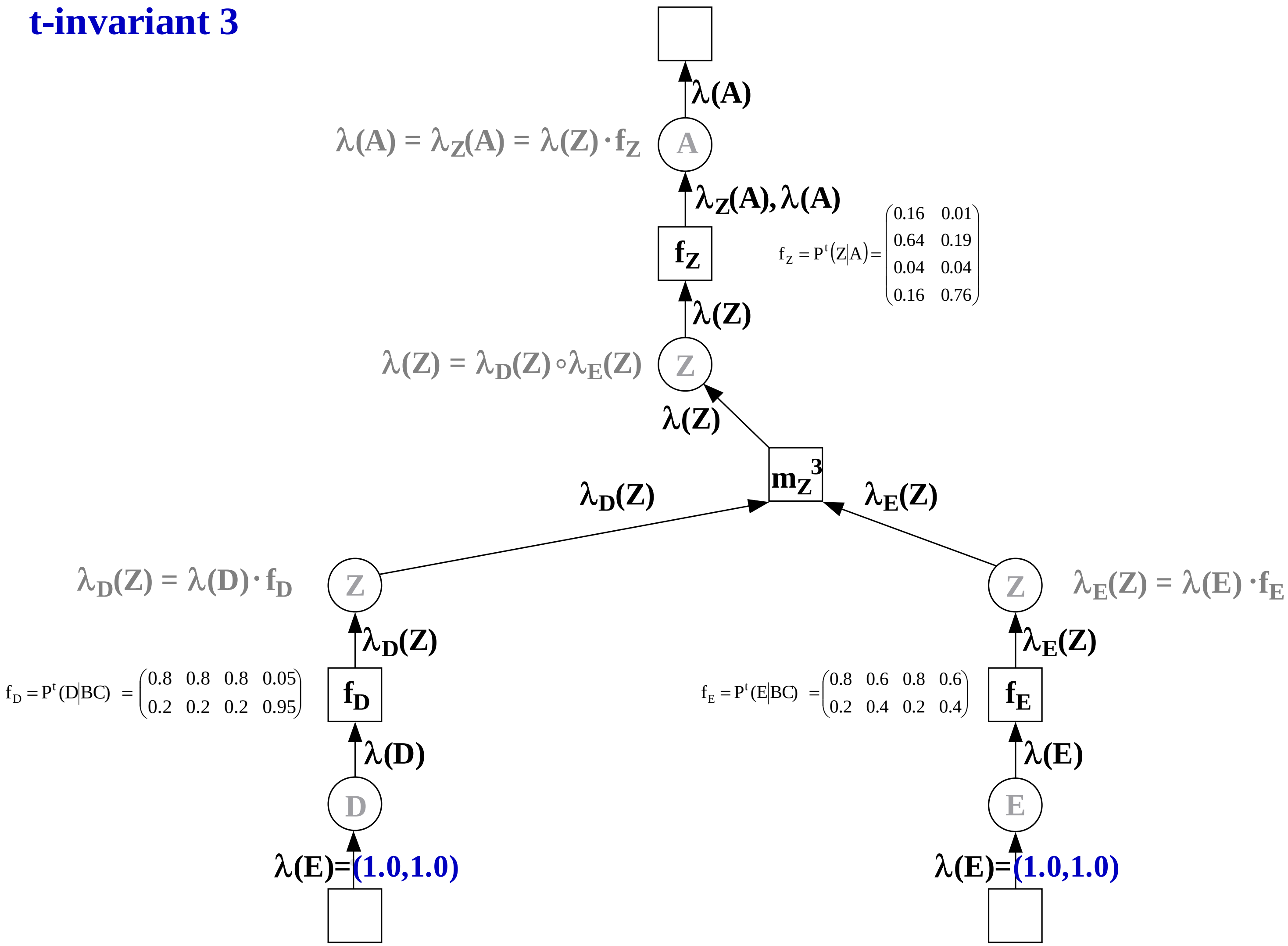
$\pi(D) = \pi_D(Z) \cdot f_D$

t-invariant 2

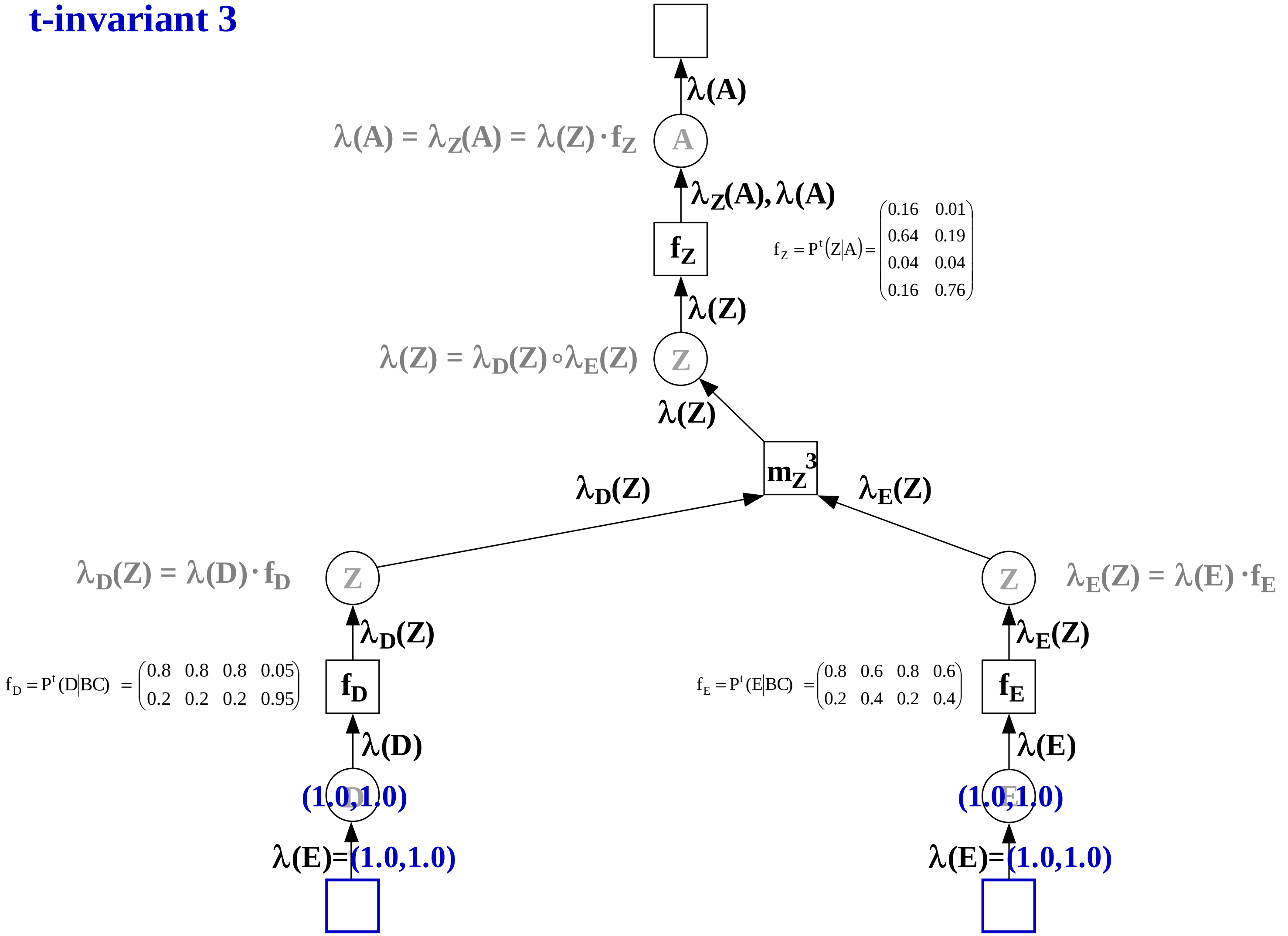


Clustering (Pearl)

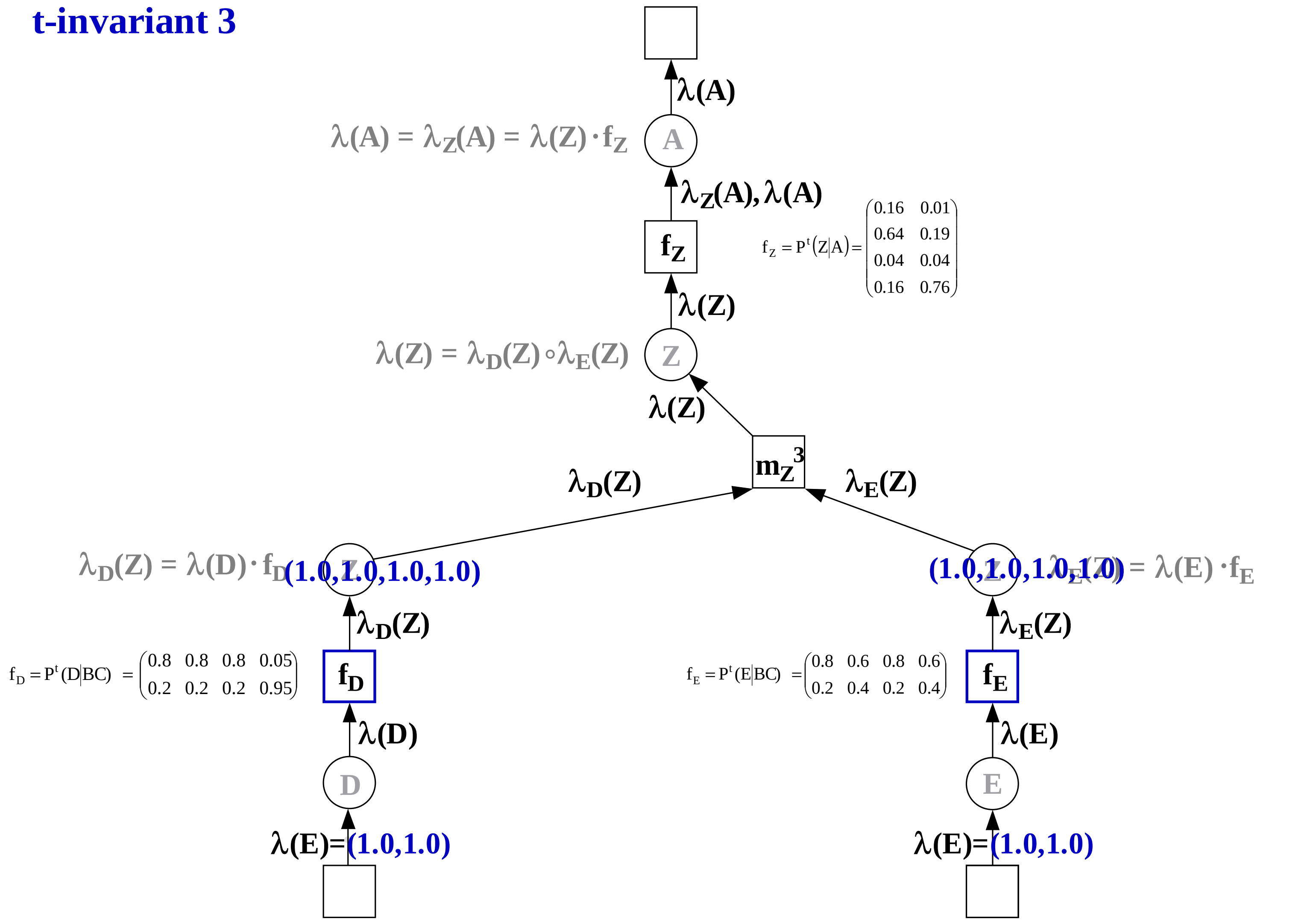
t-invariant 3



t-invariant 3

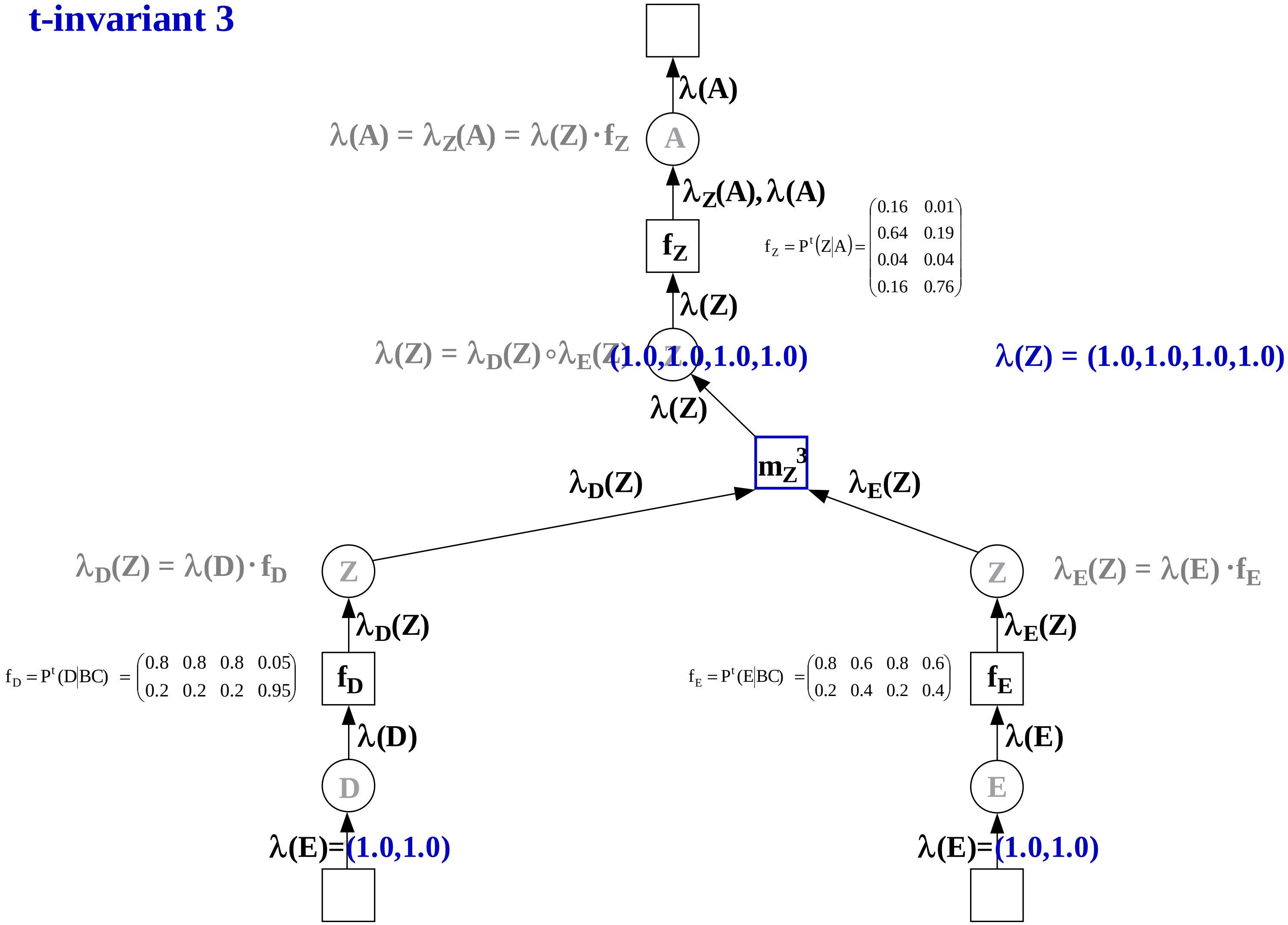


t-invariant 3



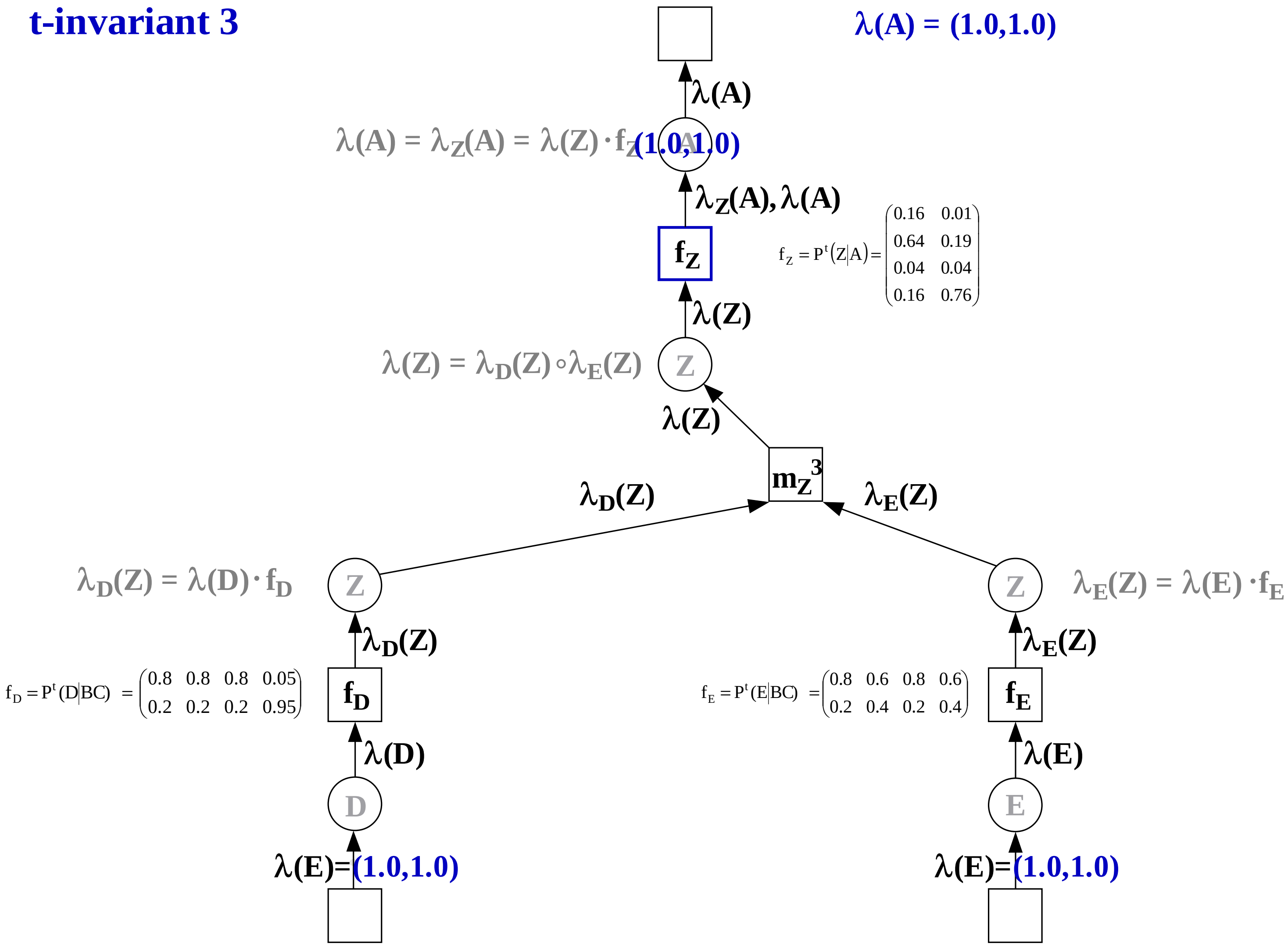
Clustering (Pearl)

t-invariant 3



t-invariant 3

$\lambda(A) = (1.0,1.0)$



Probabilities:

$P(A) = BEL(A) = \alpha\, \pi(A) \circ \lambda(A) = \alpha\, \underline{(0.2, 0.8) \circ (1, 1)} = (0.2, 0.8)$

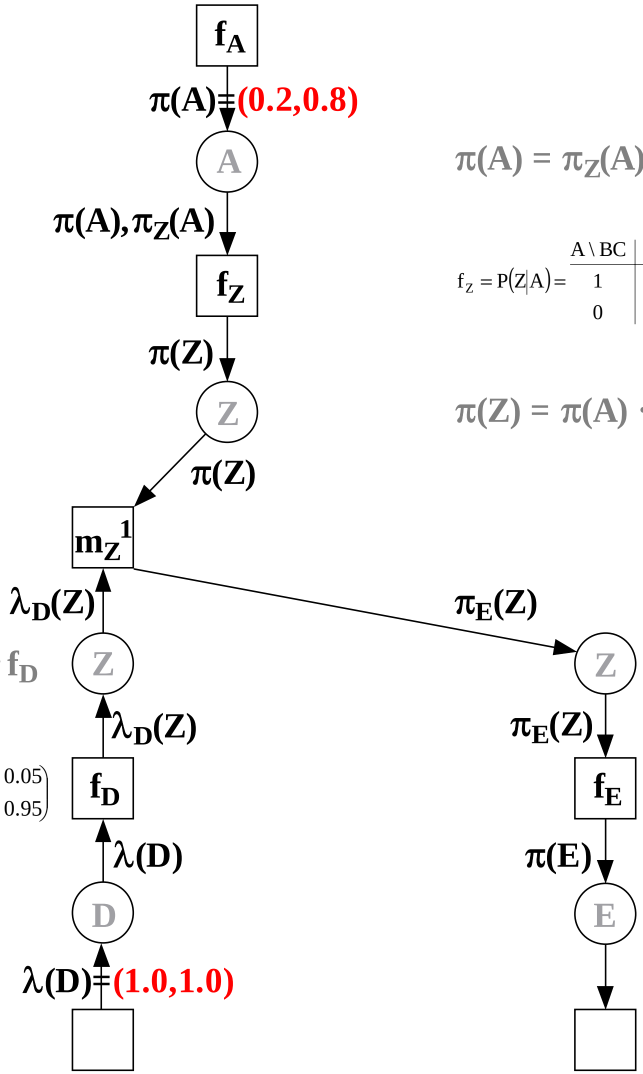
$P(Z) = BEL(Z) = \alpha\, \pi(Z) \circ \lambda(Z) = \alpha\, \underline{(0.04, 0.28, 0.04, 0.64) \circ (1, 1, 1, 1)}$
$= (0.04, 0.28, 0.04, 0.64)$

$P(B) = BEL(B) = (0.04 + 0.28, 0.04 + 0.64) = \boxed{(0.32, 0.68)}$

$P(C) = BEL(C) = (0.04 + 0.04, 0.28 + 0.64) = \boxed{(0.08, 0.92)}$

Clustering (Pearl)

t-invariant 1



$\pi(A) = \pi_Z(A) = f_A = (0.2,\ 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_D(Z) = \lambda(D) \cdot f_D$

$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$

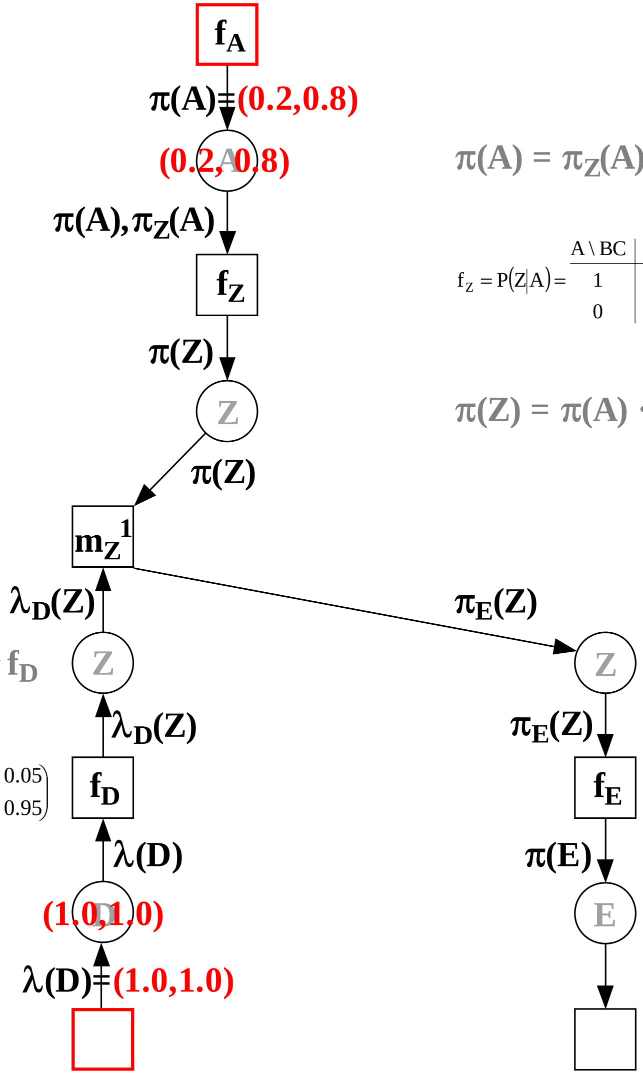
$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

$f_E = P(E|BC) =$

BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$\pi(E) = \pi_E(Z) \cdot f_E$

t-invariant 1



$\pi(A) = \pi_Z(A) = f_A = (0.2, 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_D(Z) = \lambda(D) \cdot f_D$

$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$

$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

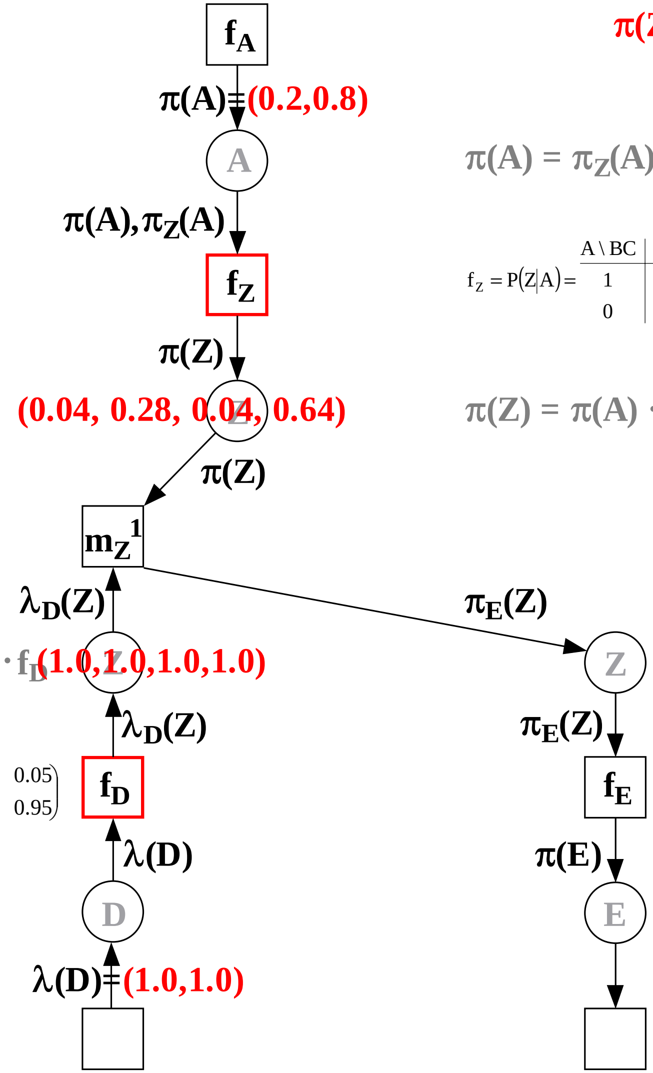
$f_E = P(E|BC) =$

BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$\pi(E) = \pi_E(Z) \cdot f_E$

t-invariant 1

$\pi(Z) = (0.04, 0.28, 0.04, 0.64)$



$\pi(A) = \pi_Z(A) = f_A = (0.2, 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_D(Z) = \lambda(D) \cdot f_D$

$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$

$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

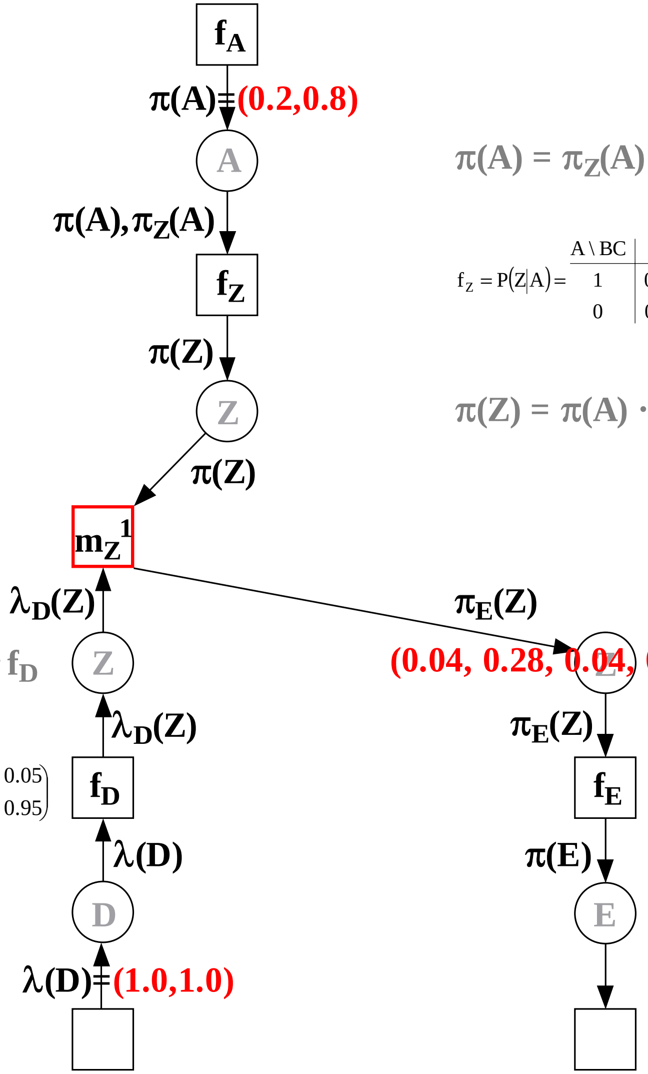
$f_E = P(E|BC) =$

BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$\pi(E) = \pi_E(Z) \cdot f_E$

Clustering (Pearl)

t-invariant 1



$\pi(A) = \pi_Z(A) = f_A = (0.2, 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_D(Z) = \lambda(D) \cdot f_D$

$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$

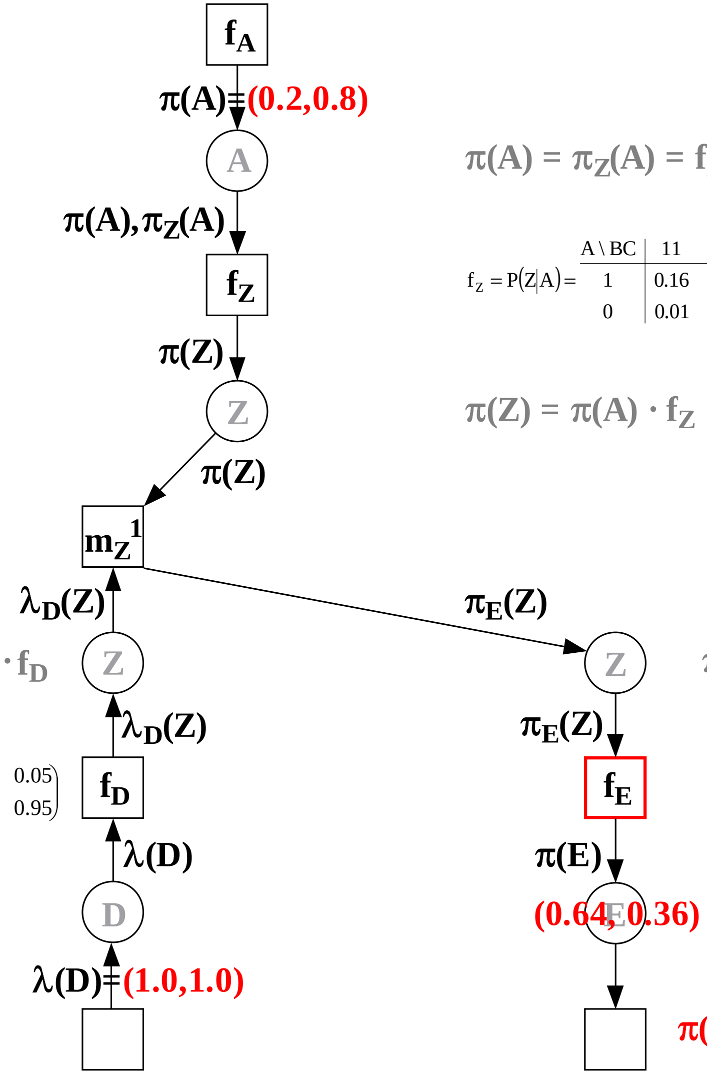
$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

$f_E = P(E|BC) =$

BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$\pi(E) = \pi_E(Z) \cdot f_E$

t-invariant 1



$\pi(A) = \pi_Z(A) = f_A = (0.2,\ 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_D(Z) = \lambda(D) \cdot f_D$

$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$

$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

$f_E = P(E|BC) =$

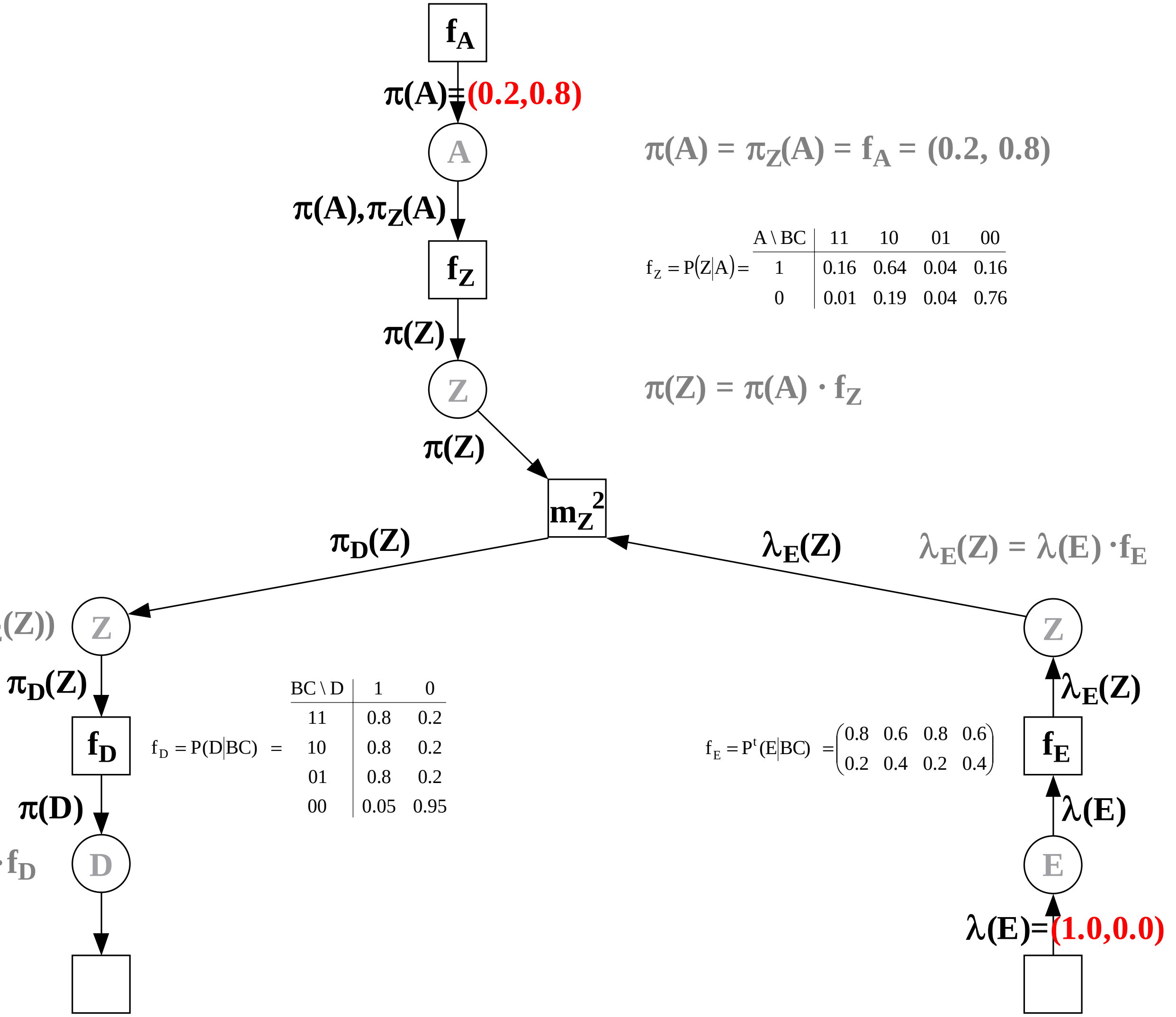
BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$\pi(E) = \pi_E(Z) \cdot f_E$

$\pi(E) = (0.64,\ 0.36)$

Clustering (Pearl)

t-invariant 2



$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

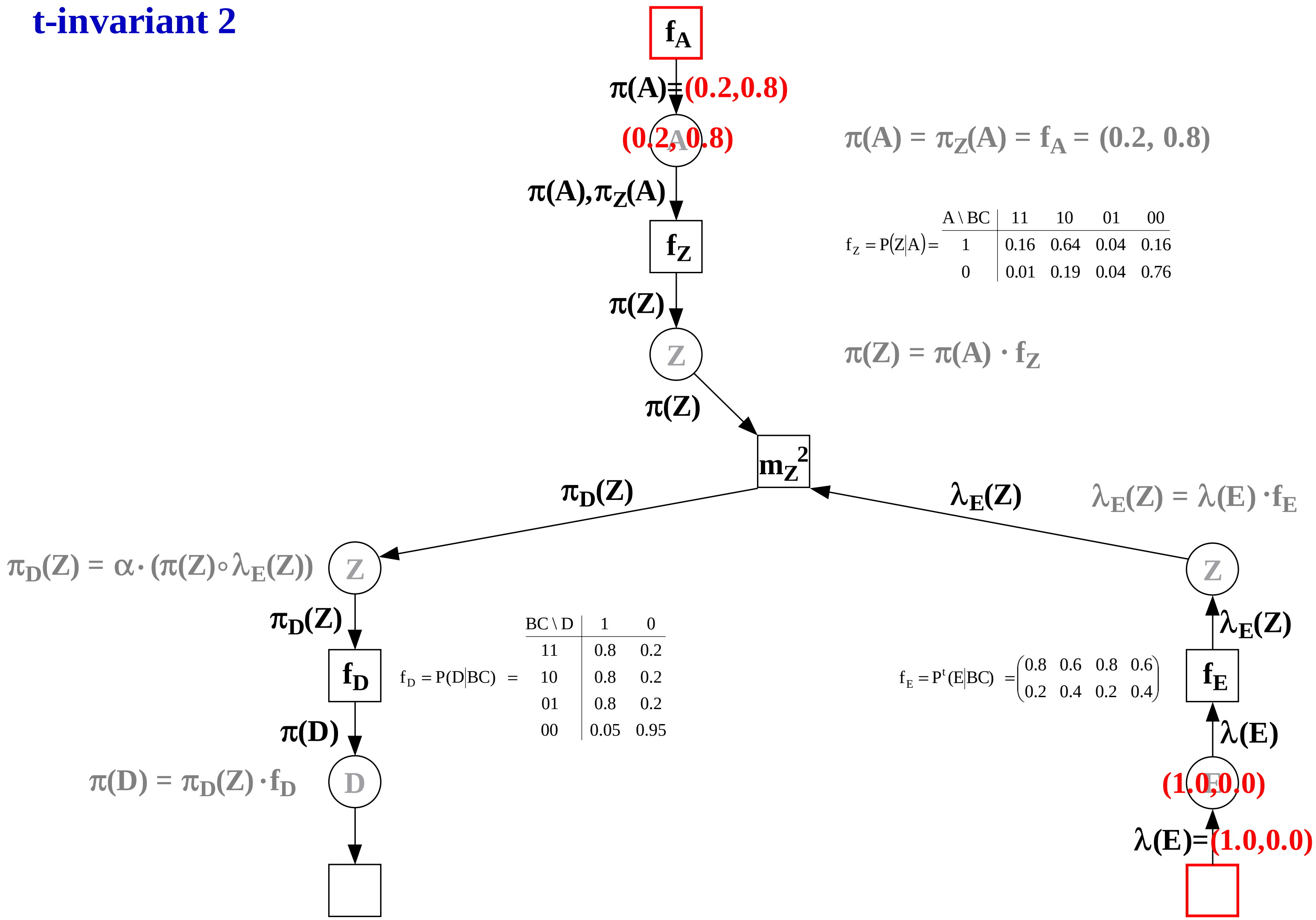
$f_D = P(D|BC) =$

BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

$$f_E = P^t(E|BC) = \begin{pmatrix} 0.8 & 0.6 & 0.8 & 0.6 \\ 0.2 & 0.4 & 0.2 & 0.4 \end{pmatrix}$$

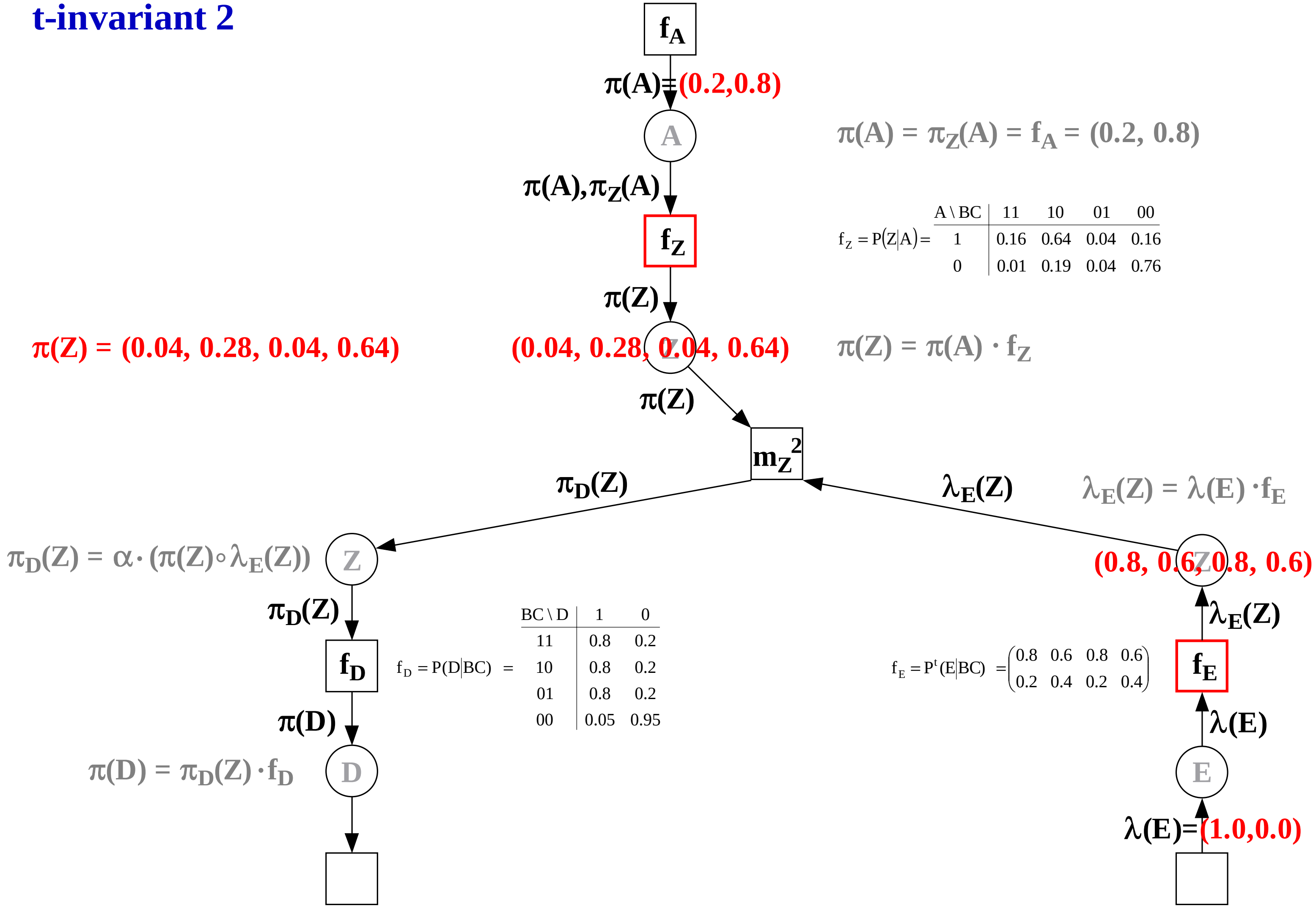
Clustering (Pearl)

t-invariant 2



Clustering (Pearl)

t-invariant 2



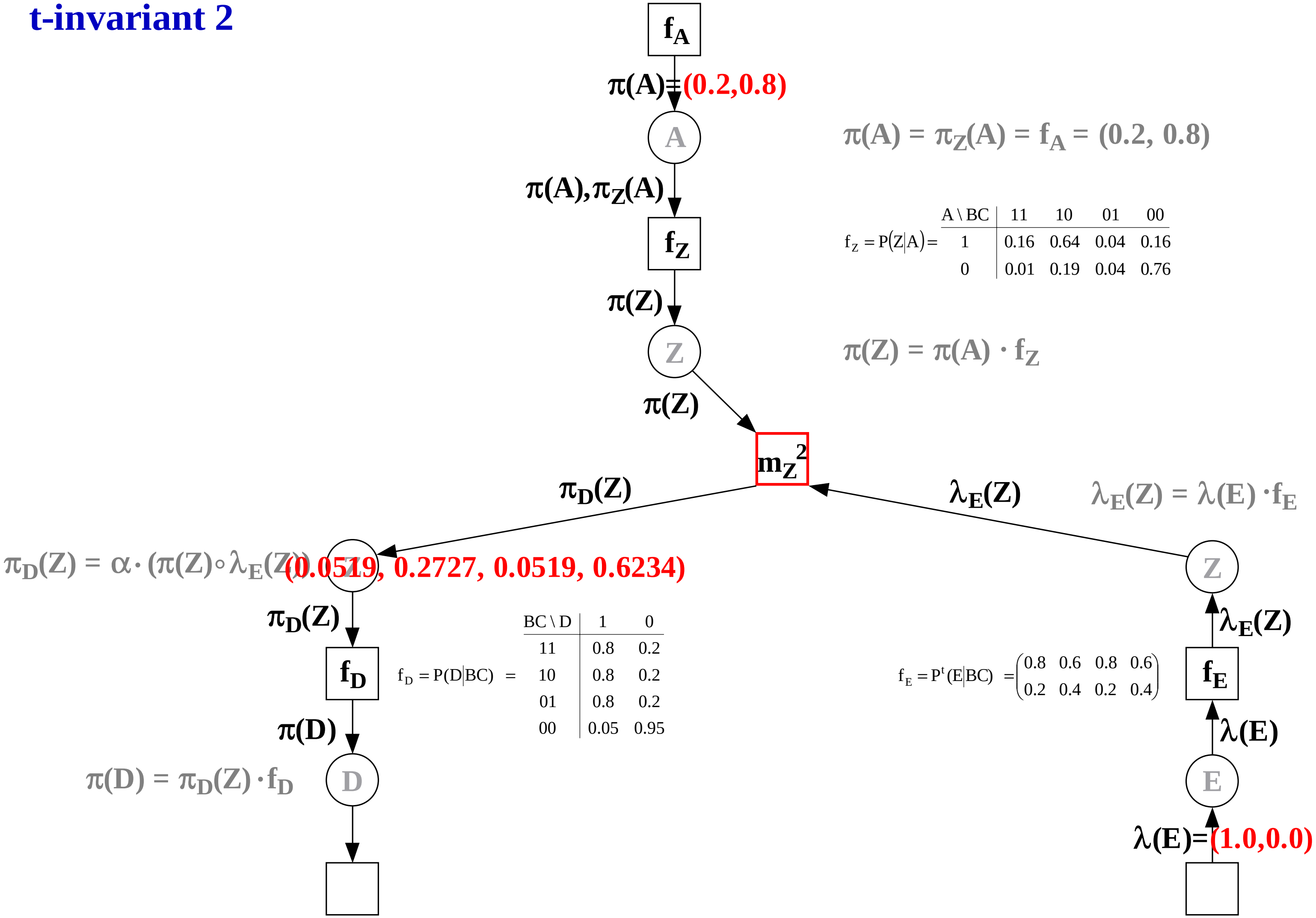
$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$f_D = P(D|BC) =$

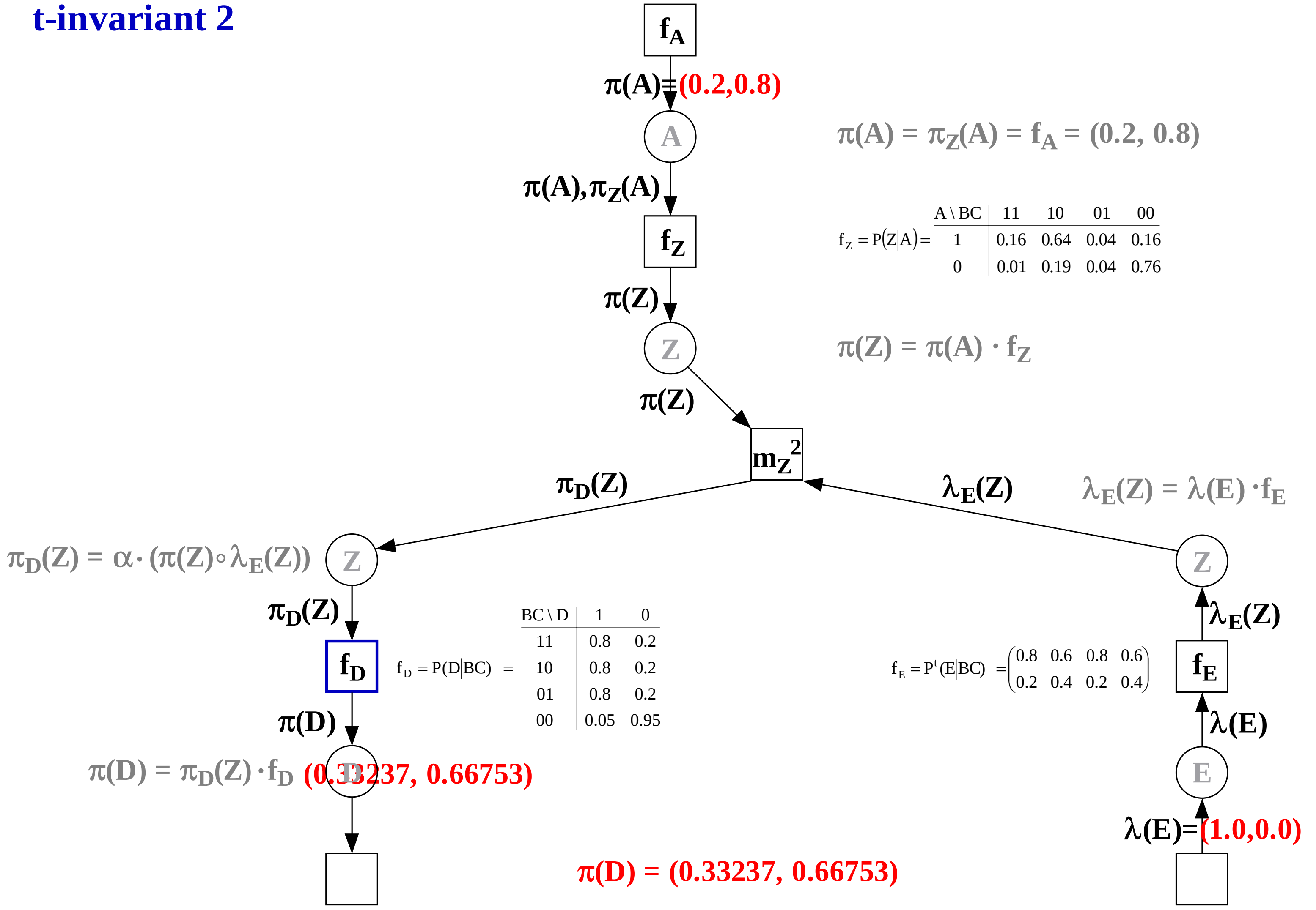
BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

t-invariant 2



Clustering (Pearl)

t-invariant 2



$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

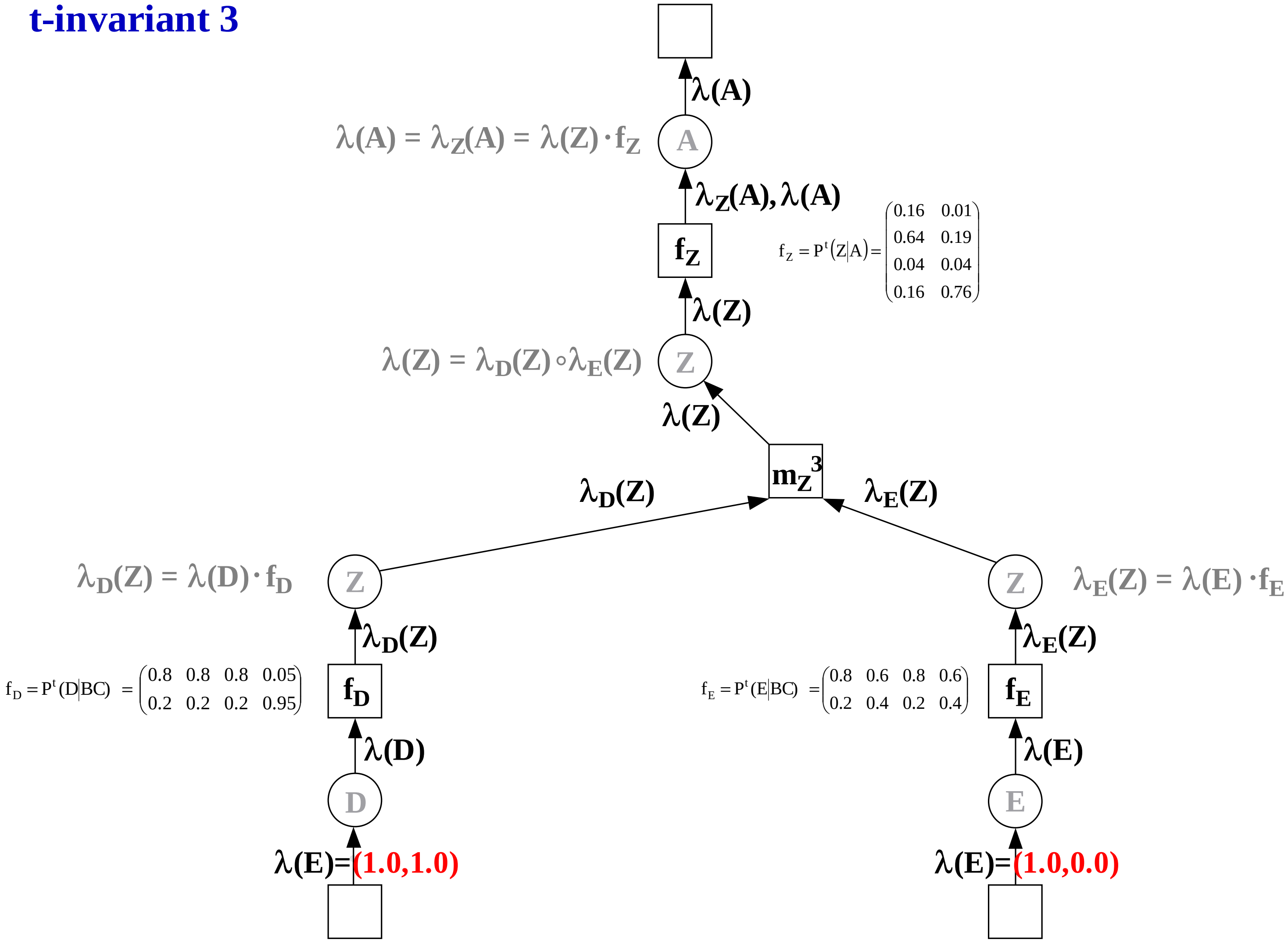
$f_D = P(D|BC) =$

BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

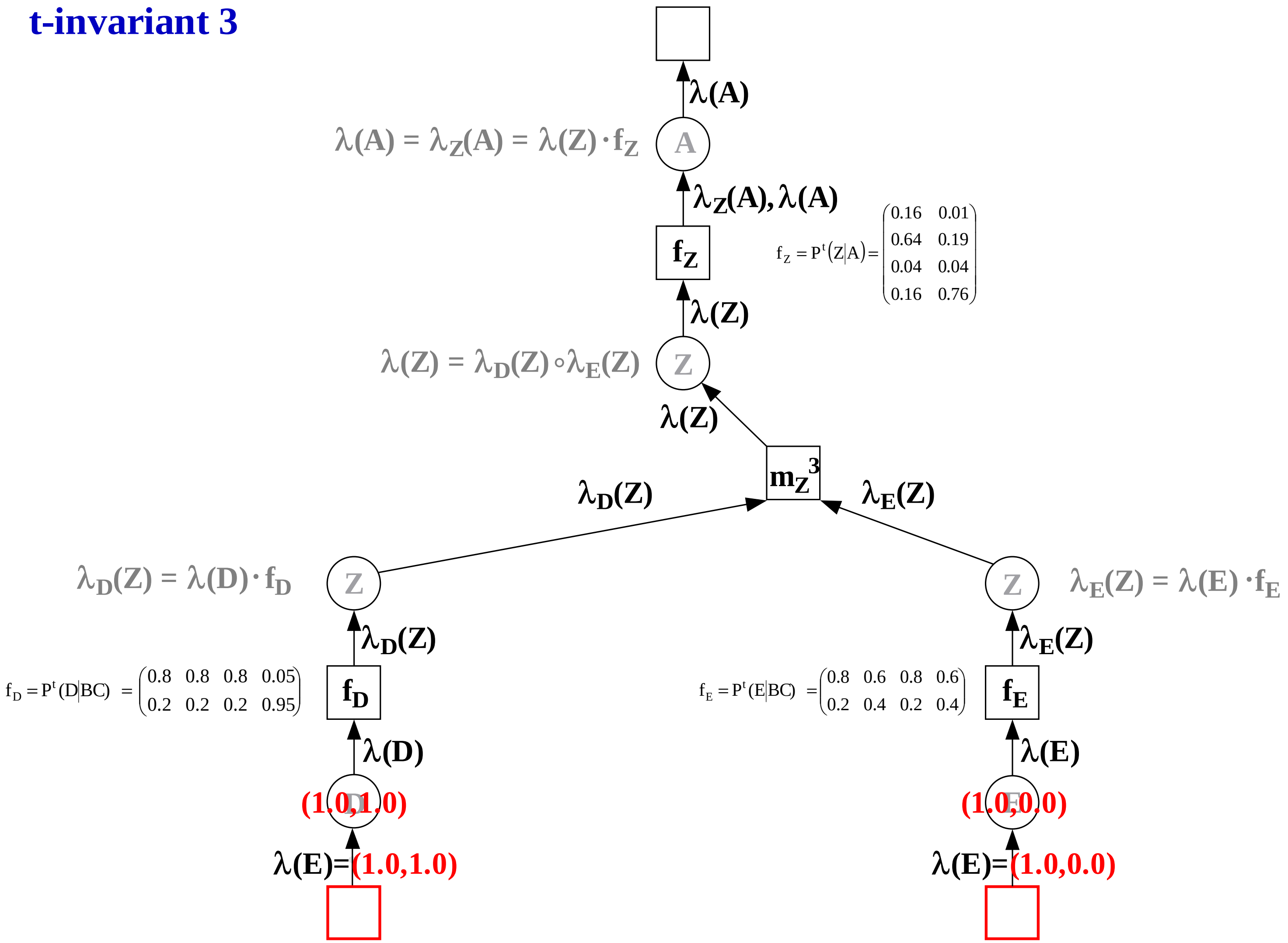
$$f_E = P^t(E|BC) = \begin{pmatrix} 0.8 & 0.6 & 0.8 & 0.6 \\ 0.2 & 0.4 & 0.2 & 0.4 \end{pmatrix}$$

Clustering (Pearl)

t-invariant 3

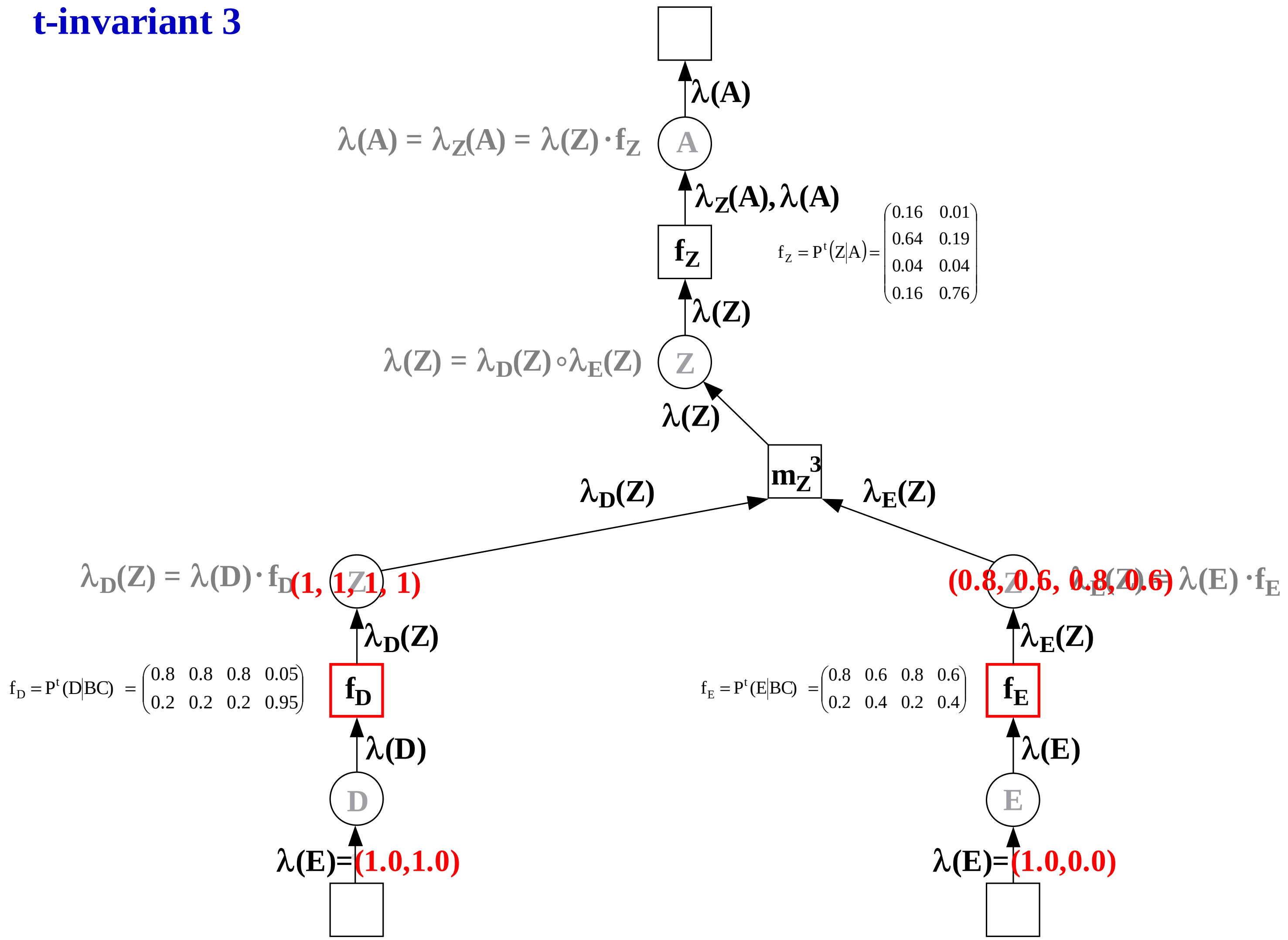


t-invariant 3



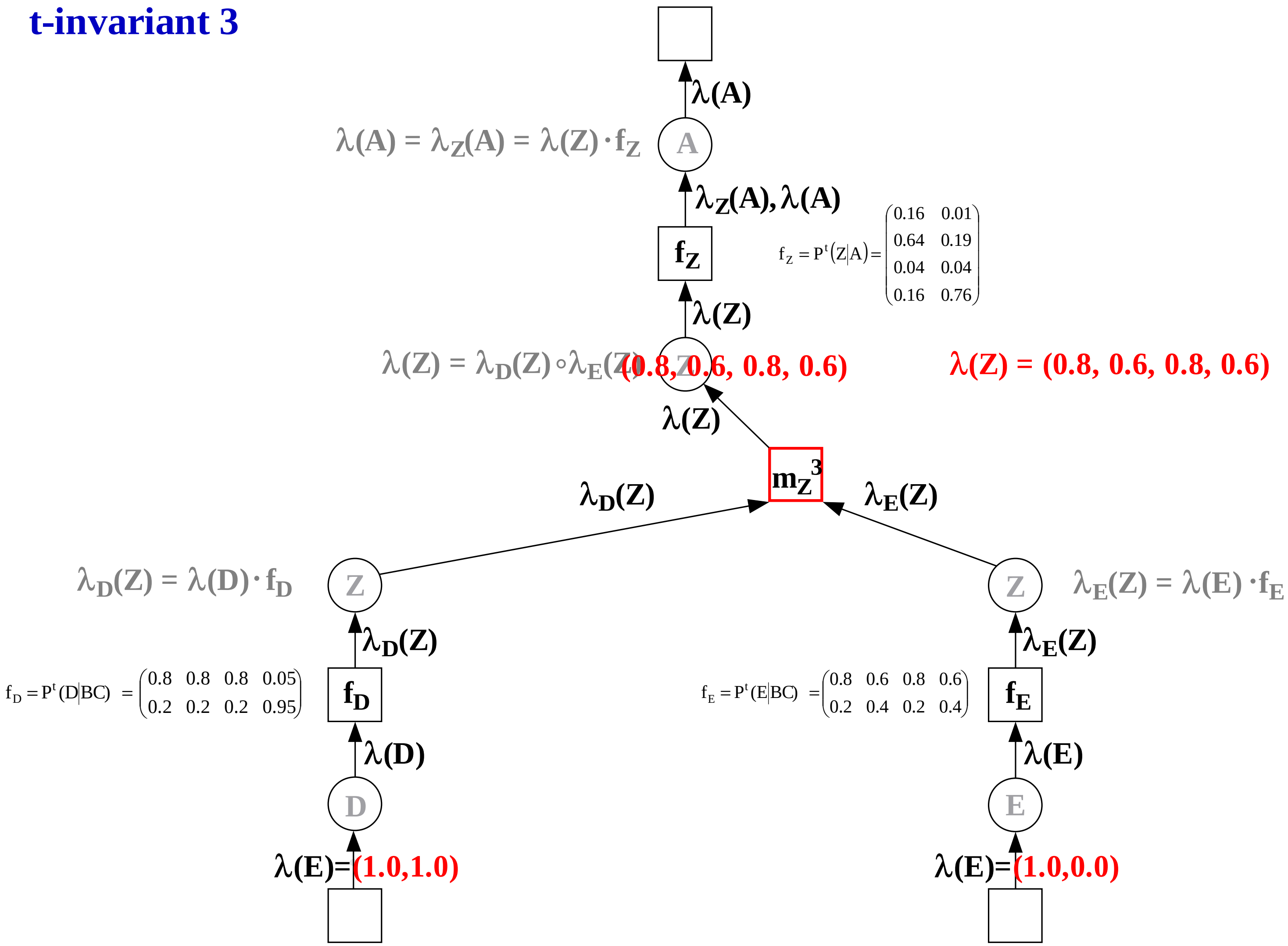
Clustering (Pearl)

t-invariant 3



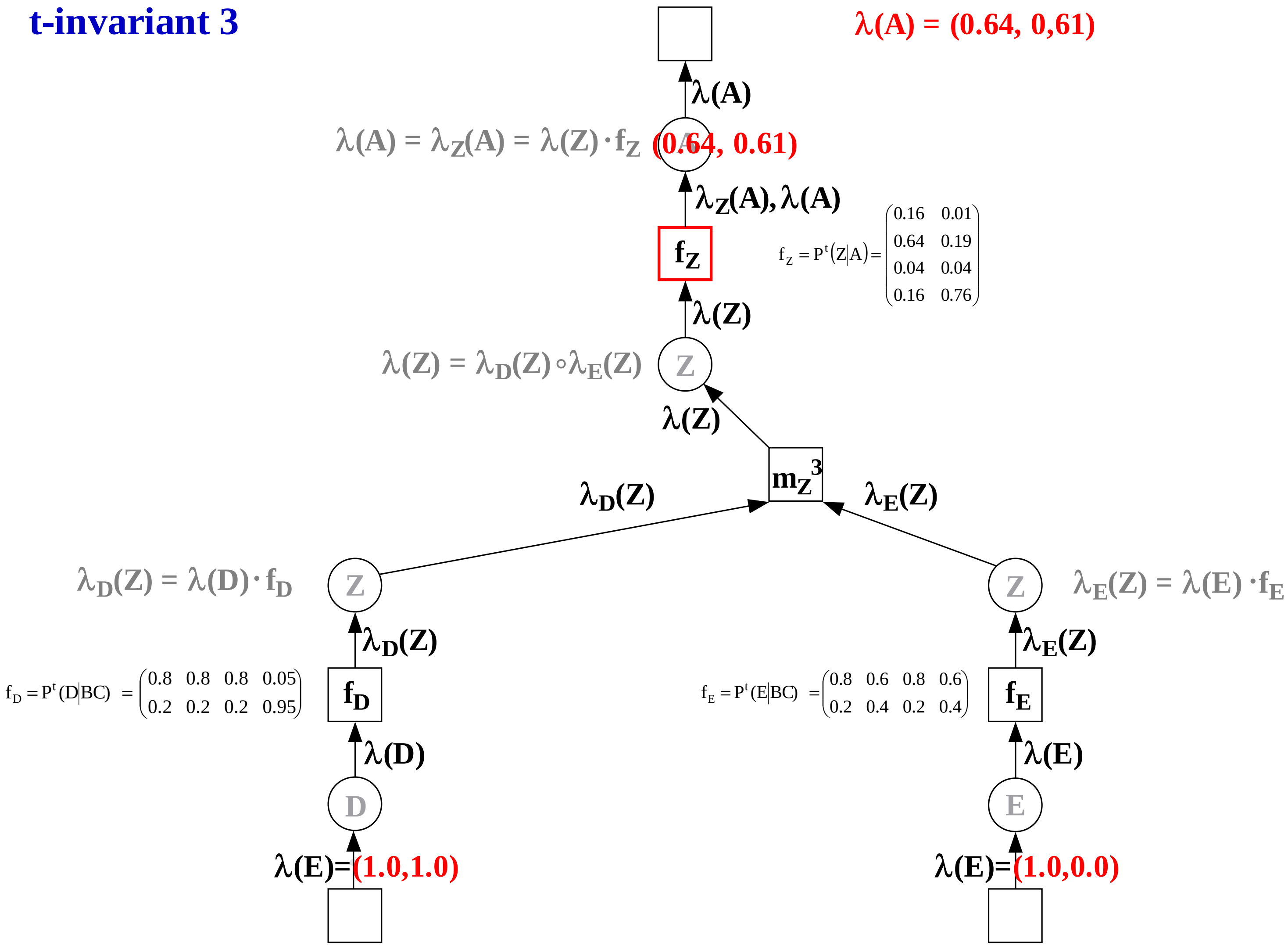
Clustering (Pearl)

t-invariant 3



t-invariant 3

$\lambda(A) = (0.64, 0,61)$



Probabilities:

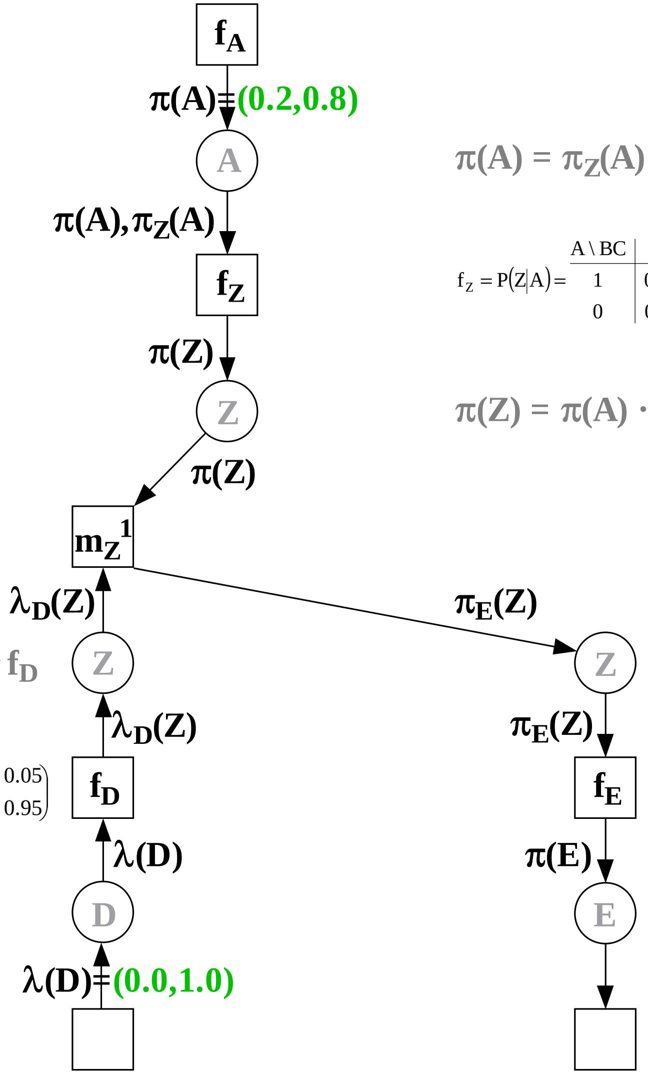
$P(A) = BEL(A) = \alpha\, \pi(A) \circ \lambda(A) = \alpha\, \underline{(0.2, 0.8) \circ (0.64, 0.61)} = (0.2078, 0.7922)$

$P(Z) = BEL(Z) = \alpha\, \pi(Z) \circ \lambda(Z) = \alpha\, \underline{(0.04, 0.28, 0.04, 0.64) \circ (0.8, 0.6, 0.8, 0.6)}$
$= (0.0519, 0.2727, 0.0519, 0.6234)$

$P(B) = BEL(B) = (0.0519 + 0.2727, 0.0519 + 0.6234) = \boxed{(0.3246, 0.6753)}$

$P(C) = BEL(C) = (0.0519 + 0.0519, 0.2727 + 0.6234) = \boxed{(0.1038, 0.8961)}$

t-invariant 1



$$\pi(A) = \pi_Z(A) = f_A = (0.2,\ 0.8)$$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$$\pi(Z) = \pi(A) \cdot f_Z$$

$$\lambda_D(Z) = \lambda(D) \cdot f_D$$

$$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$$

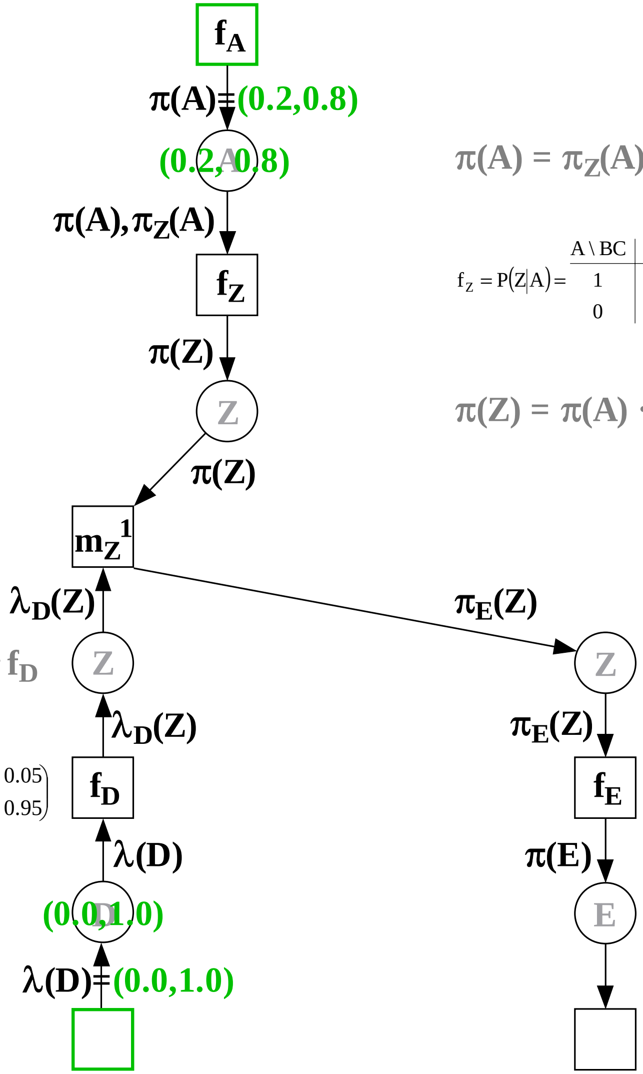
$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

$f_E = P(E|BC) =$

BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$$\pi(E) = \pi_E(Z) \cdot f_E$$

t-invariant 1



$\pi(A) = \pi_Z(A) = f_A = (0.2,\ 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_D(Z) = \lambda(D) \cdot f_D$

$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$

$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

$f_E = P(E|BC) =$

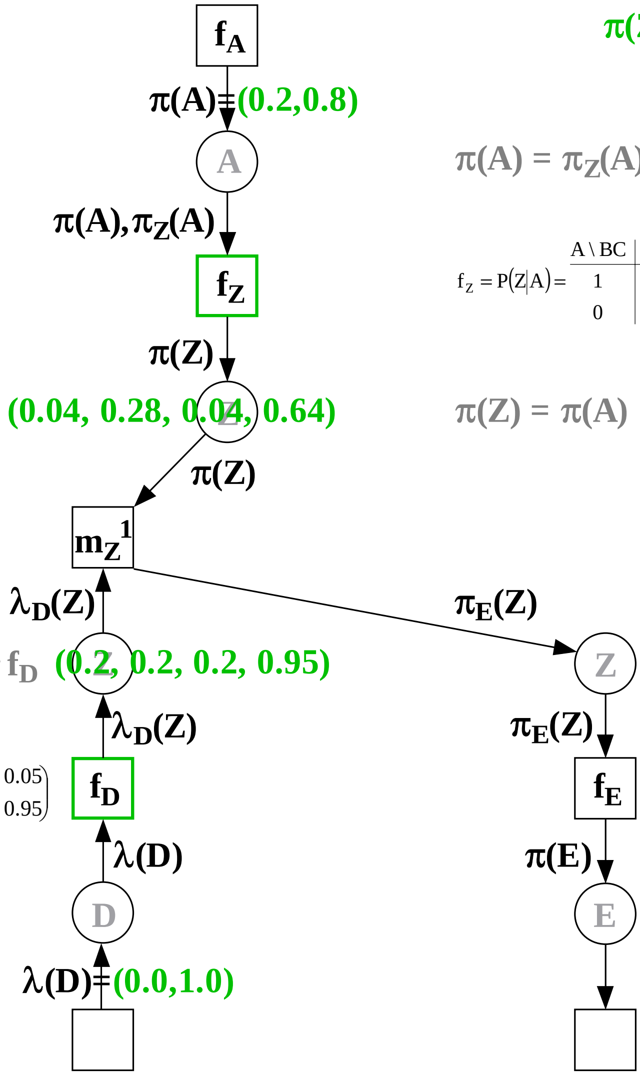
BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$\pi(E) = \pi_E(Z) \cdot f_E$

Clustering (Pearl)

t-invariant 1

$\pi(Z) = (0.04, 0.28, 0.04, 0.64)$



$\pi(A) = \pi_Z(A) = f_A = (0.2, 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_D(Z) = \lambda(D) \cdot f_D$

$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$

$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

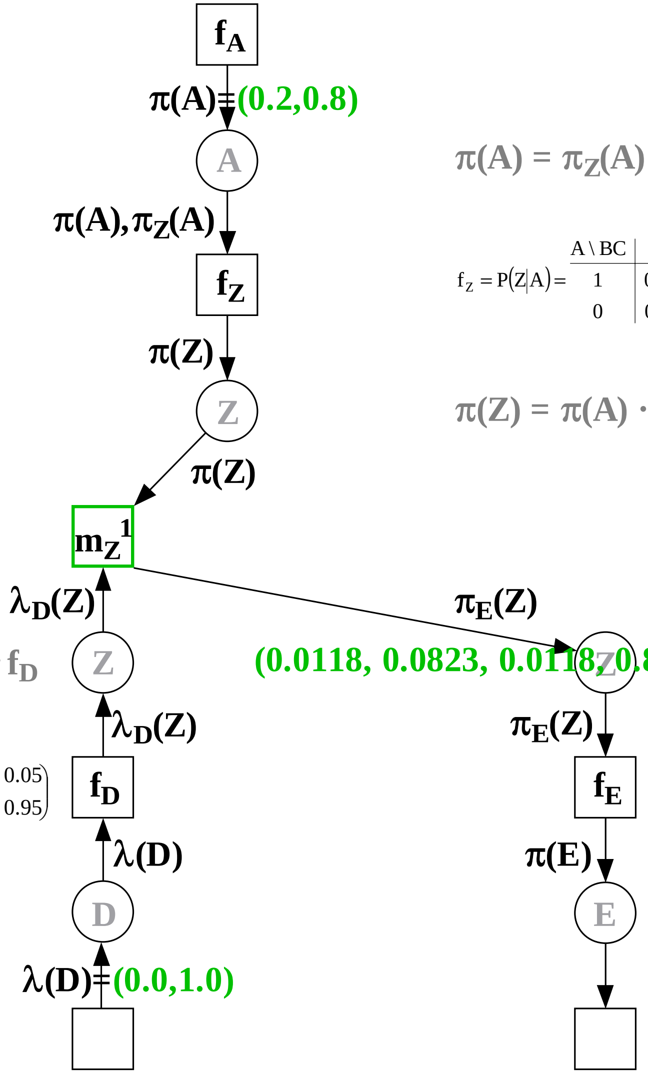
$f_E = P(E|BC) =$

BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$\pi(E) = \pi_E(Z) \cdot f_E$

Clustering (Pearl)

t-invariant 1



$\pi(A) = \pi_Z(A) = f_A = (0.2, 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_D(Z) = \lambda(D) \cdot f_D$

$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$

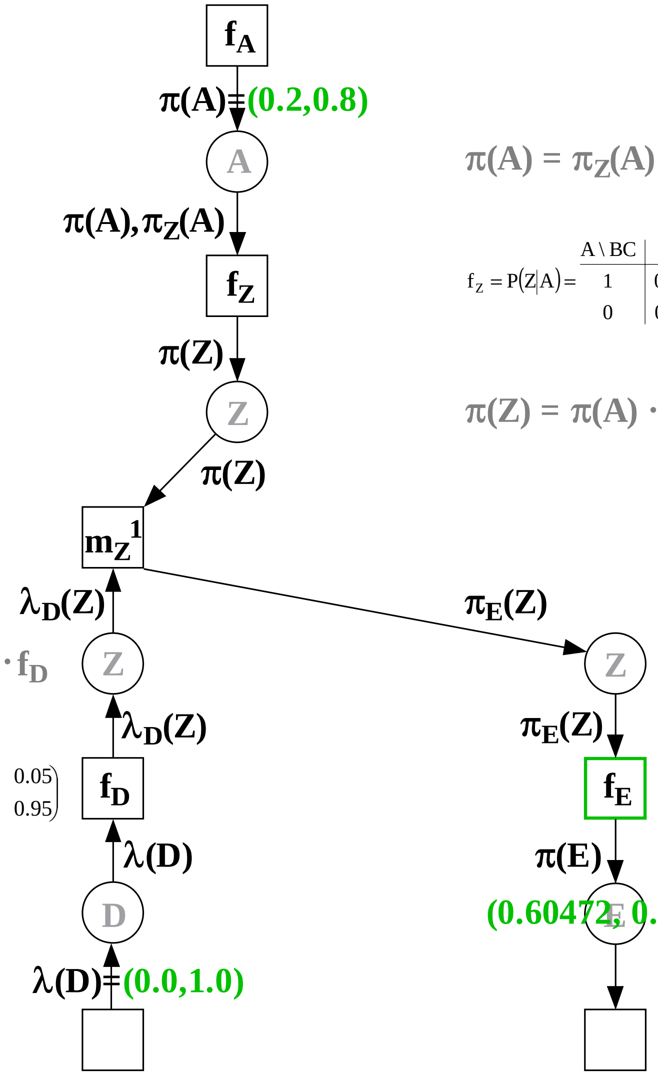
$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

$f_E = P(E|BC) =$

BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$\pi(E) = \pi_E(Z) \cdot f_E$

t-invariant 1



$\pi(A) = \pi_Z(A) = f_A = (0.2, 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(E) = (0.60472, 0.39528)$

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_D(Z) = \lambda(D) \cdot f_D$

$\pi_E(Z) = \alpha \cdot (\pi(Z) \circ \lambda_D(Z))$

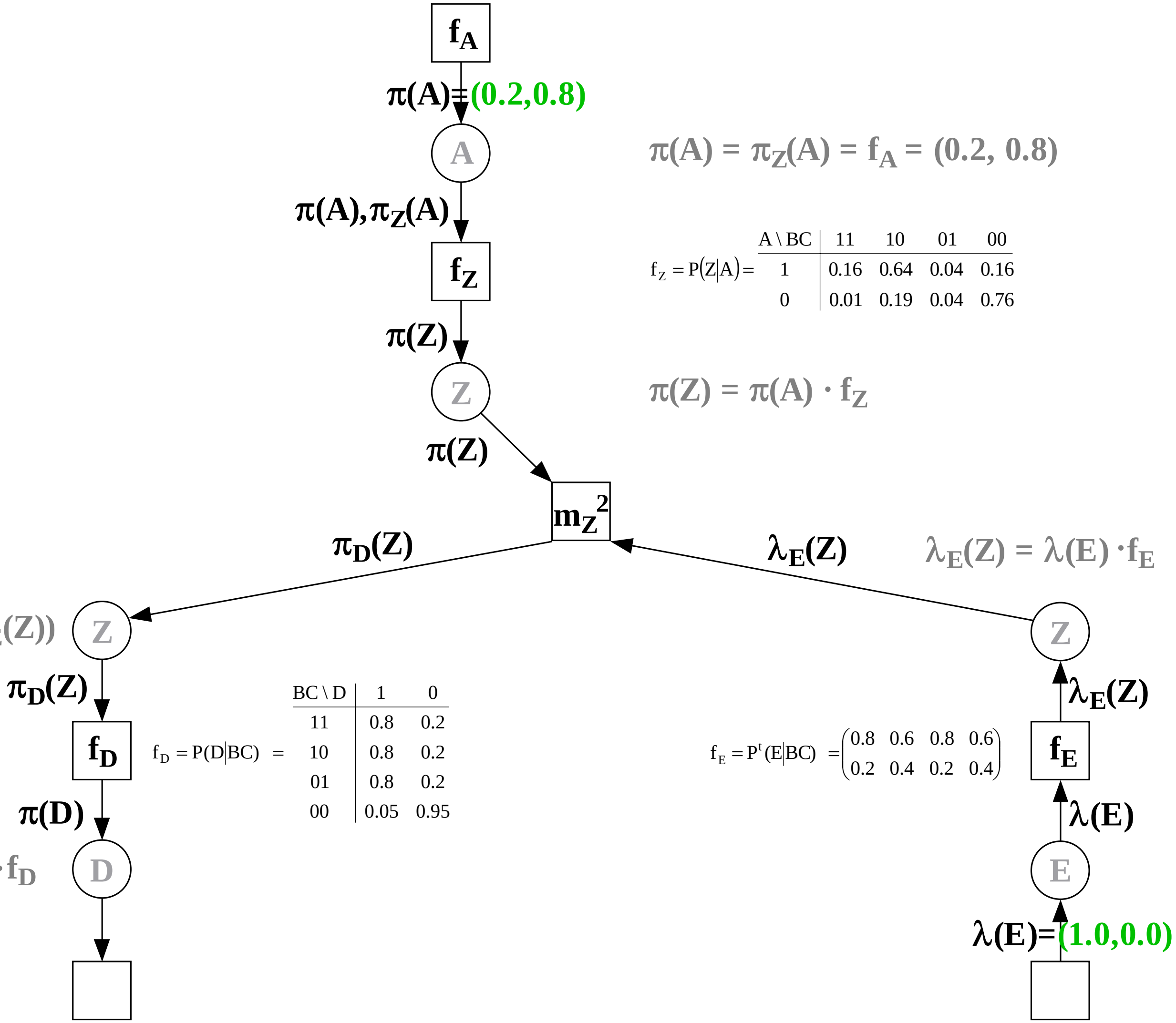
$$f_D = P^t(D|BC) = \begin{pmatrix} 0.8 & 0.8 & 0.8 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.95 \end{pmatrix}$$

$f_E = P(E|BC) =$

BC \ E	1	0
11	0.8	0.2
10	0.6	0.4
01	0.8	0.2
00	0.6	0.4

$\pi(E) = \pi_E(Z) \cdot f_E$

t-invariant 2



$\pi(A) = \pi_Z(A) = f_A = (0.2,\ 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_E(Z) = \lambda(E) \cdot f_E$

$\pi_D(Z) = \alpha \cdot (\pi(Z) \circ \lambda_E(Z))$

$f_D = P(D|BC) =$

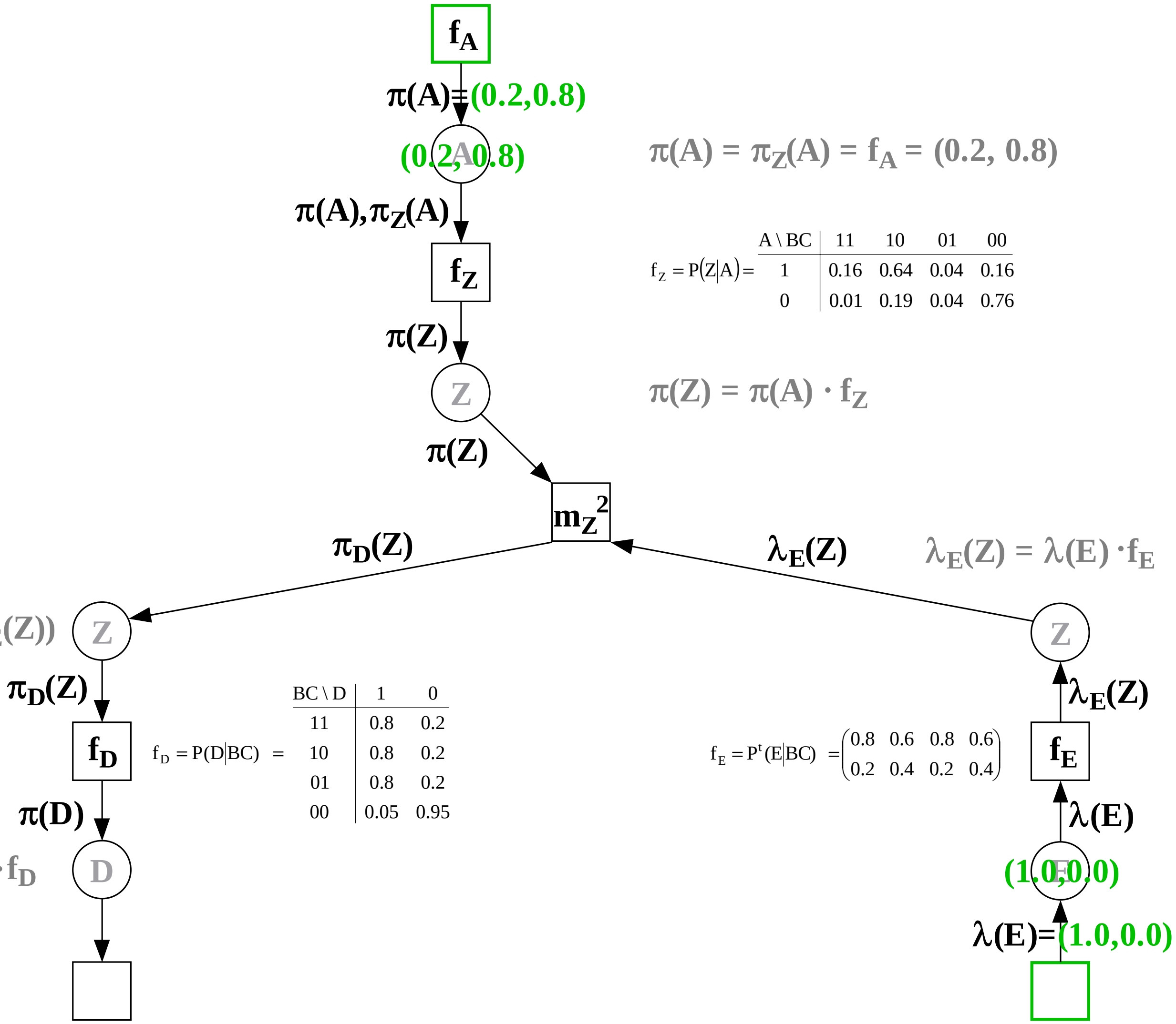
BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

$f_E = P^t(E|BC) = \begin{pmatrix} 0.8 & 0.6 & 0.8 & 0.6 \\ 0.2 & 0.4 & 0.2 & 0.4 \end{pmatrix}$

$\pi(D) = \pi_D(Z) \cdot f_D$

Clustering (Pearl)

t-invariant 2



$\pi(A) = \pi_Z(A) = f_A = (0.2,\ 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_E(Z) = \lambda(E) \cdot f_E$

$\pi_D(Z) = \alpha \cdot (\pi(Z) \circ \lambda_E(Z))$

$f_D = P(D|BC) =$

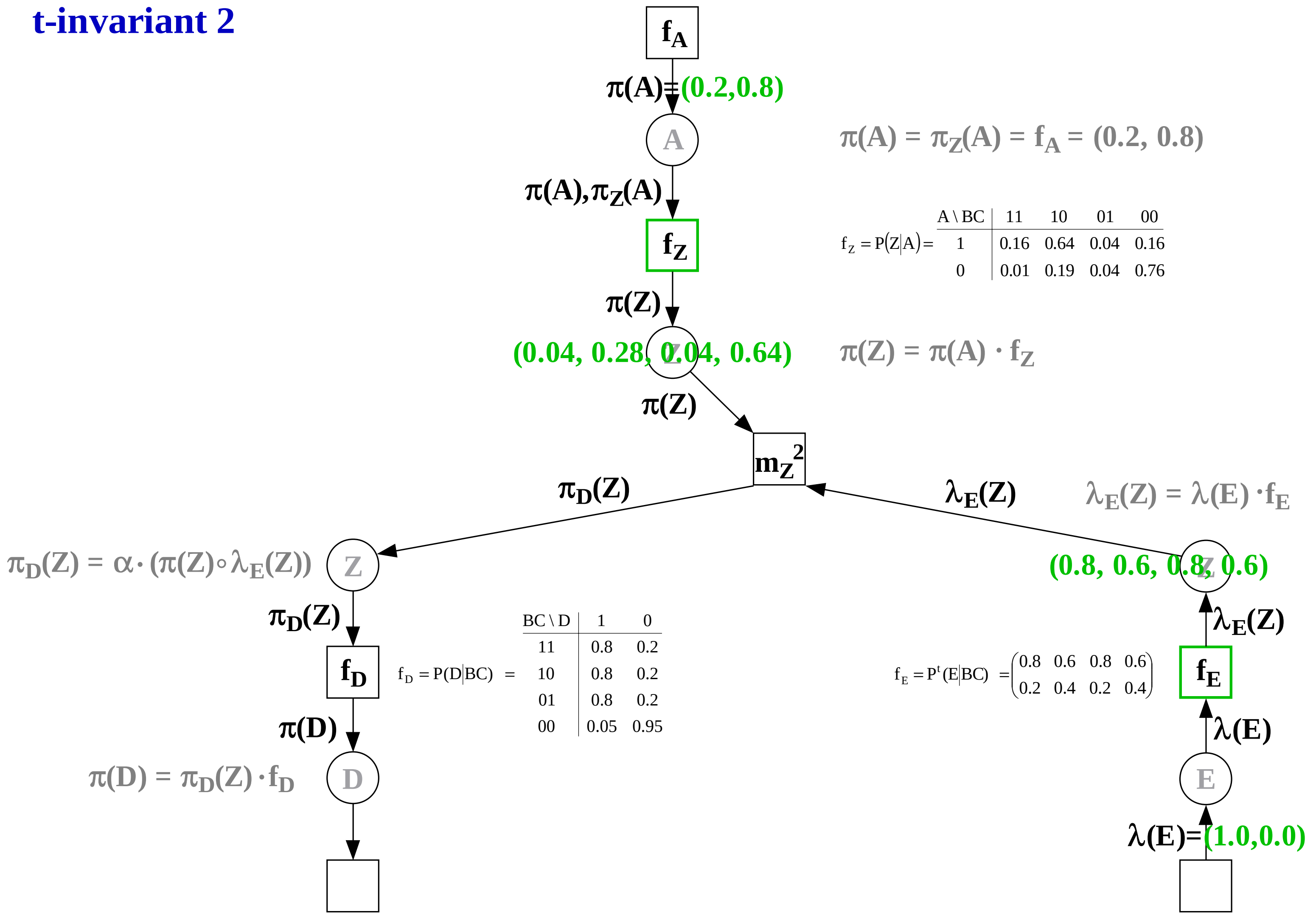
BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

$$f_E = P^t(E|BC) = \begin{pmatrix} 0.8 & 0.6 & 0.8 & 0.6 \\ 0.2 & 0.4 & 0.2 & 0.4 \end{pmatrix}$$

$\pi(D) = \pi_D(Z) \cdot f_D$

Clustering (Pearl)

t-invariant 2



$f_Z = P(Z|A) =$

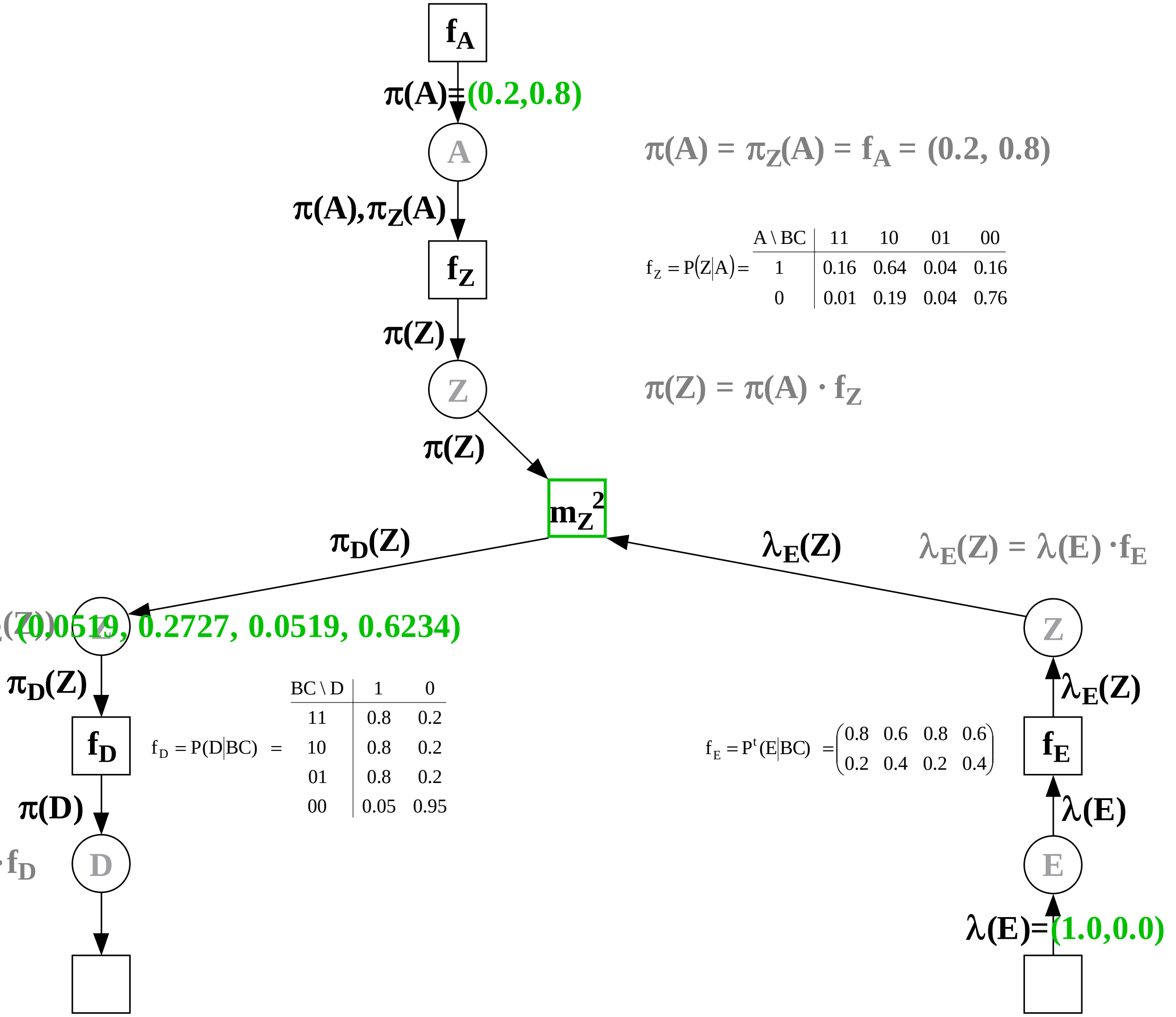
A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$f_D = P(D|BC) =$

BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

$$f_E = P^t(E|BC) = \begin{pmatrix} 0.8 & 0.6 & 0.8 & 0.6 \\ 0.2 & 0.4 & 0.2 & 0.4 \end{pmatrix}$$

t-invariant 2



$\pi(A) = \pi_Z(A) = f_A = (0.2,\ 0.8)$

$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$\pi(Z) = \pi(A) \cdot f_Z$

$\lambda_E(Z) = \lambda(E) \cdot f_E$

$\pi_D(Z) = \alpha \cdot (\pi(Z) \circ \lambda_E(Z))$

$f_D = P(D|BC) =$

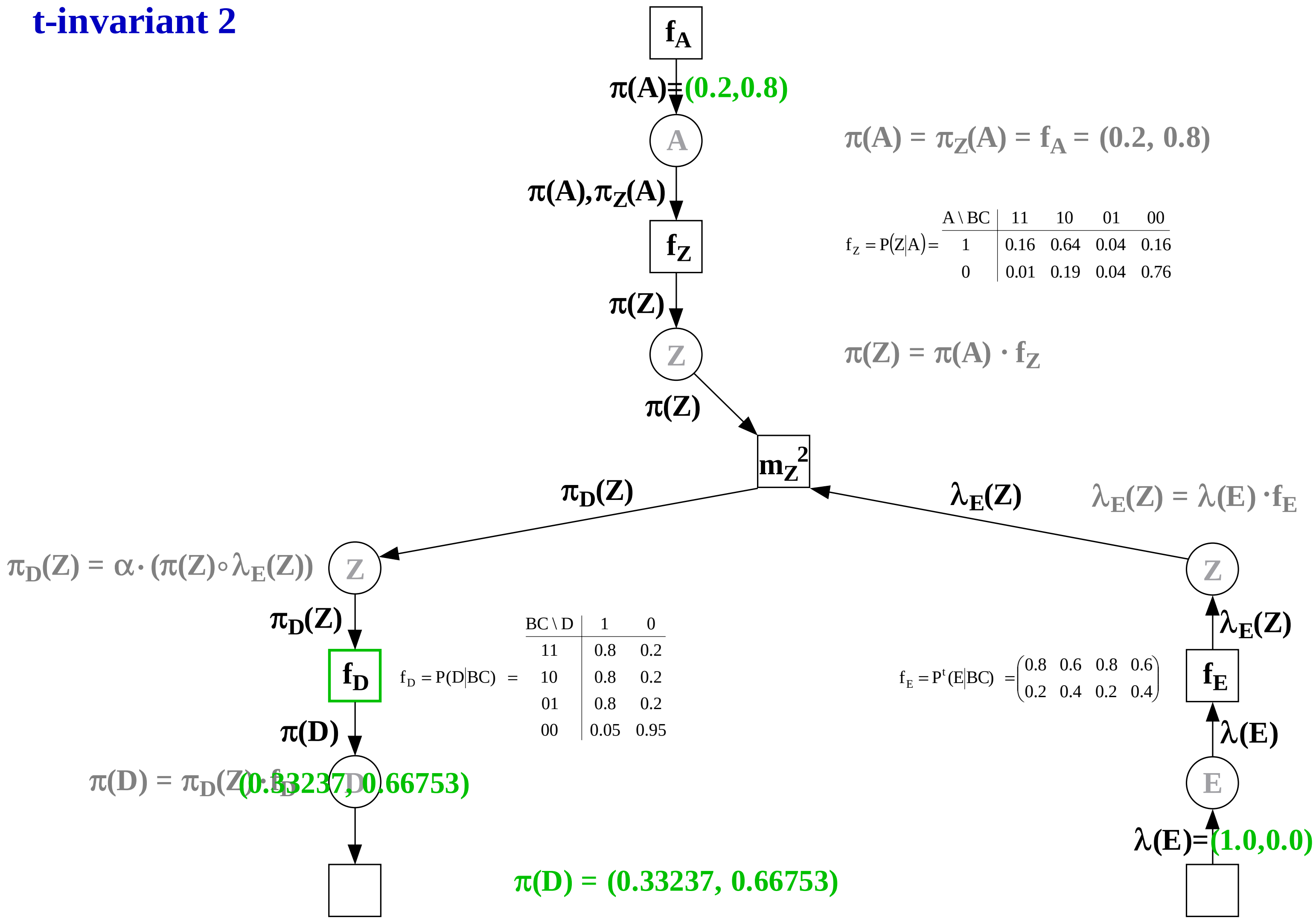
BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

$f_E = P^t(E|BC) = \begin{pmatrix} 0.8 & 0.6 & 0.8 & 0.6 \\ 0.2 & 0.4 & 0.2 & 0.4 \end{pmatrix}$

$\pi(D) = \pi_D(Z) \cdot f_D$

Clustering (Pearl)

t-invariant 2



$f_Z = P(Z|A) =$

A \ BC	11	10	01	00
1	0.16	0.64	0.04	0.16
0	0.01	0.19	0.04	0.76

$f_D = P(D|BC) =$

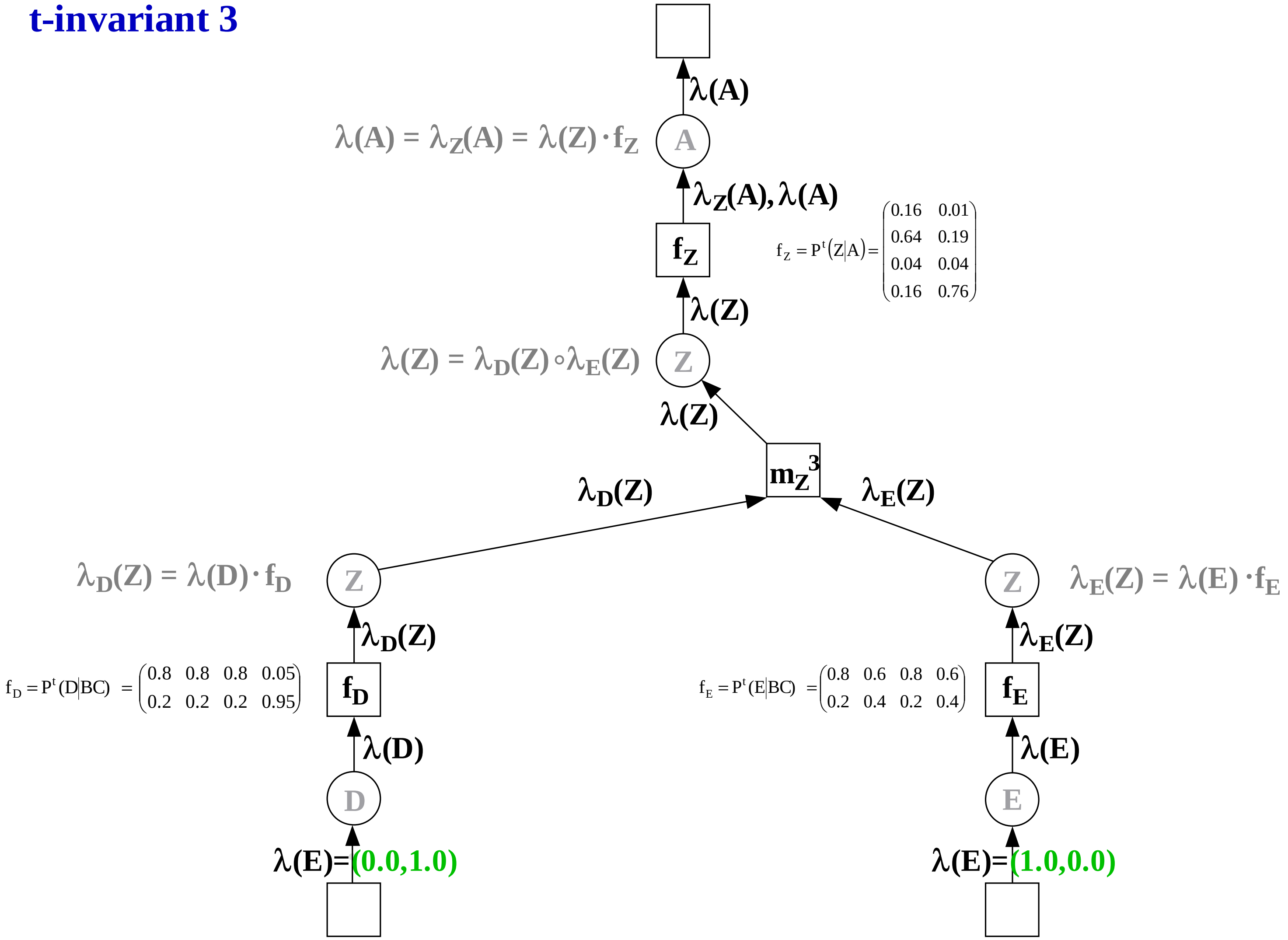
BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

$$f_E = P^t(E|BC) = \begin{pmatrix} 0.8 & 0.6 & 0.8 & 0.6 \\ 0.2 & 0.4 & 0.2 & 0.4 \end{pmatrix}$$

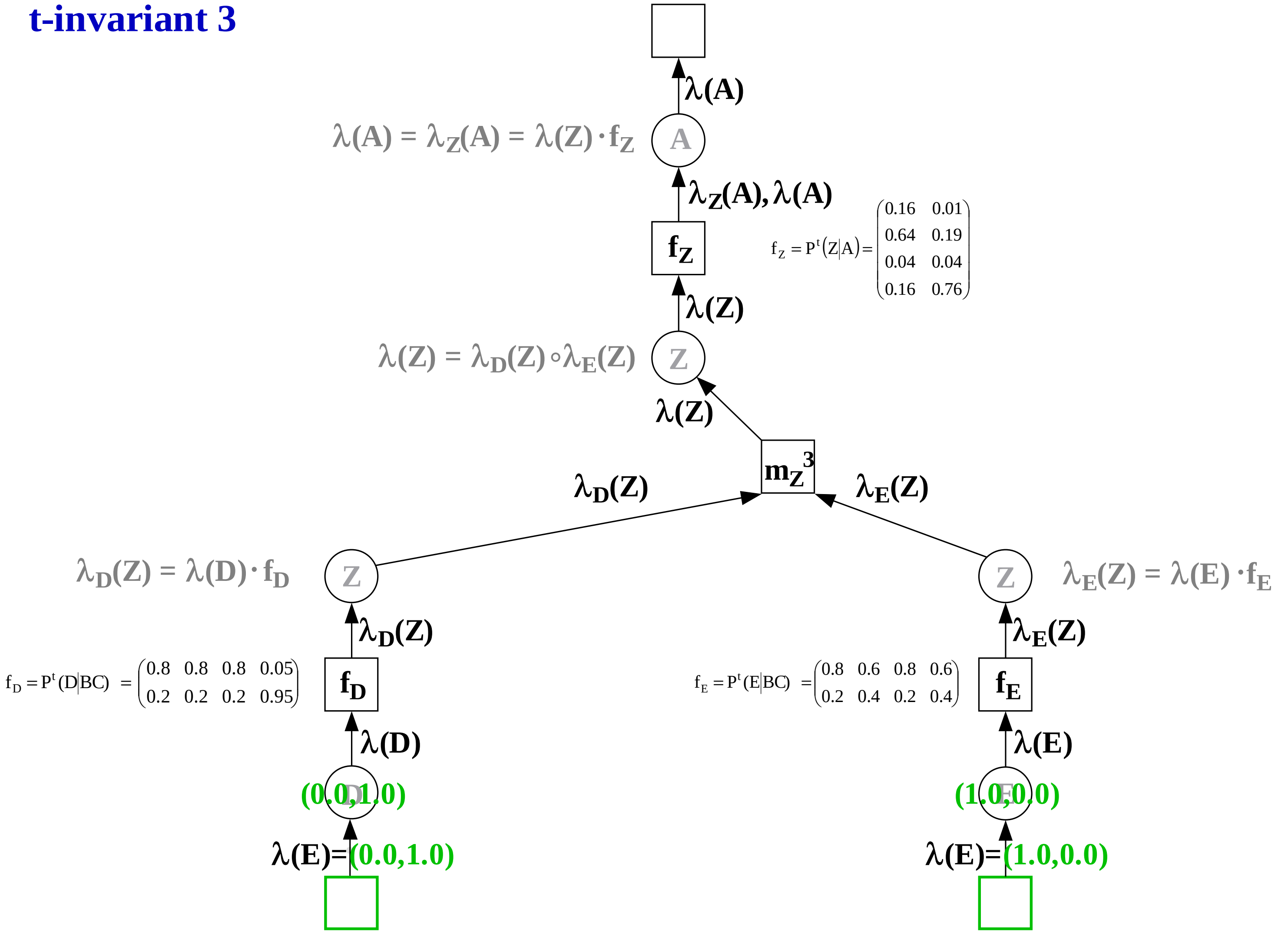
$\pi(D) = (0.33237, 0.66753)$

Clustering (Pearl)

t-invariant 3

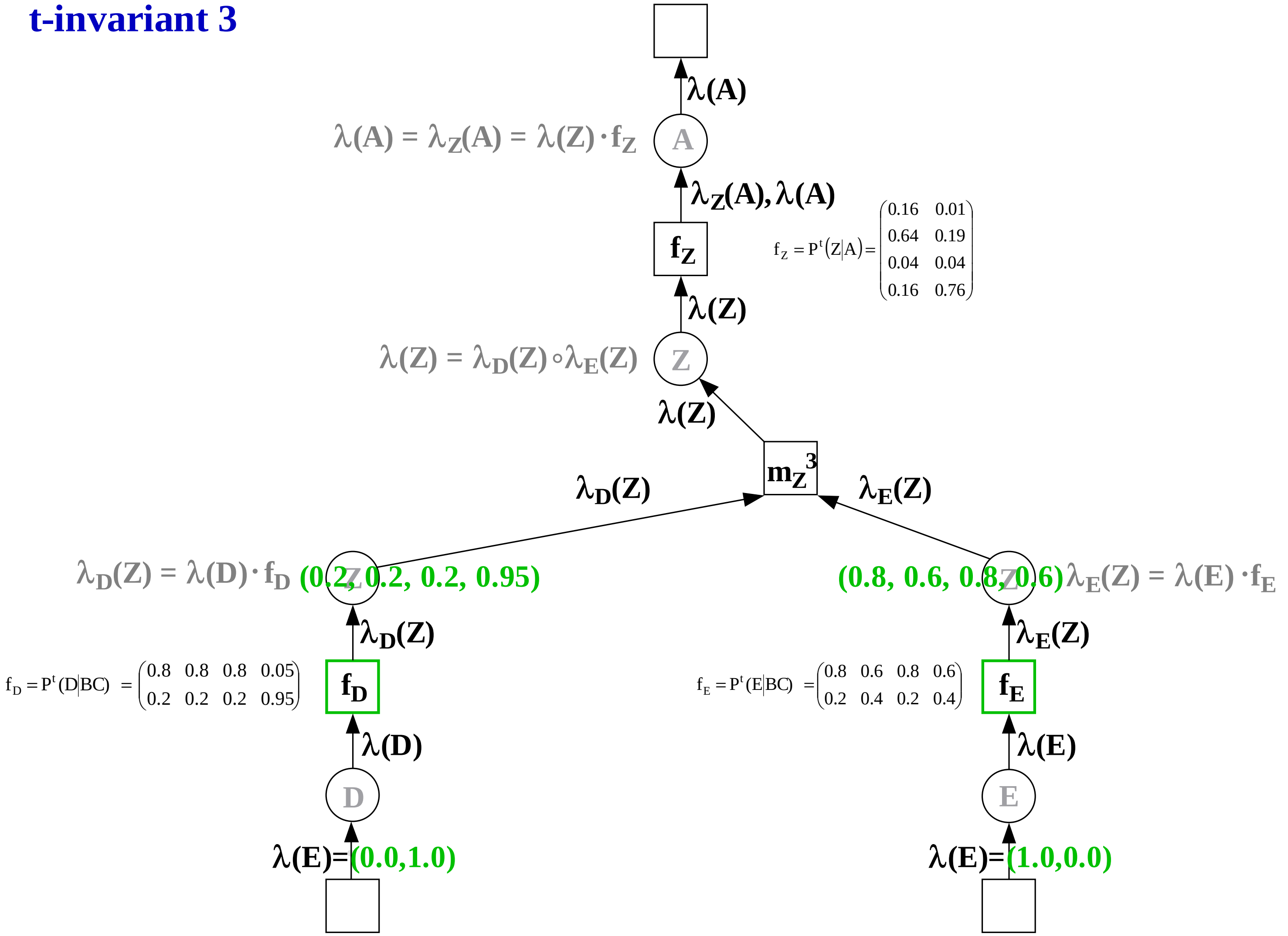


t-invariant 3



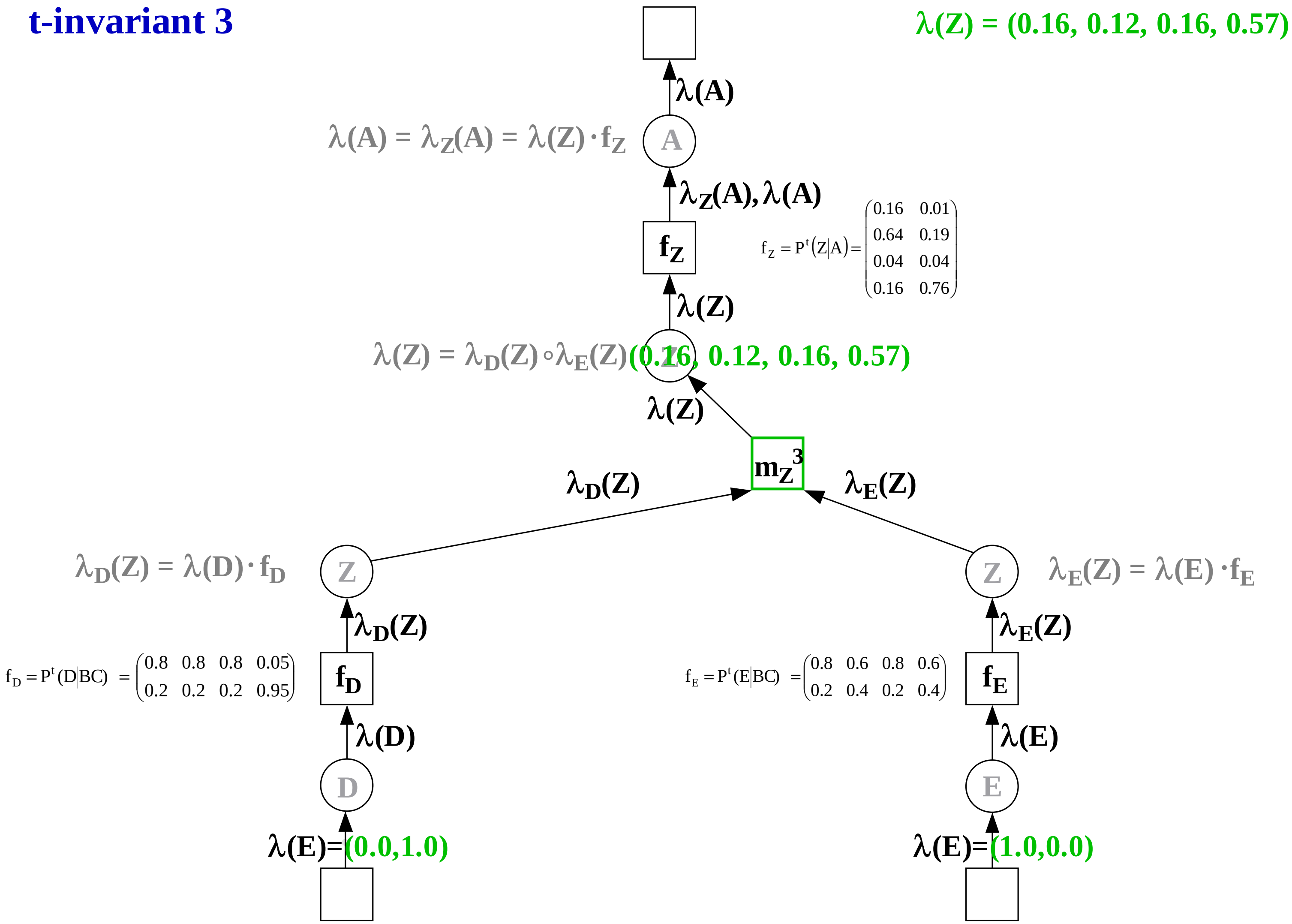
Clustering (Pearl)

t-invariant 3



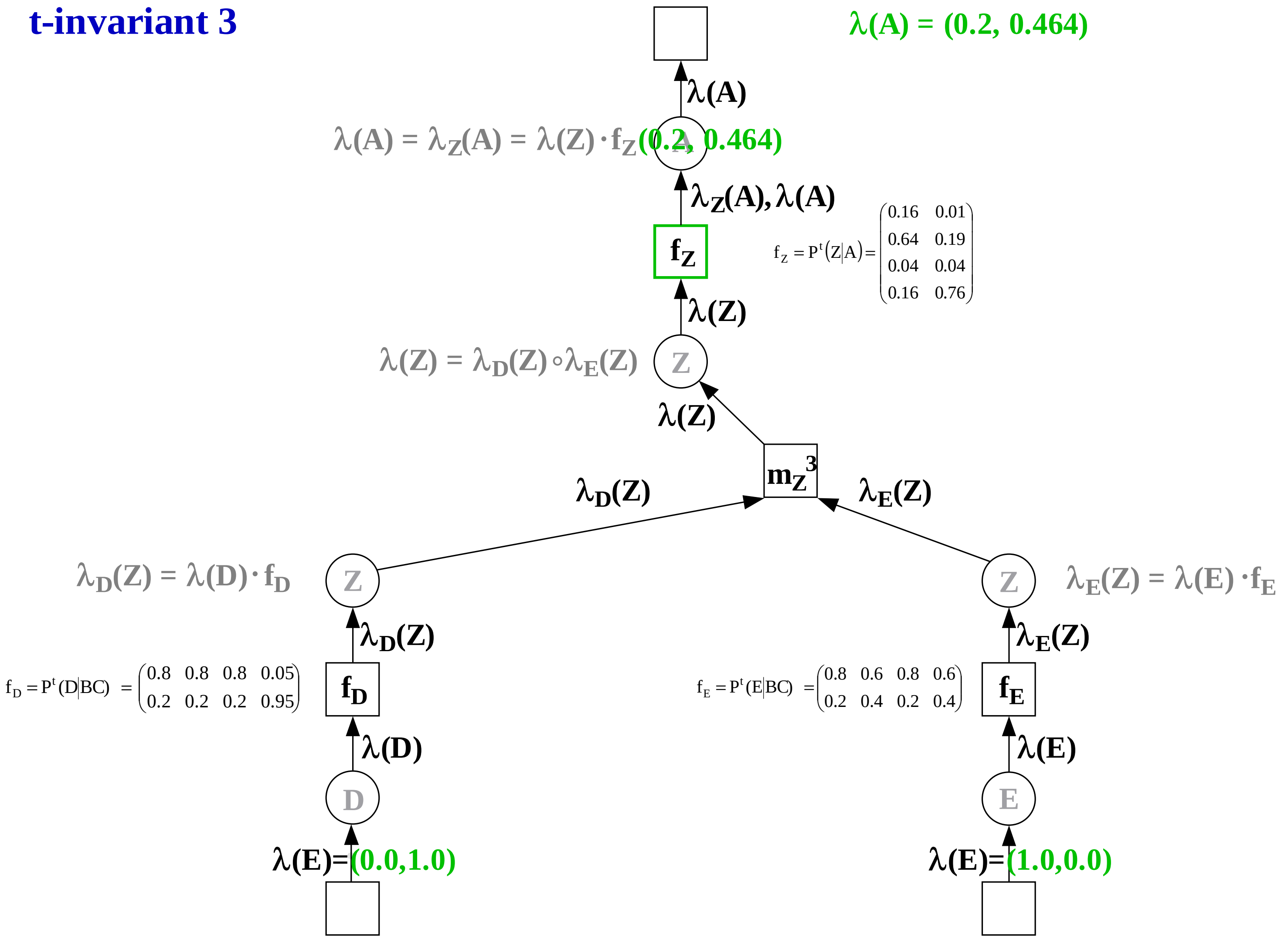
Clustering (Pearl)

t-invariant 3



t-invariant 3

$\lambda(A) = (0.2, 0.464)$



Probabilities:

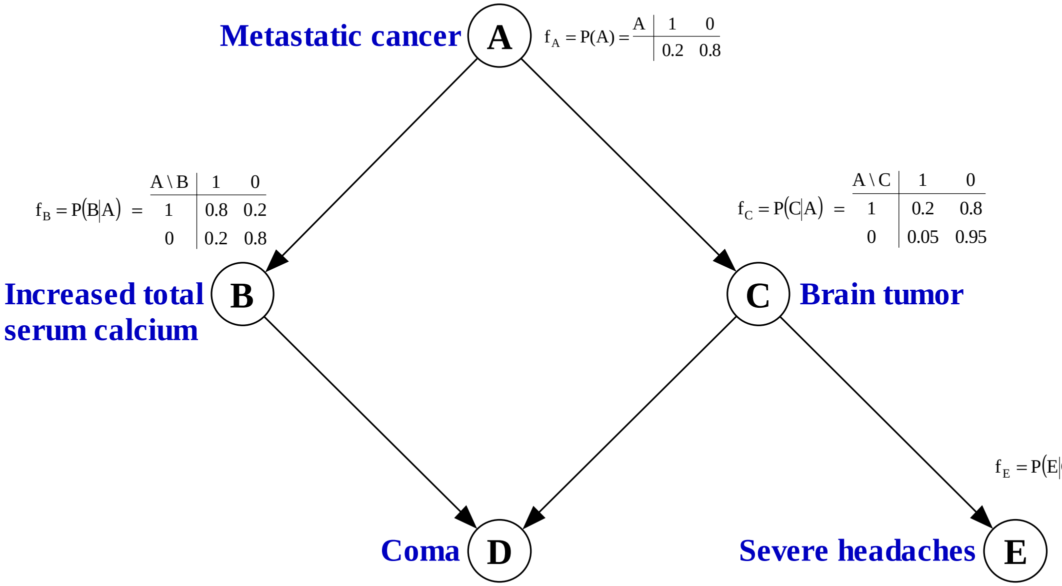
$P(A) = BEL(A) = \alpha\, \pi(A) \circ \lambda(A) = \alpha\, \underline{(0.2, 0.8) \circ (0.2, 0.464)} = (0.097, 0.903)$

$P(Z) = BEL(Z) = \alpha\, \pi(Z) \circ \lambda(Z) = \alpha\, \underline{(0.04, 0.28, 0.04, 0.64) \circ (0.16, 0.12, 0.16, 0.57)}$
$= (0.0156, 0.0817, 0.0156, 0.887)$

$P(B) = BEL(B) = (0.0156 + 0.0817, 0.0156 + 0.887) = \boxed{(0.097, 0.903)}$

$P(C) = BEL(C) = (0.0156 + 0.0156, 0.0817 + 0.887) = \boxed{(0.031, 0.969)}$

Coping with Loops (Pearl)



$f_A = P(A) =$

A	1	0
	0.2	0.8

$f_B = P(B|A) =$

A \ B	1	0
1	0.8	0.2
0	0.2	0.8

$f_C = P(C|A) =$

A \ C	1	0
1	0.2	0.8
0	0.05	0.95

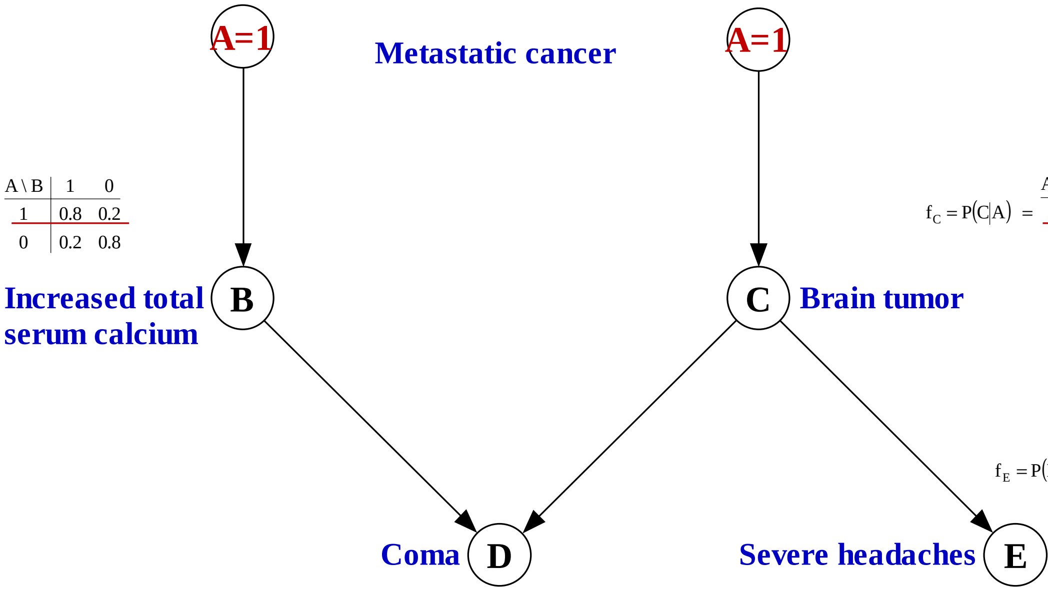
$f_E = P(E|C) =$

C \ E	1	0
1	0.8	0.2
0	0.6	0.4

$f_D = P(D|BC) =$

BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

Conditioning (Pearl)



$f_B = P(B|A) =$

A \ B	1	0
1	0.8	0.2
0	0.2	0.8

$f_C = P(C|A) =$

A \ C	1	0
1	0.2	0.8
0	0.05	0.95

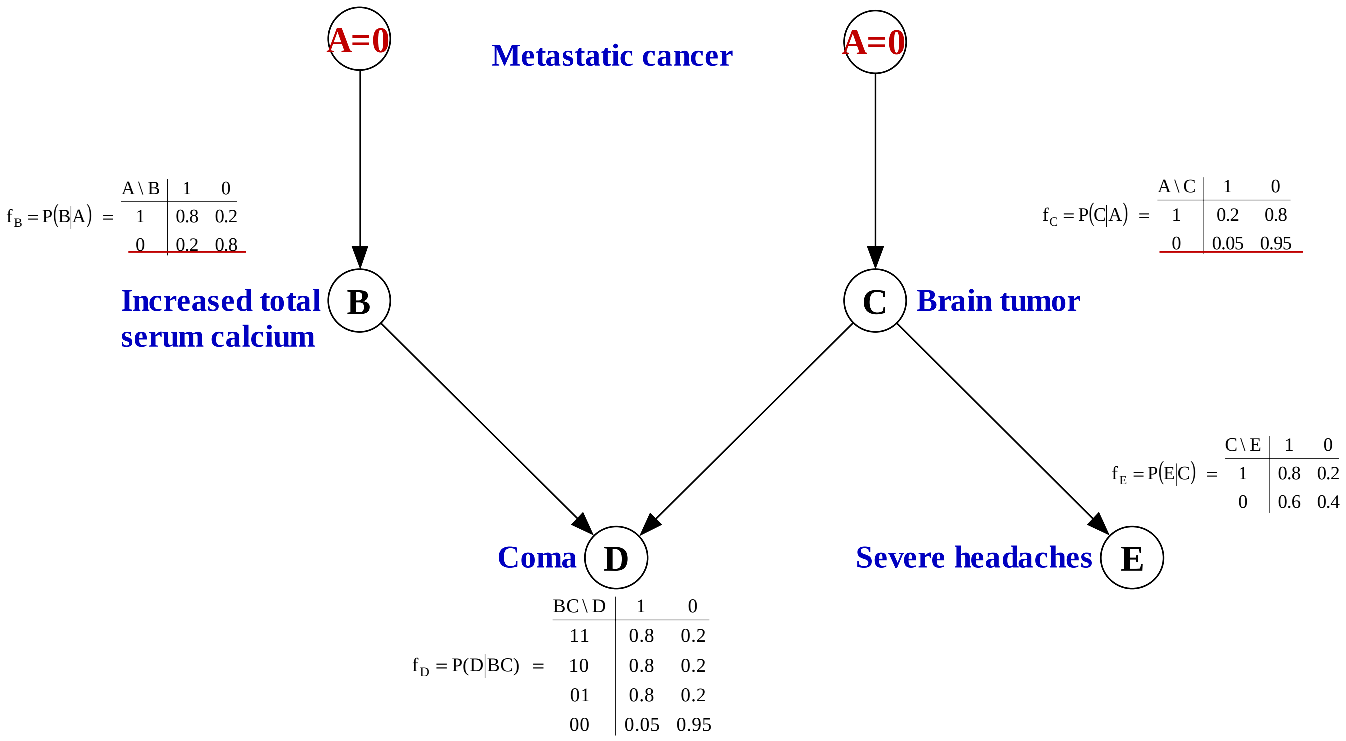
$f_E = P(E|C) =$

C \ E	1	0
1	0.8	0.2
0	0.6	0.4

$f_D = P(D|BC) =$

BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

Conditioning (Pearl)



$f_B = P(B|A) =$

A \ B	1	0
1	0.8	0.2
0	0.2	0.8

$f_C = P(C|A) =$

A \ C	1	0
1	0.2	0.8
0	0.05	0.95

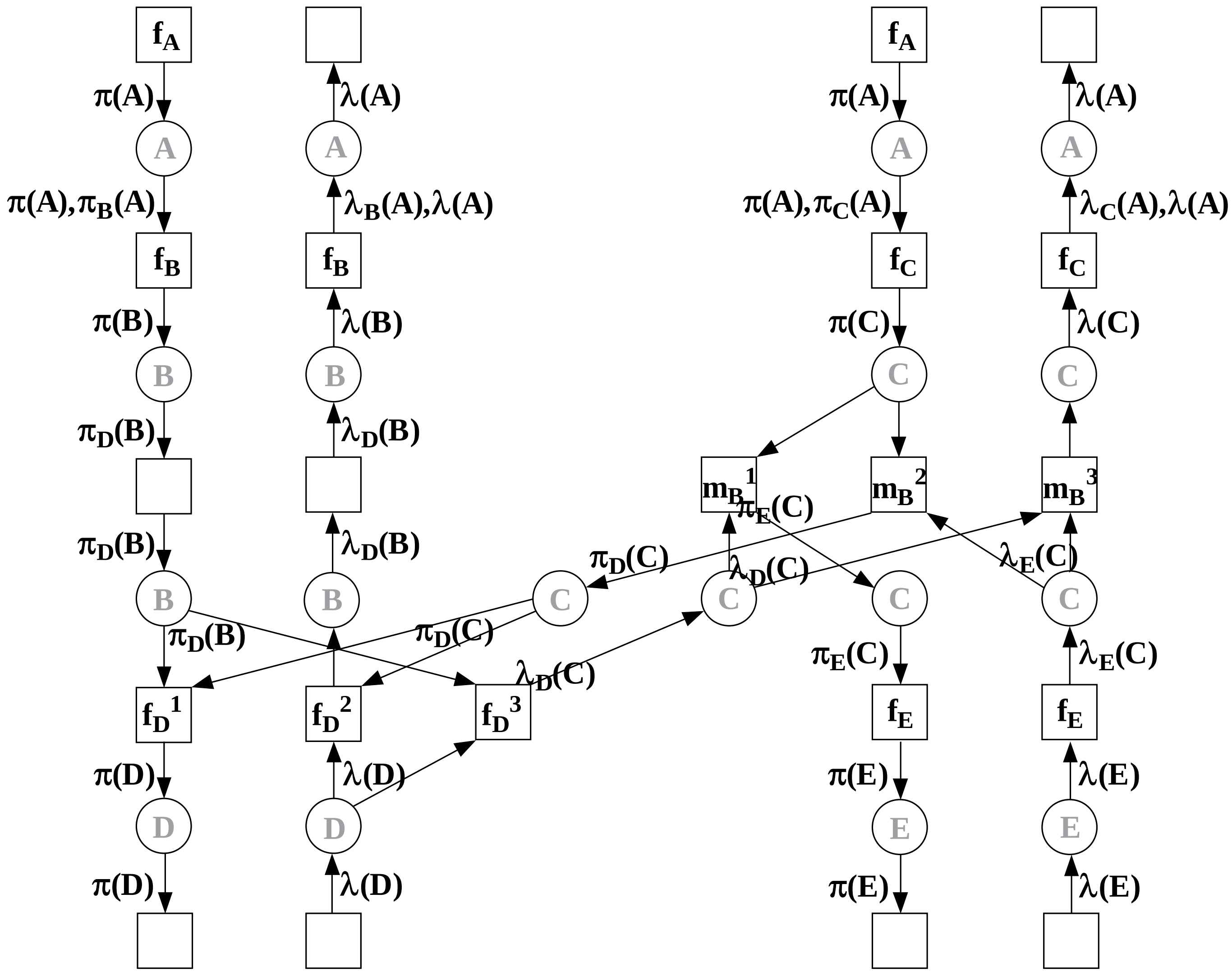
$f_E = P(E|C) =$

C \ E	1	0
1	0.8	0.2
0	0.6	0.4

$f_D = P(D|BC) =$

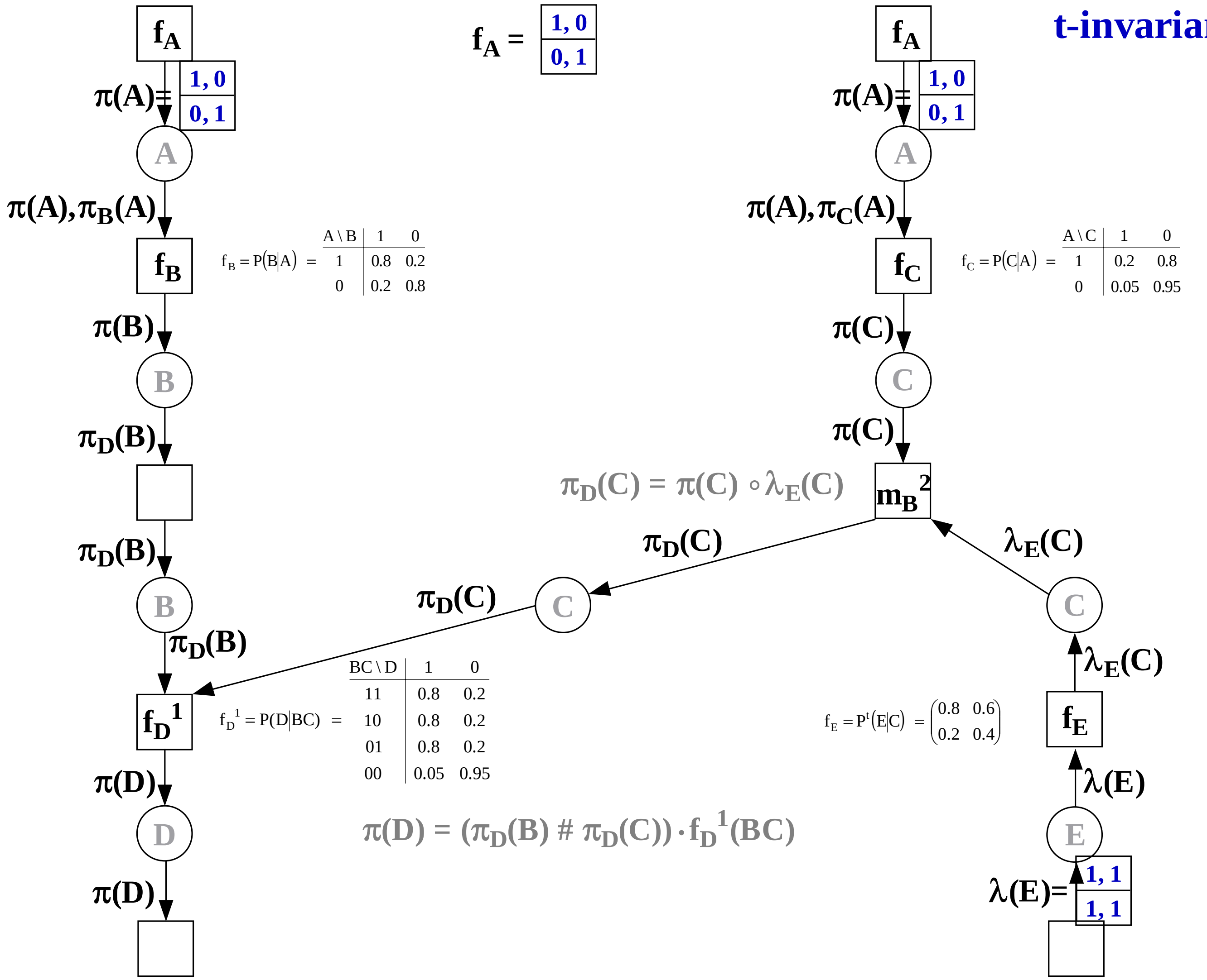
BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

Conditioning (Pearl)



Conditioning (Pearl)

t-invariant 1



$f_B = P(B|A) =$

A \ B	1	0
1	0.8	0.2
0	0.2	0.8

$f_C = P(C|A) =$

A \ C	1	0
1	0.2	0.8
0	0.05	0.95

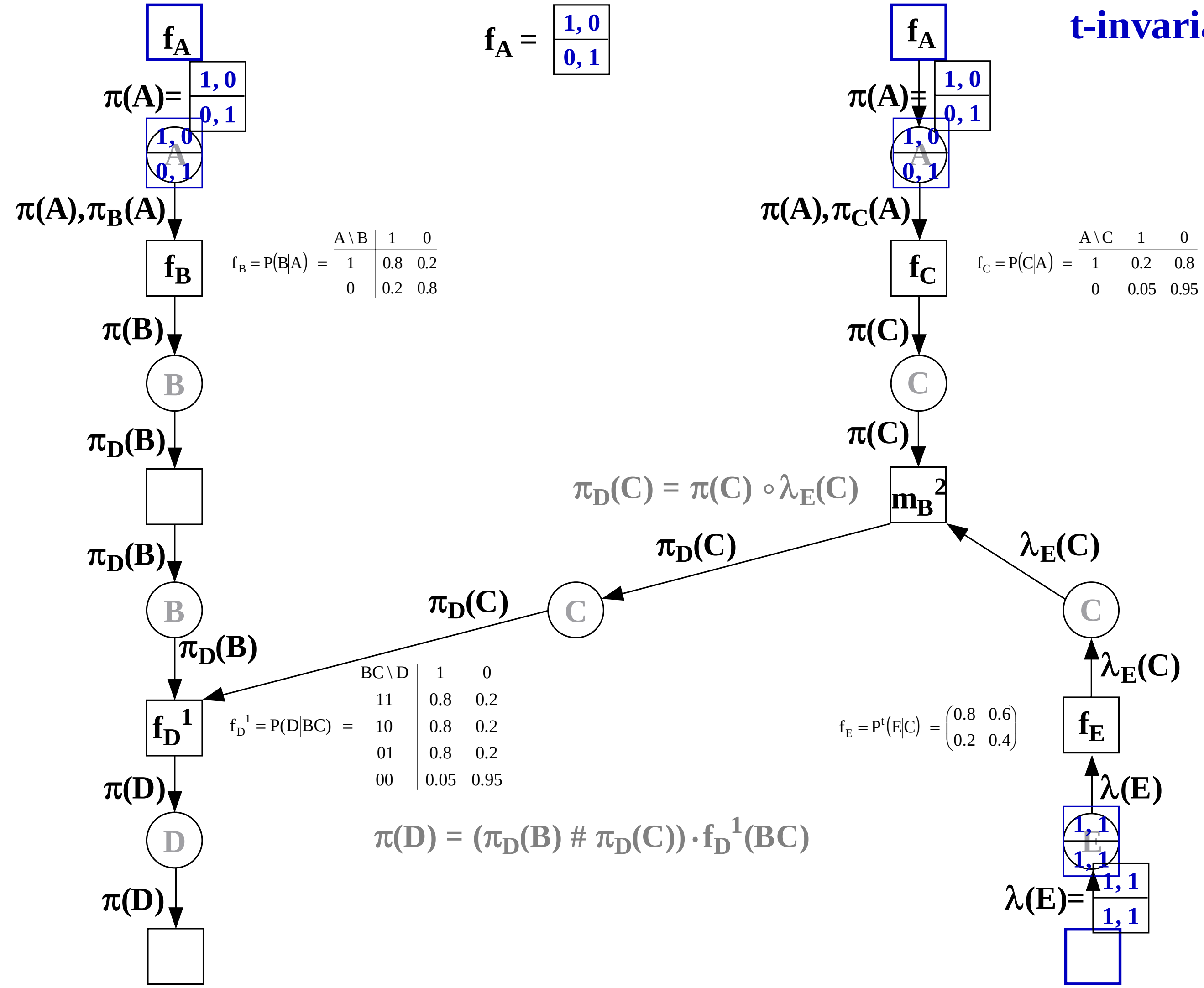
$f_D^1 = P(D|BC) =$

BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

$$f_E = P^t(E|C) = \begin{pmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{pmatrix}$$

Conditioning (Pearl)

t-invariant 1



$f_B = P(B|A) =$

A \ B	1	0
1	0.8	0.2
0	0.2	0.8

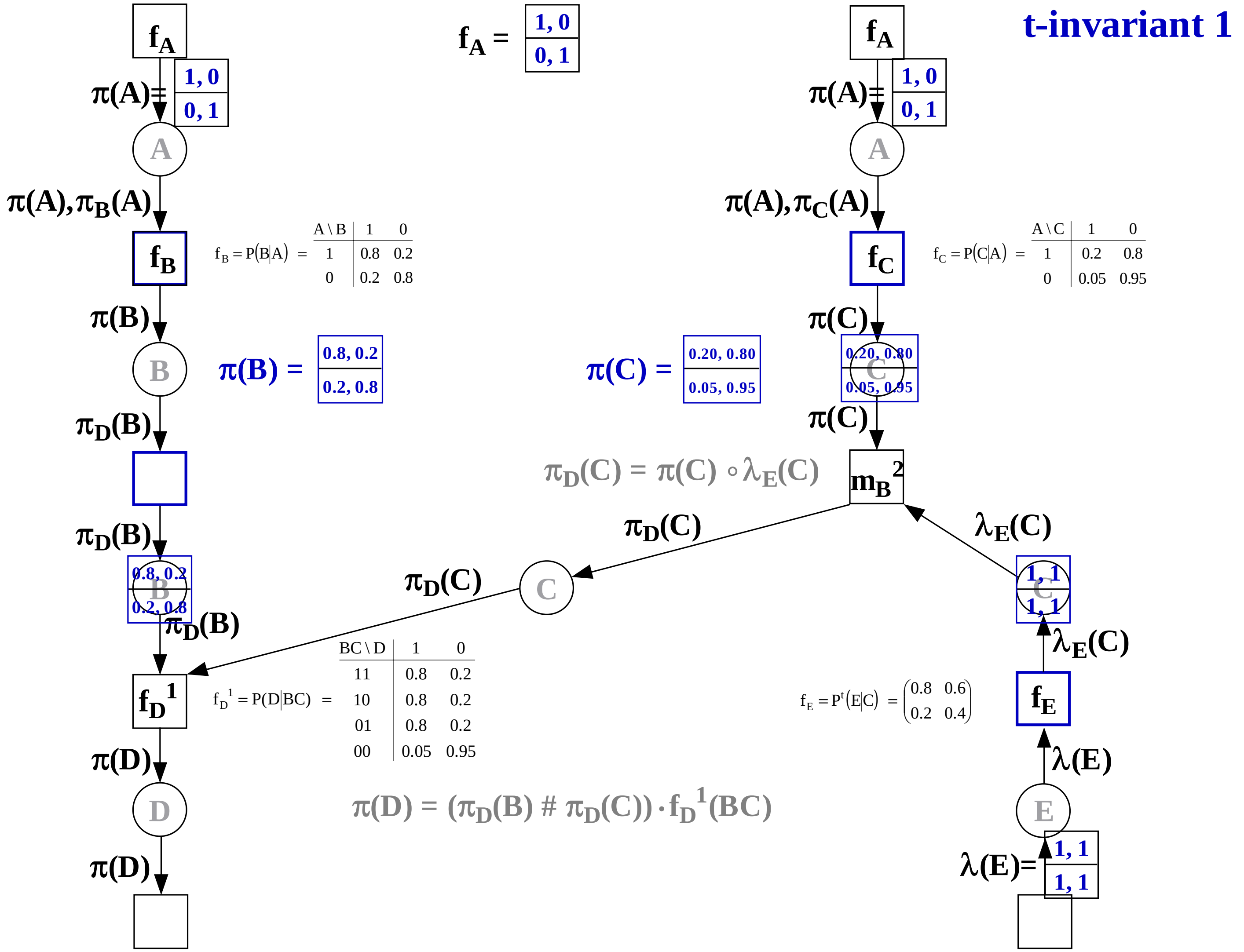
$f_C = P(C|A) =$

A \ C	1	0
1	0.2	0.8
0	0.05	0.95

$f_D^1 = P(D|BC) =$

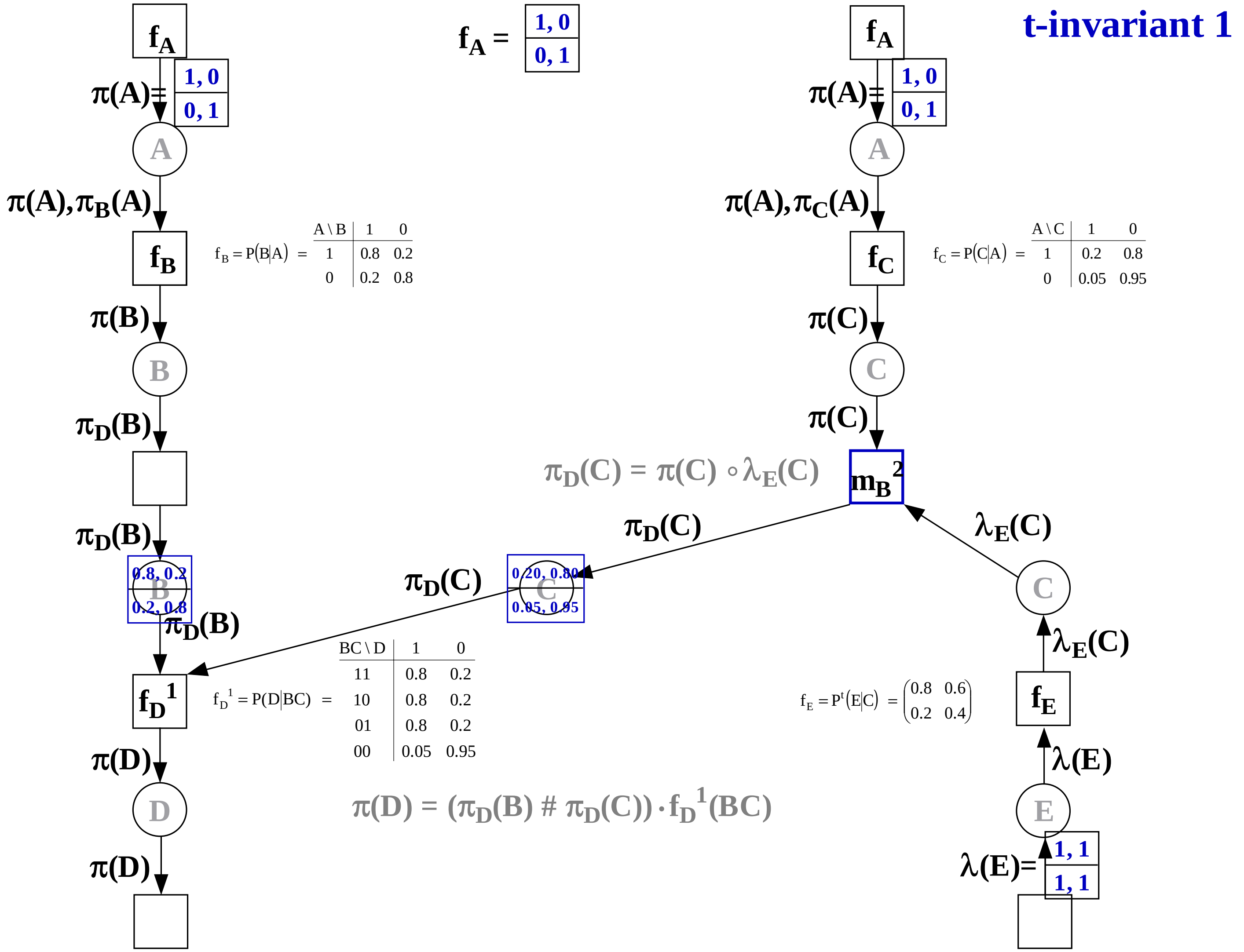
BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

$$f_E = P^t(E|C) = \begin{pmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{pmatrix}$$

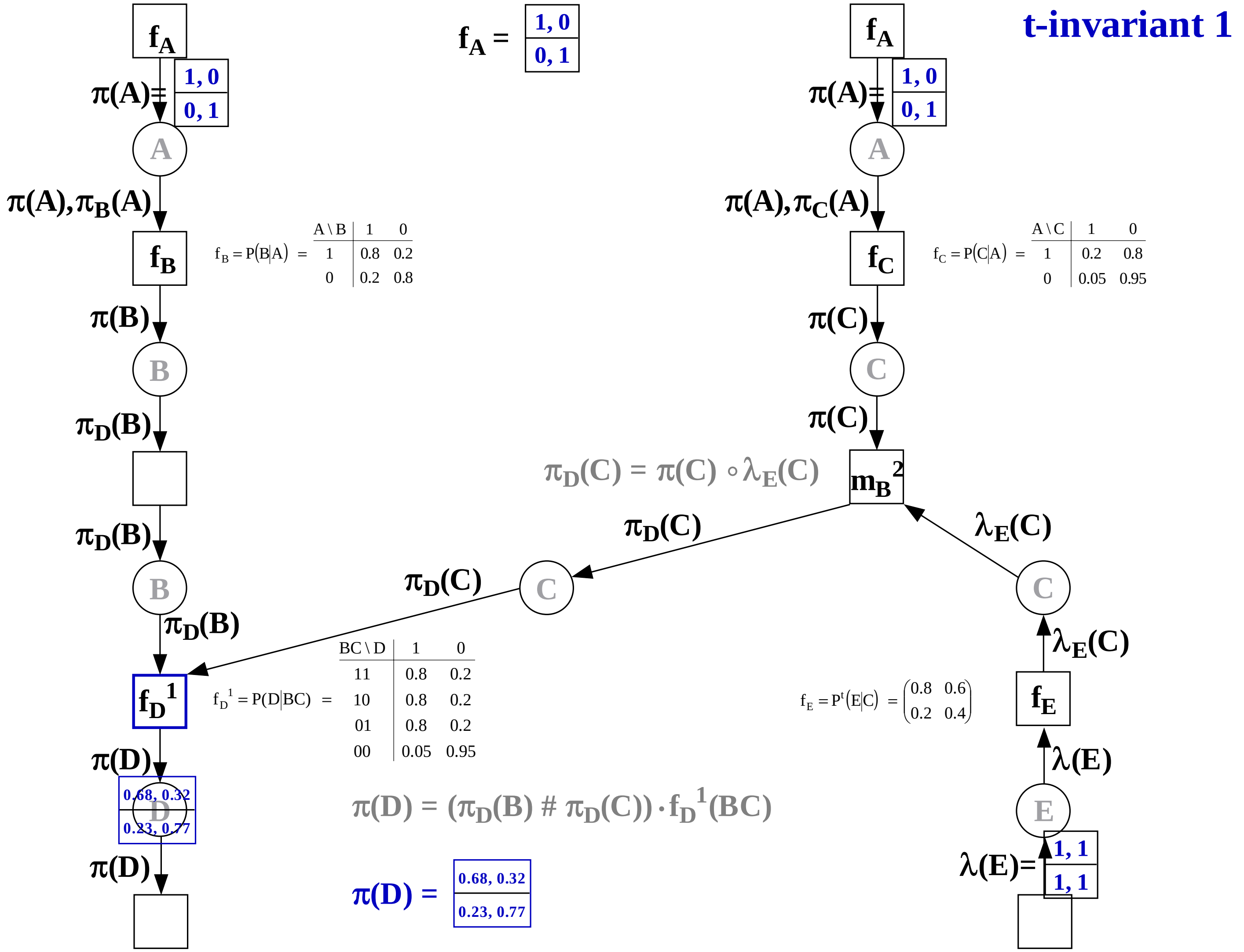


t-invariant 1
fA
π(A)=
1, 0
0, 1
A
π(A),πB(A)
fB
fB = P(B|A) =
A \ B 1 0
1 0.8 0.2
0 0.2 0.8
π(B)
B
π(B) =
0.8, 0.2
0.2, 0.8
πD(B)
0.8, 0.2
0.2, 0.8
fD1
fD1 = P(D|BC) =
BC \ D 1 0
11 0.8 0.2
10 0.8 0.2
01 0.8 0.2
00 0.05 0.95
π(D)
D
π(D) = (πD(B) # πD(C)) · fD1(BC)
fA =
fC
fC = P(C|A) =
A \ C 1 0
1 0.2 0.8
0 0.05 0.95
π(A),πC(A)
π(C)
π(C) =
0.20, 0.80
0.05, 0.95
C
mB2
πD(C) = π(C) ∘ λE(C)
πD(C)
λE(C)
1, 1
fE
fE = Pt(E|C) =
0.8 0.6
0.2 0.4
λ(E)
E
λ(E)=

Conditioning (Pearl)

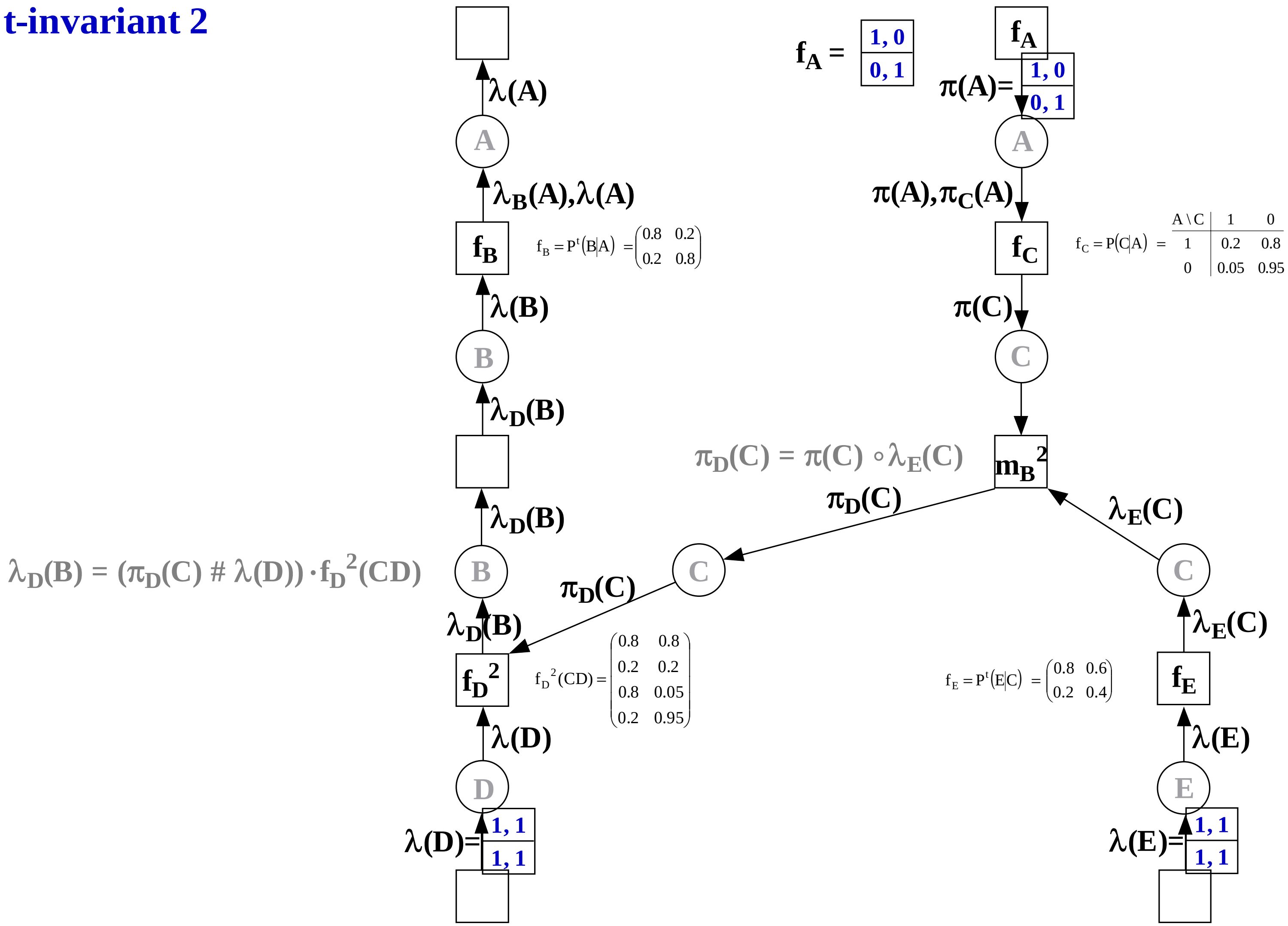


Conditioning (Pearl)



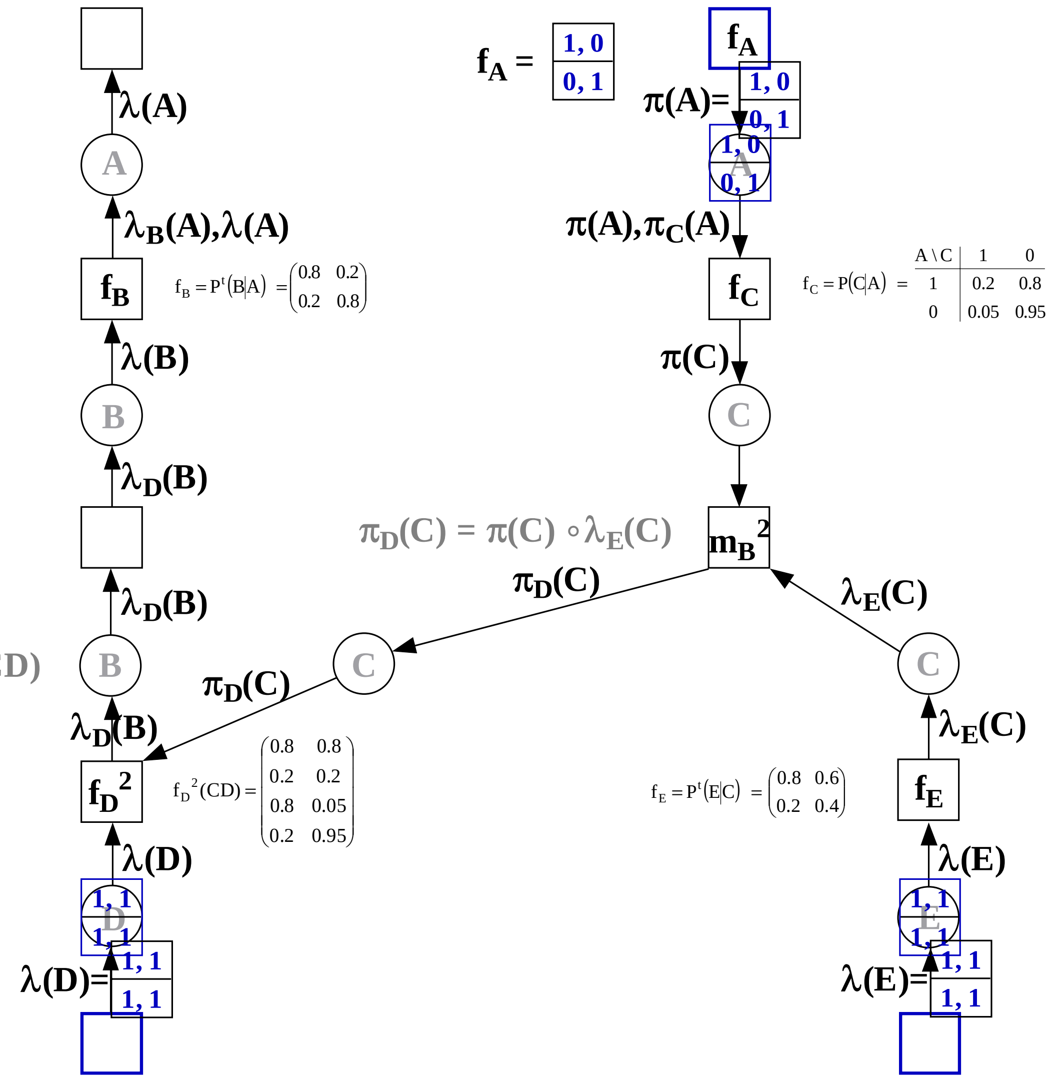
Conditioning (Pearl)

t-invariant 2



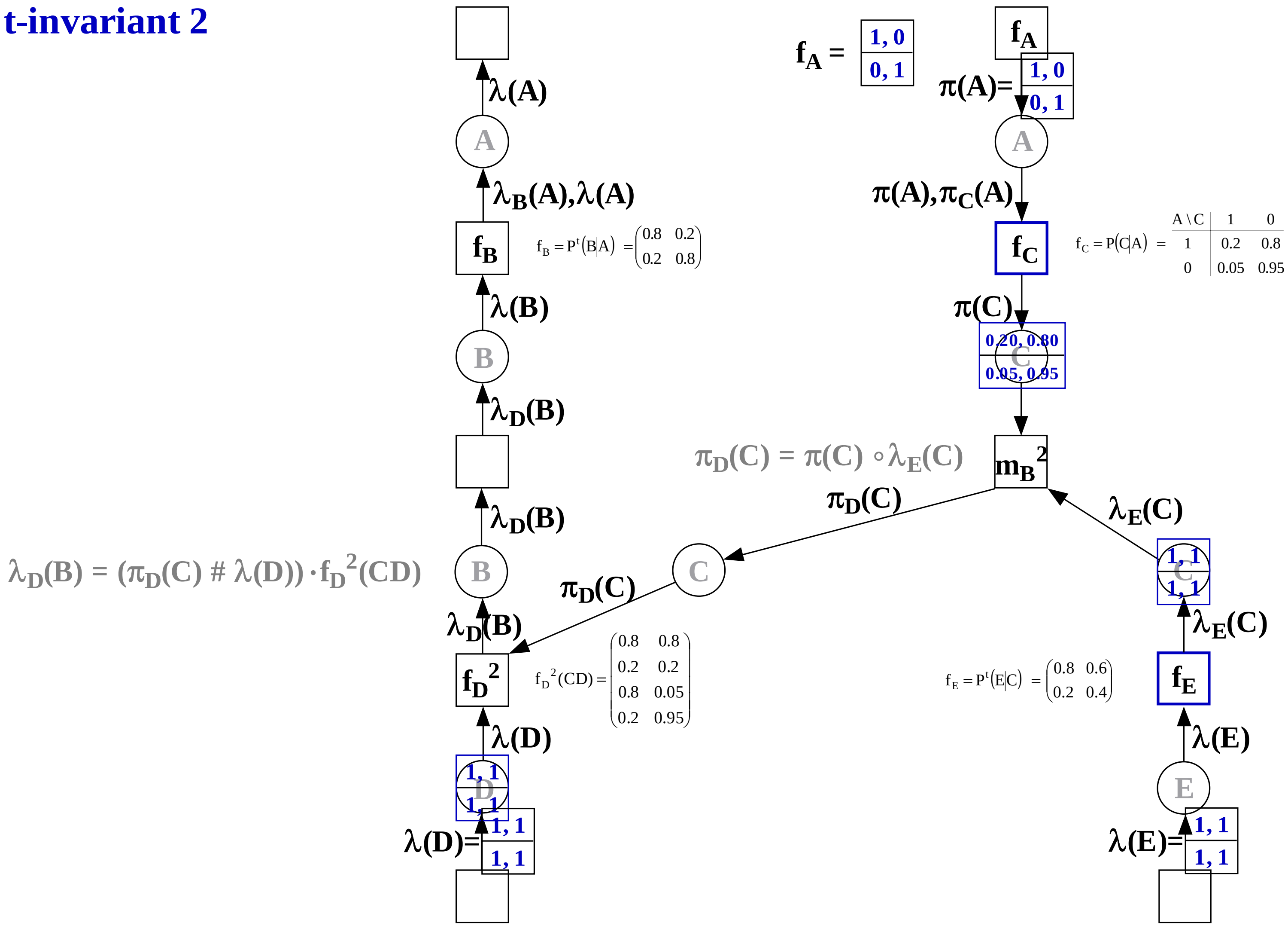
Conditioning (Pearl)

t-invariant 2



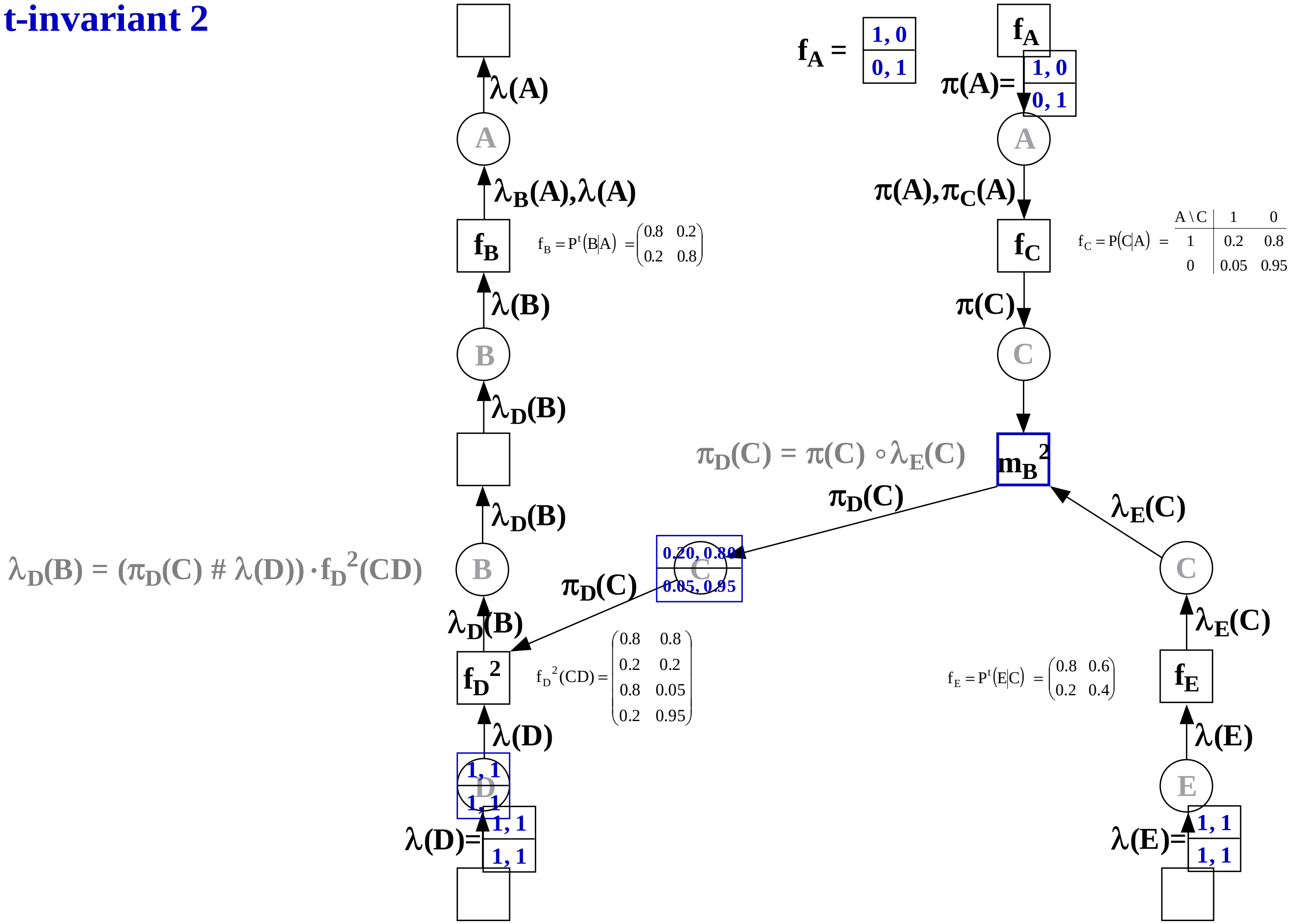
Conditioning (Pearl)

t-invariant 2

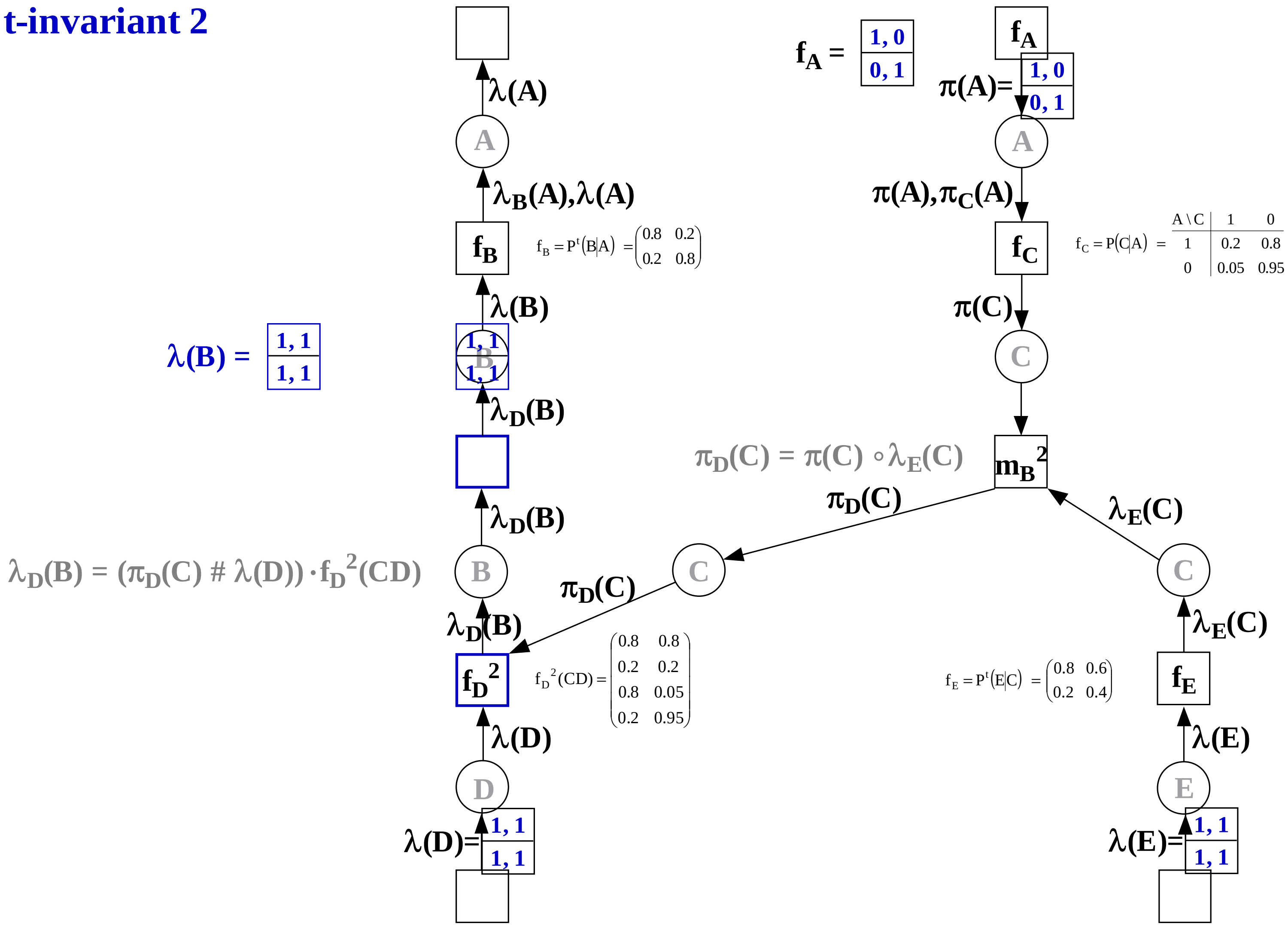


Conditioning (Pearl)

t-invariant 2



t-invariant 2

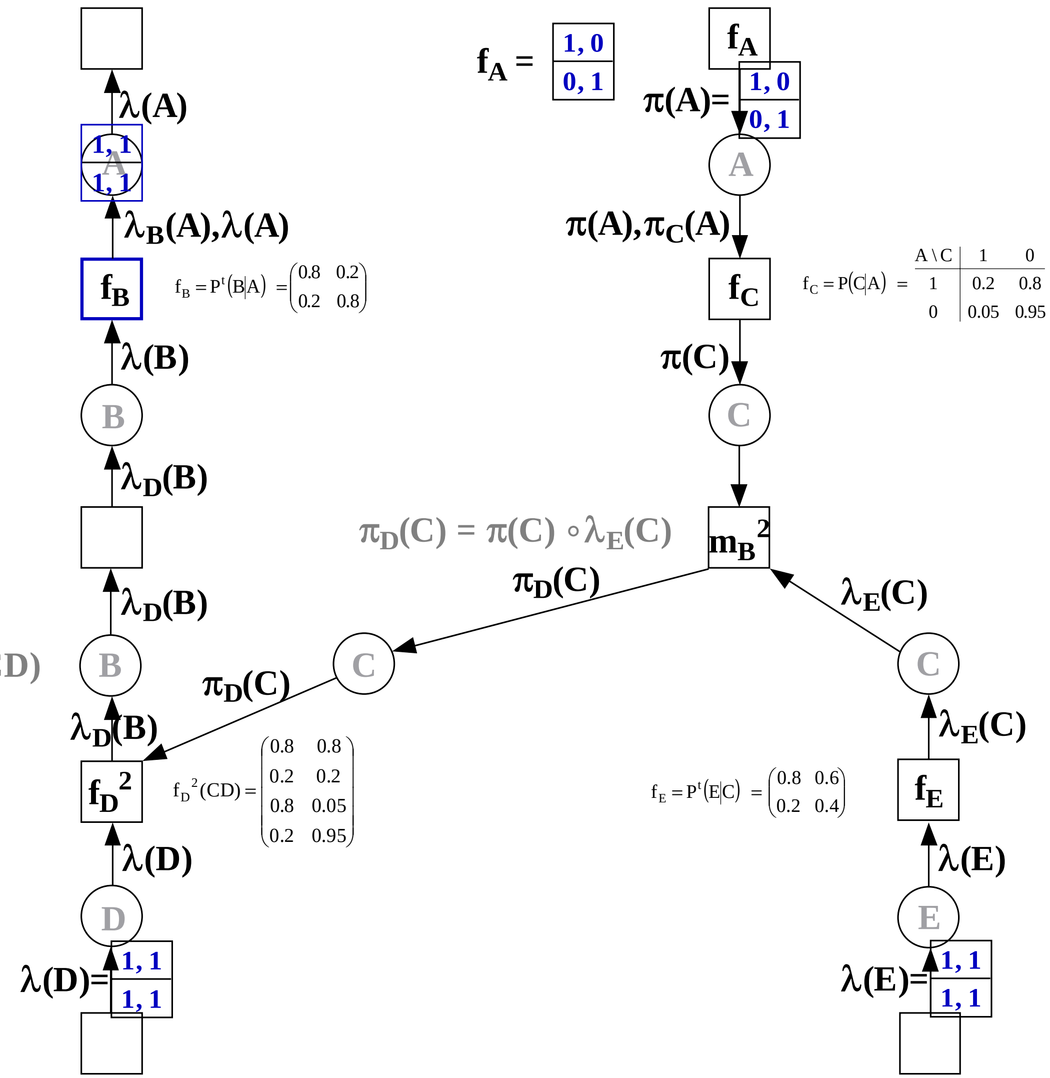


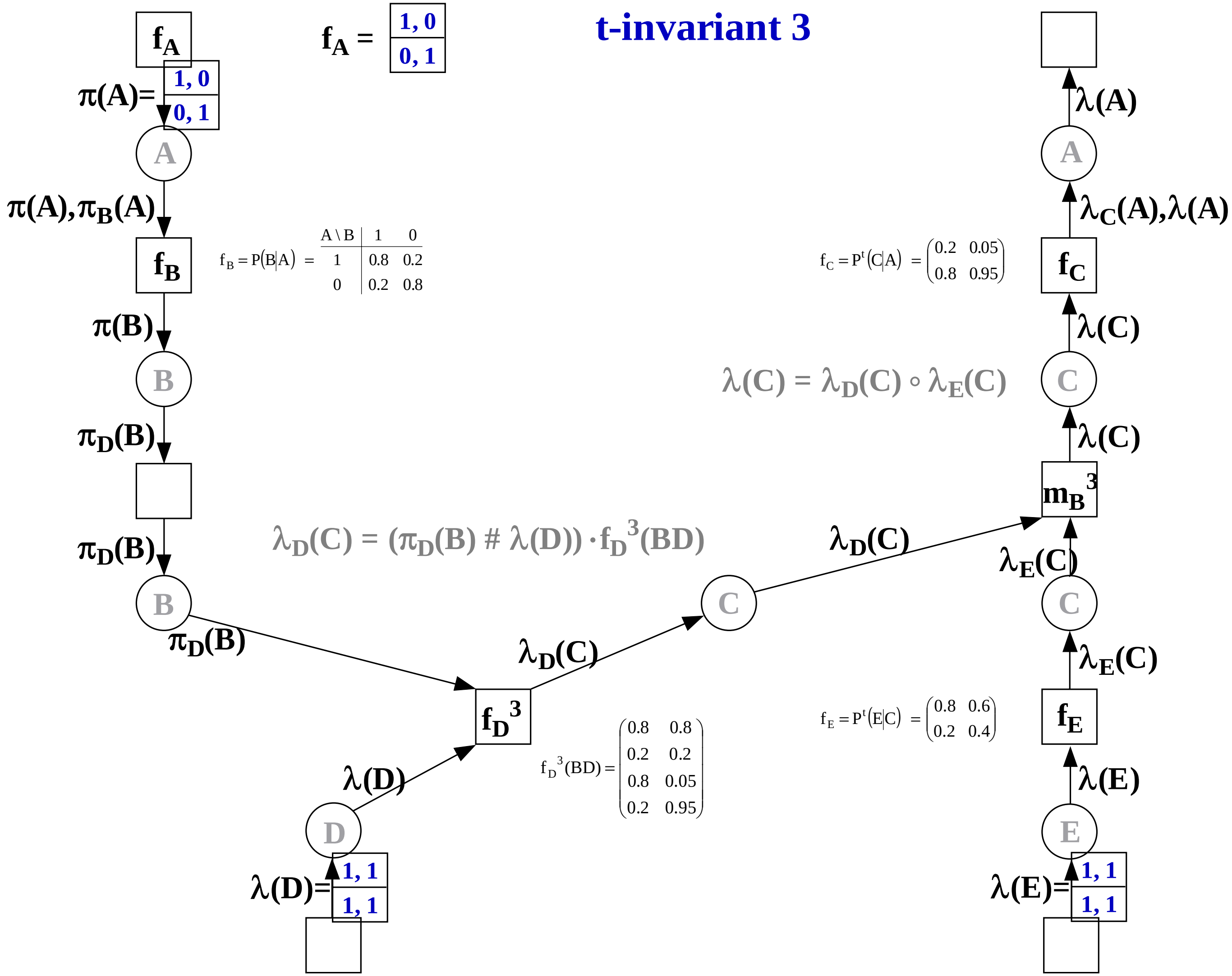
Conditioning (Pearl)

t-invariant 2

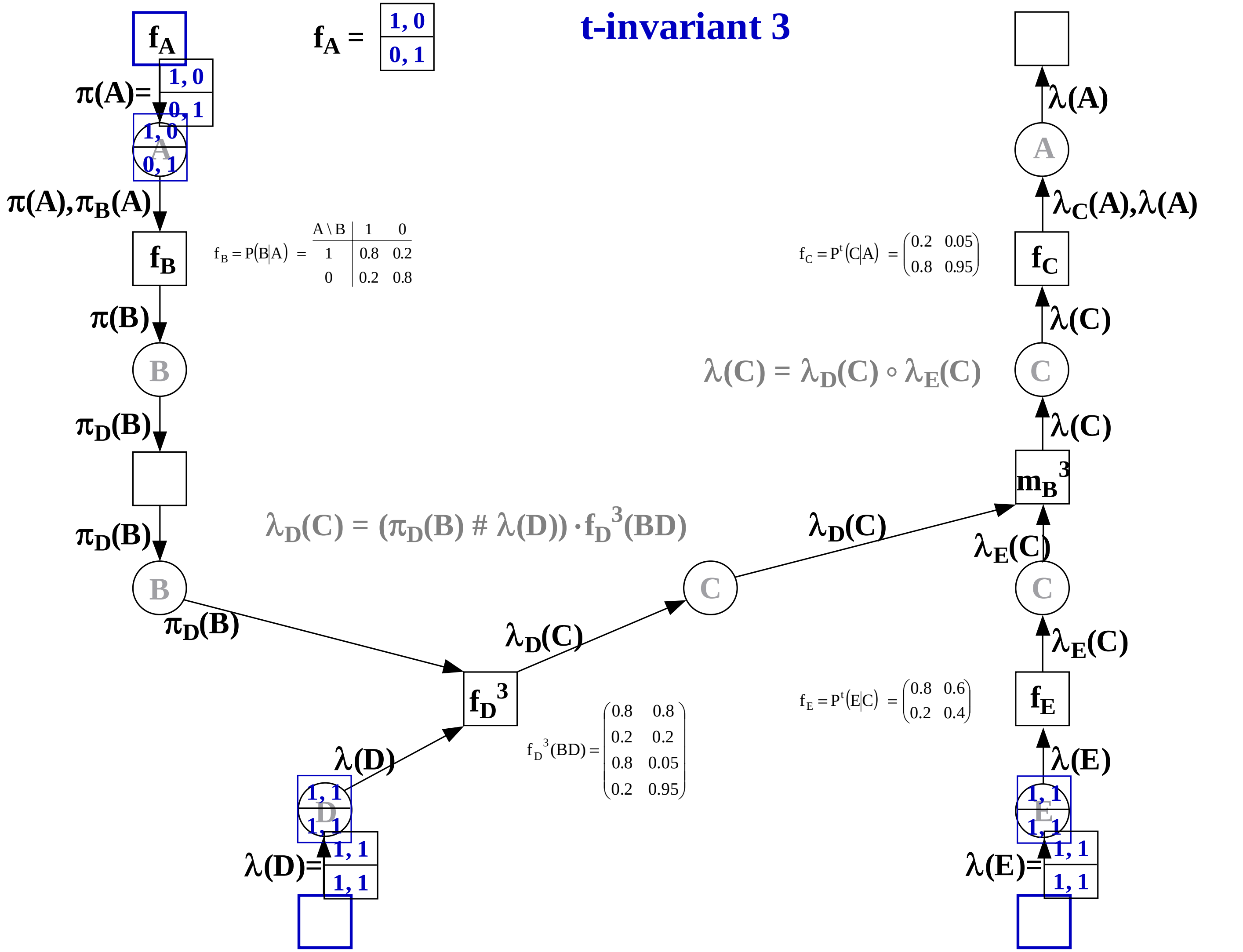
$\lambda(A) = \begin{array}{|c|}\hline 1, 1 \\ \hline 1, 1 \\ \hline\end{array}$

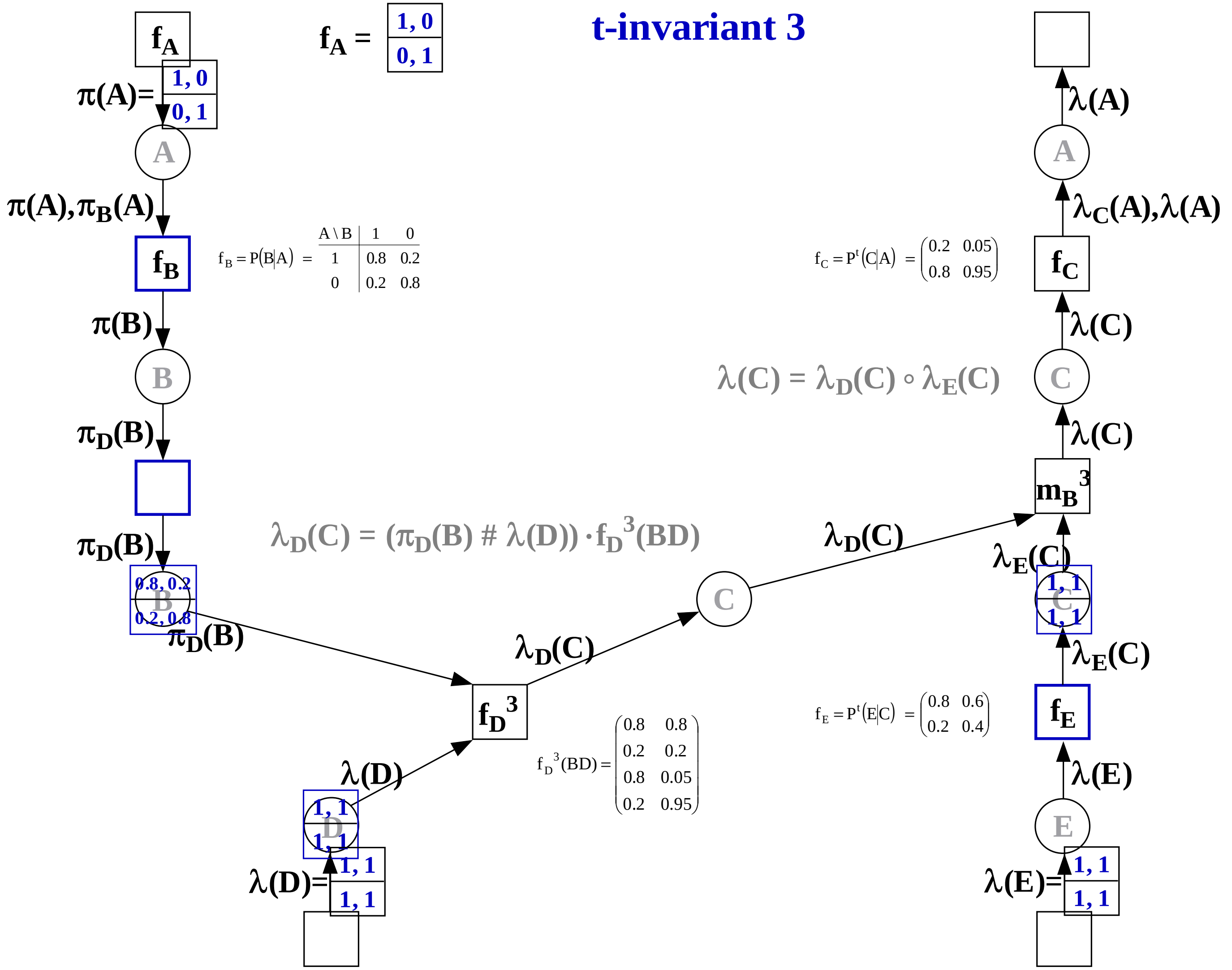
$\lambda_D(B) = (\pi_D(C) \# \lambda(D)) \cdot f_D{}^2(CD)$



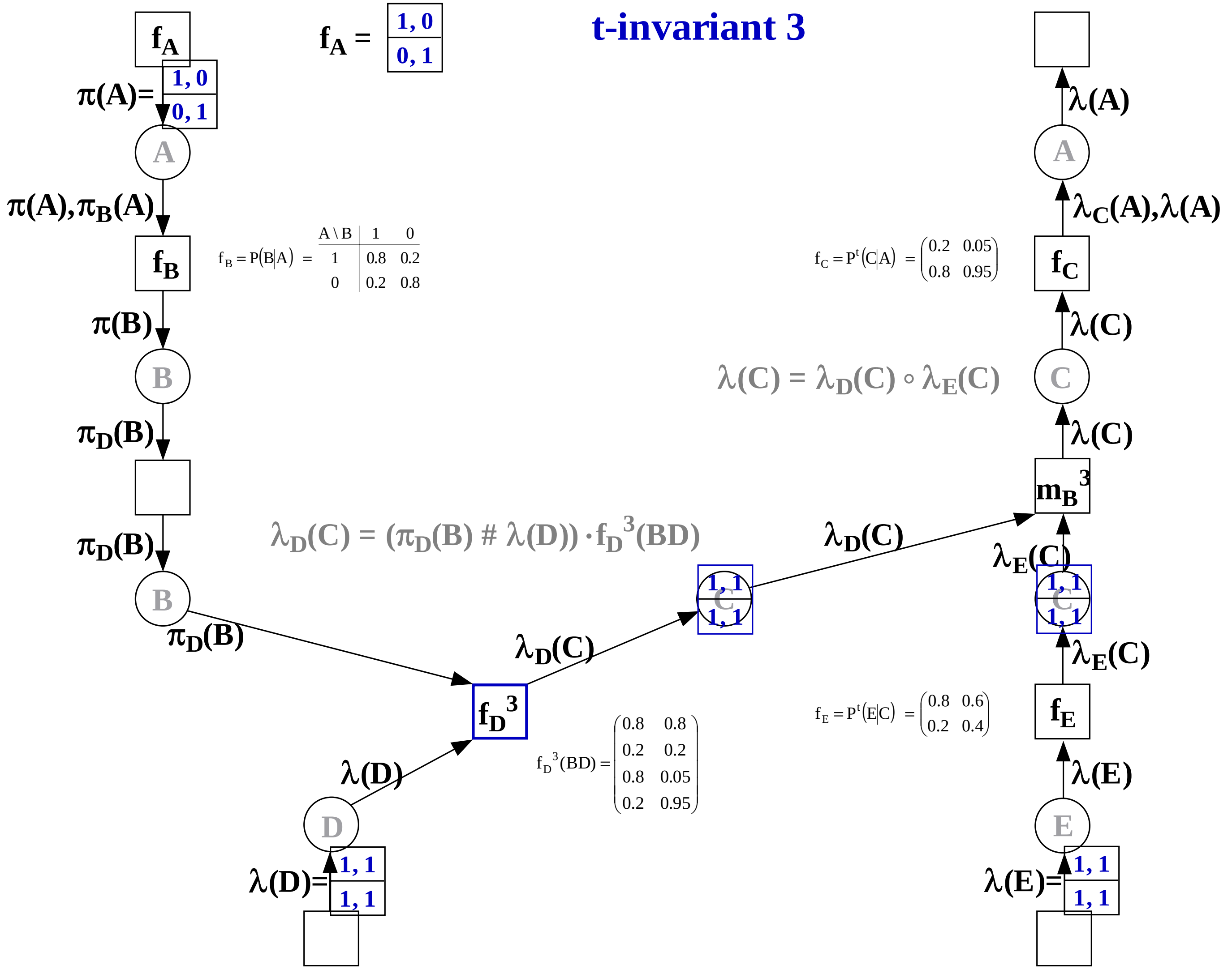

t-invariant 3
fA
π(A)=
1, 0
0, 1
fA =
1, 0
0, 1
A
π(A),πB(A)
fB
fB = P(B|A) =
A \ B | 1 | 0
1 | 0.8 | 0.2
0 | 0.2 | 0.8
π(B)
B
πD(B)
πD(B)
B
πD(B)
λD(C) = (πD(B) # λ(D)) ·fD3(BD)
fD3
fD3(BD) =
0.8 0.8
0.2 0.2
0.8 0.05
0.2 0.95
λ(D)
D
λ(D)=
1, 1
1, 1
λD(C)
C
λD(C)
λ(A)
A
λC(A),λ(A)
fC = Pt(C|A) =
0.2 0.05
0.8 0.95
fC
λ(C)
λ(C) = λD(C) ∘ λE(C)
C
λ(C)
mB3
λE(C)
C
λE(C)
fE = Pt(E|C) =
0.8 0.6
0.2 0.4
fE
λ(E)
E
λ(E)=
1, 1
1, 1

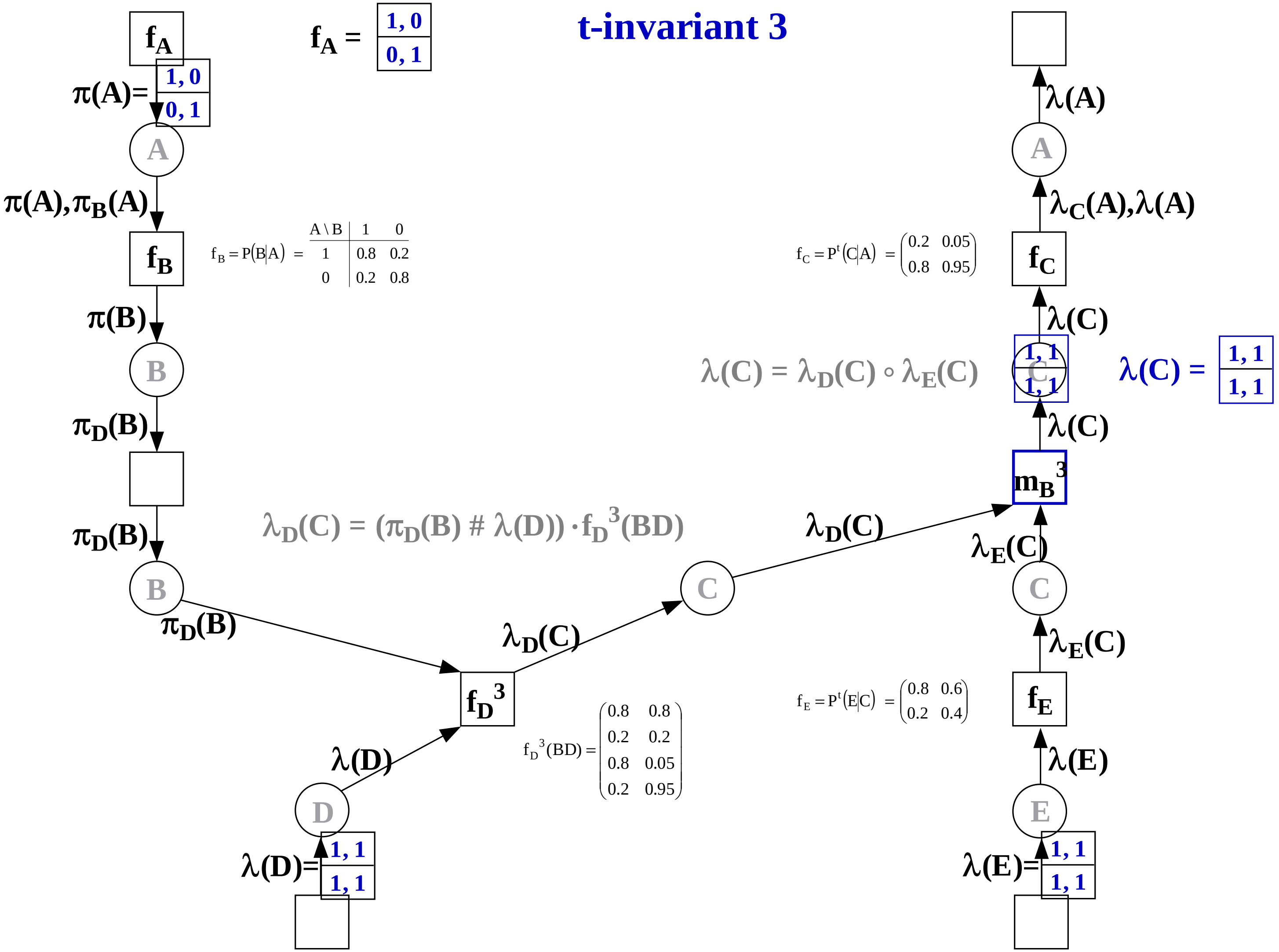
Conditioning (Pearl)



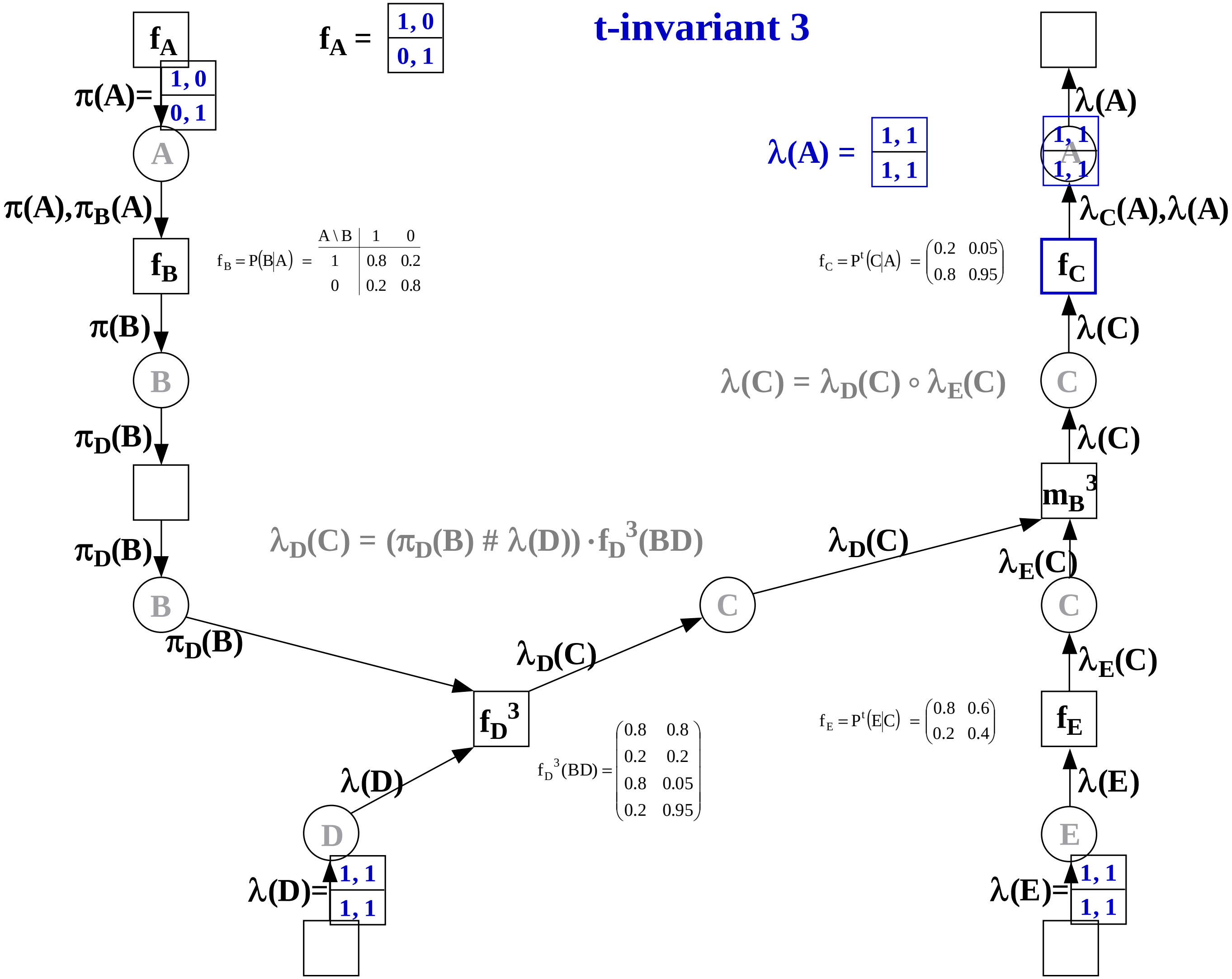

t-invariant 3
fA = 1, 0 / 0, 1
π(A)= 1, 0 / 0, 1
π(A),πB(A)
fB = P(B|A) = A\B 1 0; 1 0.8 0.2; 0 0.2 0.8
π(B)
πD(B)
λD(C) = (πD(B) # λ(D)) ·fD3(BD)
0.8, 0.2 / 0.2, 0.8
λD(C)
fD3(BD) = 0.8 0.8; 0.2 0.2; 0.8 0.05; 0.2 0.95
λ(D)
λ(D)= 1, 1 / 1, 1
λ(A)
λC(A),λ(A)
fC = Pt(C|A) = 0.2 0.05; 0.8 0.95
λ(C)
λ(C) = λD(C) ∘ λE(C)
mB3
λE(C)
fE = Pt(E|C) = 0.8 0.6; 0.2 0.4
λ(E)
λ(E)= 1, 1 / 1, 1

Conditioning (Pearl)



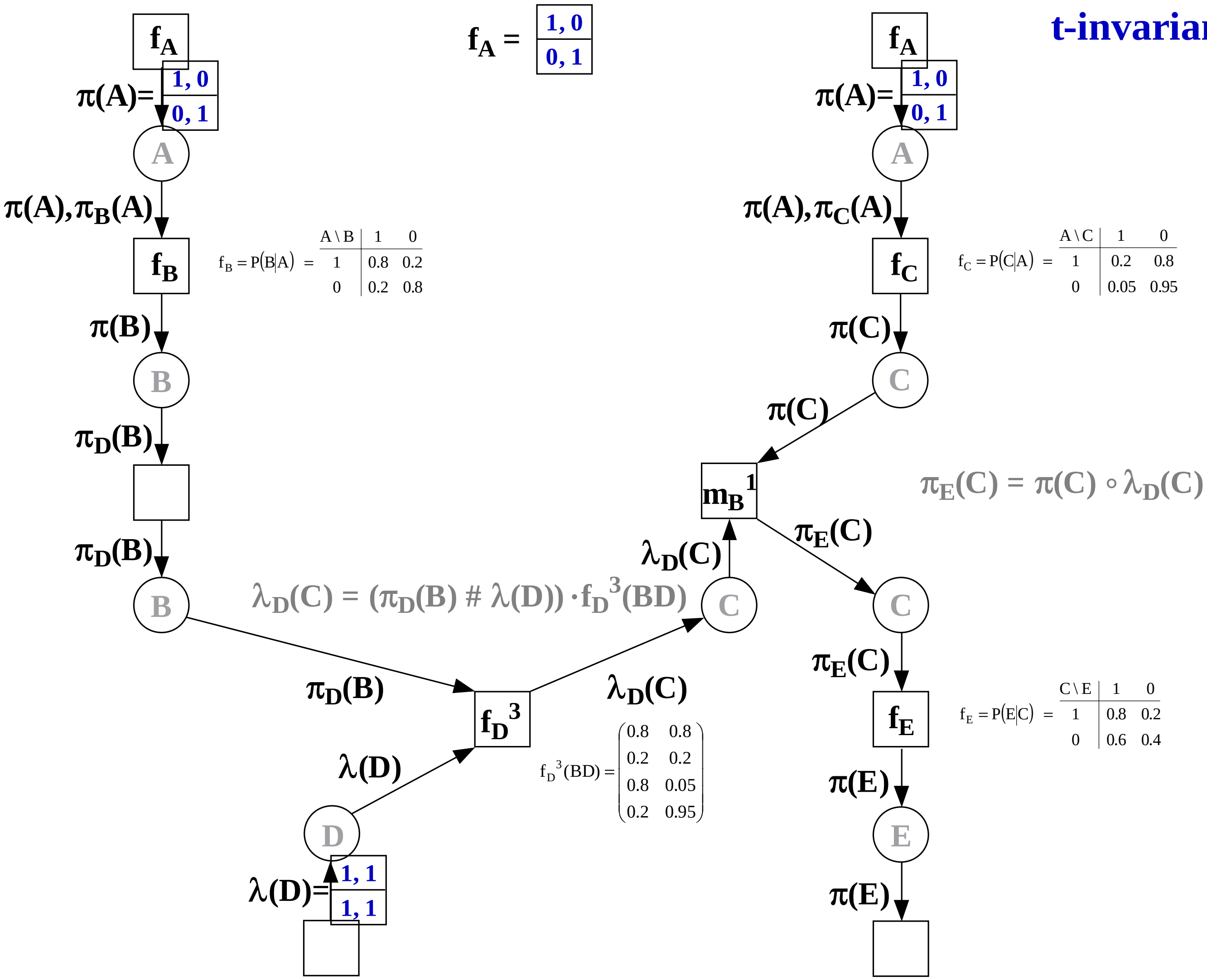

t-invariant 3
f_A = 1, 0 / 0, 1
$\pi(A)$= 1, 0 / 0, 1
$\pi(A),\pi_B(A)$
$f_B = P(B|A)$ = A \ B: 1, 0; 1: 0.8, 0.2; 0: 0.2, 0.8
$\pi(B)$
$\pi_D(B)$
$\lambda_D(C) = (\pi_D(B) \# \lambda(D)) \cdot f_D^3(BD)$
$\lambda(D)$
$\lambda(D)$= 1, 1 / 1, 1
$f_D^3(BD)$ = 0.8 0.8 / 0.2 0.2 / 0.8 0.05 / 0.2 0.95
$\lambda_D(C)$
$\lambda(A)$
$\lambda_C(A),\lambda(A)$
$f_C = P^t(C|A)$ = 0.2 0.05 / 0.8 0.95
$\lambda(C)$
$\lambda(C) = \lambda_D(C) \circ \lambda_E(C)$
$\lambda(C)$ = 1, 1 / 1, 1
m_B^3
$\lambda_E(C)$
$f_E = P^t(E|C)$ = 0.8 0.6 / 0.2 0.4
$\lambda(E)$
$\lambda(E)$= 1, 1 / 1, 1

Conditioning (Pearl)

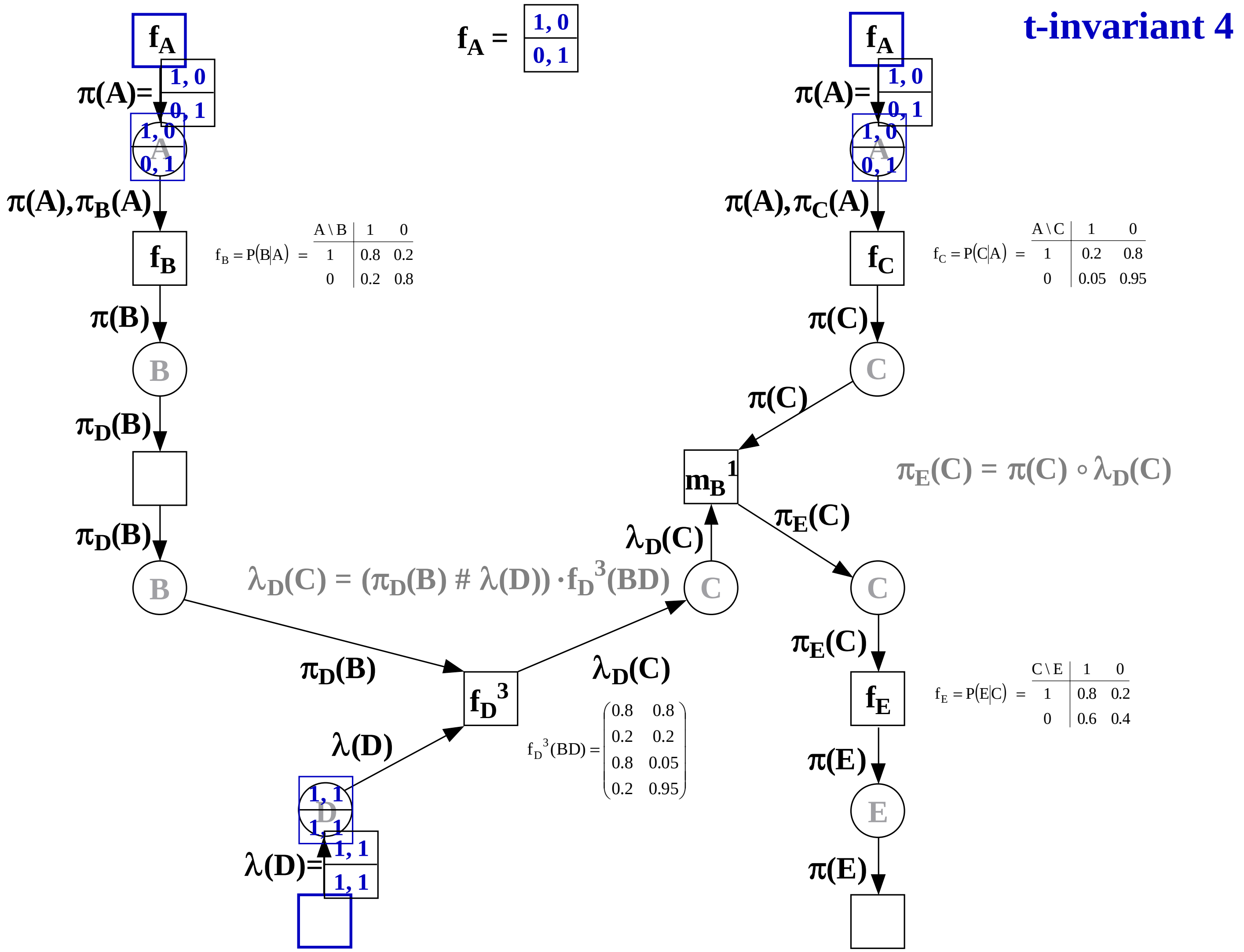


Conditioning (Pearl)

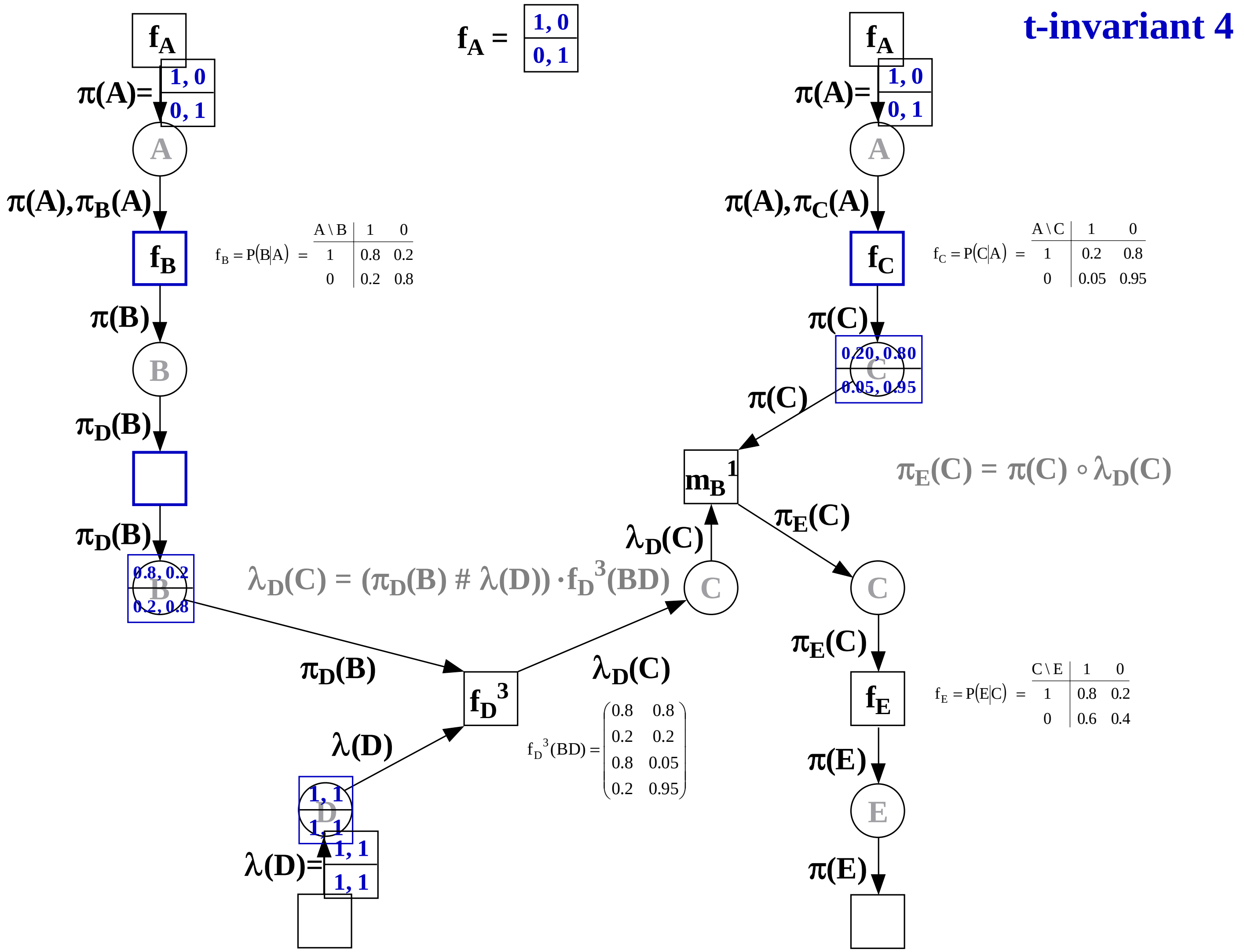
t-invariant 4



Conditioning (Pearl)

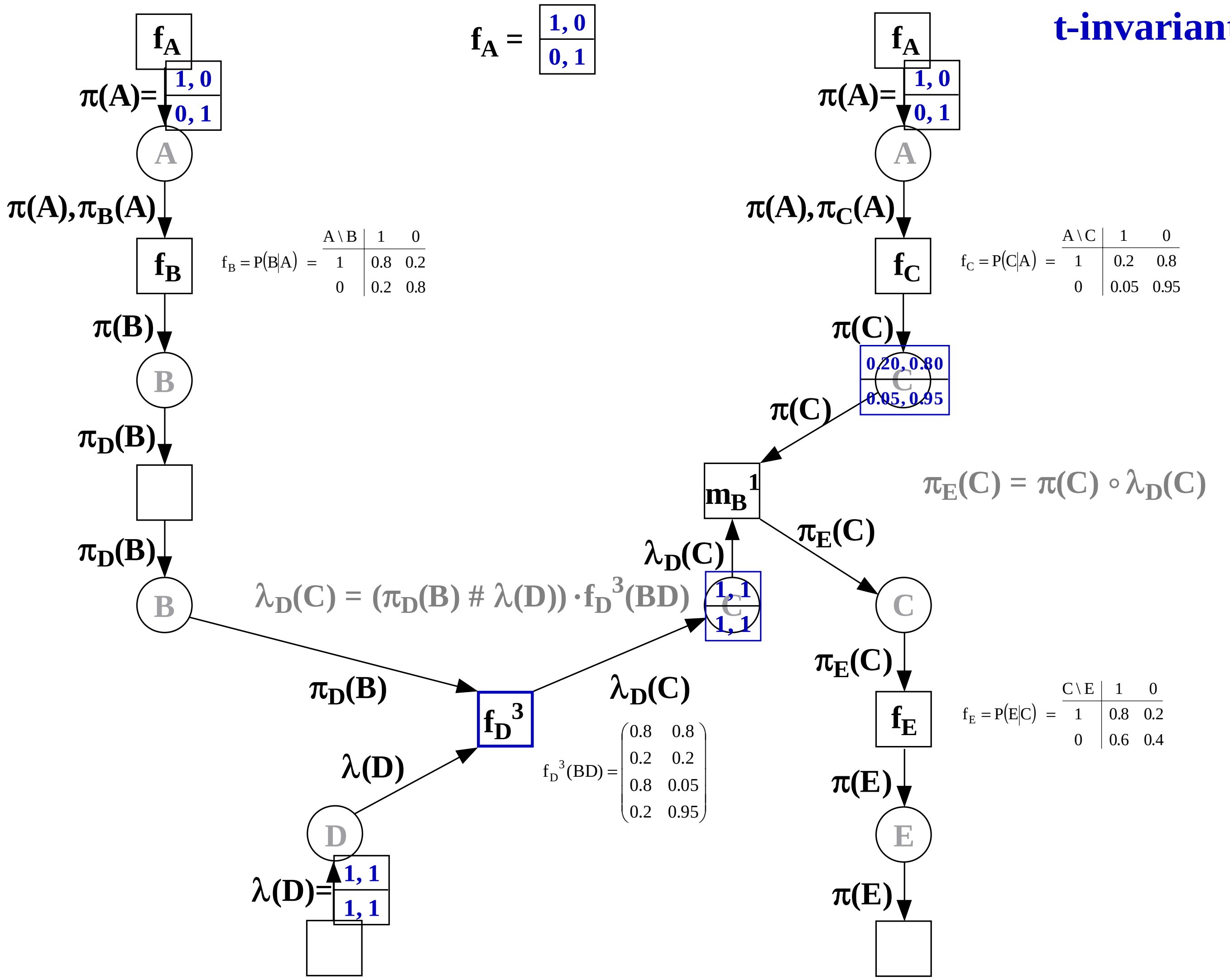


Conditioning (Pearl)

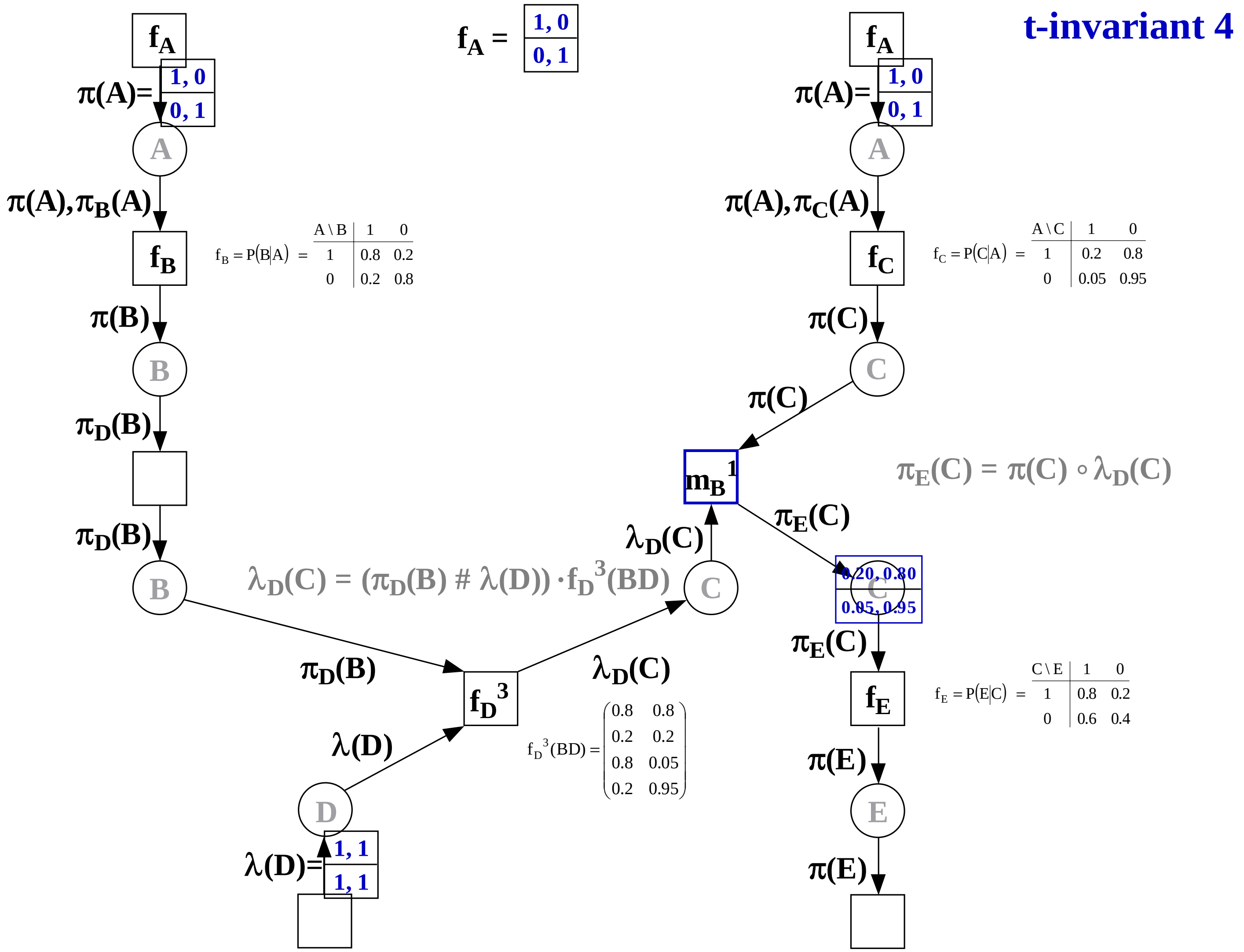


Conditioning (Pearl)

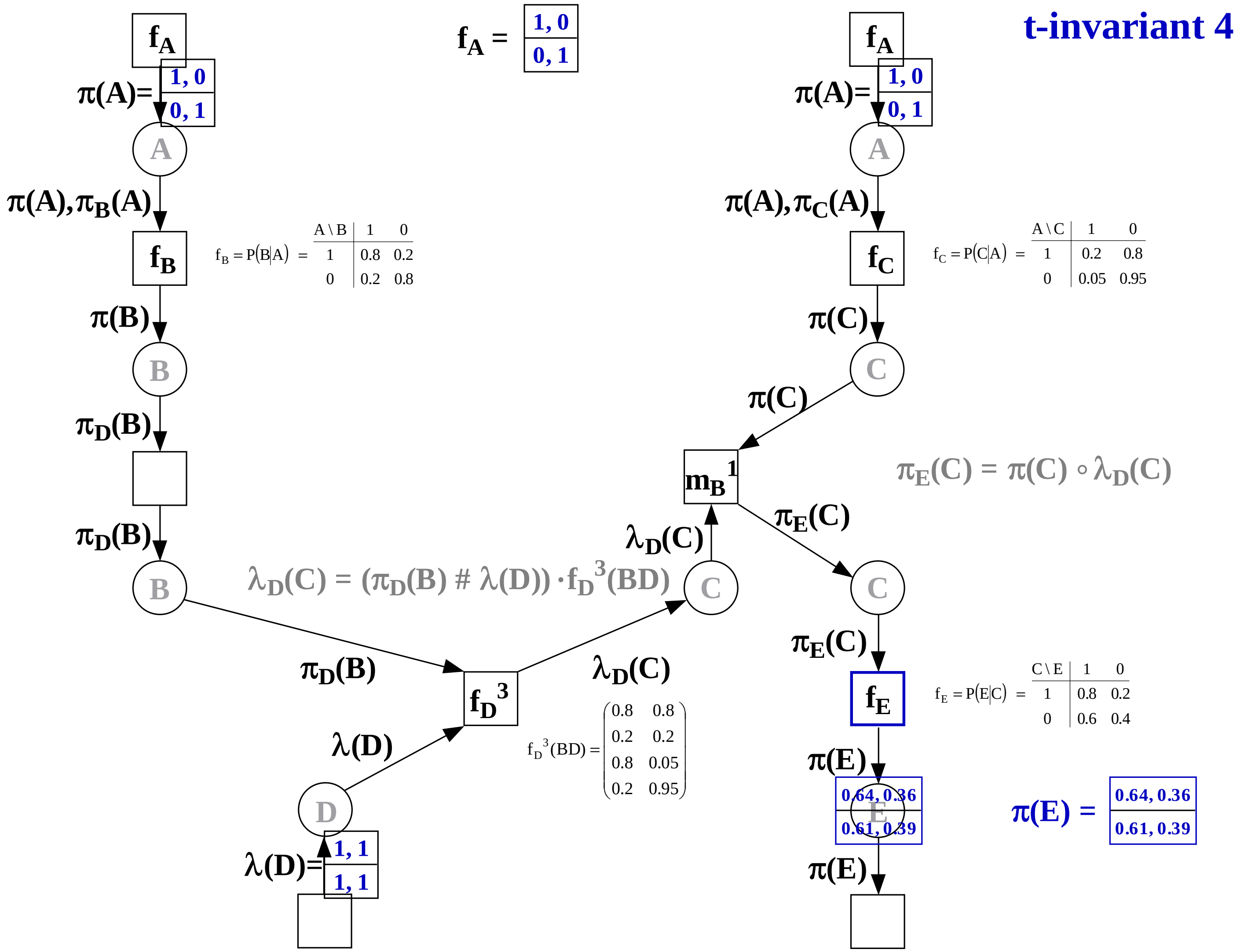
t-invariant 4



Conditioning (Pearl)



Conditioning (Pearl)



Conditioning (Pearl)

Probabilities:

P(A) = (0.2, 0.8) is the initial weight;

$$\pi(B) = \begin{array}{|c|}\hline 0.8,\ 0.2 \\ \hline 0.2,\ 0.8 \\ \hline\end{array} \qquad \lambda(B) = \begin{array}{|c|}\hline 1,\ 1 \\ \hline 1,\ 1 \\ \hline\end{array}$$

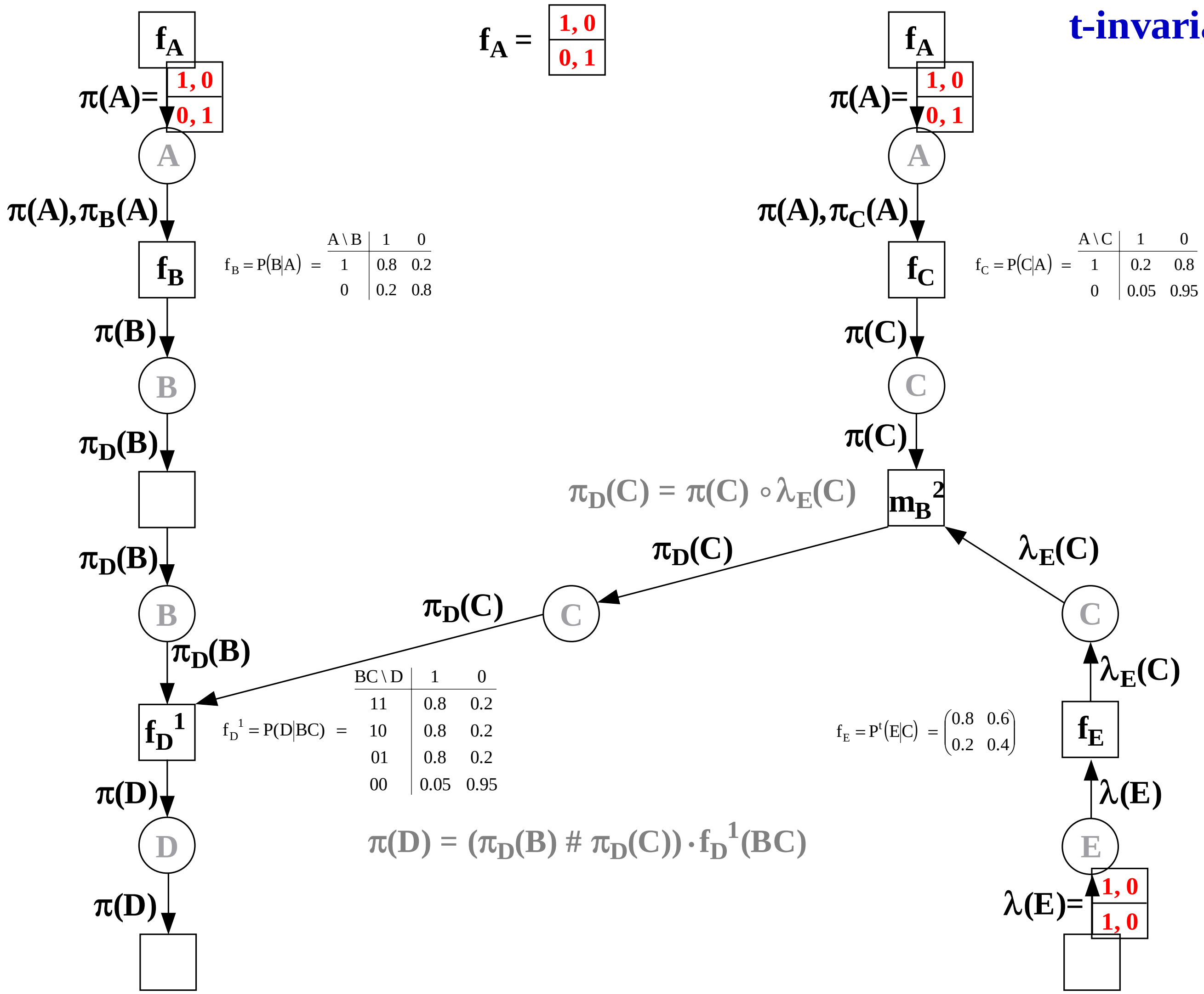
$$P(B) = BEL(B) = ((0.8, 0.2)\circ(1, 1))\cdot 0.2 + ((0.2, 0.8)\circ(1, 1))\cdot 0.8 = \boxed{(0.32, 0.68)}$$

$$\pi(C) = \begin{array}{|c|}\hline 0.20,\ 0.80 \\ \hline 0.05,\ 0.95 \\ \hline\end{array} \qquad \lambda(C) = \begin{array}{|c|}\hline 1,\ 1 \\ \hline 1,\ 1 \\ \hline\end{array}$$

$$P(C) = BEL(C) = ((0.2, 0.8)\circ(1, 1))\cdot 0.2 + ((0.05, 0.95)\circ(1, 1))\cdot 0.8 = \boxed{(0.08, 0.92)}$$

Conditioning (Pearl)

t-invariant 1



$f_B = P(B|A) =$

A \ B	1	0
1	0.8	0.2
0	0.2	0.8

$f_C = P(C|A) =$

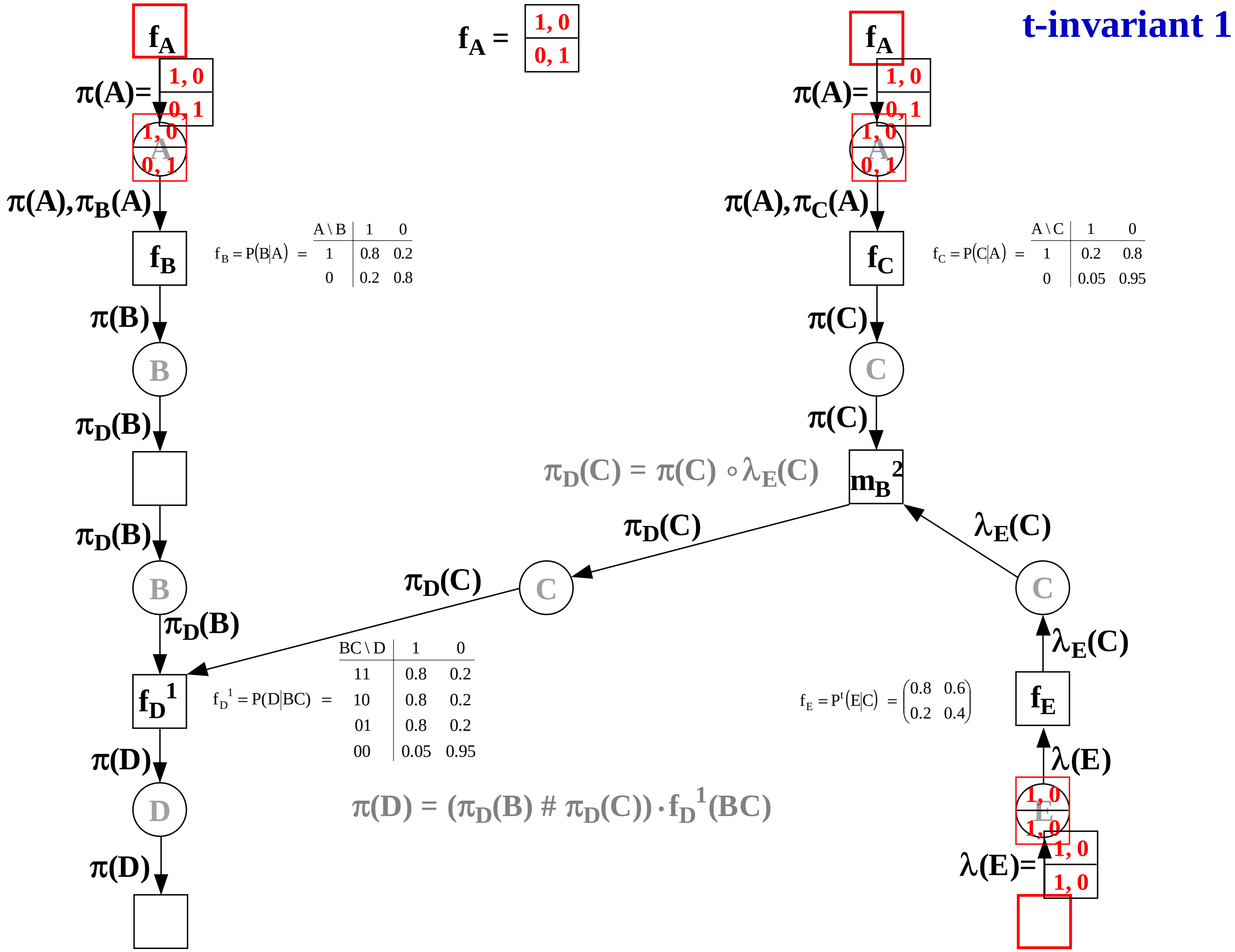
A \ C	1	0
1	0.2	0.8
0	0.05	0.95

$f_D^{\,1} = P(D|BC) =$

BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

Conditioning (Pearl)

t-invariant 1



$f_B = P(B|A) =$

A \ B	1	0
1	0.8	0.2
0	0.2	0.8

$f_C = P(C|A) =$

A \ C	1	0
1	0.2	0.8
0	0.05	0.95

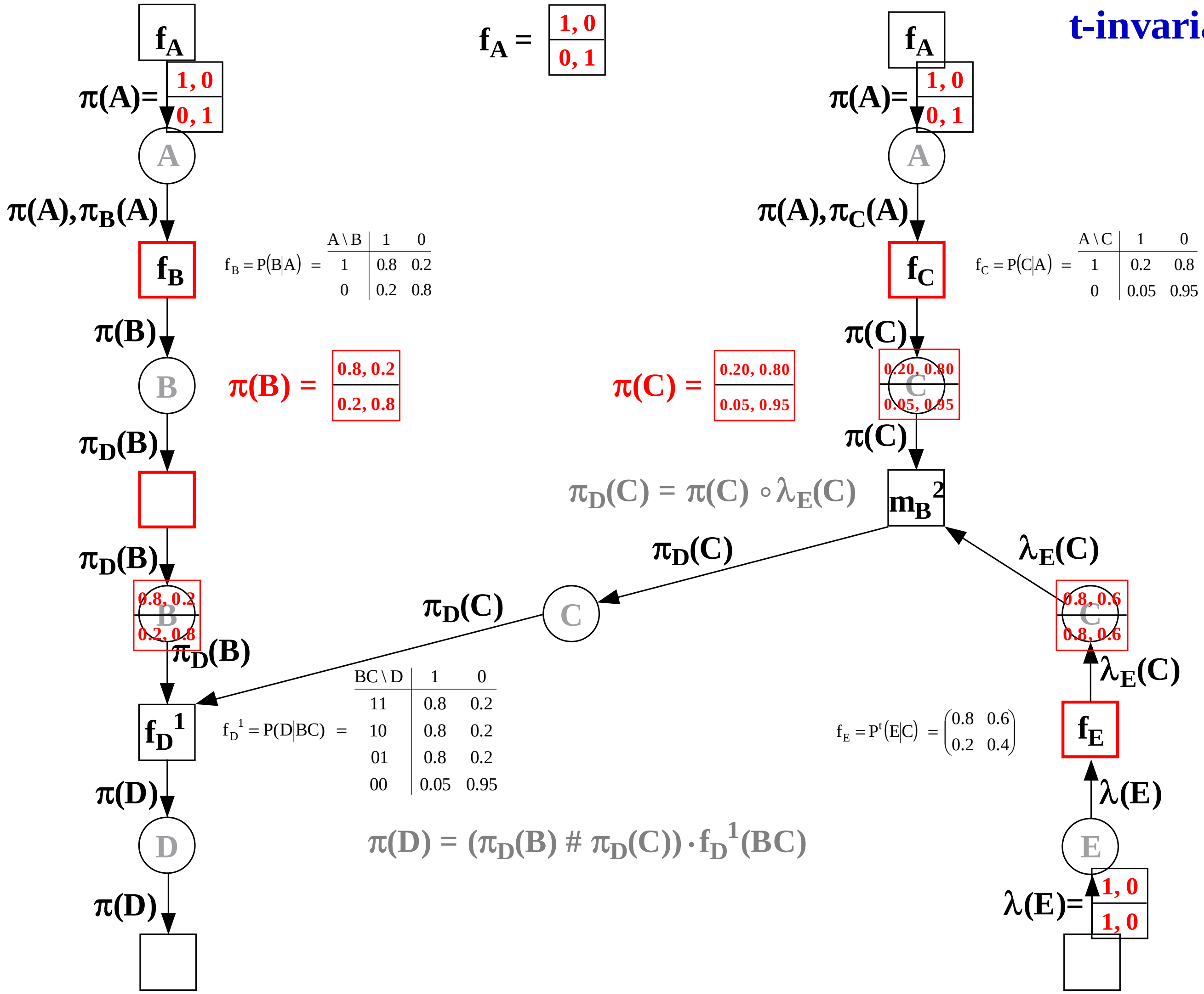
$f_D^{\ 1} = P(D|BC) =$

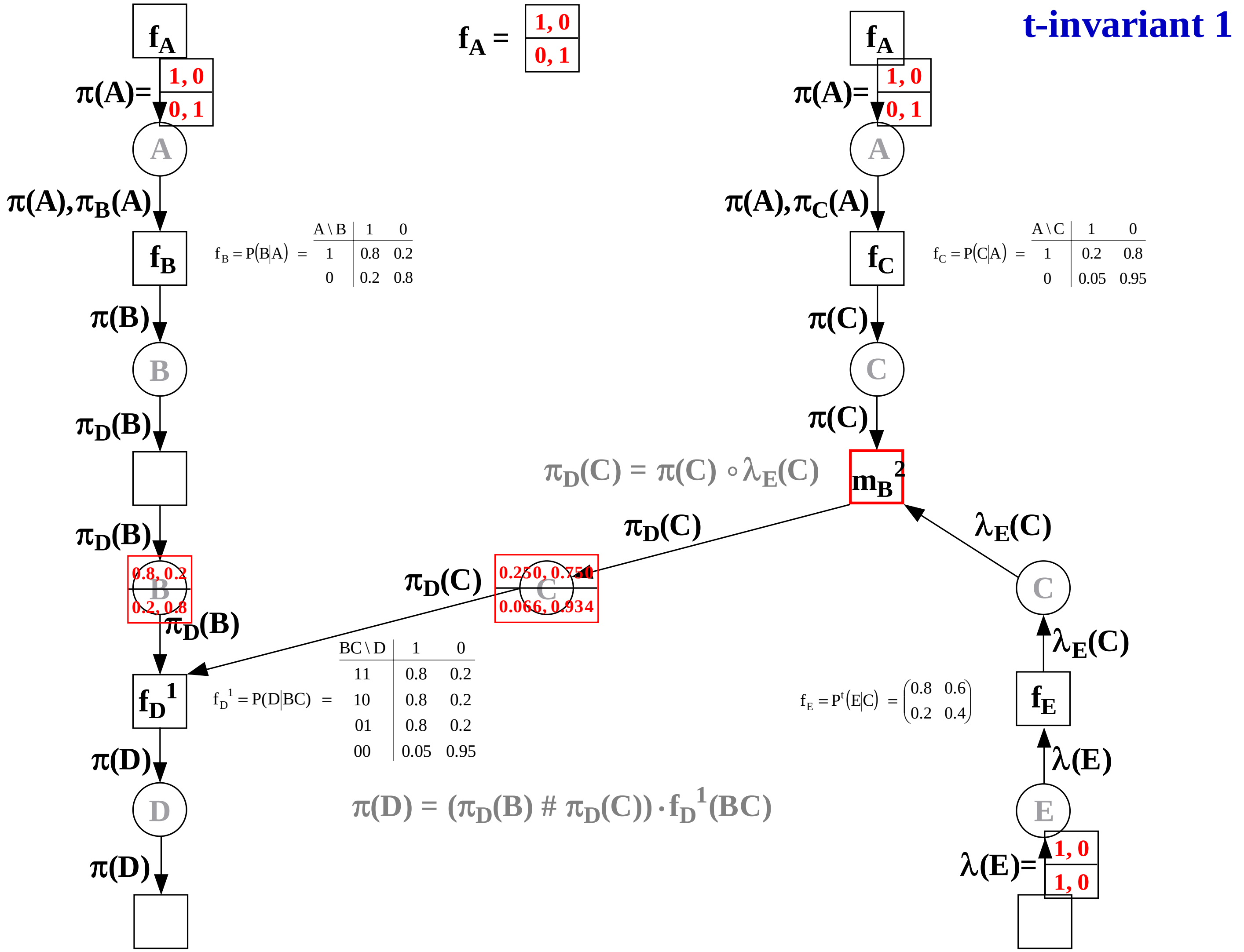
BC \ D	1	0
11	0.8	0.2
10	0.8	0.2
01	0.8	0.2
00	0.05	0.95

$$f_E = P^t(E|C) = \begin{pmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{pmatrix}$$

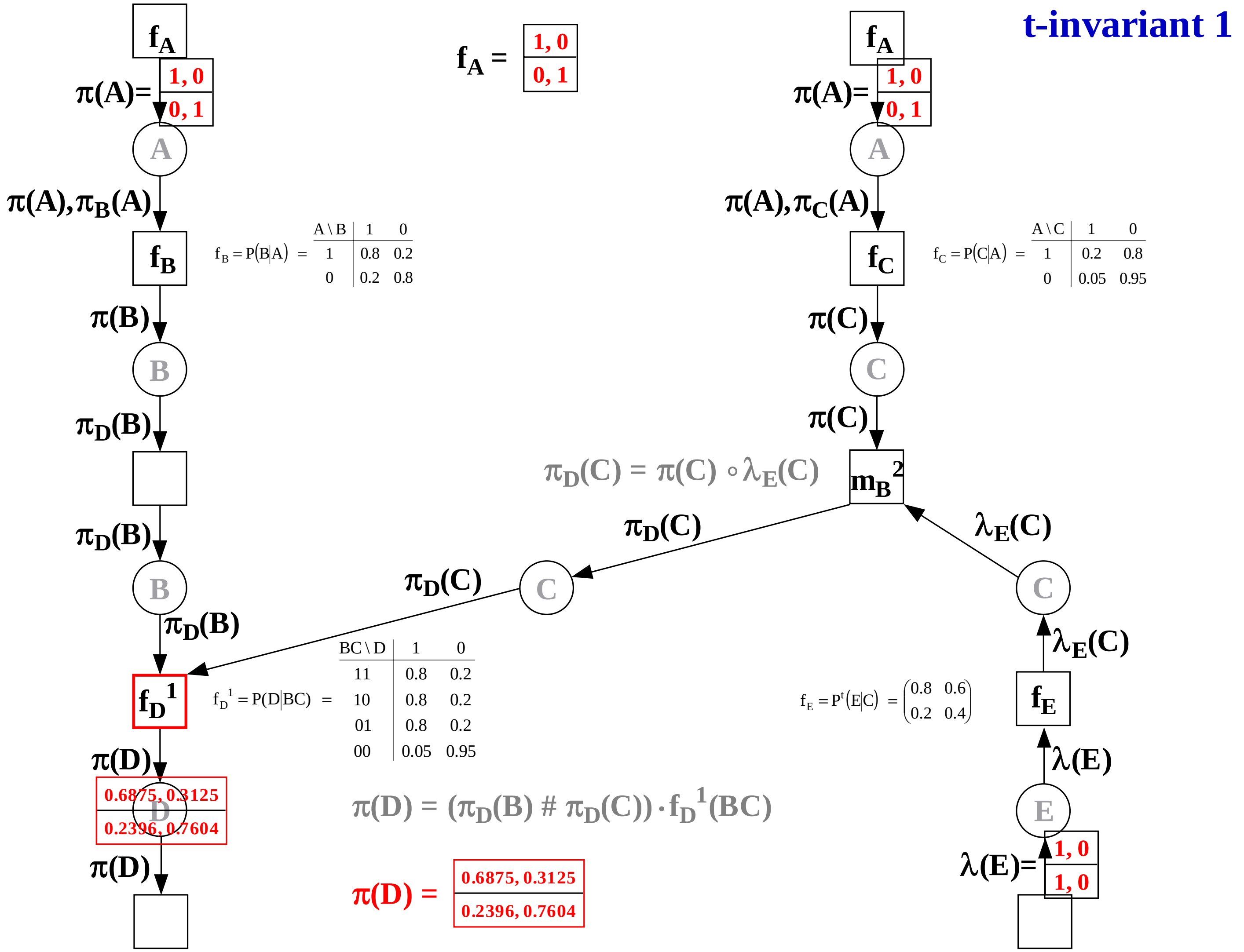
Conditioning (Pearl)

t-invariant 1

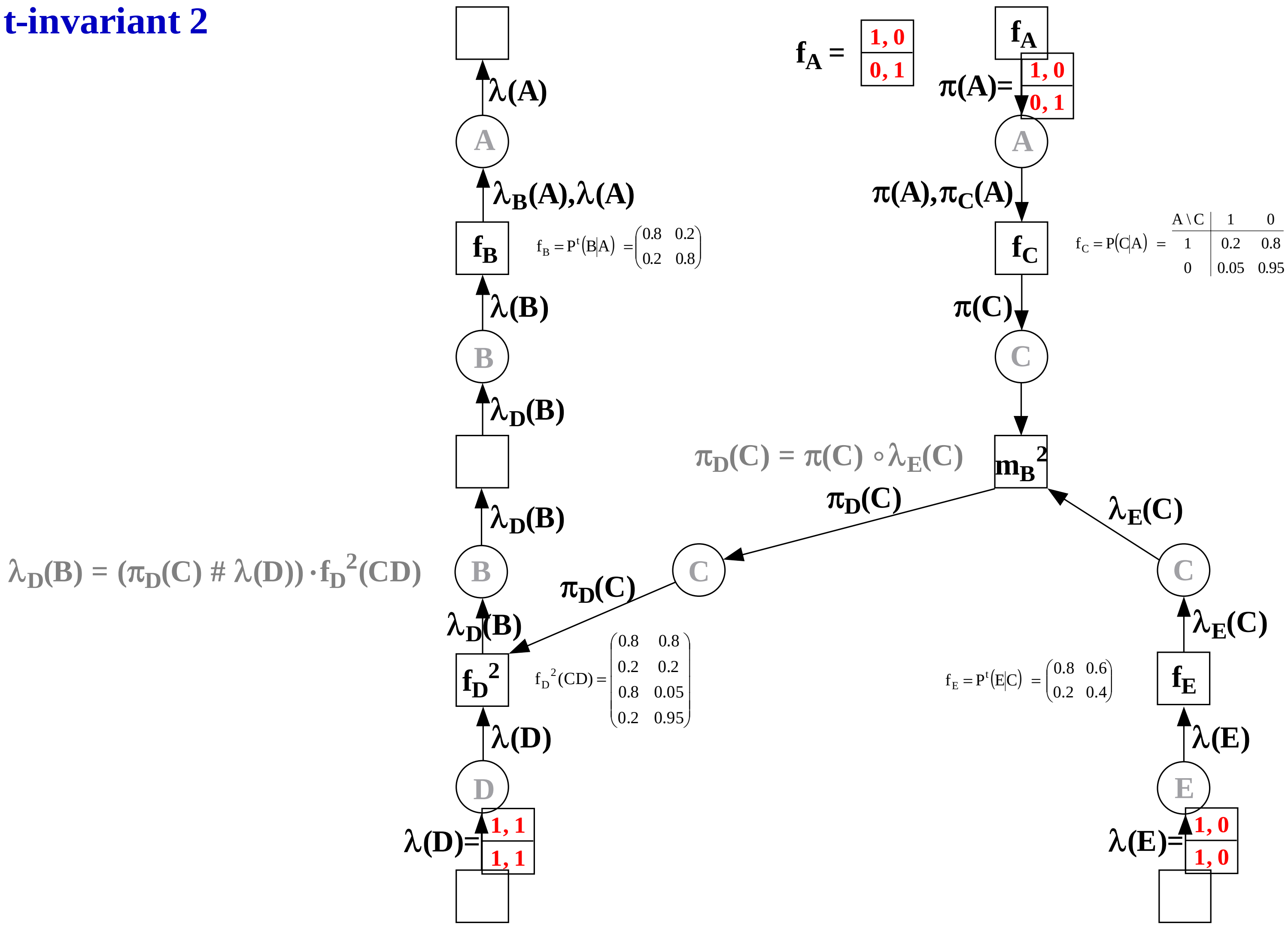



t-invariant 1
fA
π(A)=
1, 0
0, 1
A
π(A),πB(A)
fB
fB = P(B|A) =
A \ B 1 0
1 0.8 0.2
0 0.2 0.8
π(B)
B
πD(B)
πD(B)
0.8, 0.2
0.2, 0.8
πD(B)
fD1
fD1 = P(D|BC) =
BC \ D 1 0
11 0.8 0.2
10 0.8 0.2
01 0.8 0.2
00 0.05 0.95
π(D)
D
π(D)
π(D) = (πD(B) # πD(C)) · fD1(BC)
fA =
1, 0
0, 1
fA
π(A)=
1, 0
0, 1
A
π(A),πC(A)
fC
fC = P(C|A) =
A \ C 1 0
1 0.2 0.8
0 0.05 0.95
π(C)
C
π(C)
mB2
πD(C) = π(C) ∘ λE(C)
πD(C)
πD(C)
0.250, 0.750
0.066, 0.934
C
λE(C)
C
λE(C)
fE
fE = Pt(E|C) =
0.8 0.6
0.2 0.4
λ(E)
E
λ(E)=
1, 0
1, 0

Conditioning (Pearl)

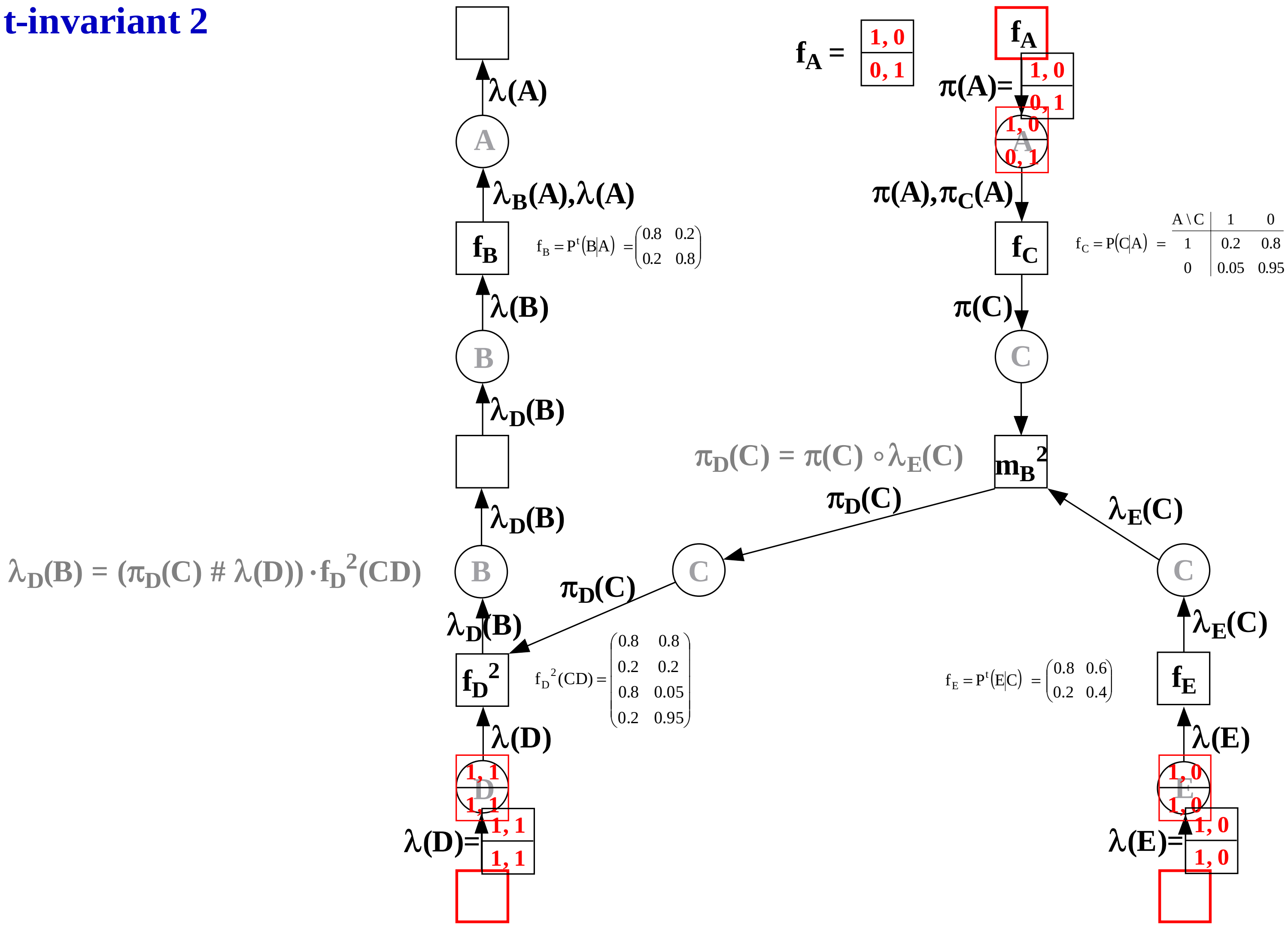


t-invariant 2



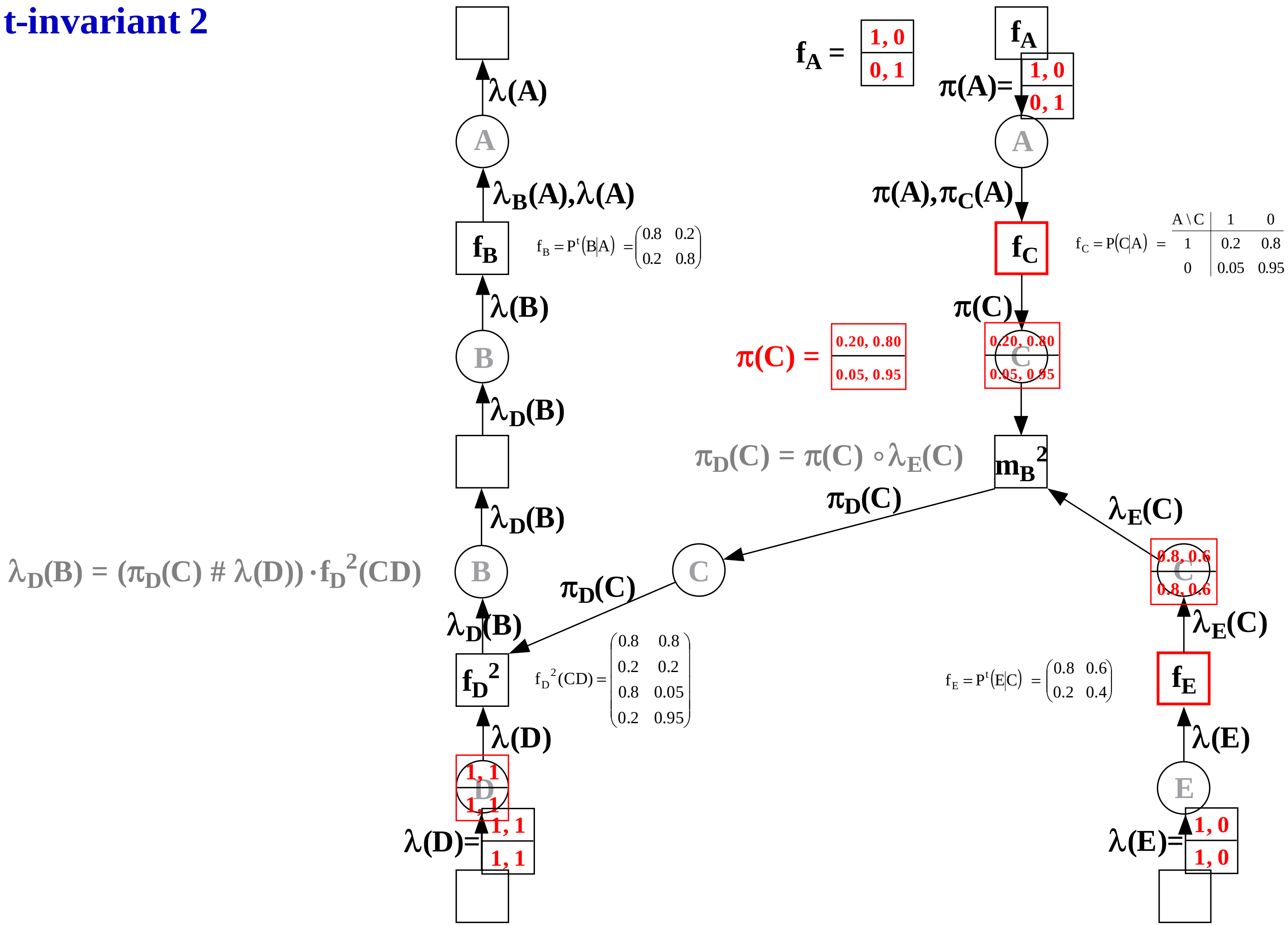
Conditioning (Pearl)

t-invariant 2



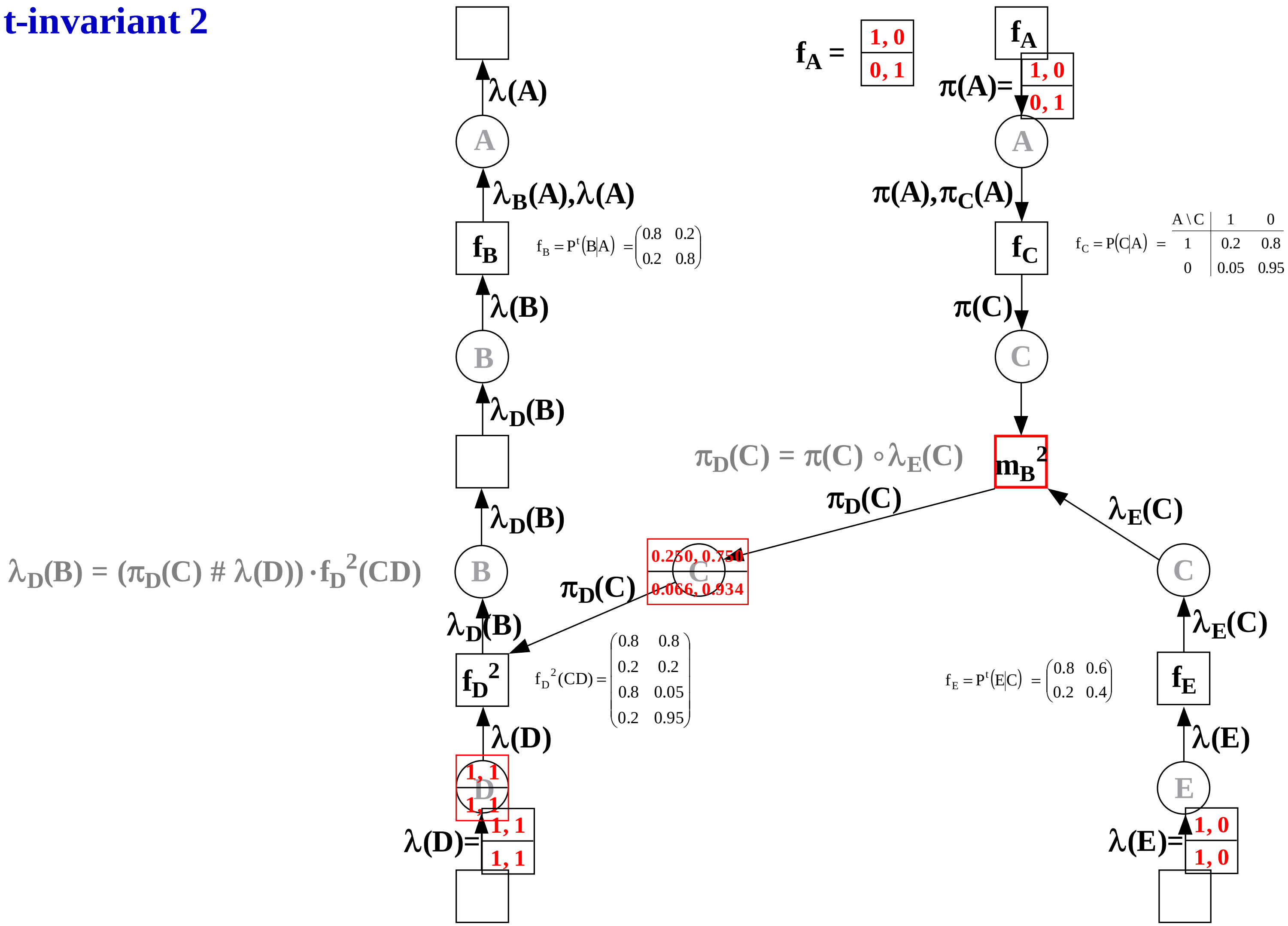
Conditioning (Pearl)

t-invariant 2



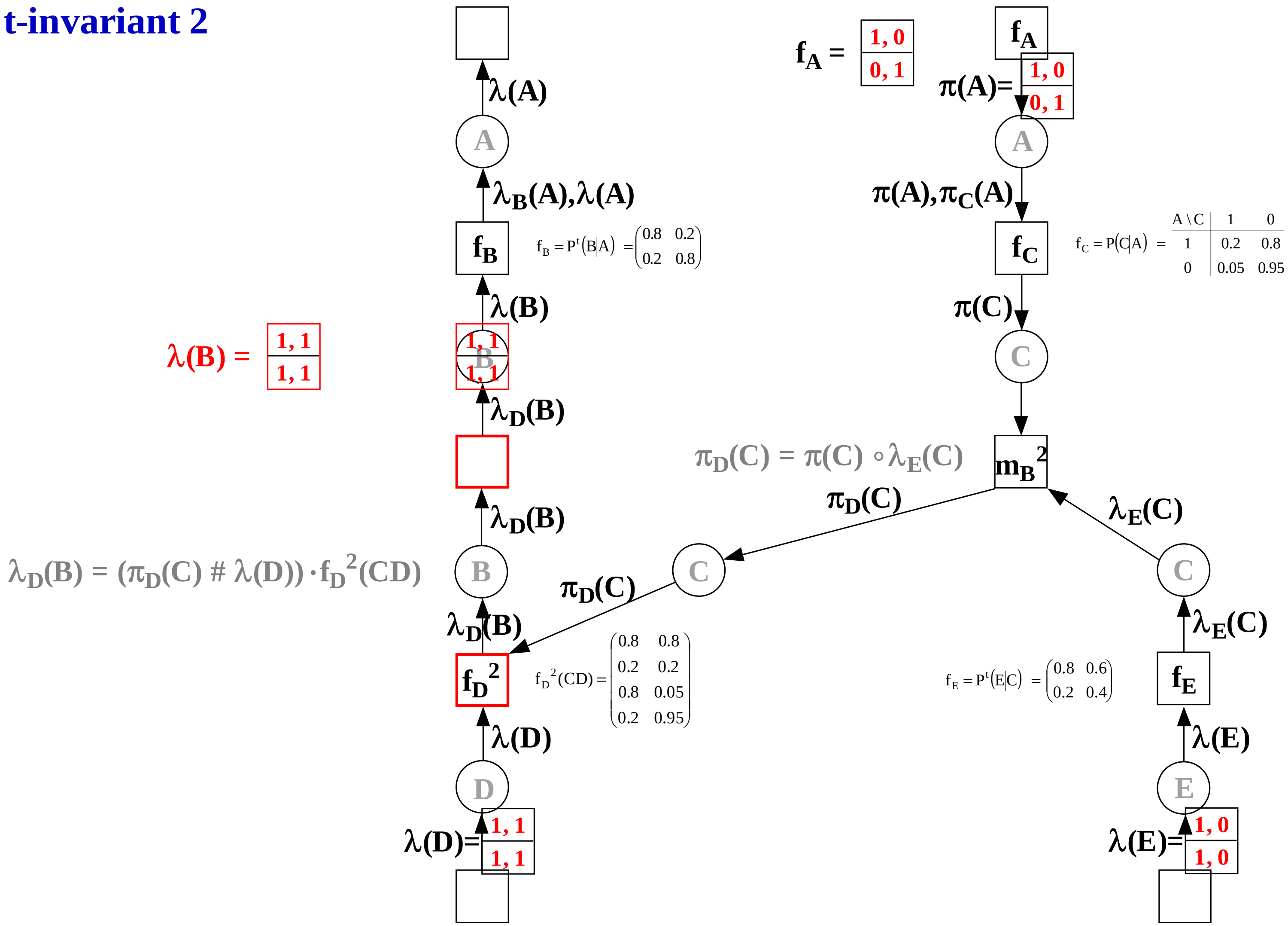
Conditioning (Pearl)

t-invariant 2



Conditioning (Pearl)

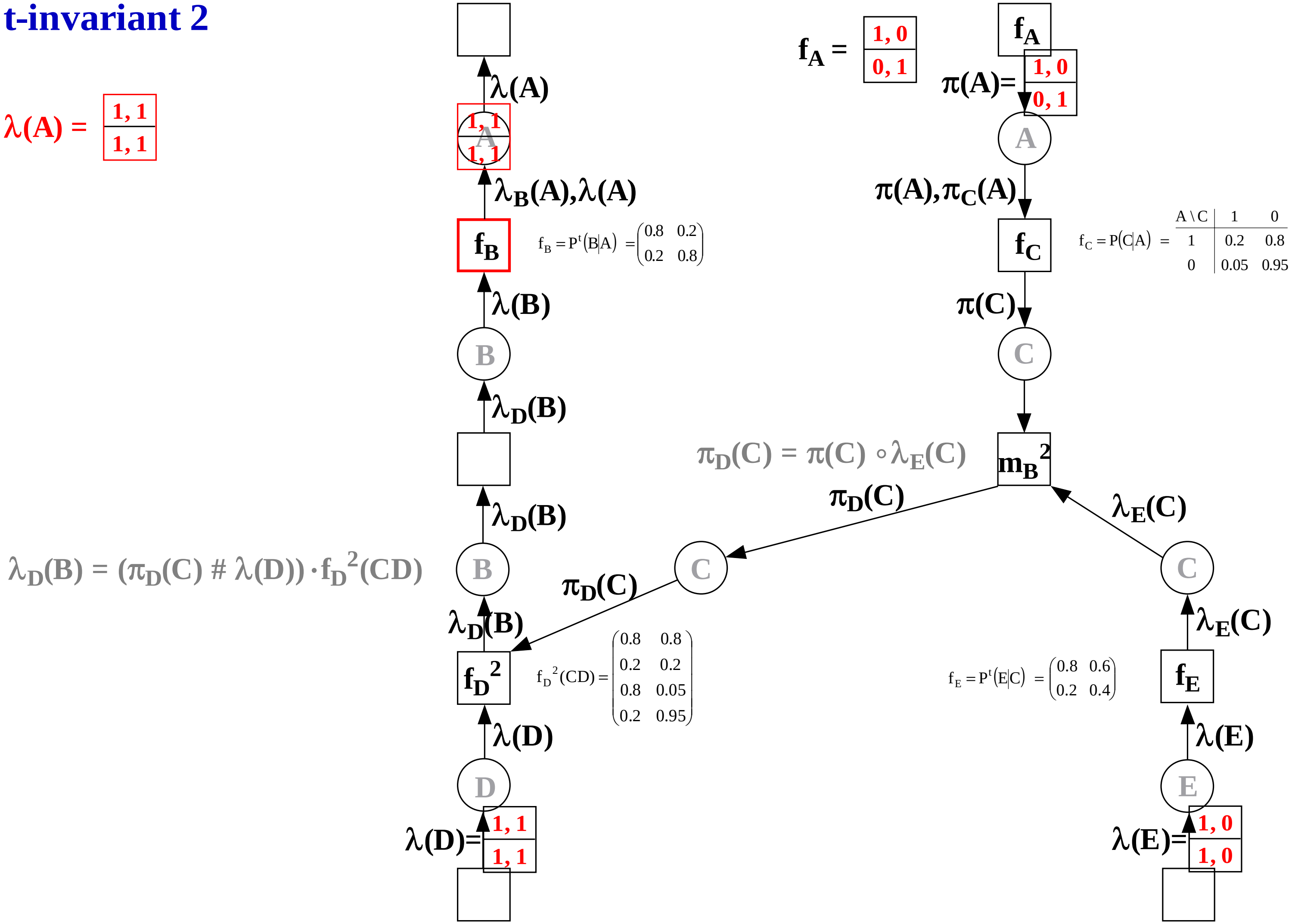
t-invariant 2



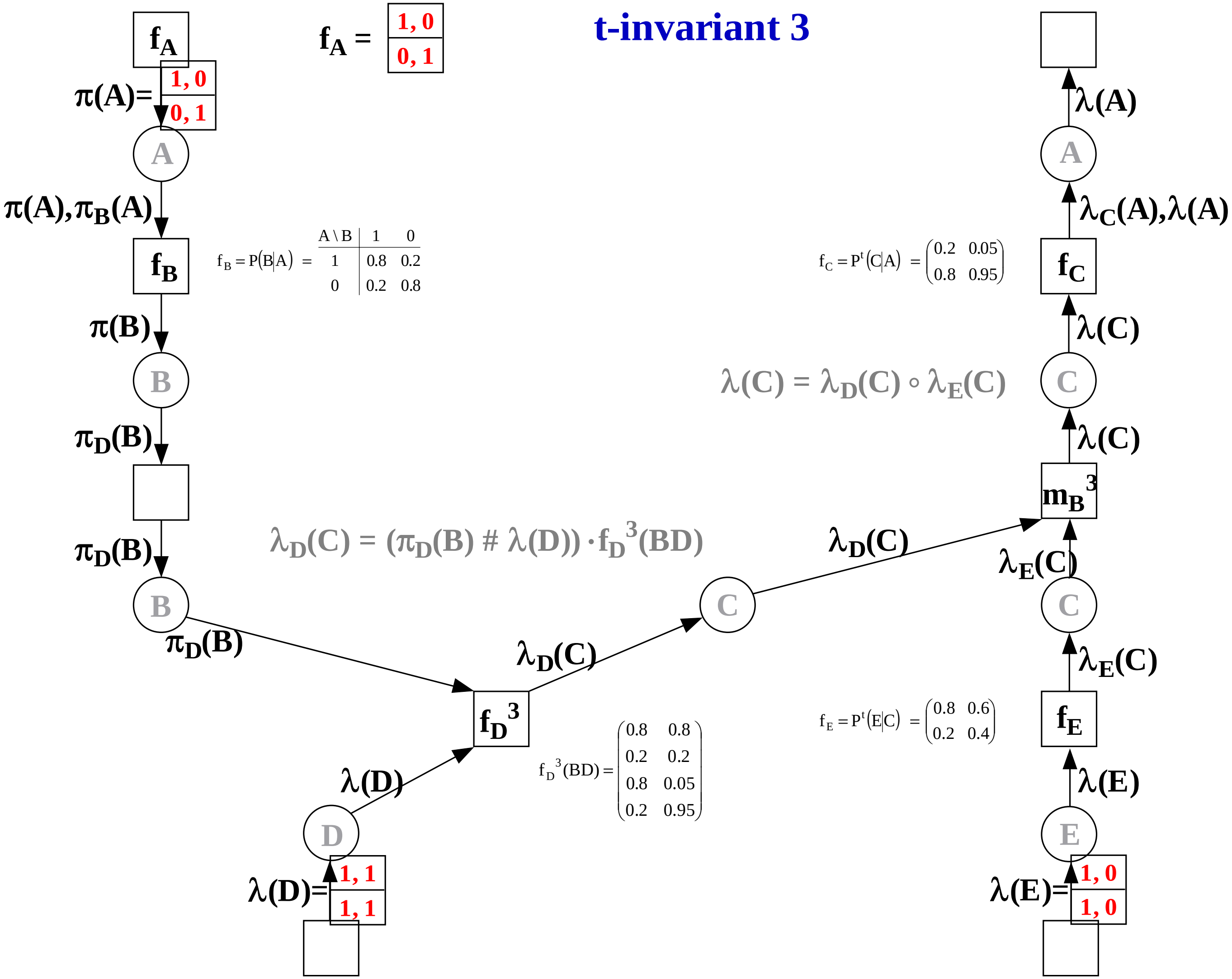
Conditioning (Pearl)

t-invariant 2

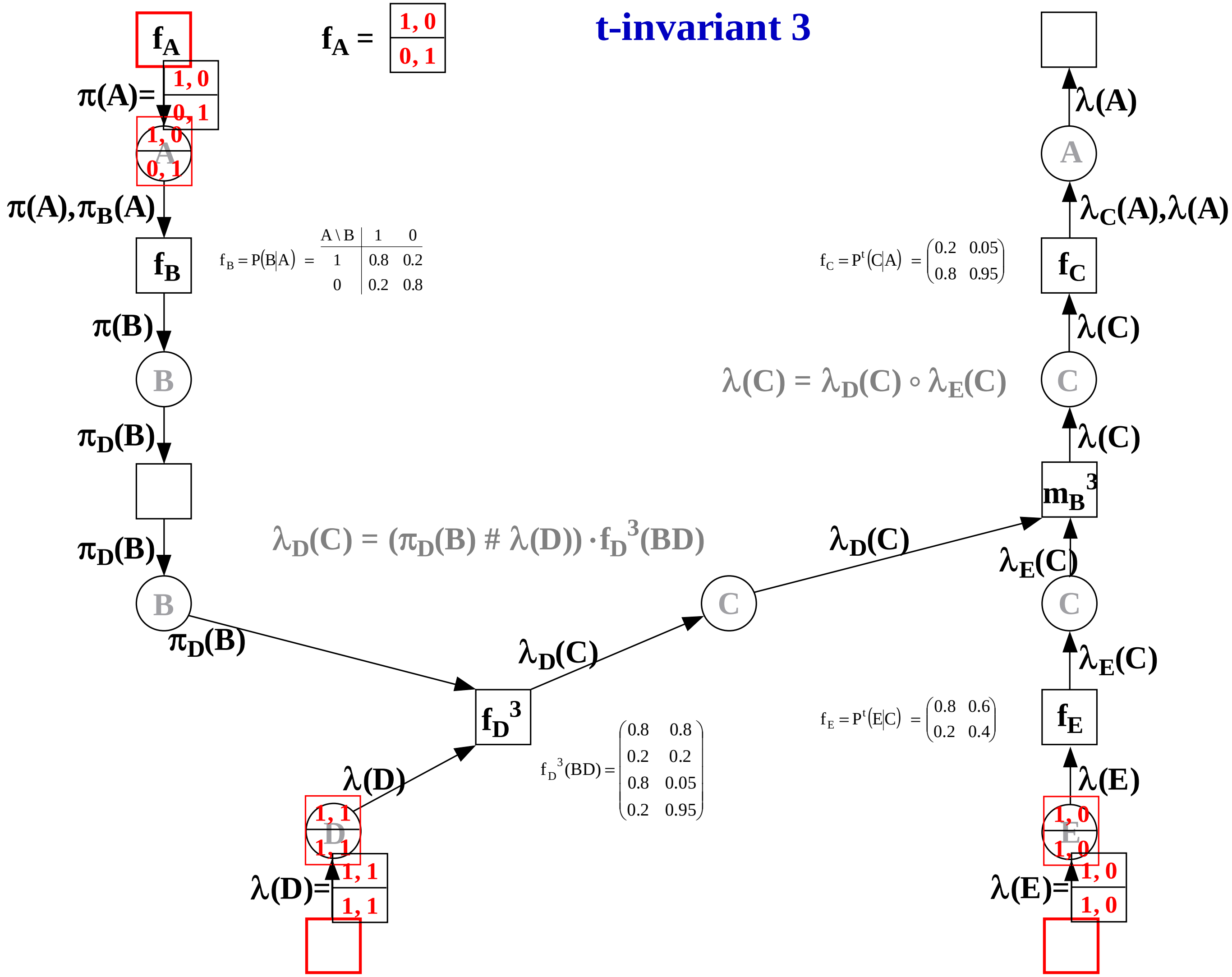
$\lambda(A) = \begin{array}{|c|}\hline 1,1 \\ \hline 1,1 \\ \hline\end{array}$



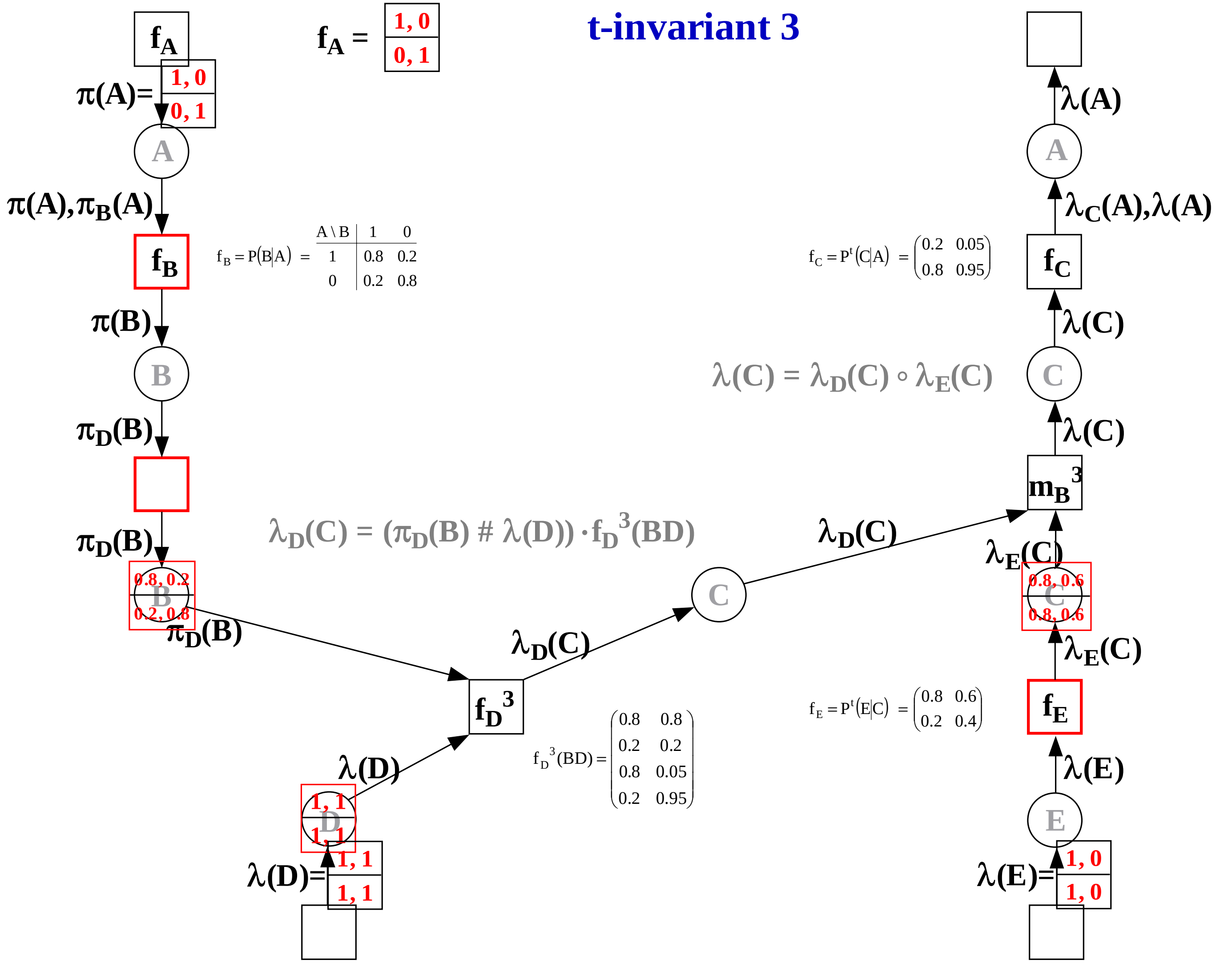
Conditioning (Pearl)



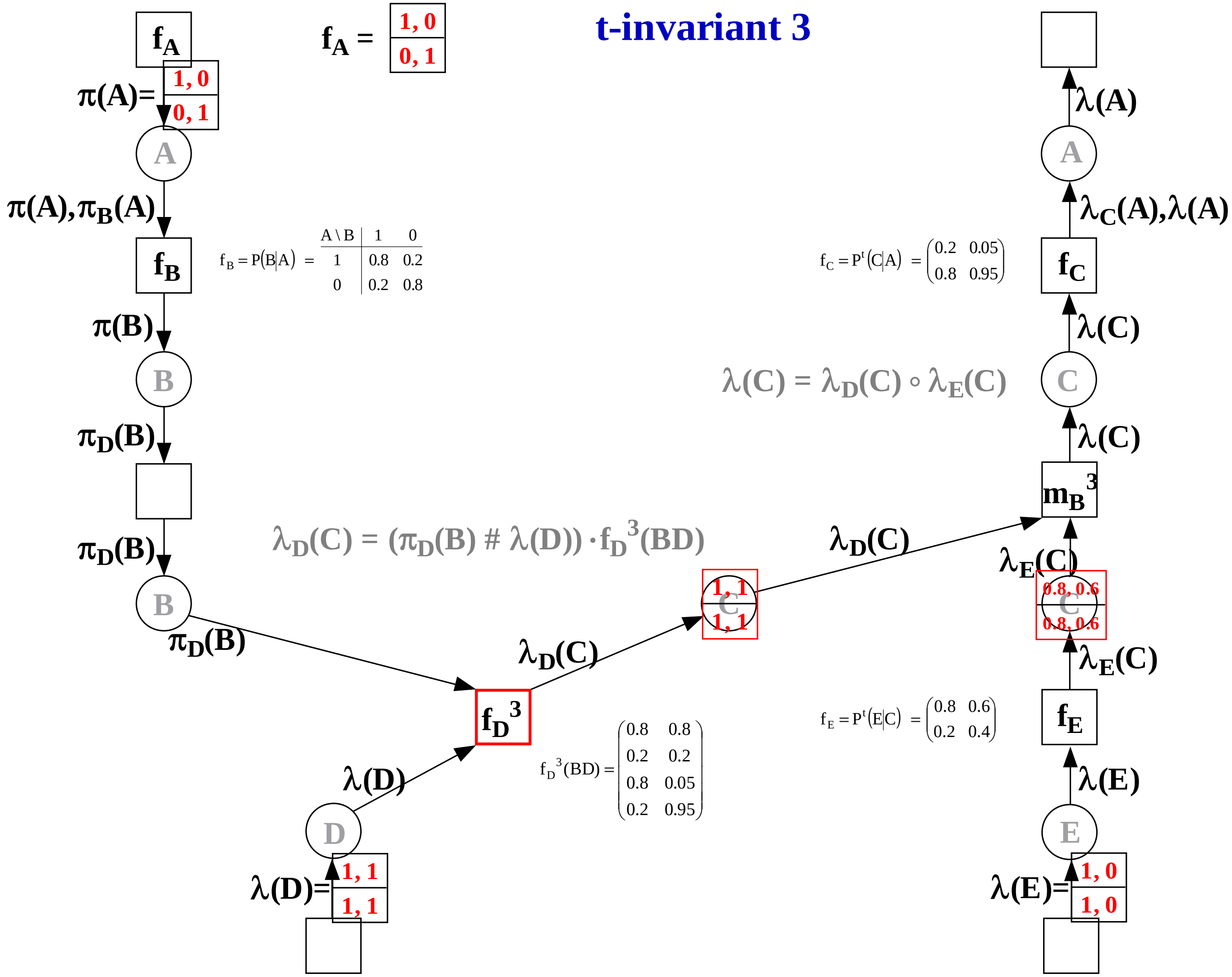
t-invariant 3



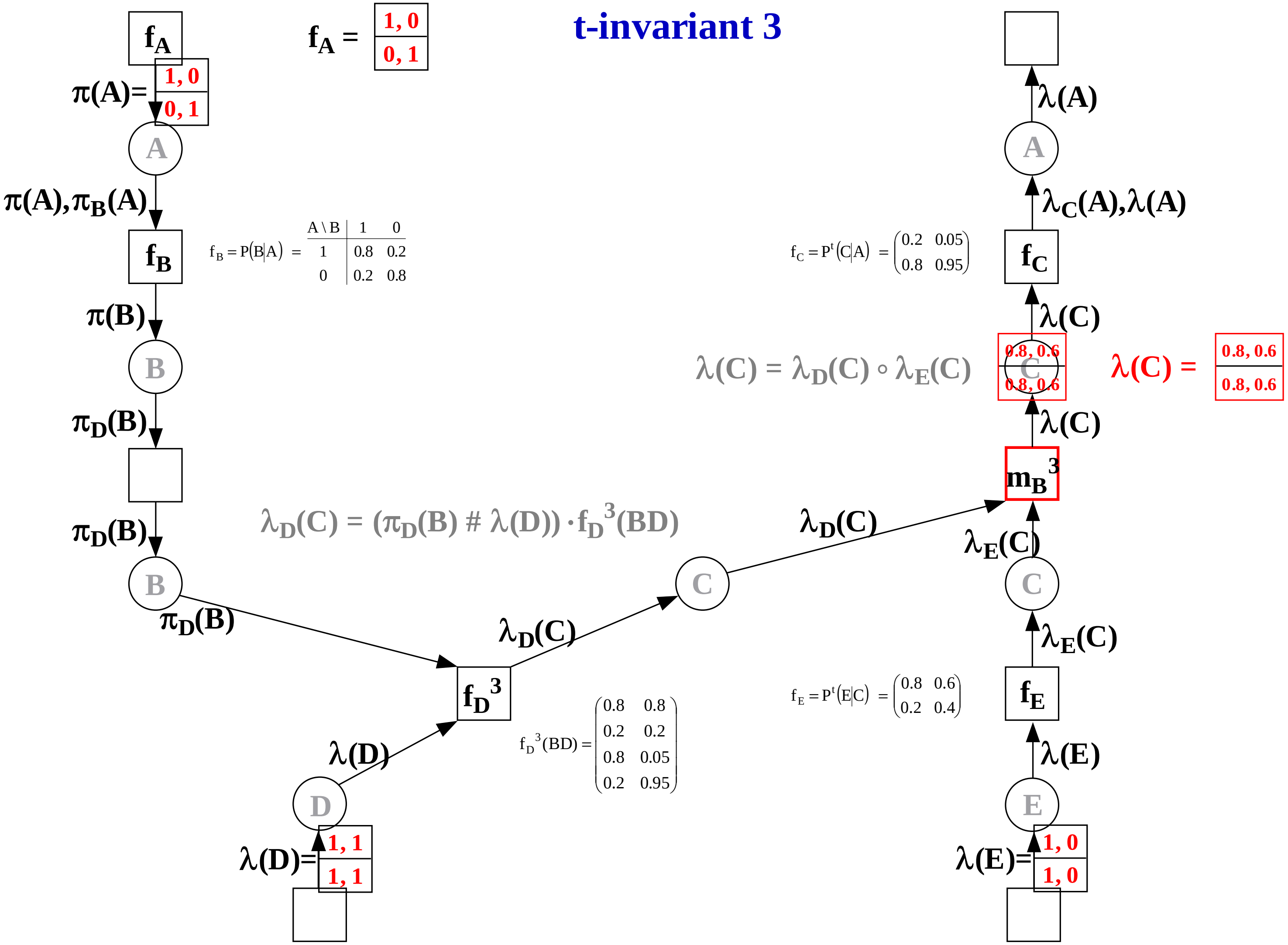
Conditioning (Pearl)



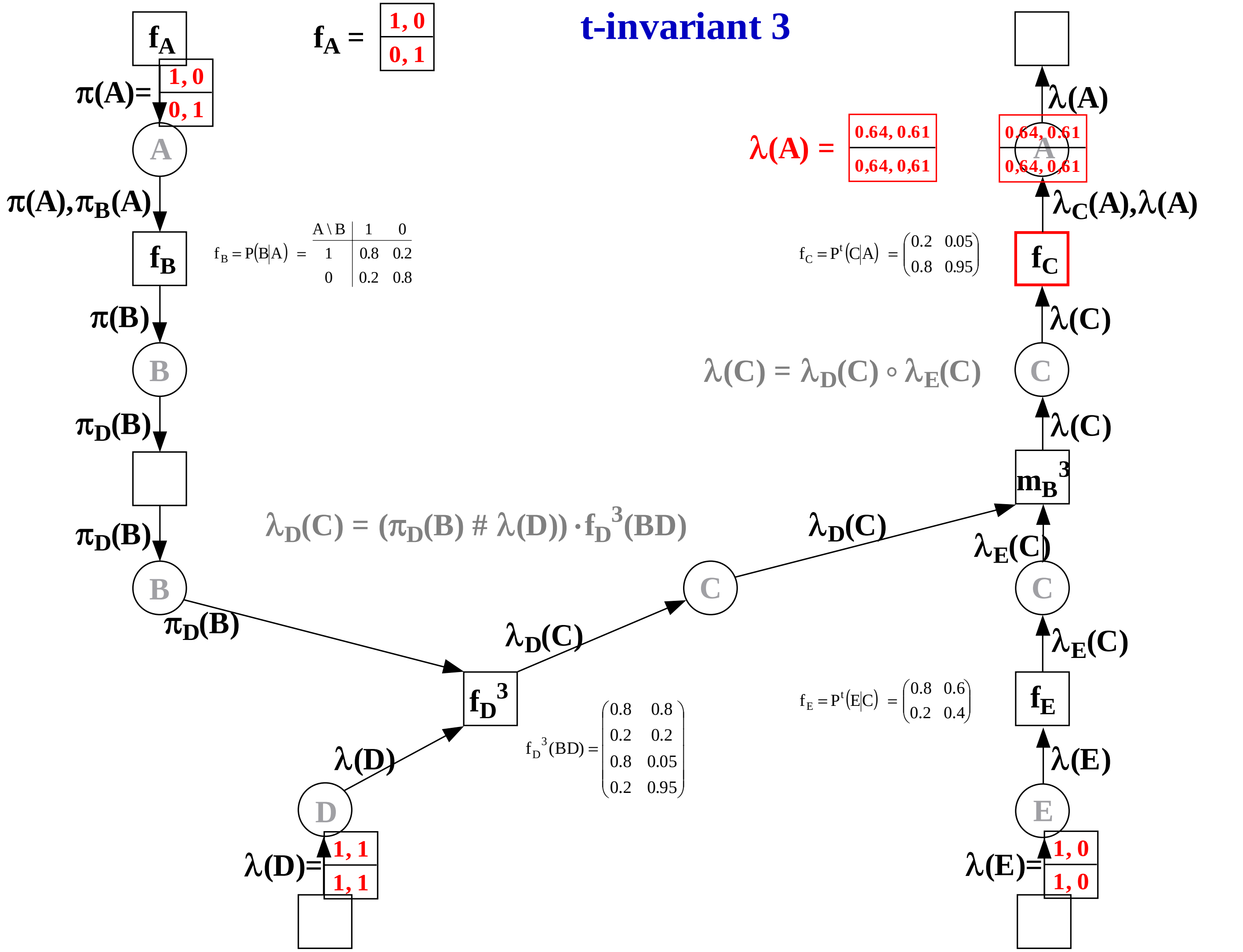
Conditioning (Pearl)



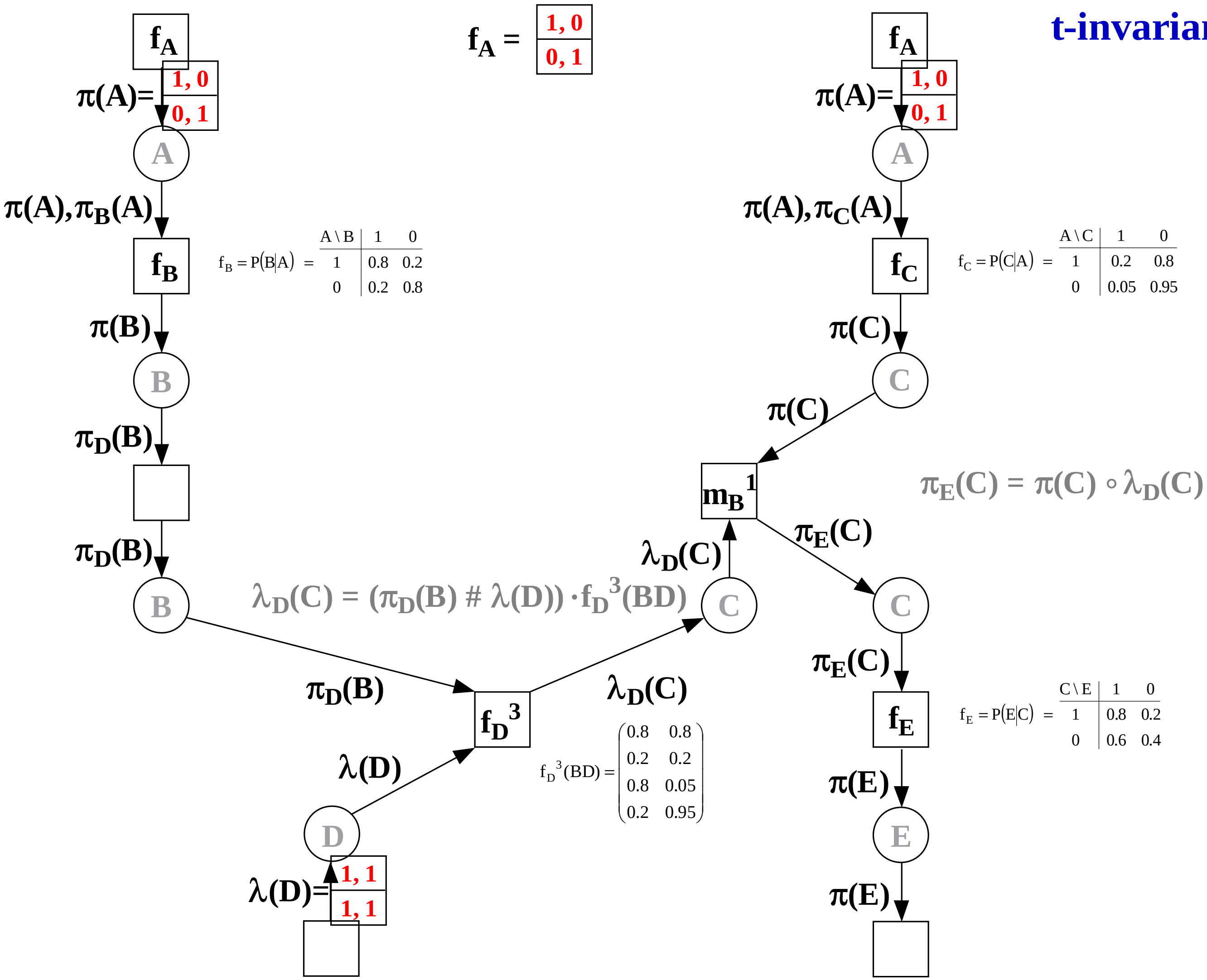
t-invariant 3



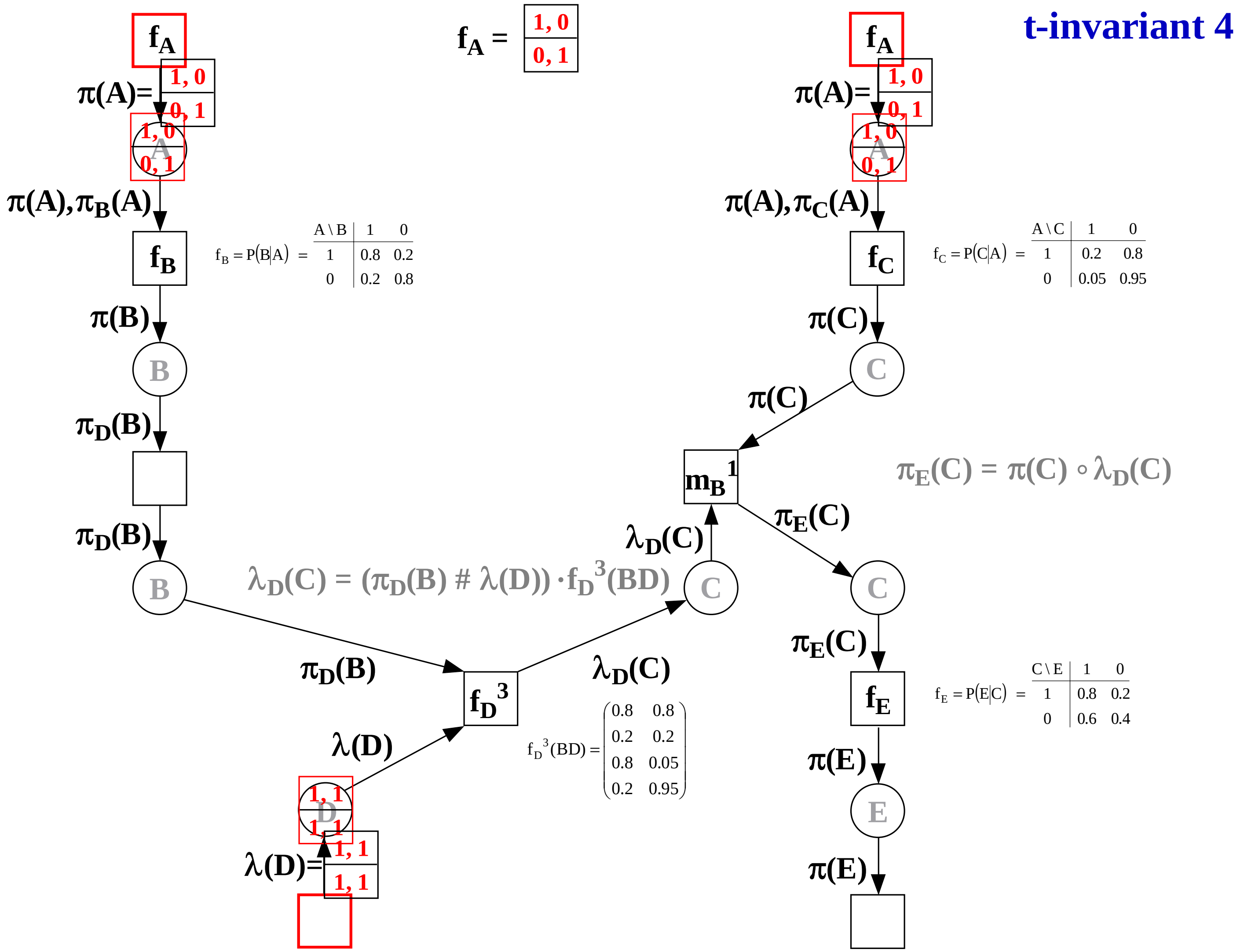
Conditioning (Pearl)



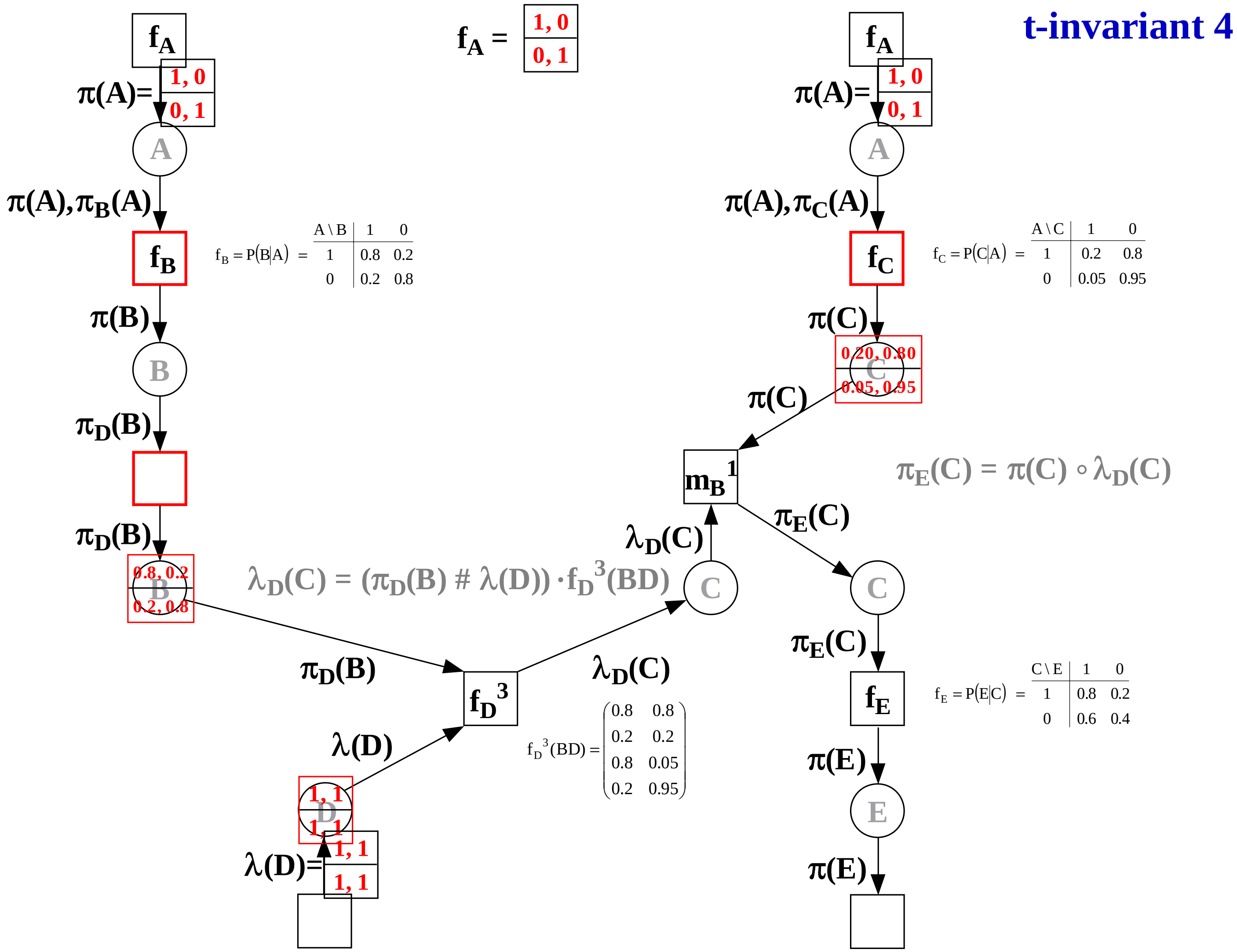
t-invariant 4



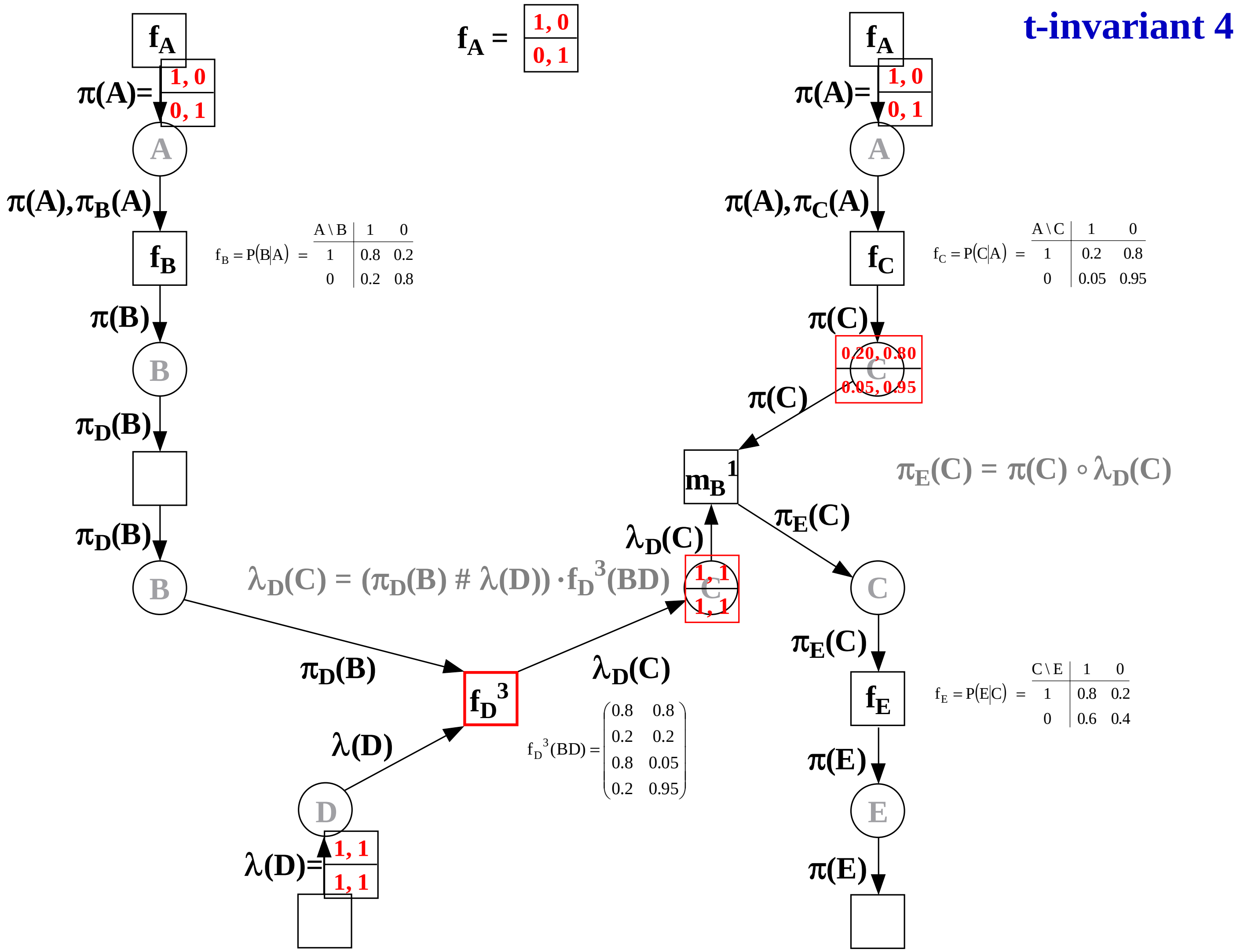
Conditioning (Pearl)

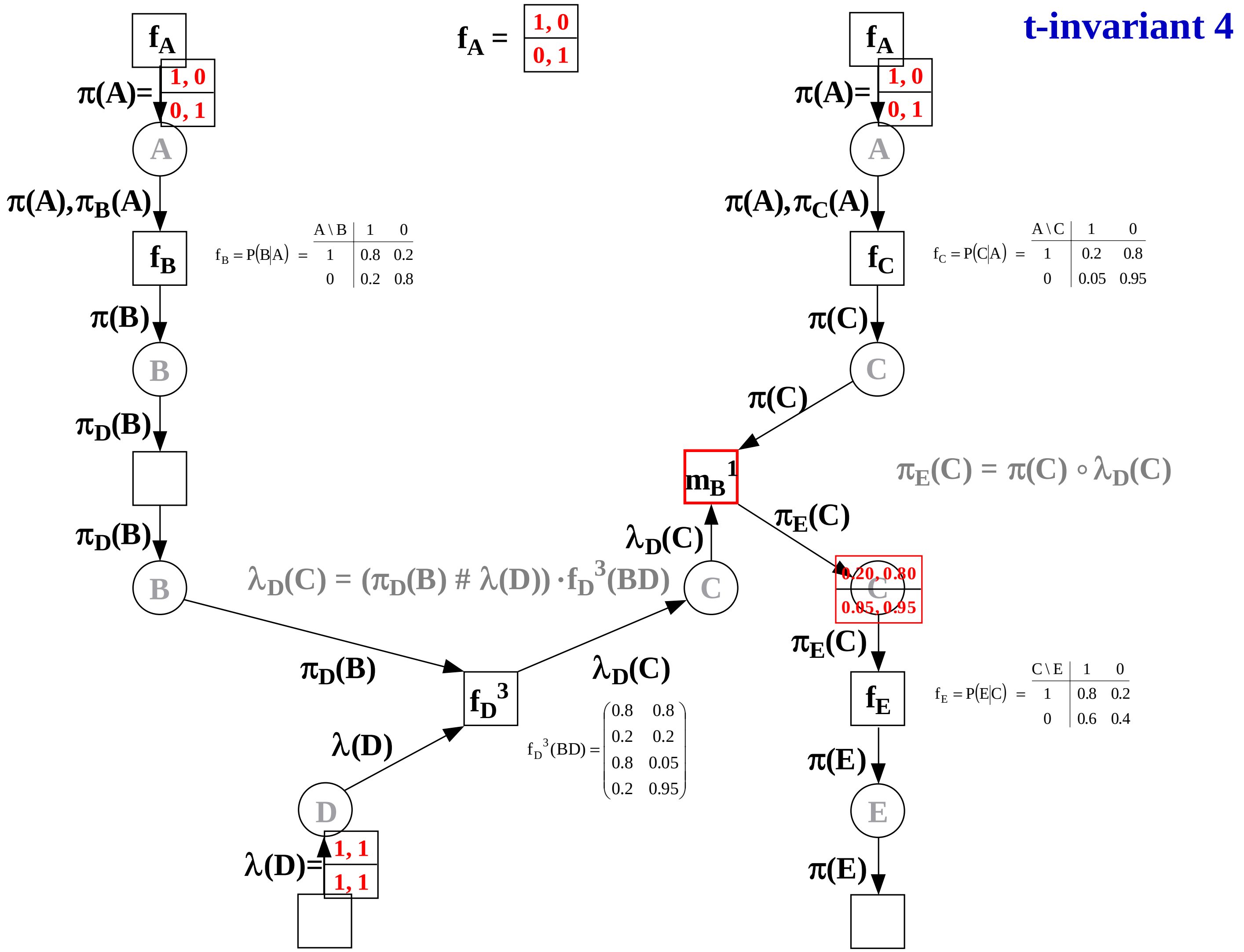


Conditioning (Pearl)

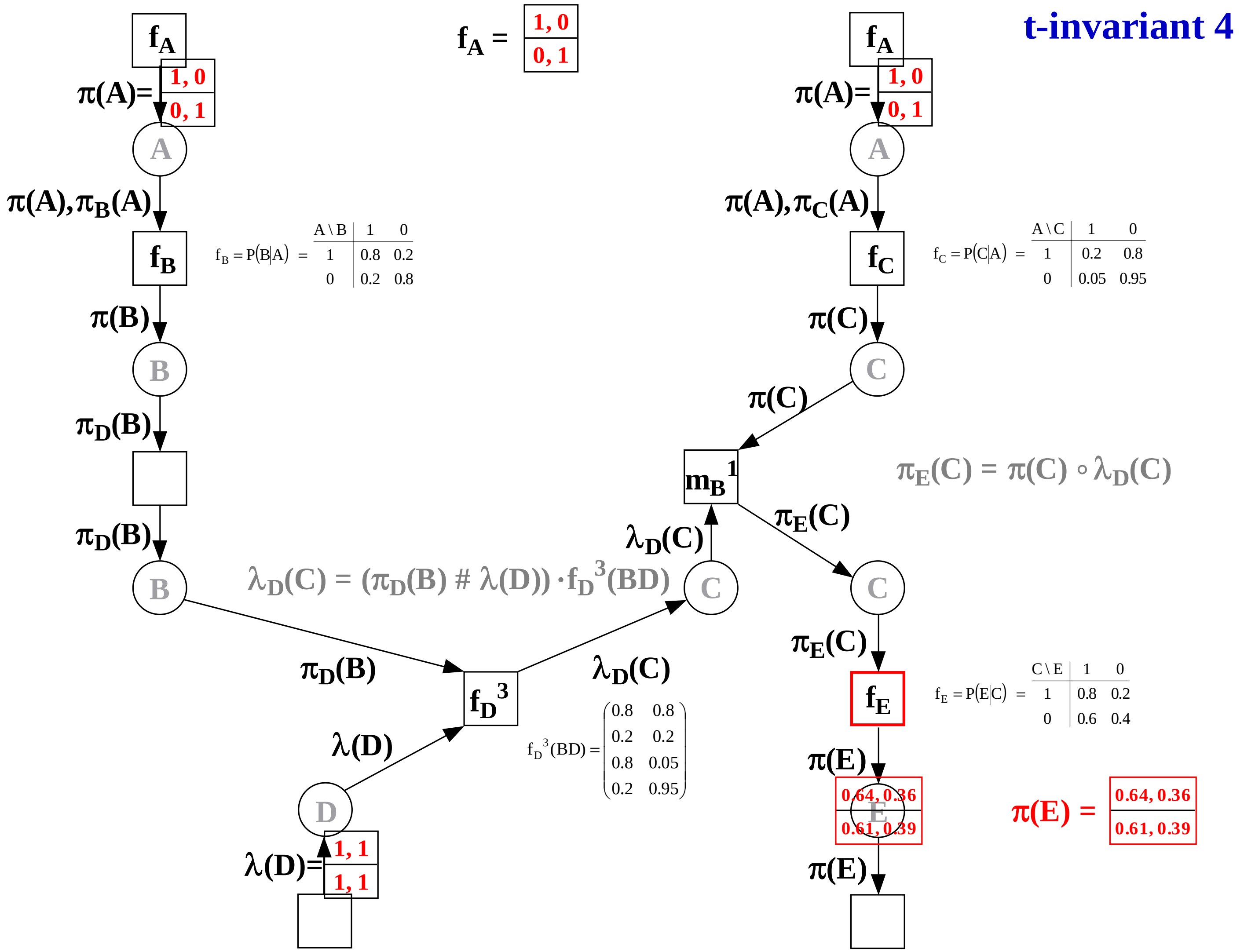


Conditioning (Pearl)





t-invariant 4
fA
π(A)=
1, 0
0, 1
A
π(A),πB(A)
fB
fB = P(B|A) =
A \ B 1 0
1 0.8 0.2
0 0.2 0.8
π(B)
B
πD(B)
πD(B)
B
λD(C) = (πD(B) # λ(D)) ·fD3(BD)
πD(B)
fD3
λ(D)
D
λ(D)=
1, 1
1, 1
λD(C)
fD3(BD) =
0.8 0.8
0.2 0.2
0.8 0.05
0.2 0.95
C
λD(C)
mB1
fA =
1, 0
0, 1
fA
π(A)=
1, 0
0, 1
A
π(A),πC(A)
fC
fC = P(C|A) =
A \ C 1 0
1 0.2 0.8
0 0.05 0.95
π(C)
C
π(C)
πE(C) = π(C) ∘ λD(C)
πE(C)
0.20, 0.80
0.05, 0.95
C
πE(C)
fE
fE = P(E|C) =
C \ E 1 0
1 0.8 0.2
0 0.6 0.4
π(E)
E
π(E)


t-invariant 4
f_A = 1, 0 / 0, 1
f_A
π(A)= 1, 0 / 0, 1
A
π(A),$π_B$(A)
f_B
$f_B = P(B|A)$ = A\B 1 0; 1 0.8 0.2; 0 0.2 0.8
π(B)
B
$π_D$(B)
$π_D$(B)
B
$λ_D(C) = (π_D(B) \# λ(D)) \cdot f_D^3(BD)$
$π_D$(B)
f_D^3
$f_D^3(BD)$ = 0.8 0.8; 0.2 0.2; 0.8 0.05; 0.2 0.95
λ(D)
D
λ(D)= 1, 1 / 1, 1
$λ_D$(C)
C
$λ_D$(C)
m_B^1
f_A
π(A)= 1, 0 / 0, 1
A
π(A),$π_C$(A)
f_C
$f_C = P(C|A)$ = A\C 1 0; 1 0.2 0.8; 0 0.05 0.95
π(C)
C
π(C)
$π_E(C) = π(C) \circ λ_D(C)$
$π_E$(C)
C
$π_E$(C)
f_E
$f_E = P(E|C)$ = C\E 1 0; 1 0.8 0.2; 0 0.6 0.4
π(E)
E 0.64, 0.36 / 0.61, 0.39
π(E)
π(E) = 0.64, 0.36 / 0.61, 0.39

Conditioning (Pearl)

Probabilities: $\pi(E) = \begin{array}{|c|}\hline 0.64, 0.36 \\ \hline 0.61, 0.39 \\ \hline\end{array}$

$$P(A \mid E=1) = \alpha\, P(E=1 \mid A) \cdot P(A) = \alpha\, (0.64 \cdot 0.2,\ 0.61 \cdot 0.8) = (0.208,\ 0.792)$$

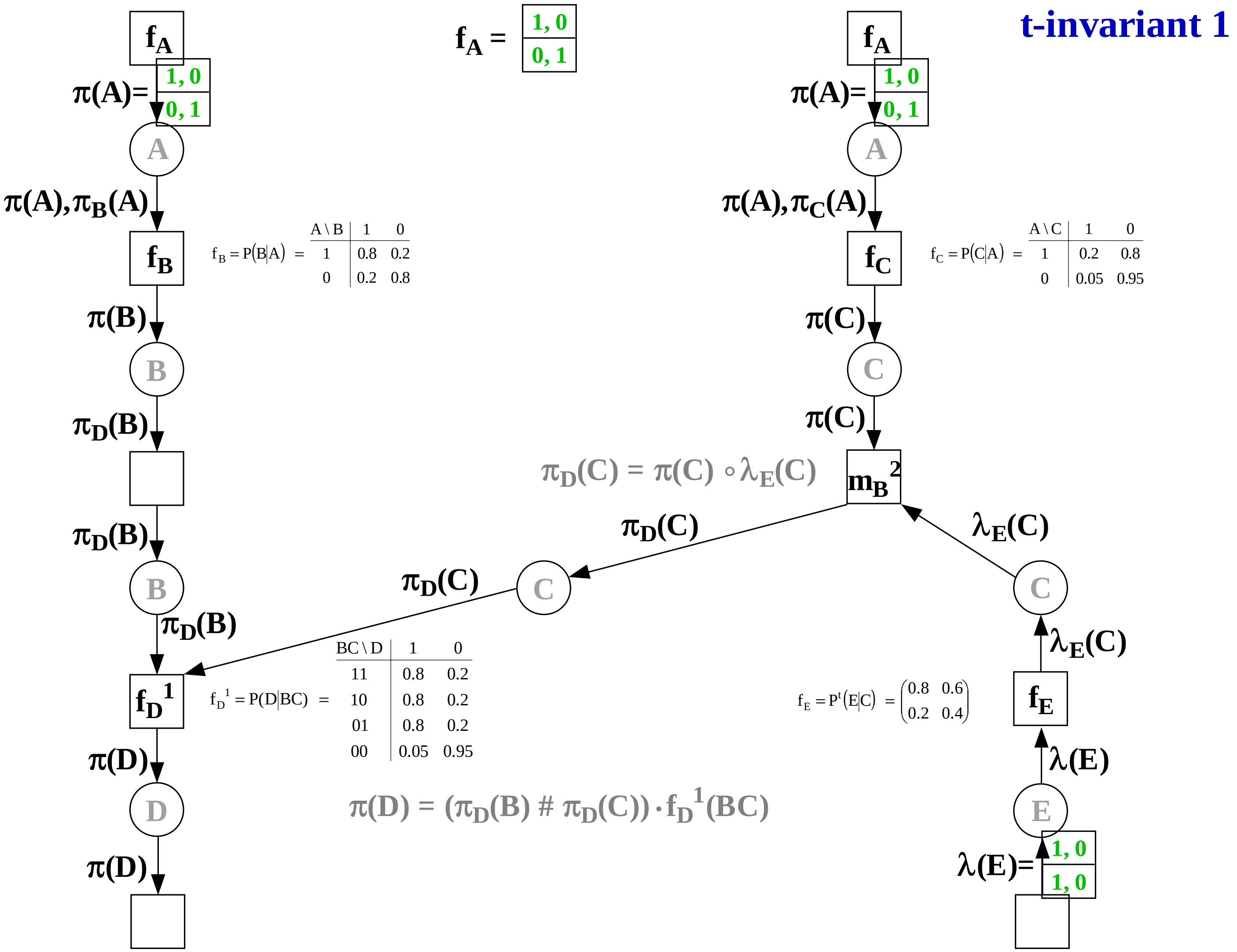
$\pi(B) = \begin{array}{|c|}\hline 0.8, 0.2 \\ \hline 0.2, 0.8 \\ \hline\end{array}$ $\lambda(B) = \begin{array}{|c|}\hline 1, 1 \\ \hline 1, 1 \\ \hline\end{array}$

$$P(B) = BEL(B) = ((0.8,\ 0.2) \circ (1,\ 1)) \cdot 0.208 + ((0.2,\ 0.8) \circ (1,\ 1)) \cdot 0.792$$
$$= \boxed{(0.3248,\ 0.6752)}$$

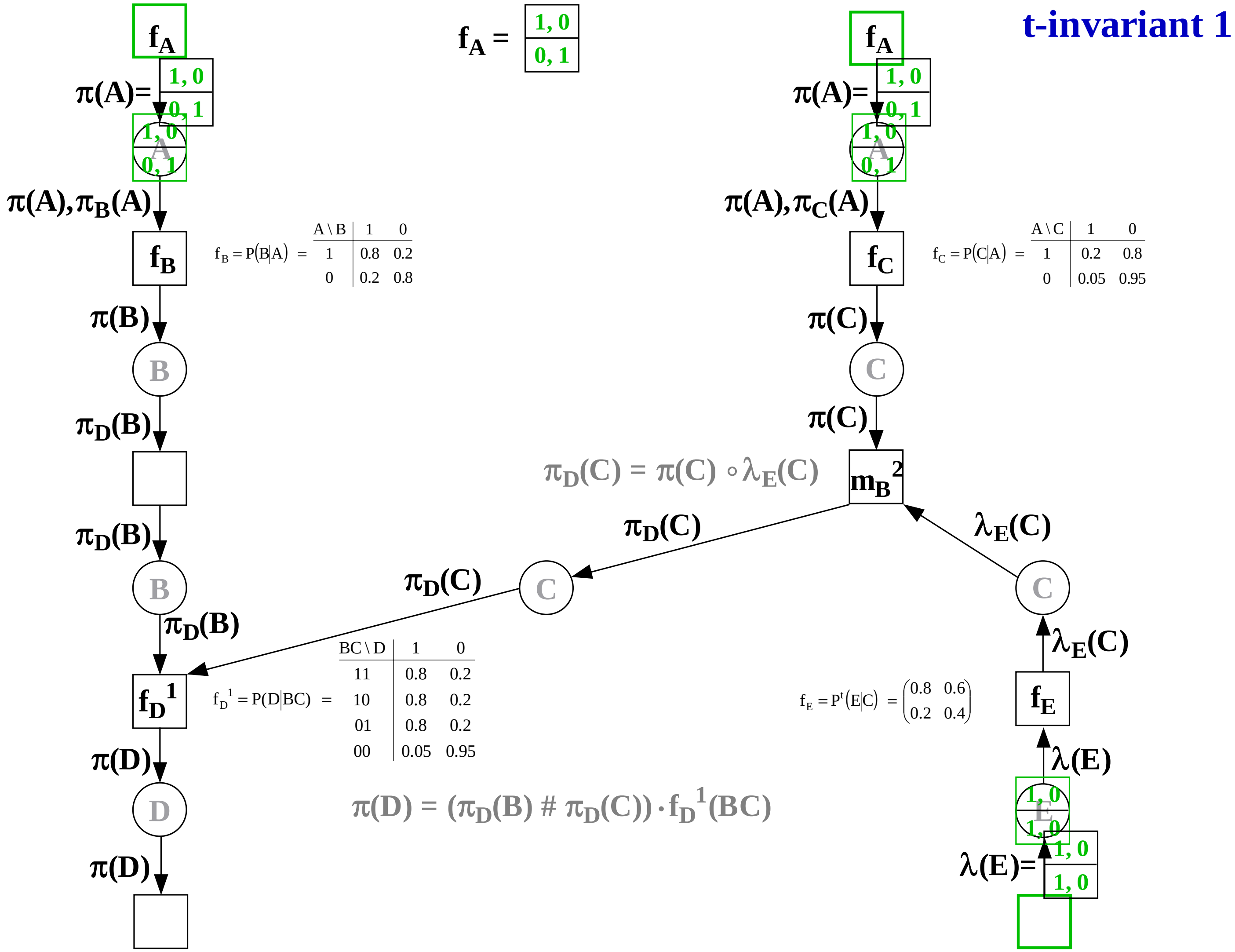
$\pi(C) = \begin{array}{|c|}\hline 0.20, 0.80 \\ \hline 0.05, 0.95 \\ \hline\end{array}$ $\lambda(C) = \begin{array}{|c|}\hline 0.8, 0.6 \\ \hline 0.8, 0.6 \\ \hline\end{array}$

$$P(C) = BEL(C) = ((0.2,\ 0.8) \circ (0.8,\ 0.6)) \cdot 0.208 + ((0.05,\ 0.95) \circ (0.8,\ 0.6)) \cdot 0.792$$
$$= \boxed{(0.104,\ 0.896)}$$

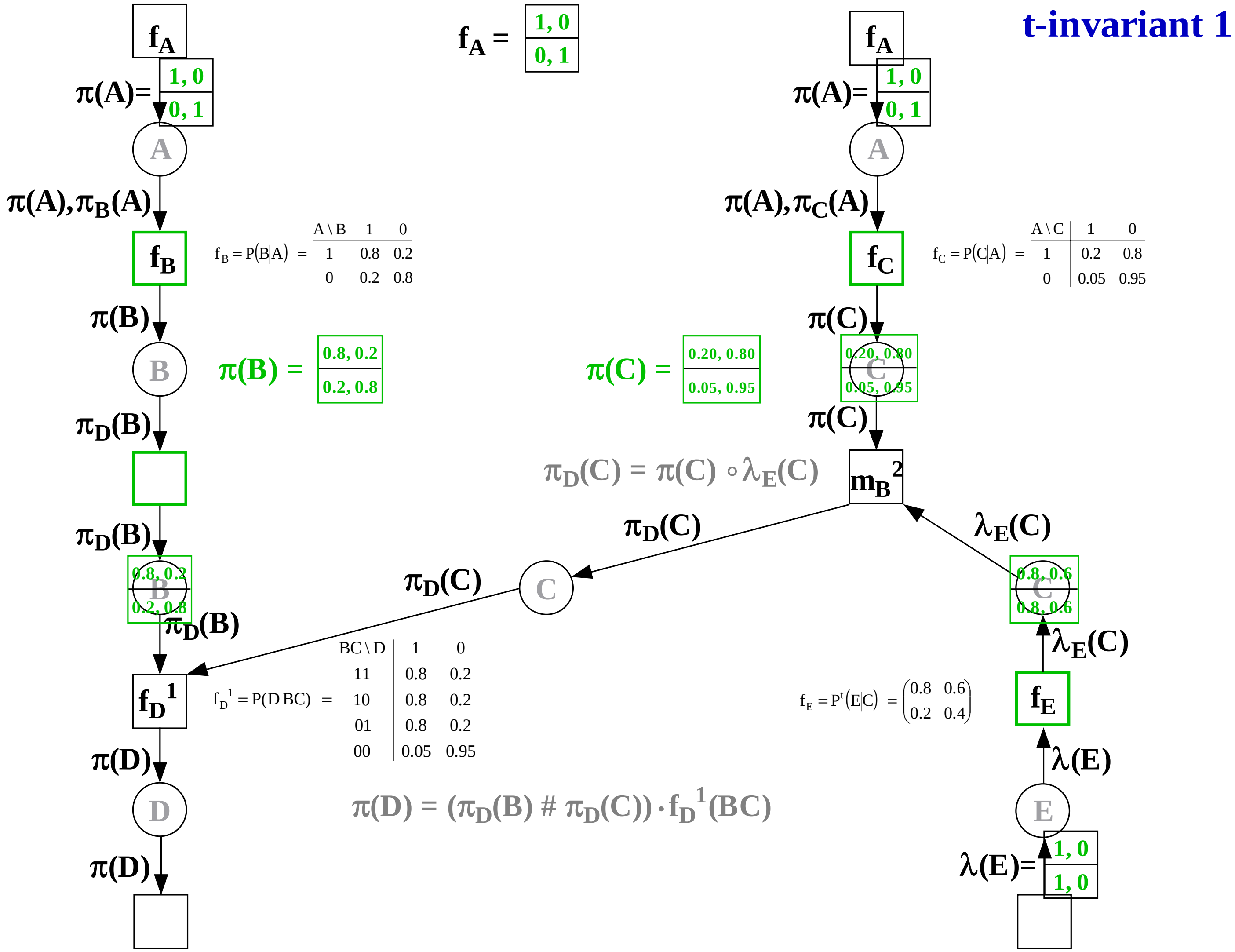
Conditioning (Pearl)



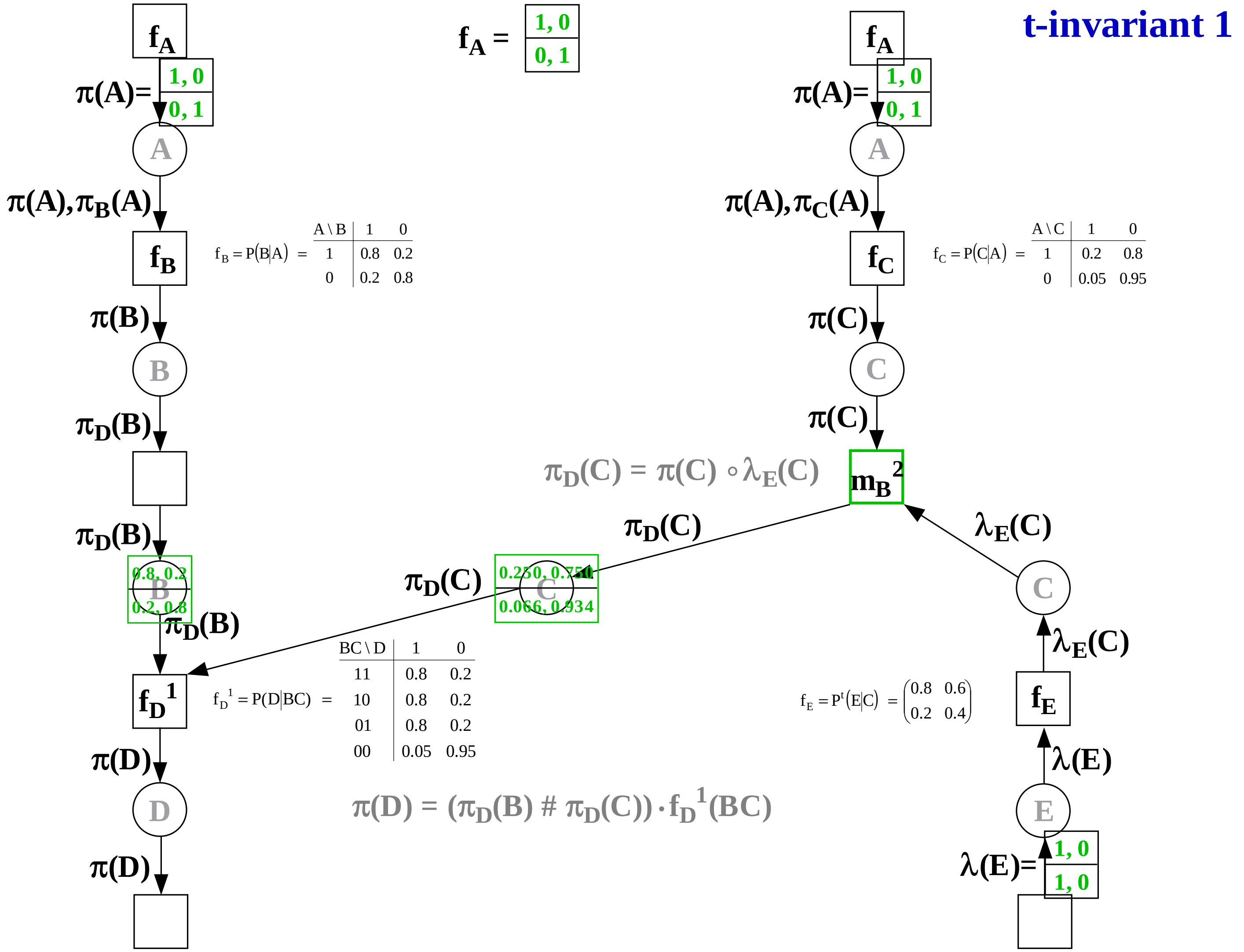
Conditioning (Pearl)



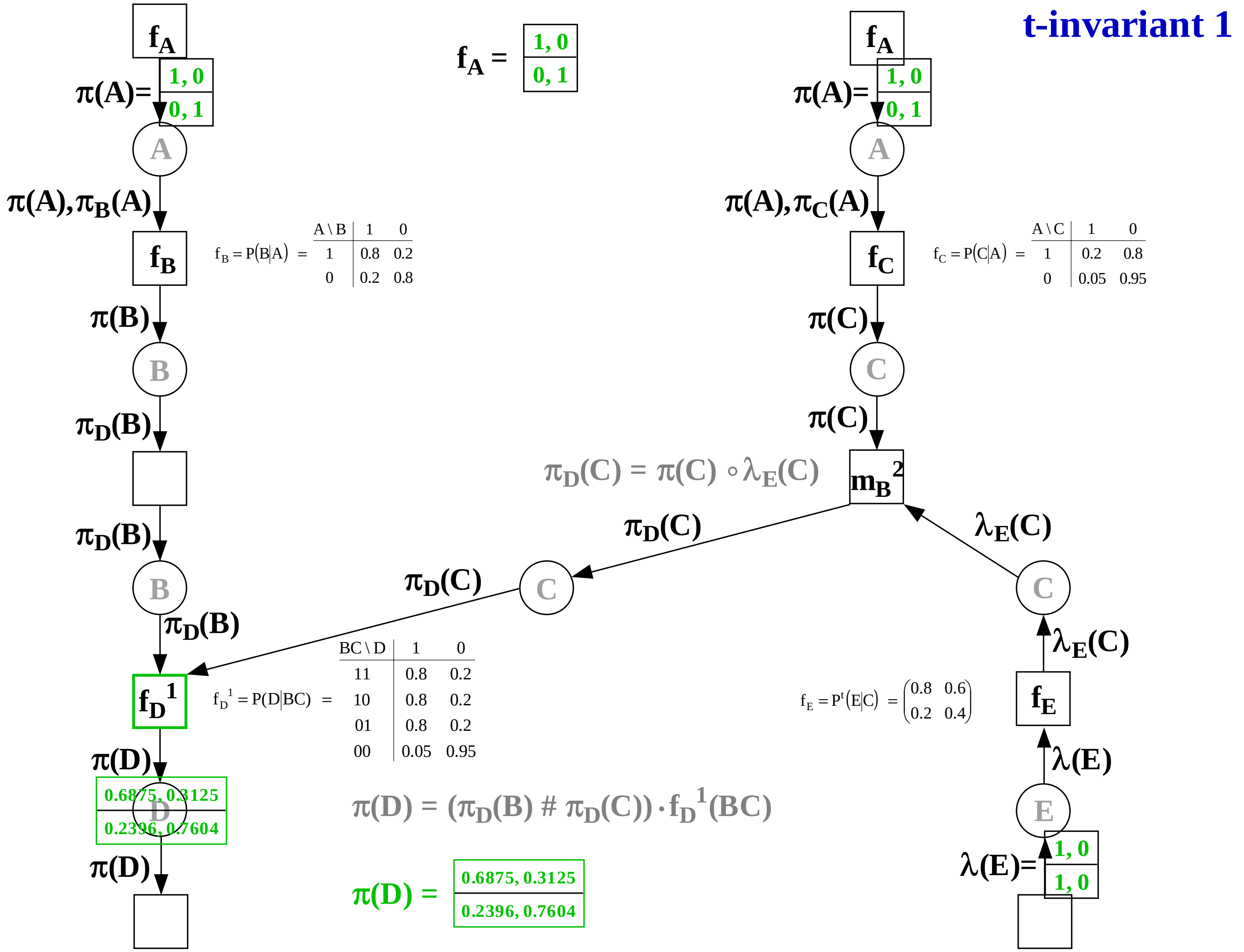
Conditioning (Pearl)



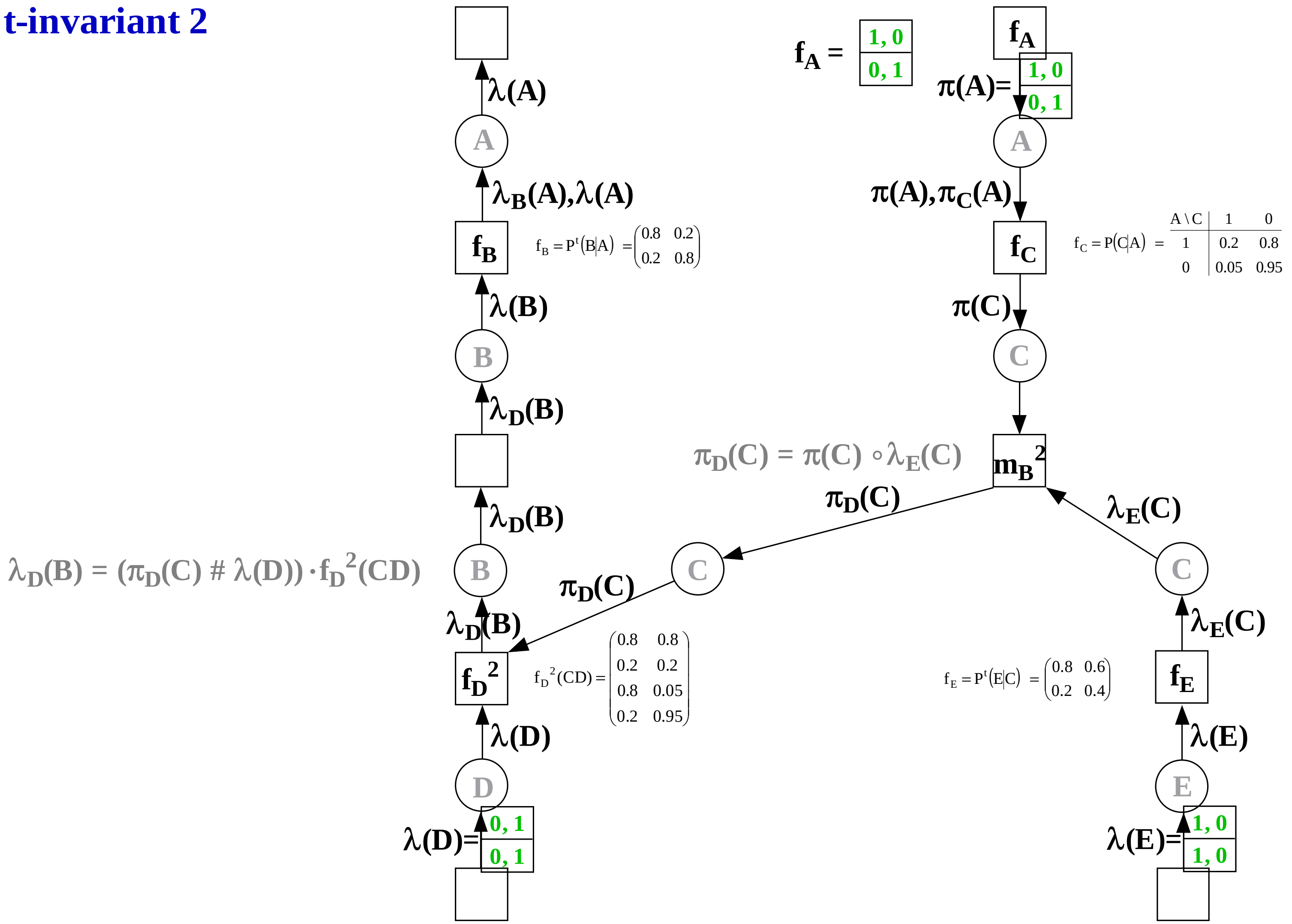
Conditioning (Pearl)



Conditioning (Pearl)

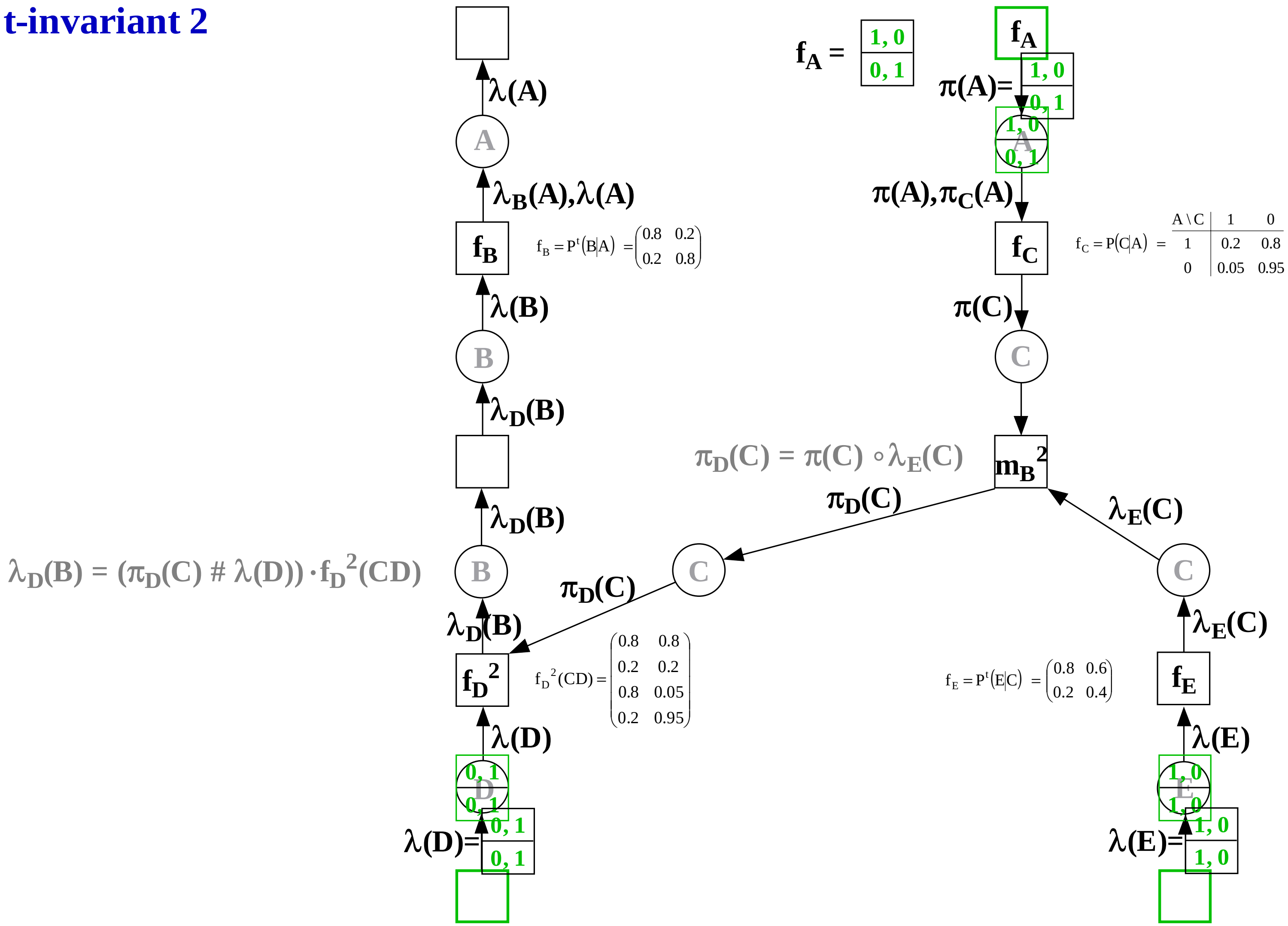


t-invariant 2



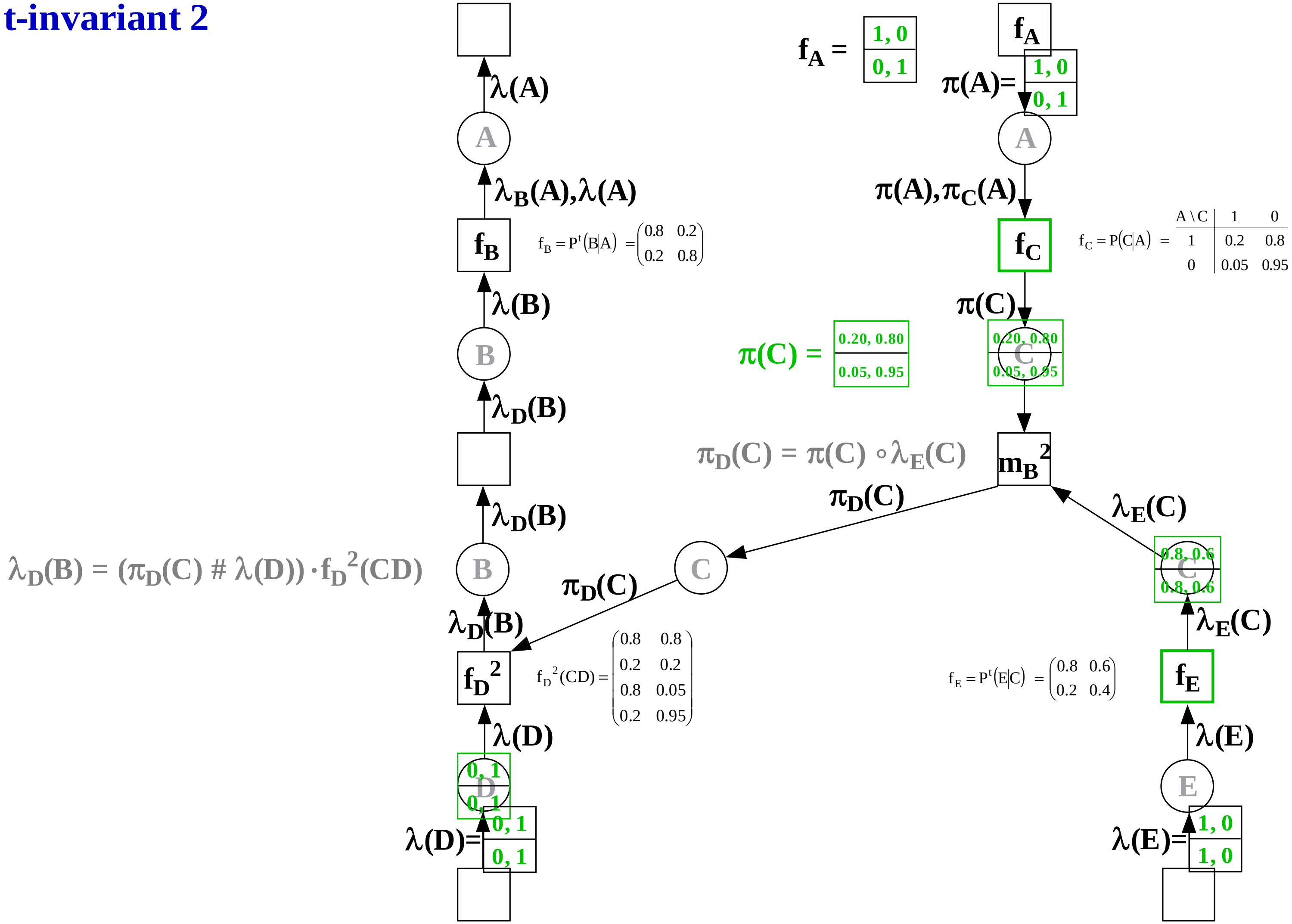
Conditioning (Pearl)

t-invariant 2

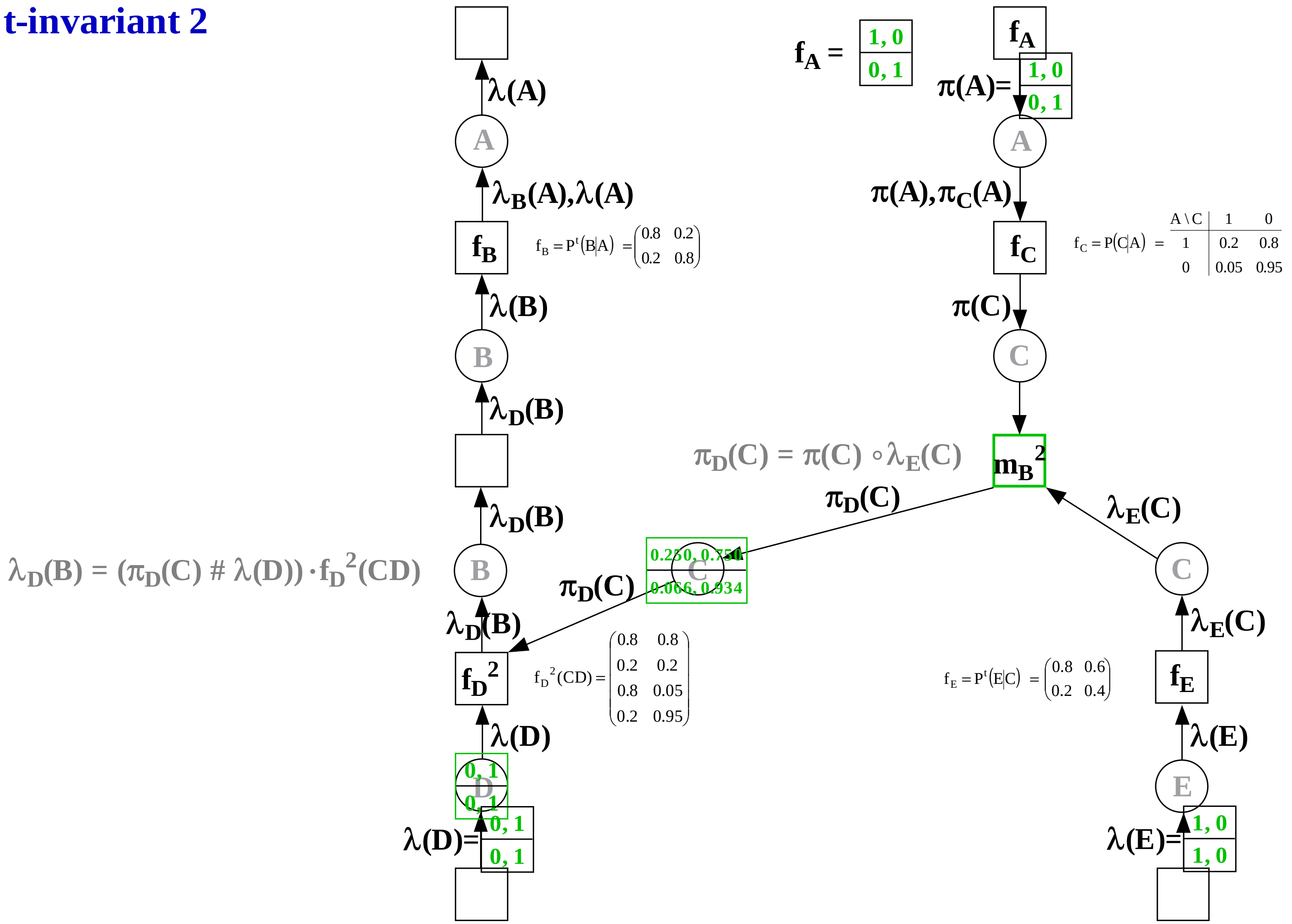


Conditioning (Pearl)

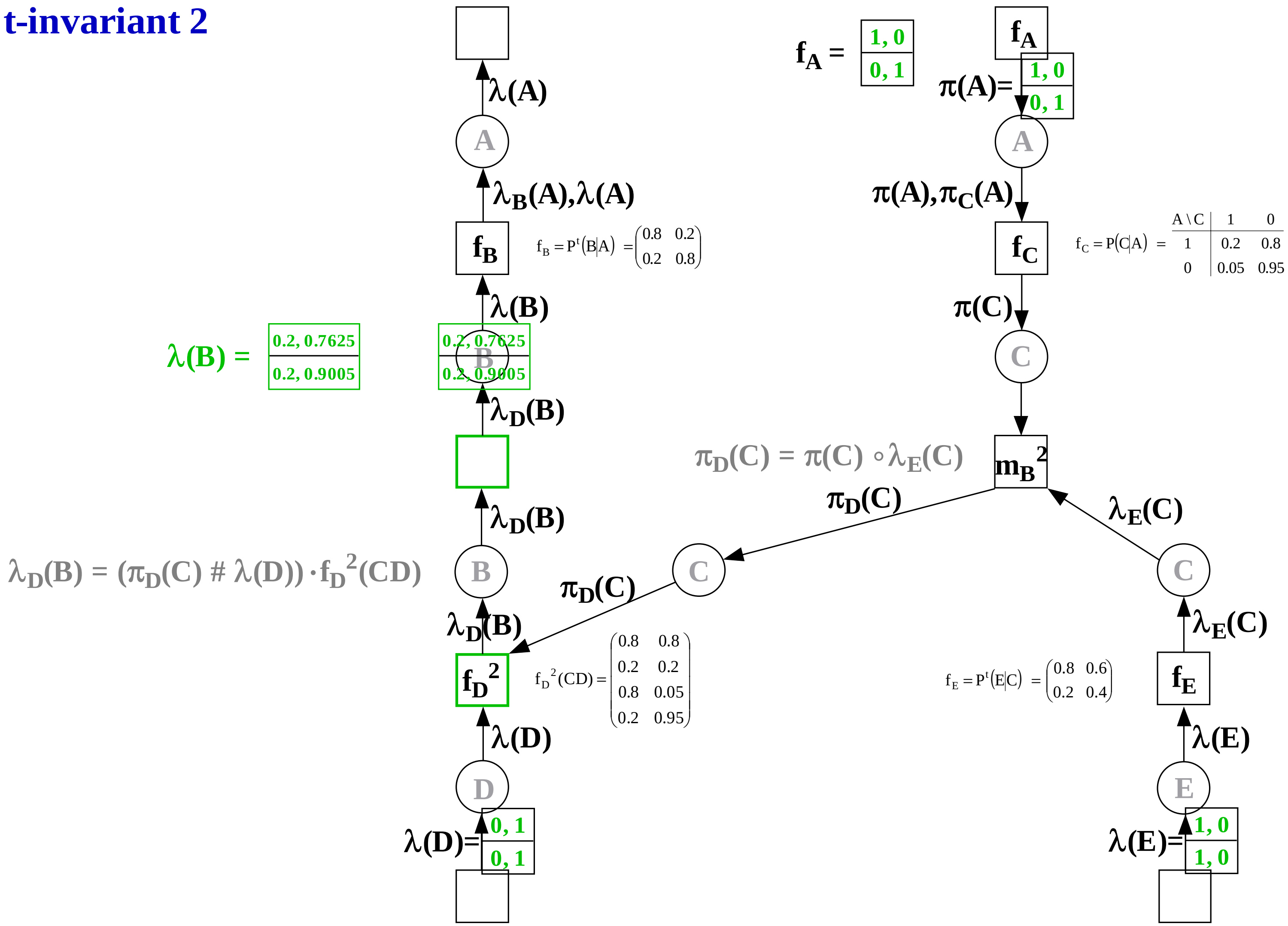
t-invariant 2



t-invariant 2

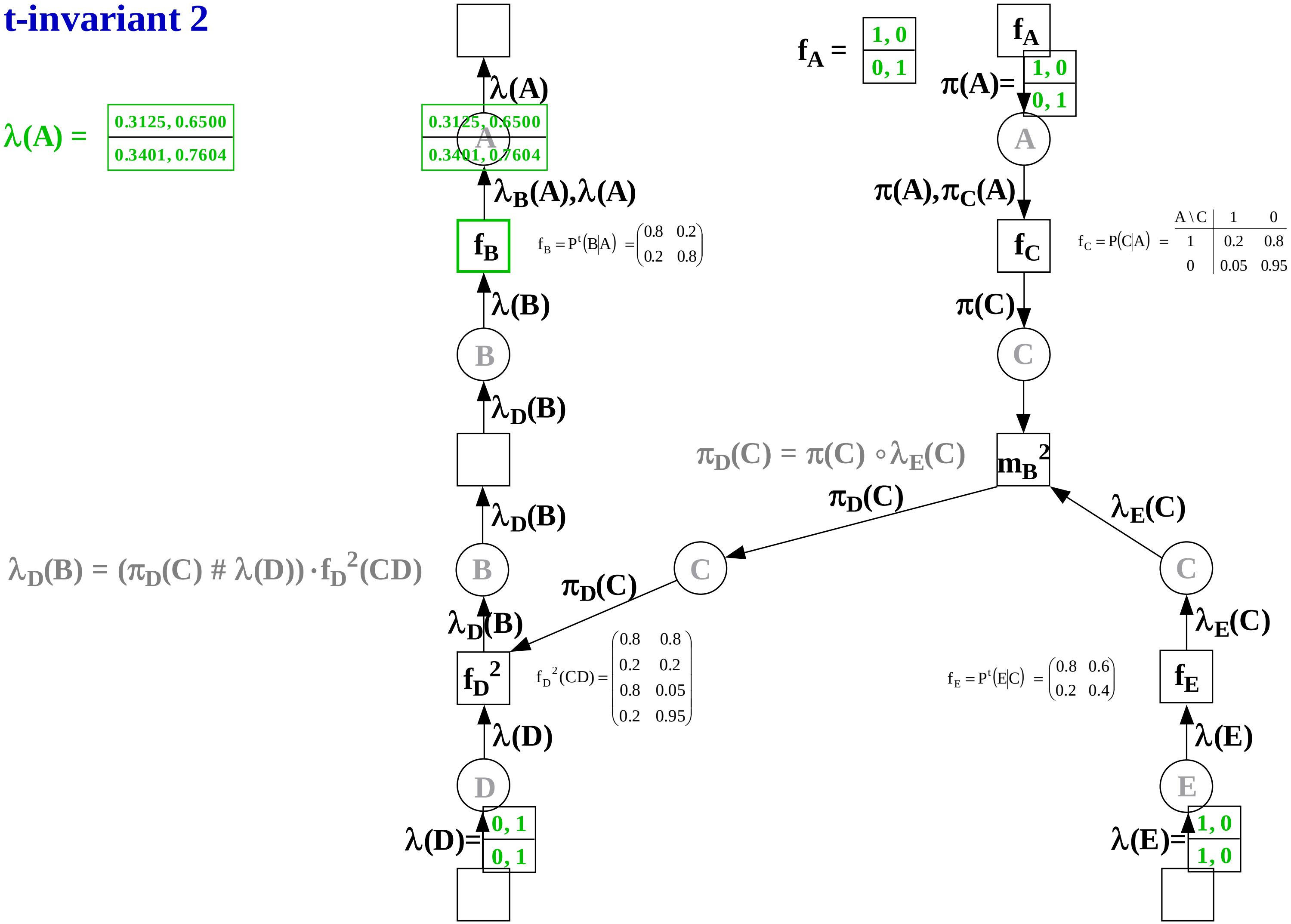


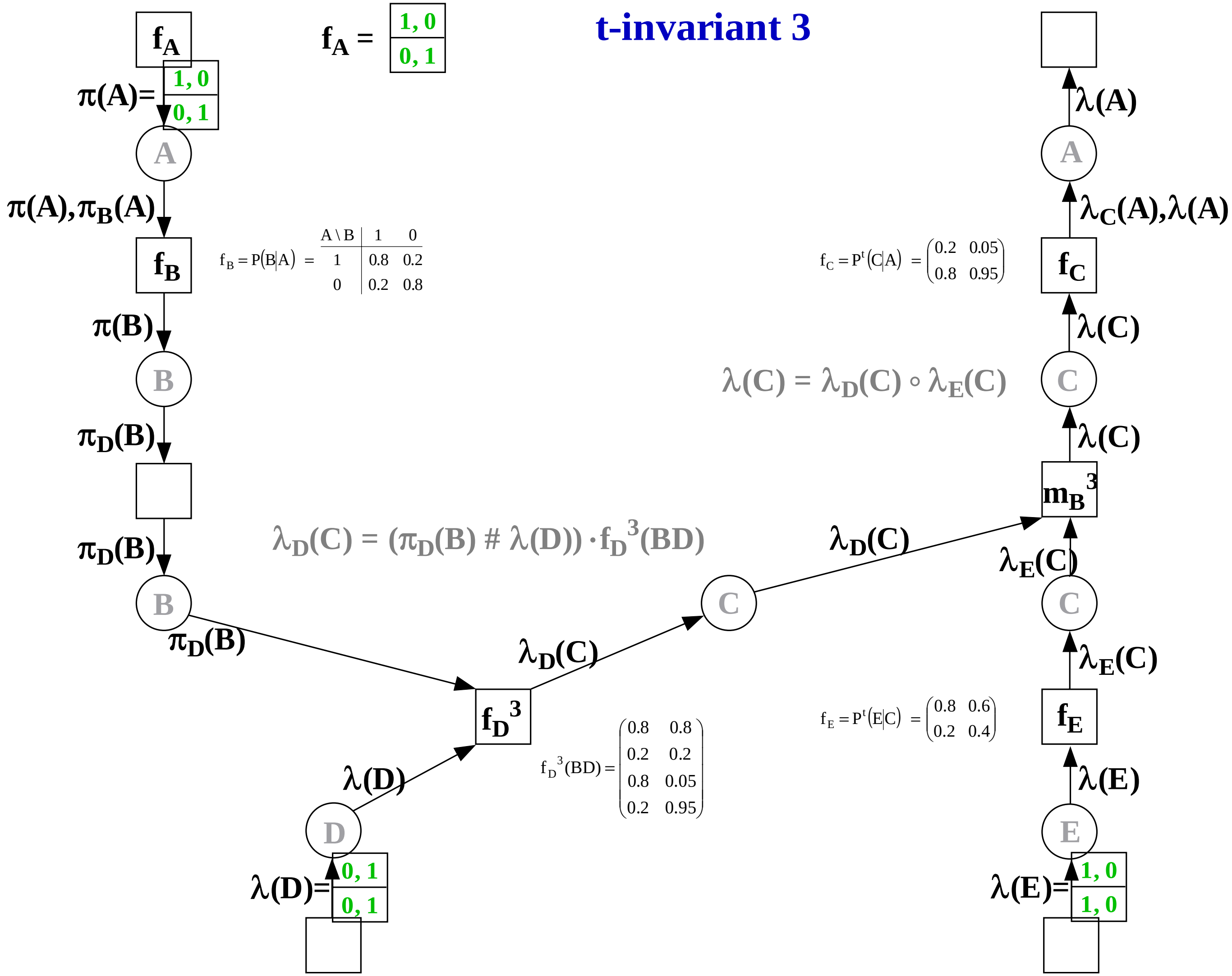
t-invariant 2



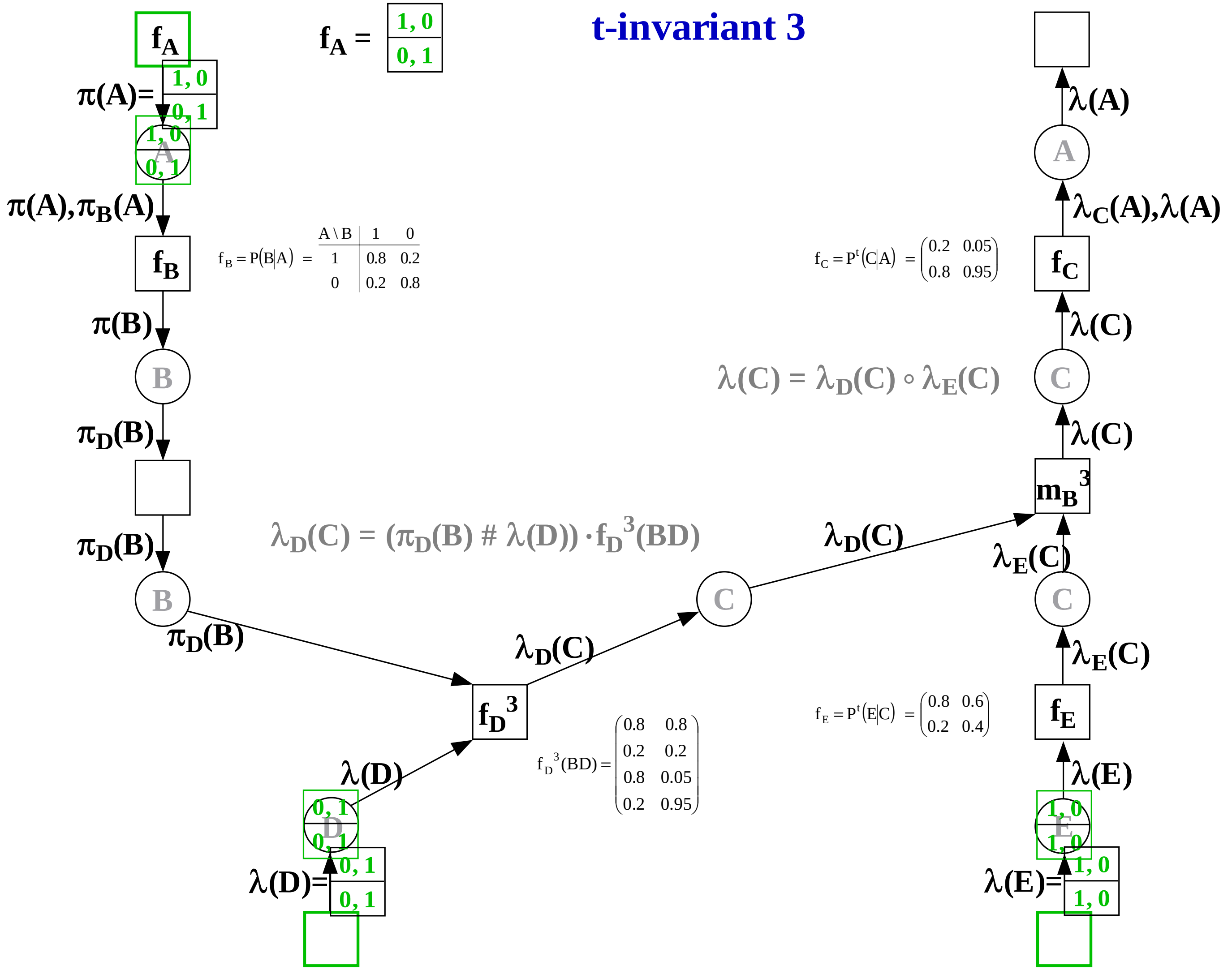
Conditioning (Pearl)

t-invariant 2





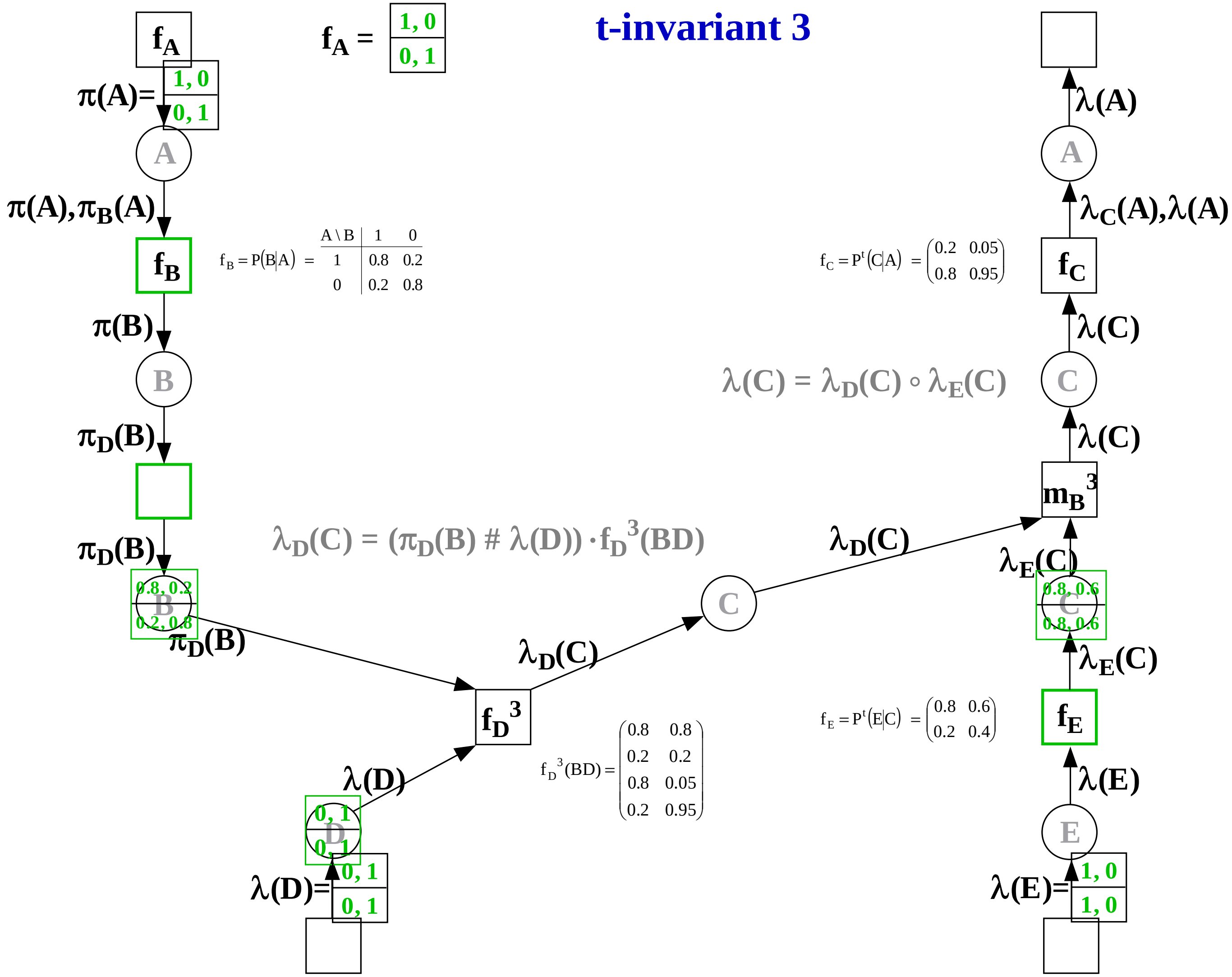
t-invariant 3
f_A
f_A = 1, 0 / 0, 1
$\pi(A)$= 1, 0 / 0, 1
A
$\pi(A), \pi_B(A)$
f_B
$f_B = P(B|A)$ = A \ B: 1, 0; 1: 0.8, 0.2; 0: 0.2, 0.8
$\pi(B)$
B
$\pi_D(B)$
$\pi_D(B)$
B
$\pi_D(B)$
$\lambda_D(C) = (\pi_D(B) \# \lambda(D)) \cdot f_D^3(BD)$
f_D^3
$\lambda_D(C)$
$f_D^3(BD) = \begin{pmatrix} 0.8 & 0.8 \\ 0.2 & 0.2 \\ 0.8 & 0.05 \\ 0.2 & 0.95 \end{pmatrix}$
$\lambda(D)$
D
$\lambda(D)$= 0, 1 / 0, 1
C
$\lambda_D(C)$
m_B^3
$\lambda_E(C)$
C
$\lambda_E(C)$
f_E
$f_E = P^t(E|C) = \begin{pmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{pmatrix}$
$\lambda(E)$
E
$\lambda(E)$= 1, 0 / 1, 0
$\lambda(C)$
C
$\lambda(C) = \lambda_D(C) \circ \lambda_E(C)$
$\lambda(C)$
f_C
$f_C = P^t(C|A) = \begin{pmatrix} 0.2 & 0.05 \\ 0.8 & 0.95 \end{pmatrix}$
$\lambda_C(A), \lambda(A)$
A
$\lambda(A)$



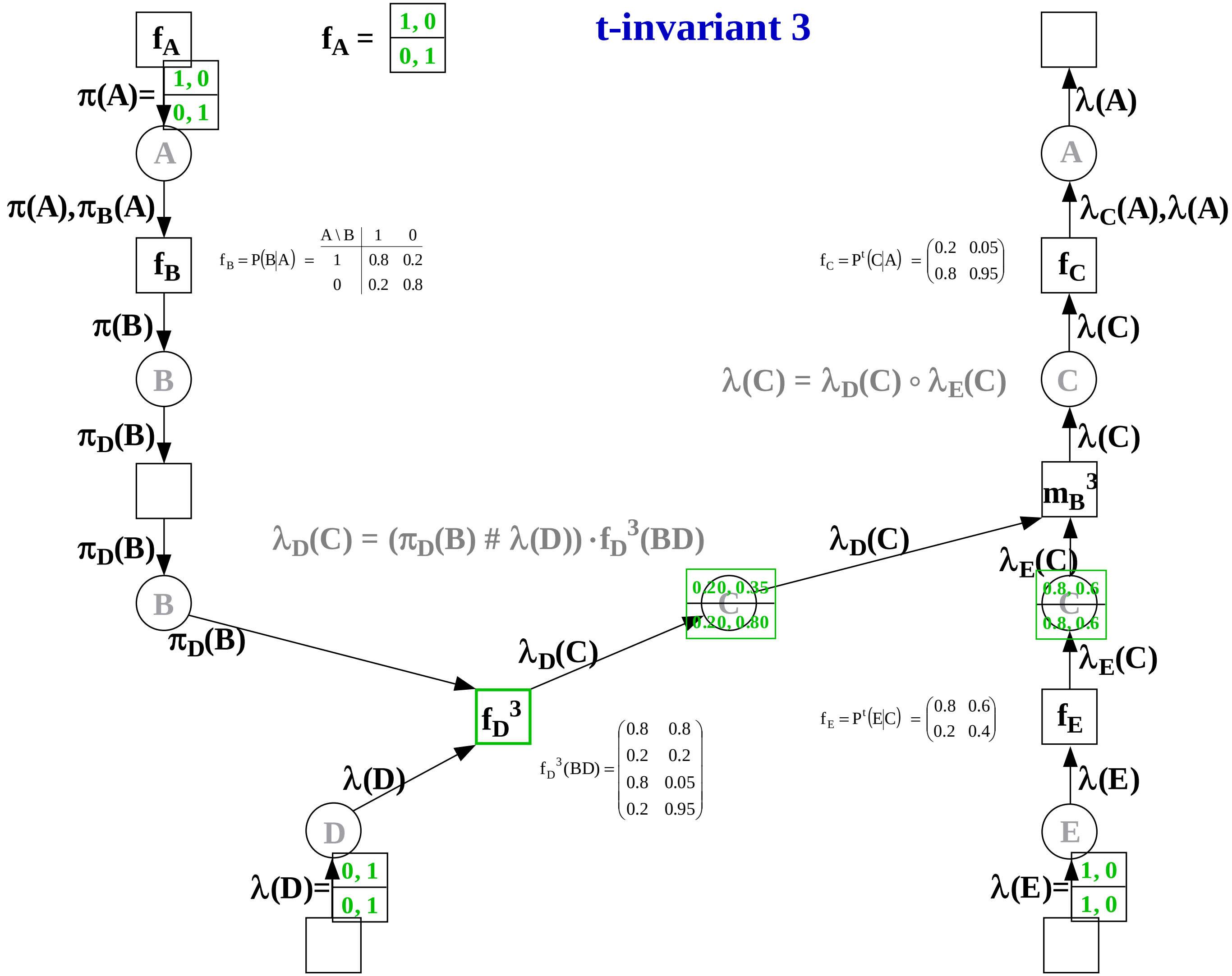
t-invariant 3
fA
fA =
1, 0
0, 1
π(A)=
1, 0
0, 1
A
π(A),πB(A)
fB
fB = P(B|A) =
A \ B 1 0
1 0.8 0.2
0 0.2 0.8
π(B)
B
πD(B)
πD(B)
B
πD(B)
λD(C) = (πD(B) # λ(D)) ·fD3(BD)
fD3
λD(C)
C
λD(C)
λ(D)
fD3(BD) =
0.8 0.8
0.2 0.2
0.8 0.05
0.2 0.95
0, 1
0, 1
D
λ(D)=
0, 1
0, 1
λ(A)
A
λC(A),λ(A)
fC
fC = Pt(C|A) =
0.2 0.05
0.8 0.95
λ(C)
C
λ(C) = λD(C) ∘ λE(C)
λ(C)
mB3
λE(C)
C
λE(C)
fE
fE = Pt(E|C) =
0.8 0.6
0.2 0.4
λ(E)
1, 0
1, 0
E
λ(E)=
1, 0
1, 0

Conditioning (Pearl)

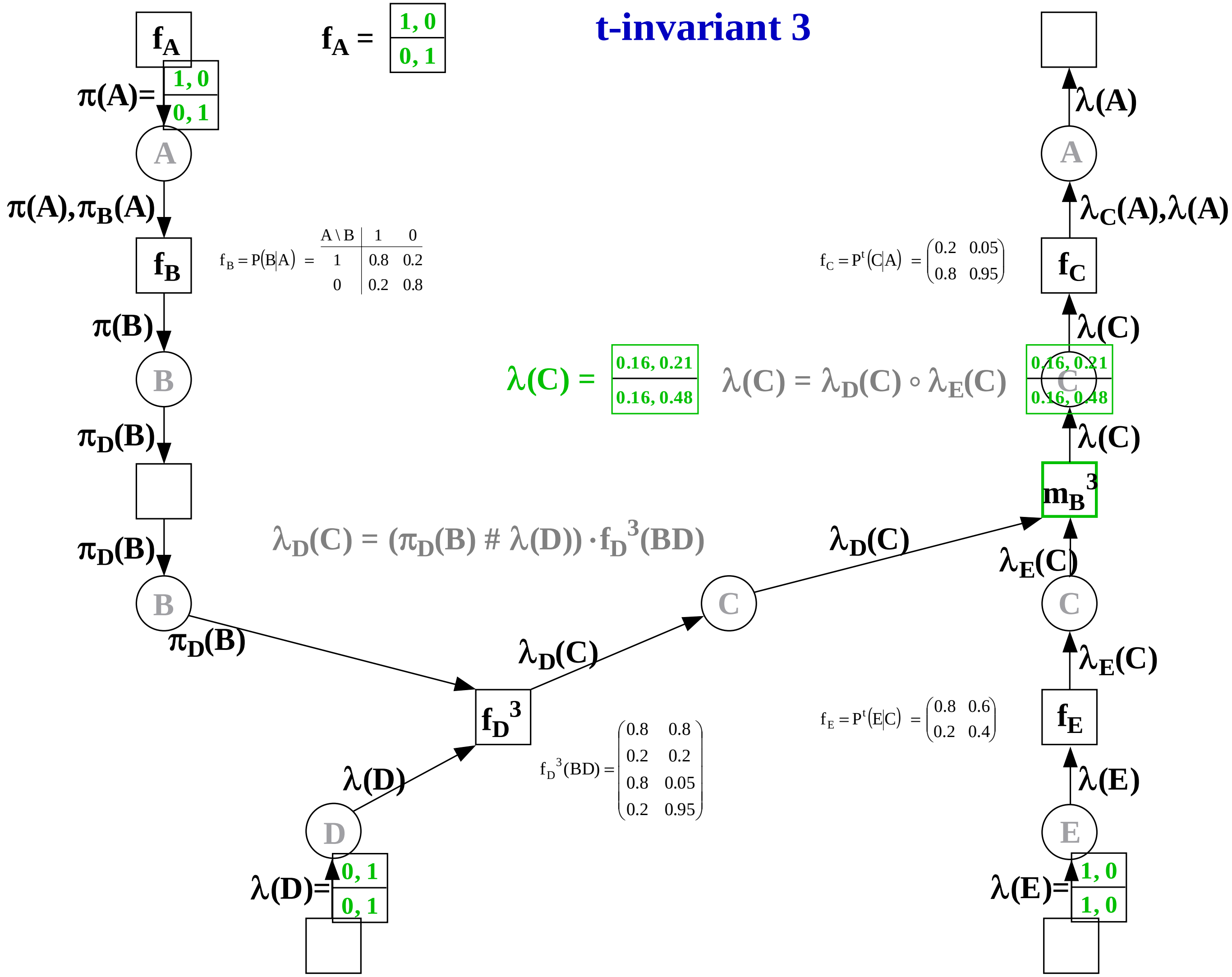
t-invariant 3

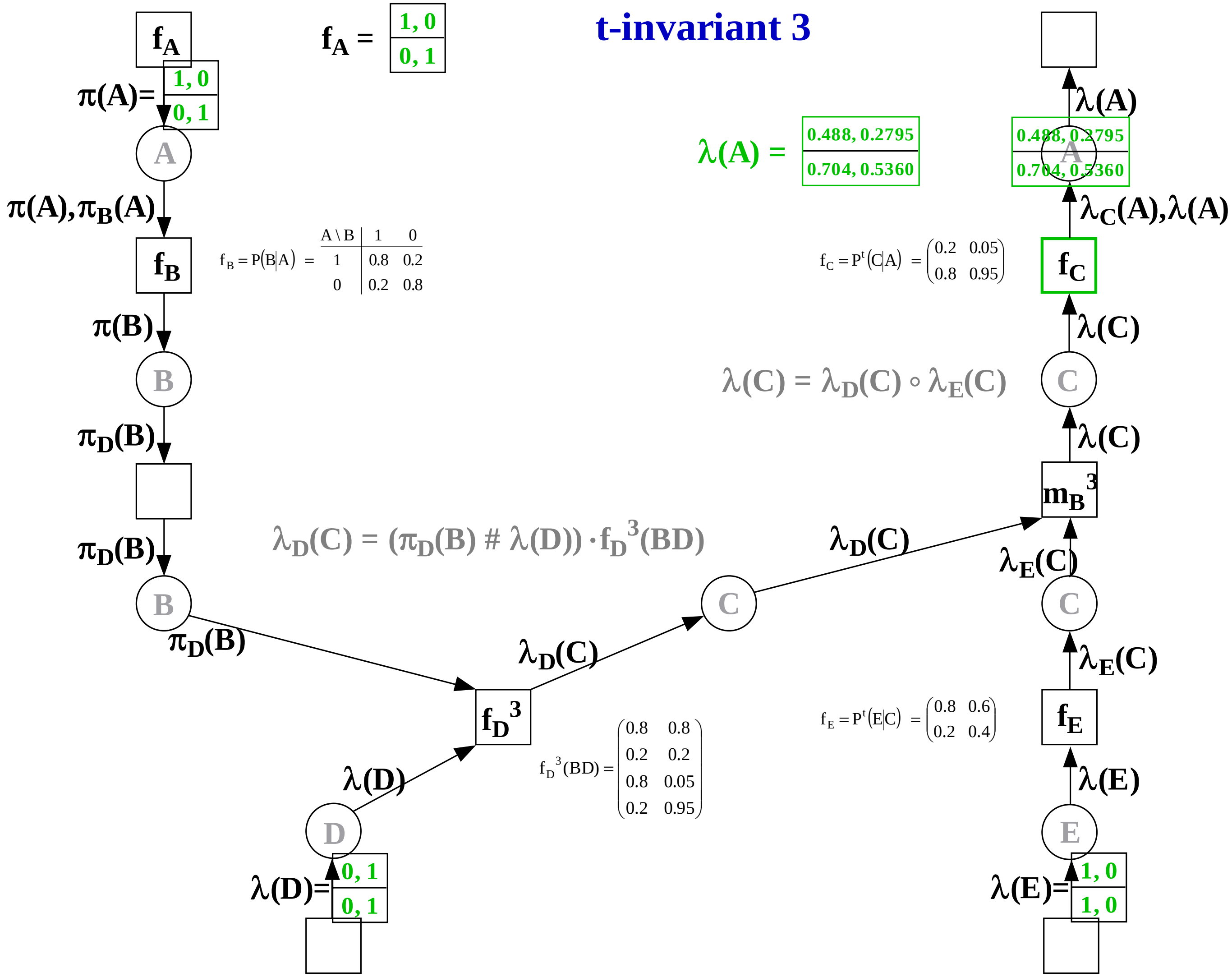


Conditioning (Pearl)



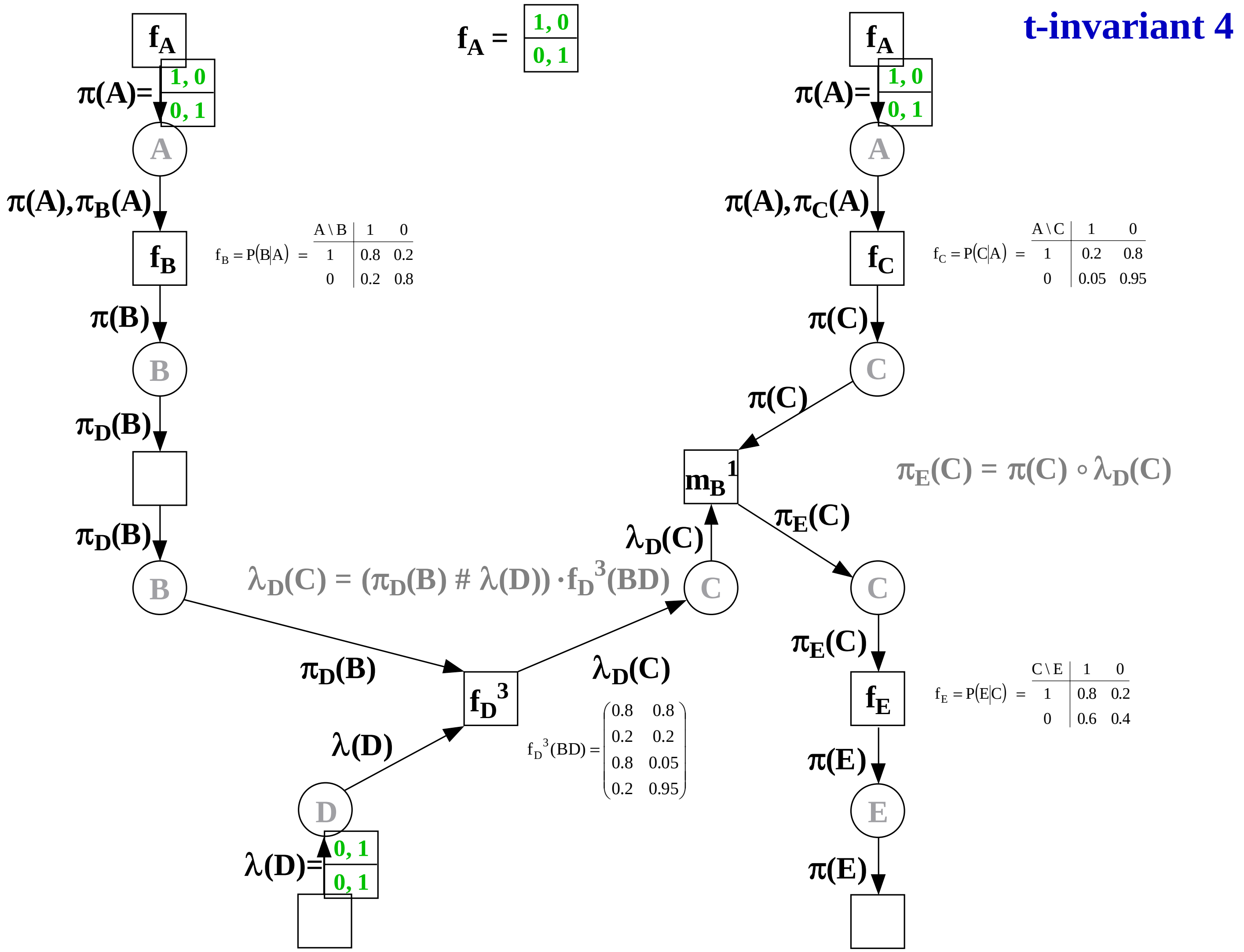
Conditioning (Pearl)





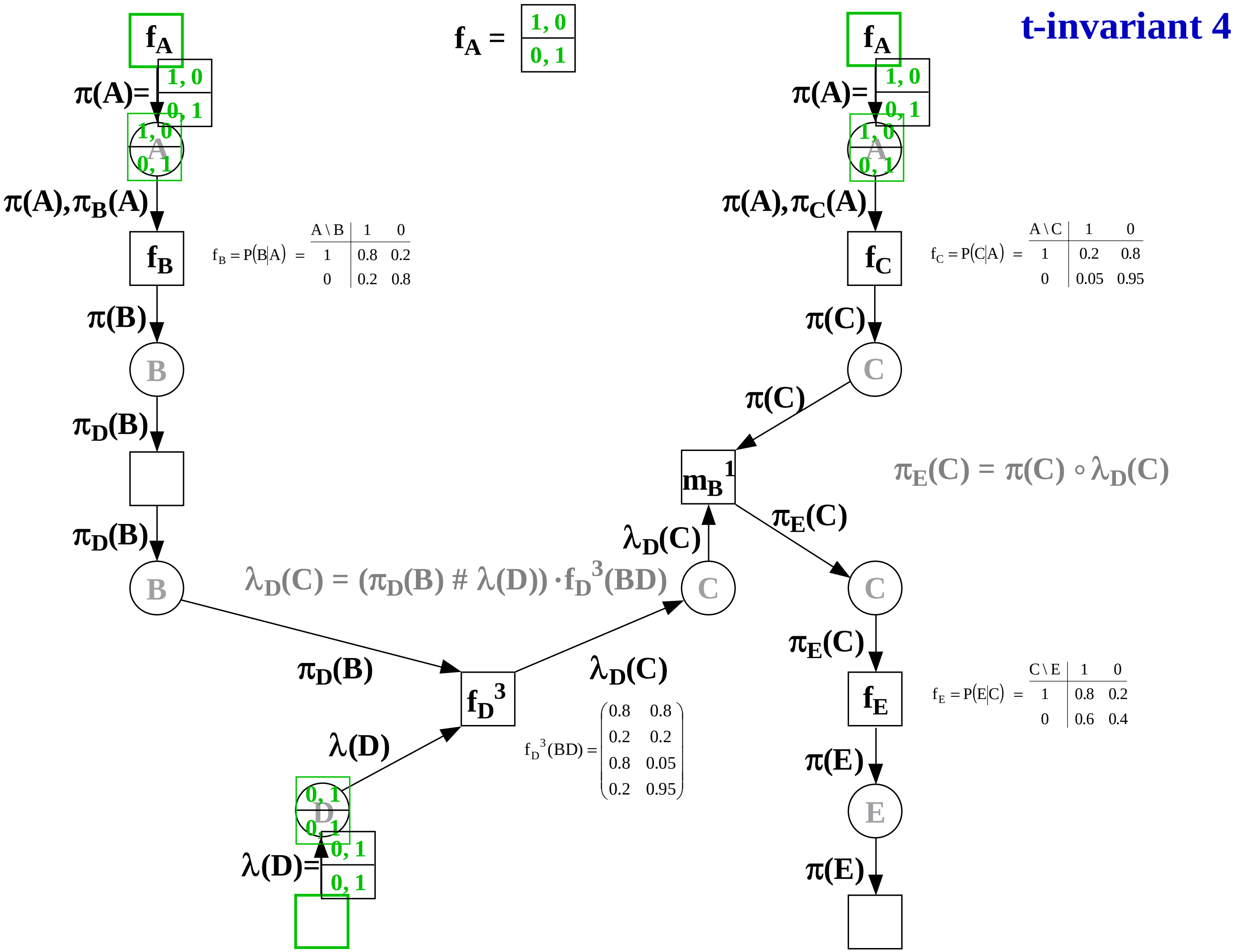
t-invariant 3
fA
fA =
1, 0
0, 1
π(A)=
1, 0
0, 1
A
π(A),πB(A)
fB
fB = P(B|A) =
A \ B
1
0
1
0.8
0.2
0
0.2
0.8
π(B)
B
πD(B)
πD(B)
B
πD(B)
λD(C) = (πD(B) # λ(D)) ·fD3(BD)
fD3
fD3(BD) =
0.8
0.8
0.2
0.2
0.8
0.05
0.2
0.95
λD(C)
λ(D)
D
λ(D)=
0, 1
0, 1
C
λD(C)
λ(A) =
0.488, 0.2795
0.704, 0.5360
λ(A)
A
λC(A),λ(A)
fC = Pt(C|A) =
0.2
0.05
0.8
0.95
fC
λ(C)
λ(C) = λD(C) ∘ λE(C)
C
λ(C)
mB3
λE(C)
C
λE(C)
fE = Pt(E|C) =
0.8
0.6
0.2
0.4
fE
λ(E)
E
λ(E)=
1, 0
1, 0

Conditioning (Pearl)

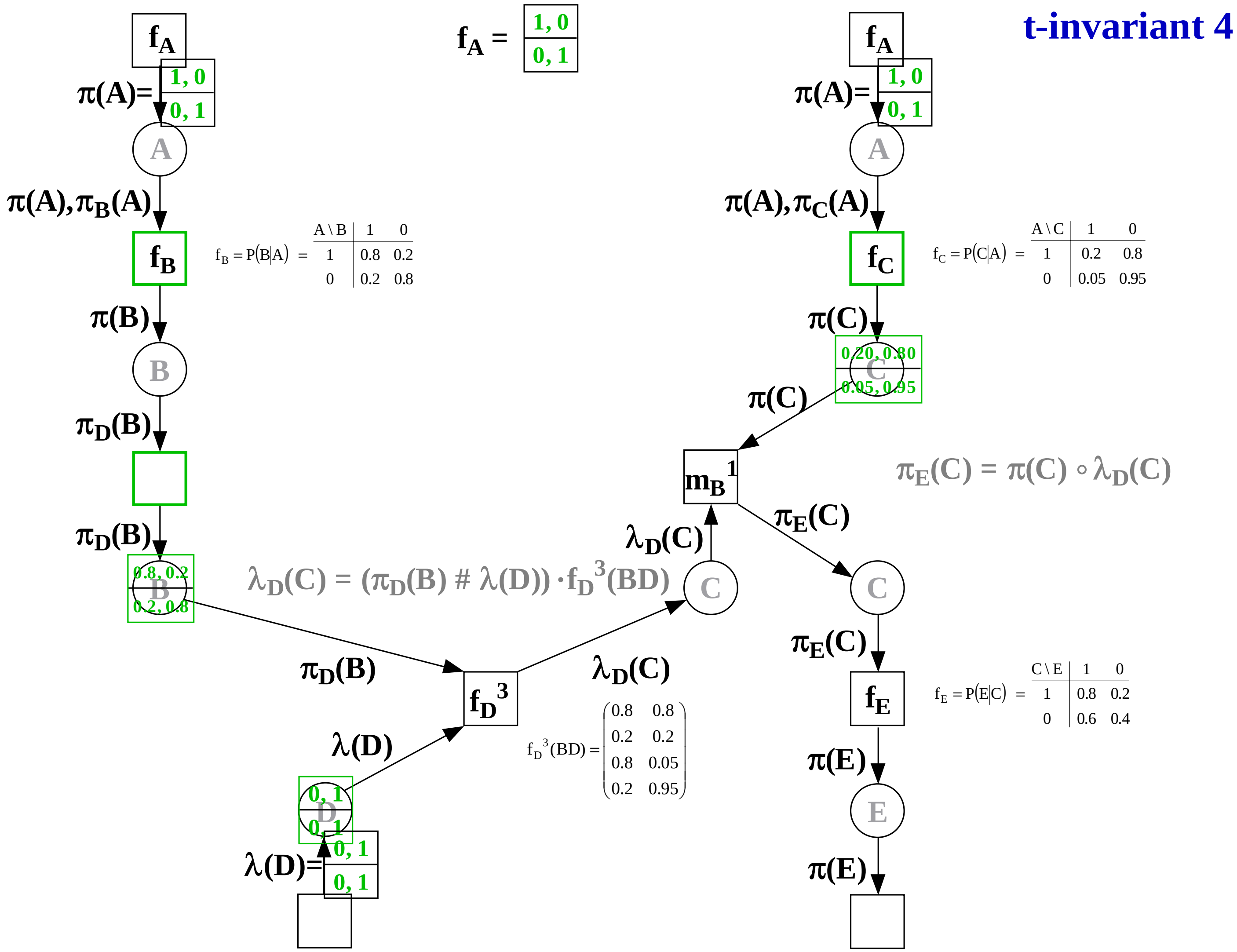


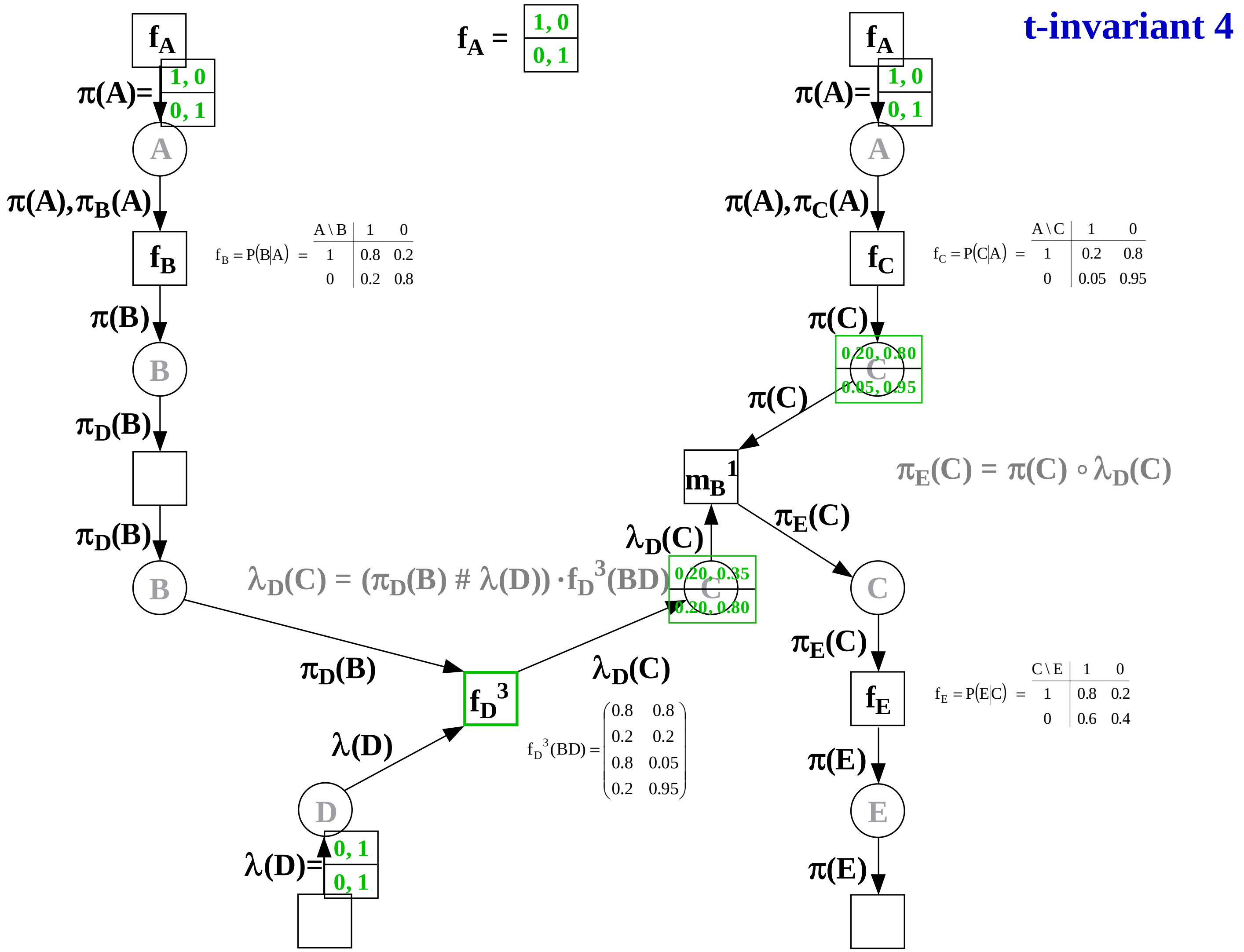
Conditioning (Pearl)

t-invariant 4



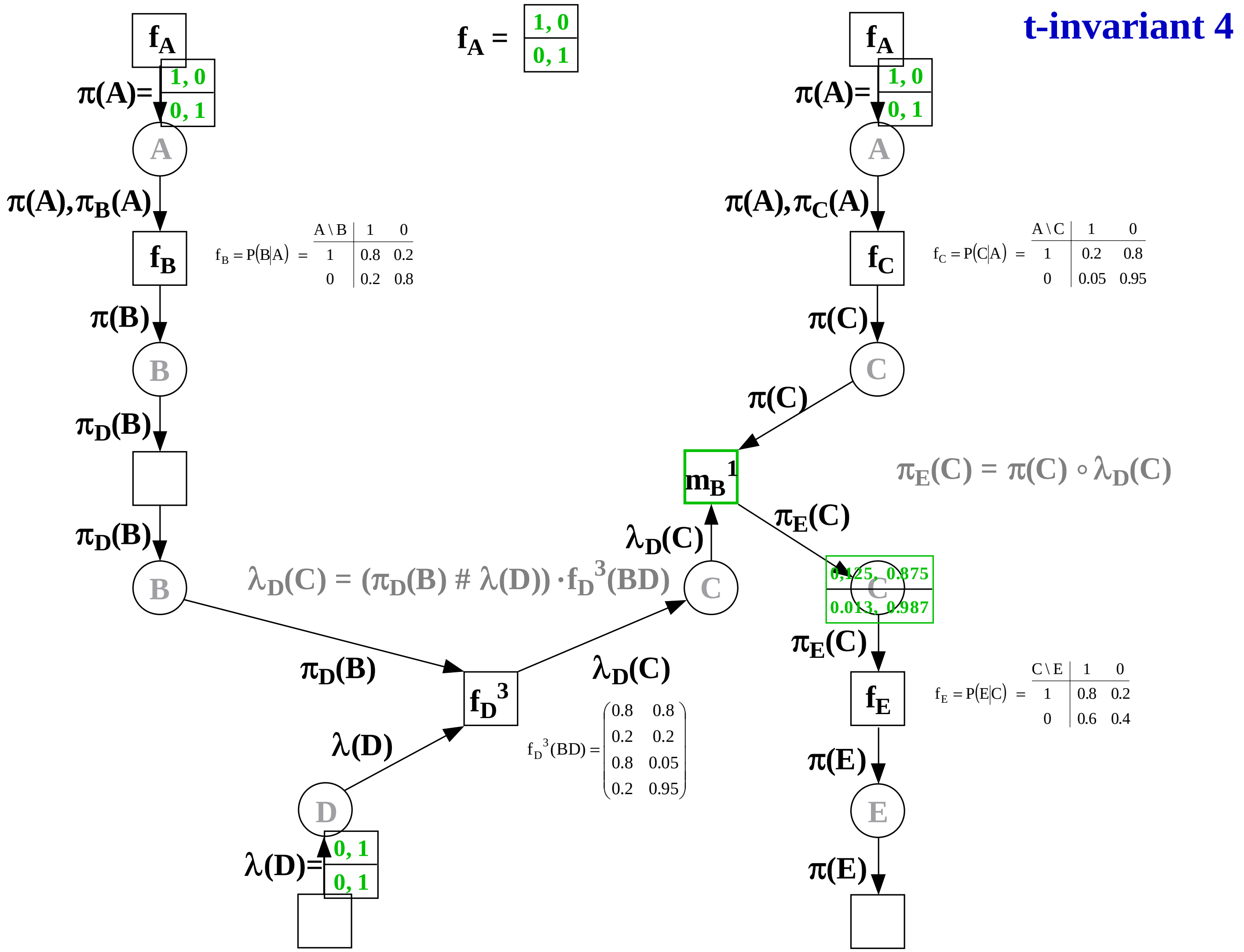
Conditioning (Pearl)



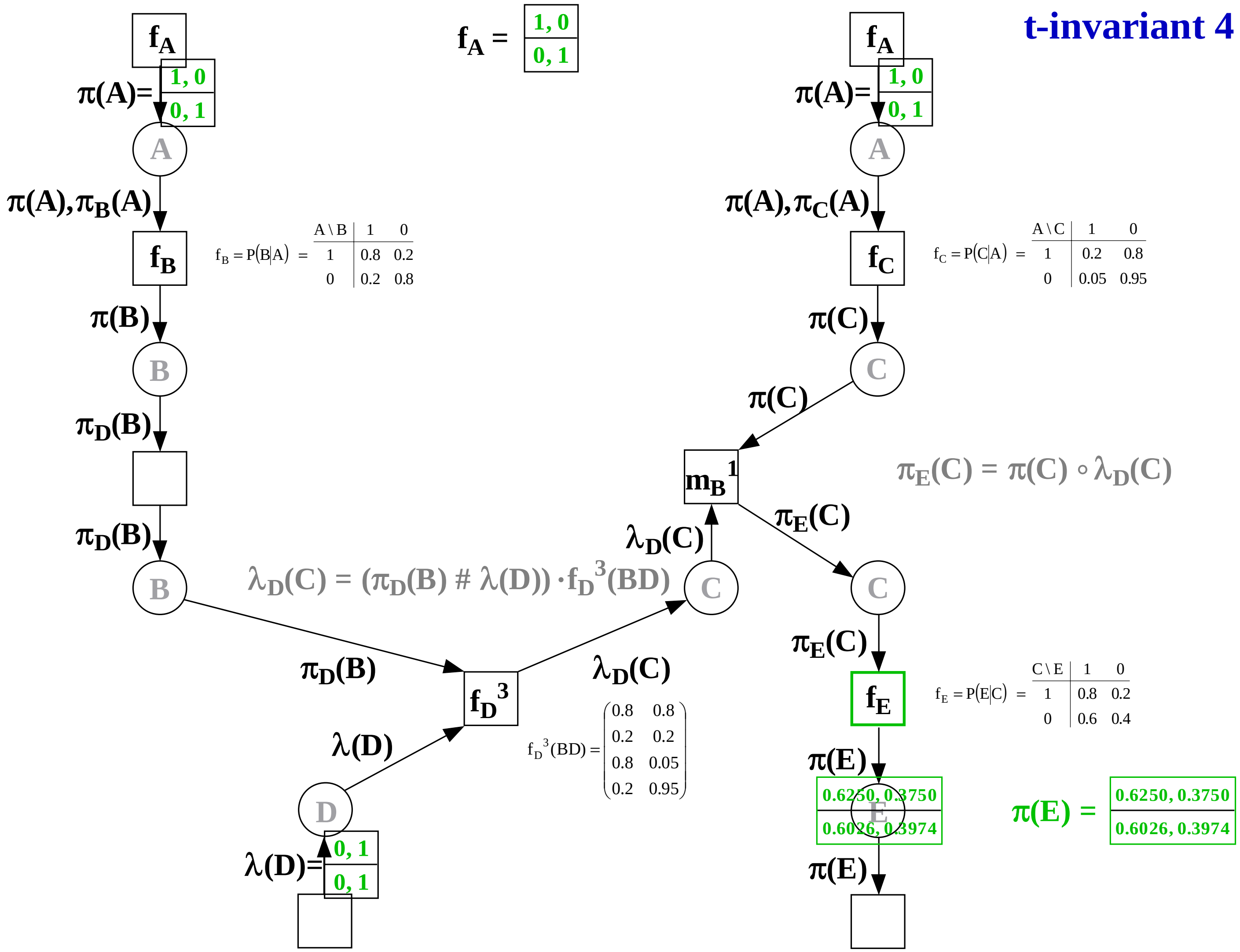


t-invariant 4
fA = 1, 0 / 0, 1
π(A)= 1, 0 / 0, 1
π(A),πB(A)
fB = P(B|A) =
A \ B | 1 | 0
1 | 0.8 | 0.2
0 | 0.2 | 0.8
π(B)
πD(B)
π(A),πC(A)
fC = P(C|A) =
A \ C | 1 | 0
1 | 0.2 | 0.8
0 | 0.05 | 0.95
π(C)
0.20, 0.80 / 0.05, 0.95
πE(C) = π(C) ∘ λD(C)
πE(C)
λD(C)
λD(C) = (πD(B) # λ(D)) ·fD3(BD)
0.20, 0.35 / 0.20, 0.80
fE = P(E|C) =
C \ E | 1 | 0
1 | 0.8 | 0.2
0 | 0.6 | 0.4
π(E)
fD3(BD) =
0.8 0.8
0.2 0.2
0.8 0.05
0.2 0.95
λ(D)
λ(D)= 0, 1 / 0, 1

Conditioning (Pearl)



Conditioning (Pearl)



Probabilities:

$\pi(D) = \begin{array}{|c|}\hline 0.6875,\ 0.3125 \\ \hline 0.2396,\ 0.7604 \\ \hline\end{array}$

$$P(A \mid E=1, D=0) = \alpha\, P(D=0 \mid A, E=1) \cdot P(A \mid E=1)$$
$$= \alpha\, (0.3125 \cdot 0.208,\ 0.7604 \cdot 0.792) = (0.0975,\ 0.9025)$$

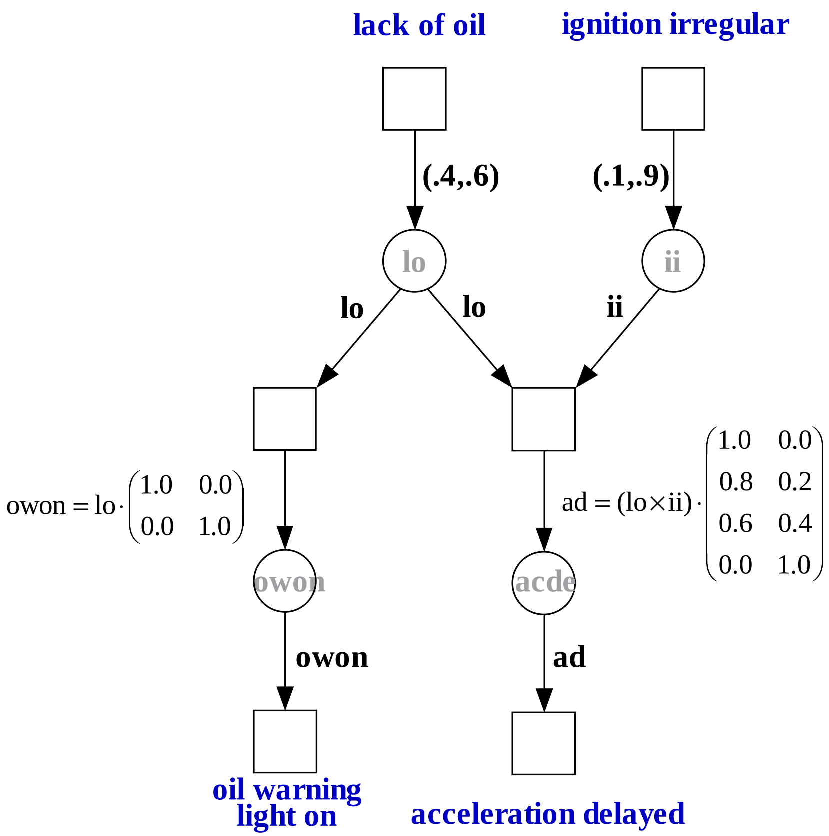
$\pi(B) = \begin{array}{|c|}\hline 0.8,\ 0.2 \\ \hline 0.2,\ 0.8 \\ \hline\end{array}$ $\quad \lambda(B) = \begin{array}{|c|}\hline 0.2,\ 0.7625 \\ \hline 0.2,\ 0.9005 \\ \hline\end{array}$

$$P(B) = BEL(B) = ((0.8,\ 0.2) \circ (0.2,\ 0.7625)) \cdot 0.0975$$
$$+ ((0.2,\ 0.8) \circ (0.2,\ 0.9005)) \cdot 0.9025 = \boxed{(0.098,\ 0.902)}$$

$\pi(C) = \begin{array}{|c|}\hline 0.20,\ 0.80 \\ \hline 0.05,\ 0.95 \\ \hline\end{array}$ $\quad \lambda(C) = \begin{array}{|c|}\hline 0.16,\ 0.21 \\ \hline 0.16,\ 0.48 \\ \hline\end{array}$

$$P(C) = BEL(C) = ((0.20,\ 0.80) \circ (0.16,\ 0.21)) \cdot 0.0975$$
$$+ ((0.05,\ 0.95) \circ (0.16,\ 0.48)) \cdot 0.9025 = \boxed{(0.031,\ 0.969)}$$

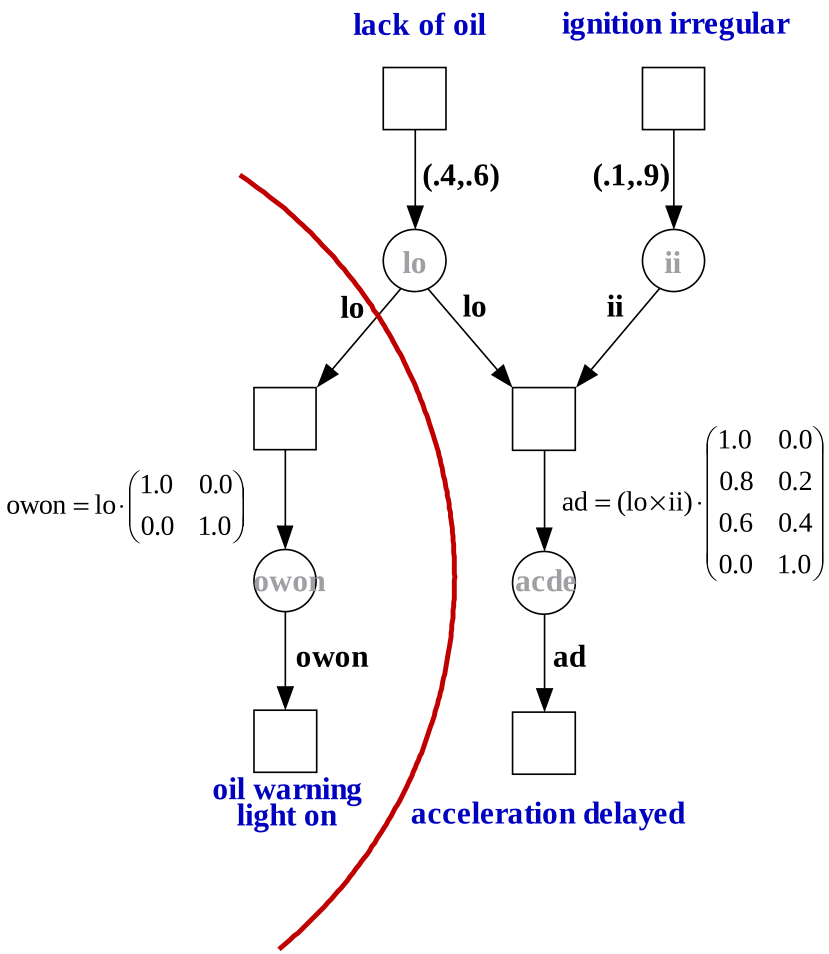
Folded Probability Propagation Net



What is the most probable explanation of "acceleration delayed" ?

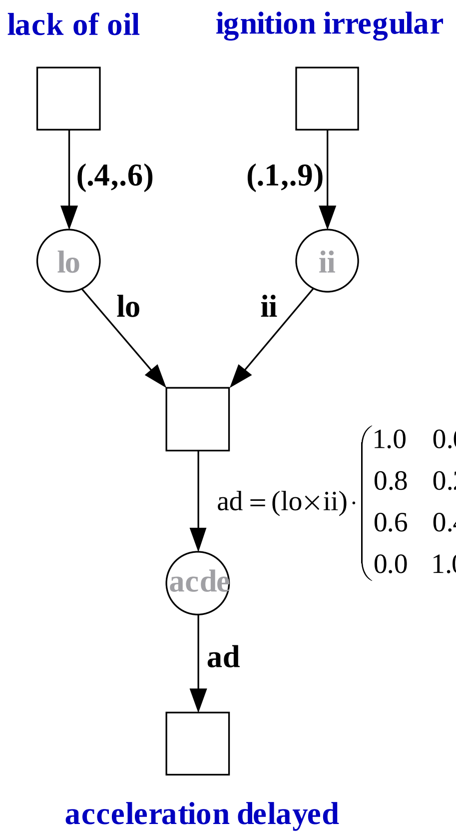
the former result is **{lo, igno}** with probability **0.288**

Folded Probability Propagation Net



What is the most probable explanation of "acceleration delayed" ?

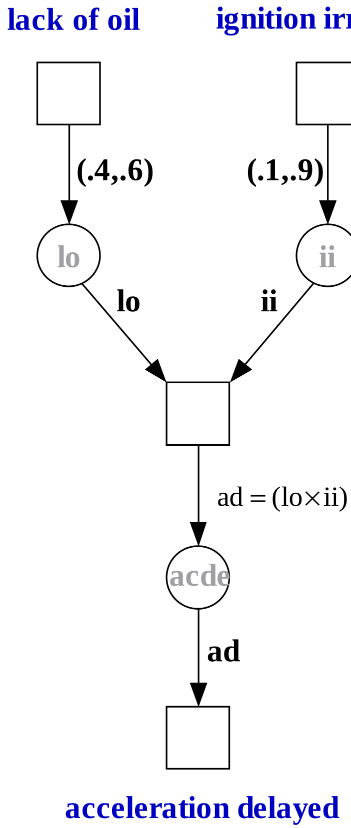
Folded Probability Propagation Net



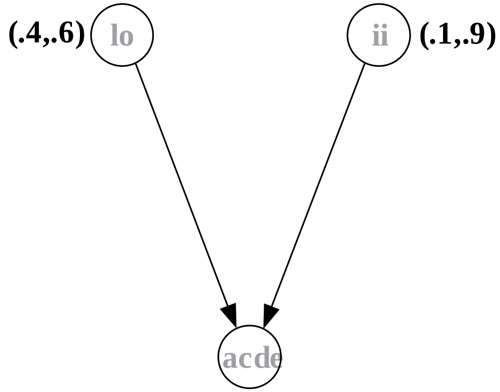
$$ad = (lo \times ii) \cdot \begin{pmatrix} 1.0 & 0.0 \\ 0.8 & 0.2 \\ 0.6 & 0.4 \\ 0.0 & 1.0 \end{pmatrix}$$

What is the most probable explanation of "acceleration delayed" ?

Folded Probability Propagation Net

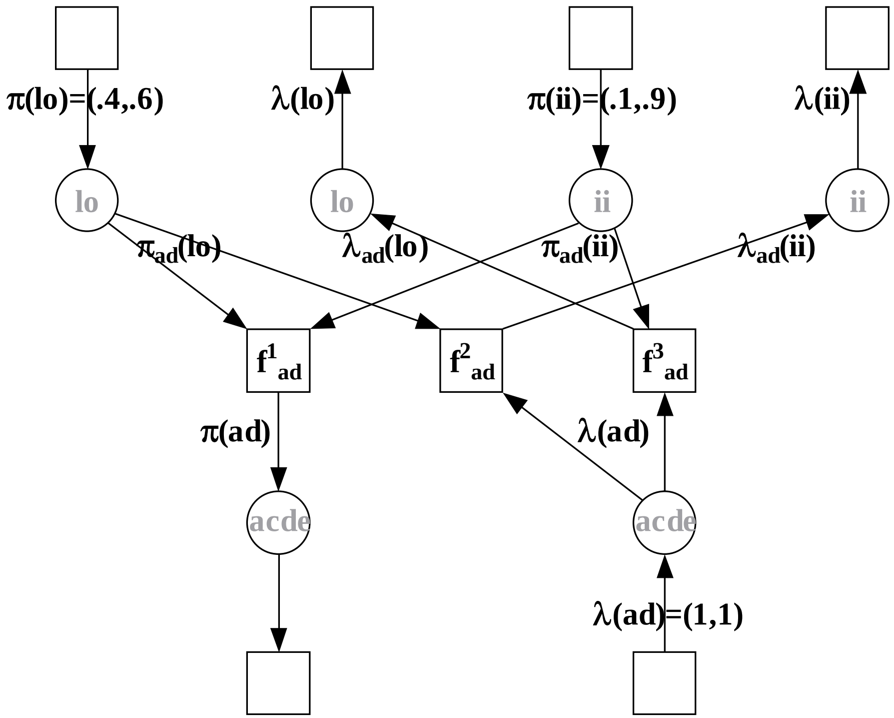


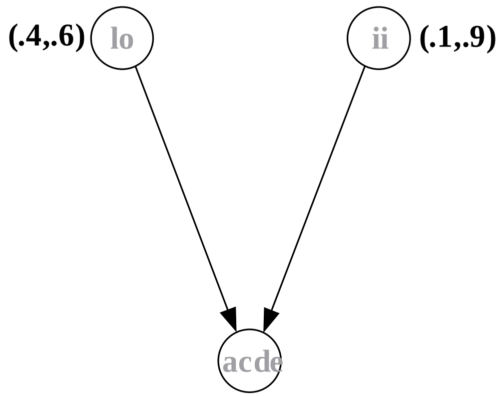
$$ad = (lo \times ii) \cdot \begin{pmatrix} 1.0 & 0.0 \\ 0.8 & 0.2 \\ 0.6 & 0.4 \\ 0.0 & 1.0 \end{pmatrix}$$



		acde	
lo	ii	1	0
1	1	1.0	0.0
1	0	0.8	0.2
0	1	0.6	0.4
0	0	0.0	1.0

the corresponding Bayesian network

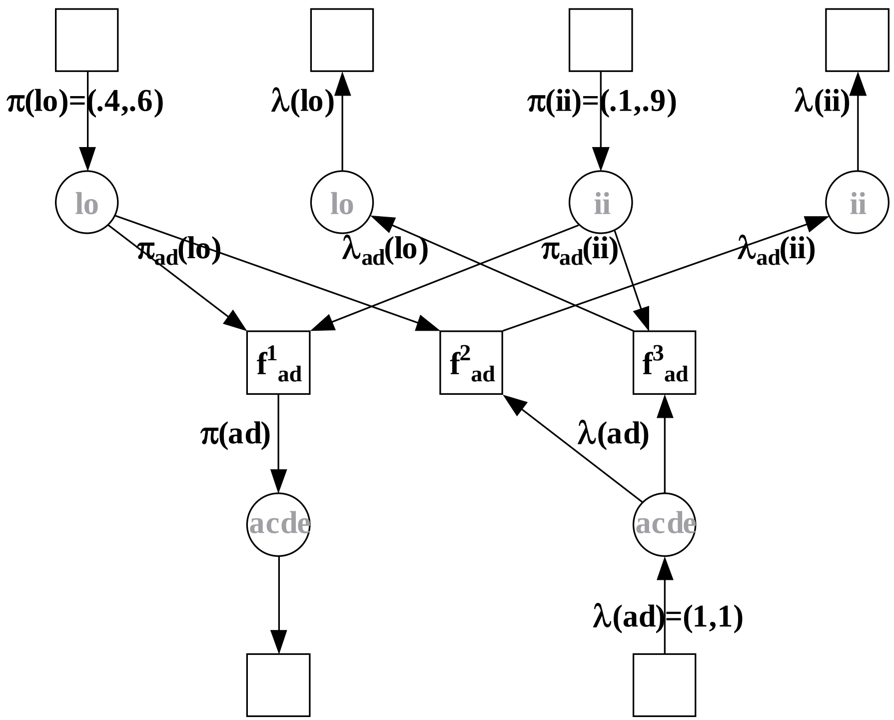




		acde	
lo	ii	1	0
1	1	1.0	0.0
1	0	0.8	0.2
0	1	0.6	0.4
0	0	0.0	1.0

the corresponding Bayesian network

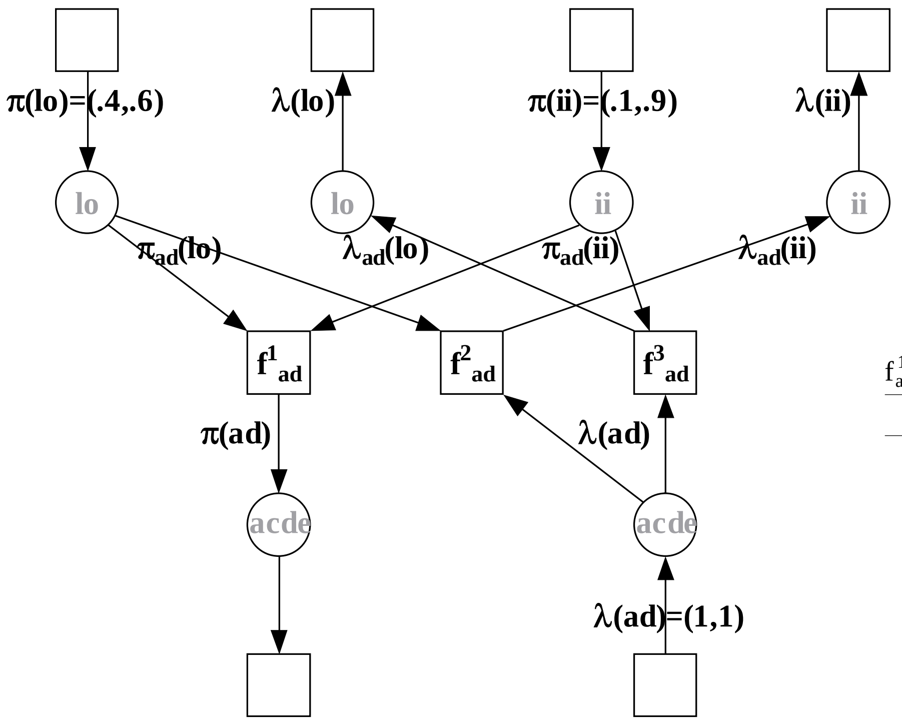
Petri Net Representation



f^1_{ad}		ad	
lo	ii	1	0
1	1	1.0	0.0
1	0	0.8	0.2
0	1	0.6	0.4
0	0	0.0	1.0

f^1_{ad} (lo, ii)			
ad	lo	ii	
1	1	1	1.0
1	1	0	0.8
1	0	1	0.6
1	0	0	0.0
0	1	1	0.0
0	1	0	0.2
0	0	1	0.4
0	0	0	1.0

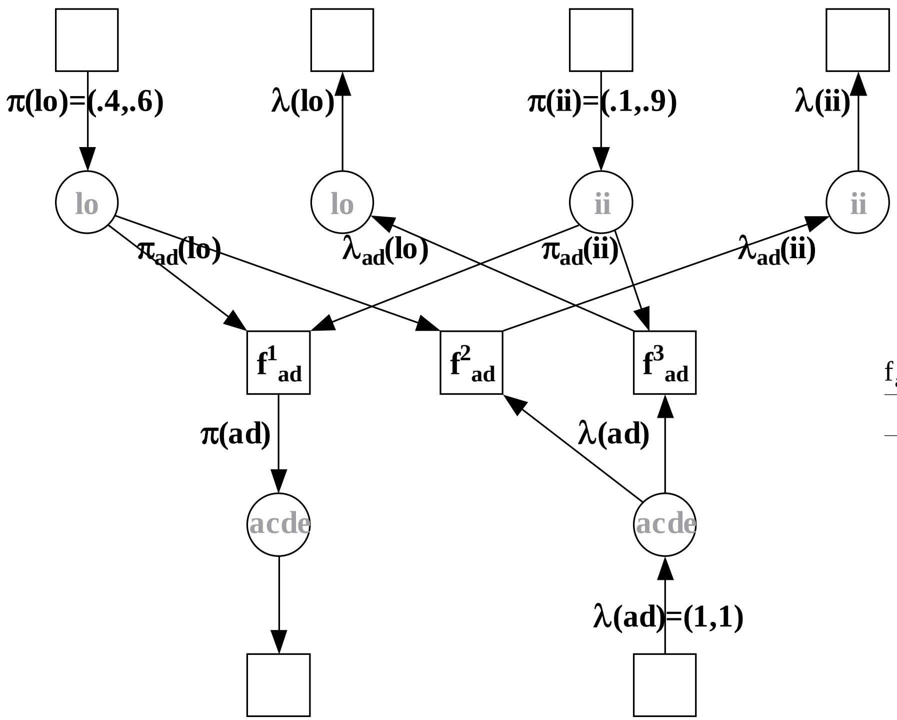
Petri Net Representation



$f^1_{ad}(lo, ii)$			
ad	lo	ii	
1	1	1	1.0
1	1	0	0.8
1	0	1	0.6
1	0	0	0.0
0	1	1	0.0
0	1	0	0.2
0	0	1	0.4
0	0	0	1.0

$f^2_{ad}(lo, ad)$			
ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	0	1	0.6
0	0	1	0.4
1	1	0	0.8
0	1	0	0.2
1	0	0	0.0
0	0	0	1.0

Petri Net Representation



$f^3_{ad}(ii, ad)$

ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	1	0	0.8
0	1	0	0.2
1	0	1	0.6
0	0	1	0.4
1	0	0	0.0
0	0	0	1.0

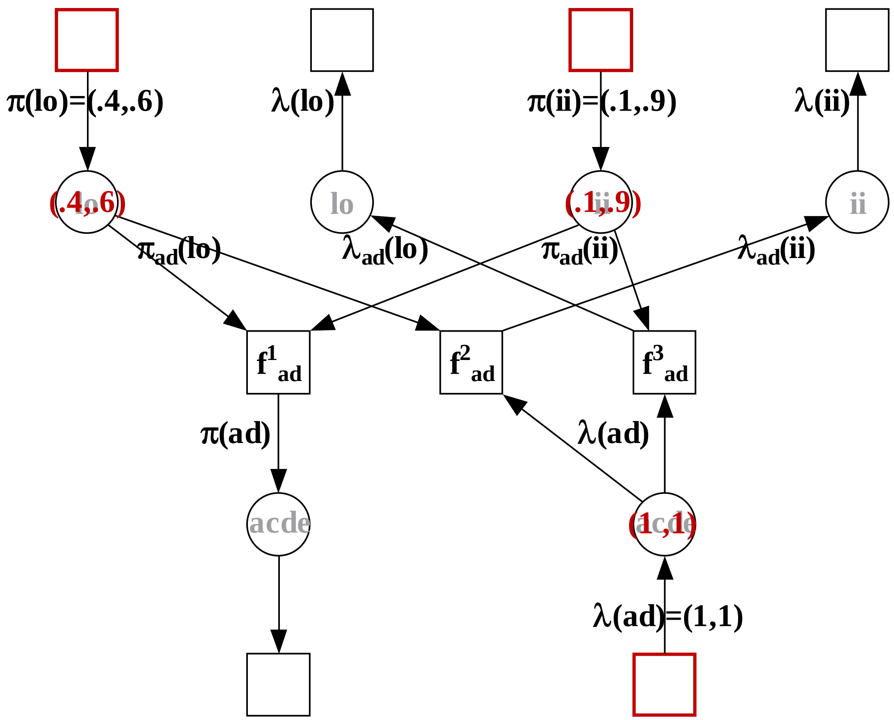
$f^1_{ad}(lo, ii)$

ad	lo	ii	
1	1	1	1.0
1	1	0	0.8
1	0	1	0.6
1	0	0	0.0
0	1	1	0.0
0	1	0	0.2
0	0	1	0.4
0	0	0	1.0

$f^2_{ad}(lo, ad)$

ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	0	1	0.6
0	0	1	0.4
1	1	0	0.8
0	1	0	0.2
1	0	0	0.0
0	0	0	1.0

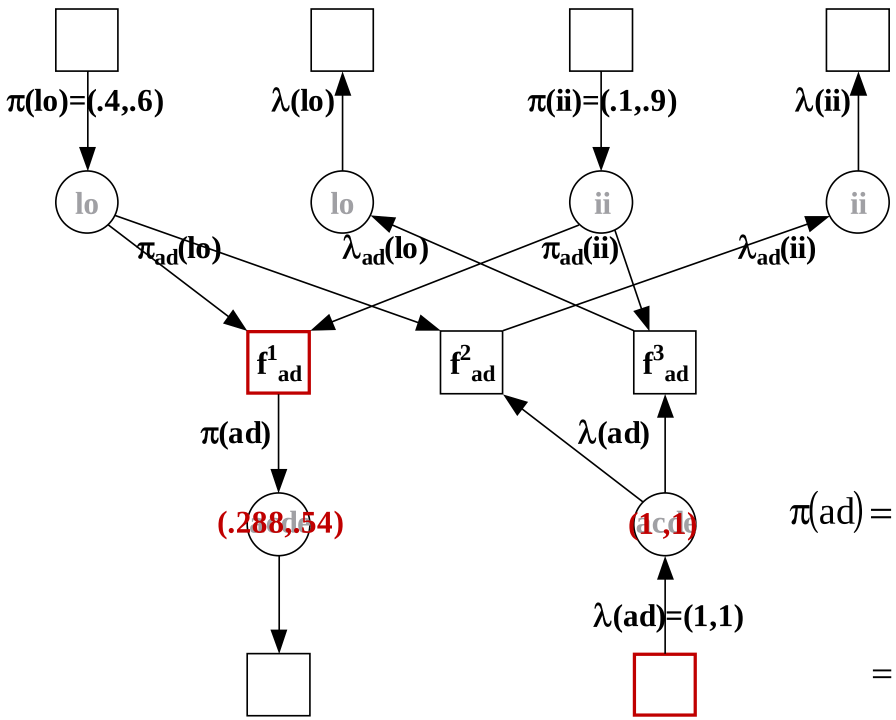
Petri Net Representation



$f^1_{ad}(lo, ii)$

ad	lo	ii	
1	1	1	1.0
1	1	0	0.8
1	0	1	0.6
1	0	0	0.0
0	1	1	0.0
0	1	0	0.2
0	0	1	0.4
0	0	0	1.0

Petri Net Representation



$f^1_{ad}(lo, ii)$

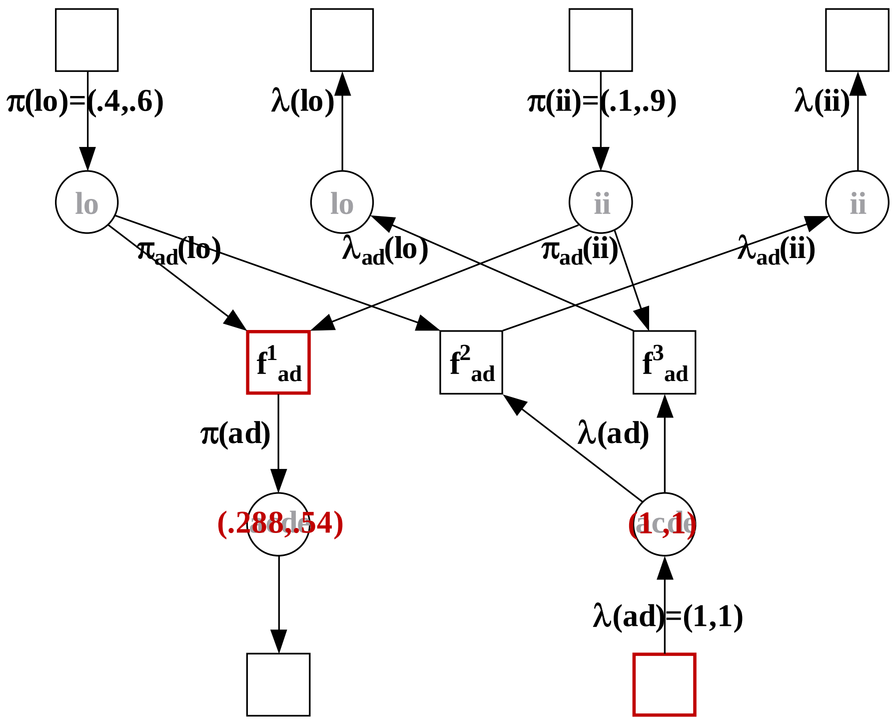
ad	lo	ii	
1	1	1	1.0
1	1	0	0.8
1	0	1	0.6
1	0	0	0.0
0	1	1	0.0
0	1	0	0.2
0	0	1	0.4
0	0	0	1.0

$$\pi(ad) = (\pi_{ad}(lo)) \times (\pi_{ad}(ii)) \bullet f^1_{ad} = ((0.4, 0.6) \times (0.1, 0.9)) \bullet f^1_{ad}$$

$$= (0.04, 0.36, 0.06, 0.54) \bullet \begin{pmatrix} 1.0 & 0.0 \\ 0.8 & 0.2 \\ 0.6 & 0.4 \\ 0.0 & 1.0 \end{pmatrix}$$

$$= (\max\{0.04, 0.288, 0.036, 0.0\}, \ \max\{0.0, 0.072, 0.024, 0.54\}) = (0.288, 0.54)$$

Petri Net Representation

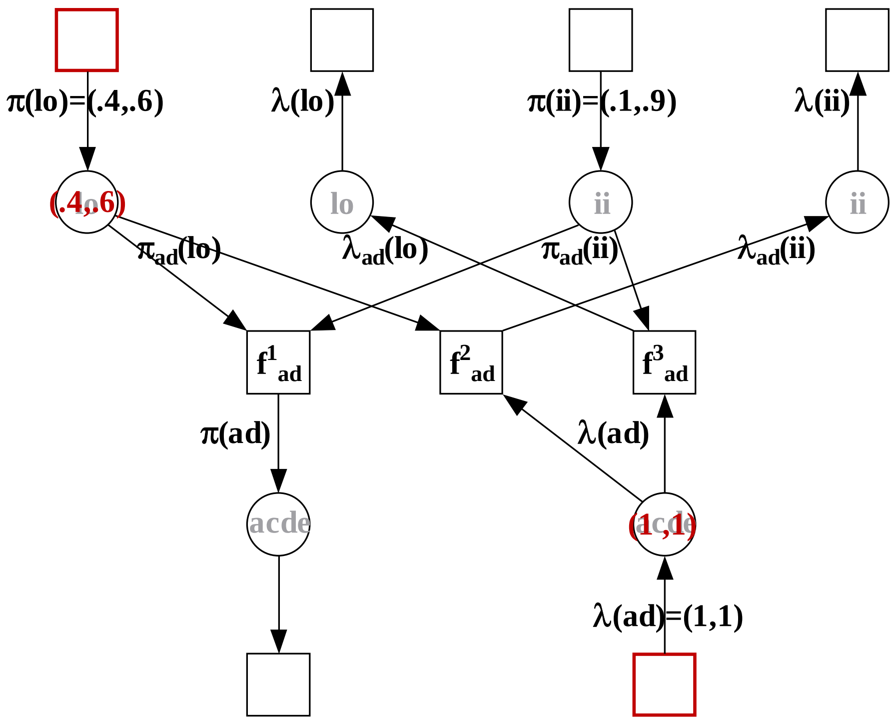


$f^1_{ad}(lo, ii)$

ad	lo	ii	
1	1	1	1.0
1	1	0	0.8
1	0	1	0.6
1	0	0	0.0
0	1	1	0.0
0	1	0	0.2
0	0	1	0.4
0	0	0	1.0

$$P^*(ad) = \pi(ad) \circ \lambda(ad) = (0.288, 0.54) \circ (1,1) = (0.288, \boxed{0.54})$$

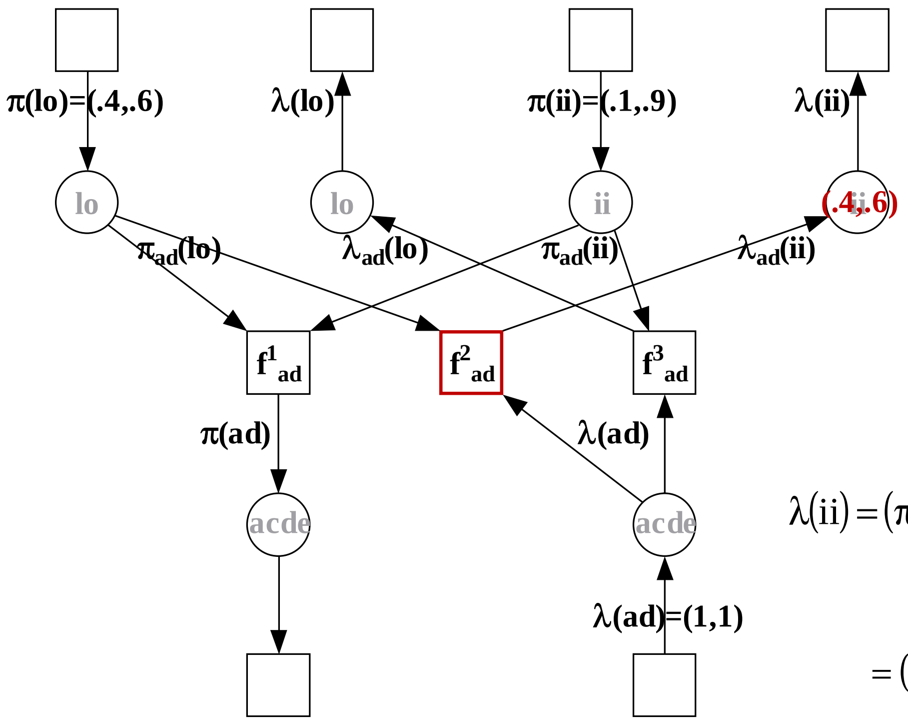
Petri Net Representation



			f^2_{ad}(lo, ad)
ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	0	1	0.6
0	0	1	0.4
1	1	0	0.8
0	1	0	0.2
1	0	0	0.0
0	0	0	1.0

Petri Net Representation

			$f^{2}_{ad}(lo, ad)$
ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	0	1	0.6
0	0	1	0.4
1	1	0	0.8
0	1	0	0.2
1	0	0	0.0
0	0	0	1.0

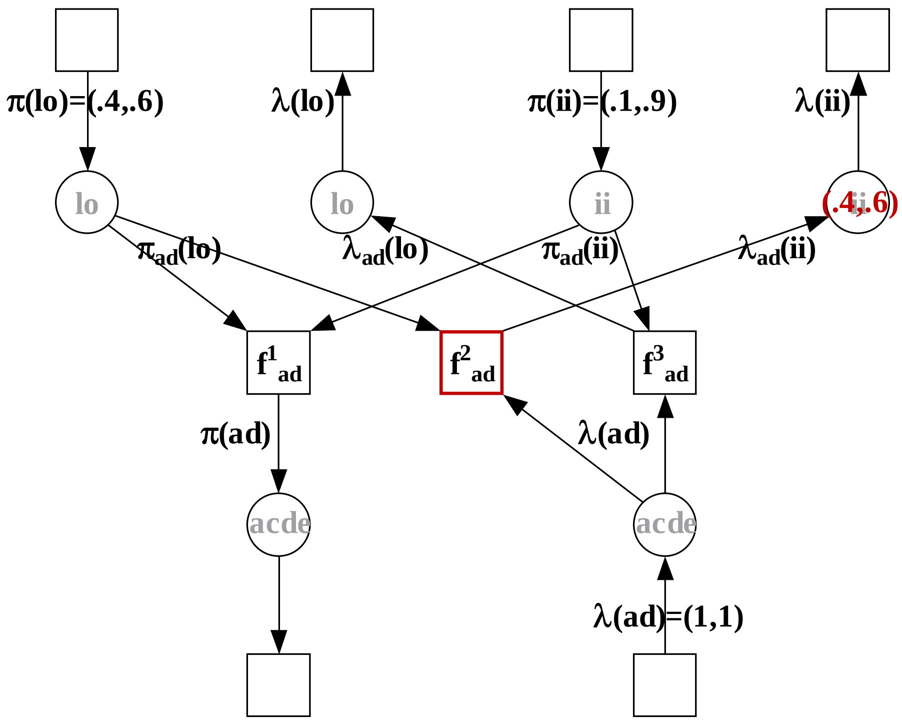


$$\lambda(ii) = (\pi_{ad}(lo)) \times (\lambda(ad)) \bullet f^{2}_{ad} = ((0.4, 0.6) \times (1.0, 1.0)) \bullet f^{2}_{ad}$$

$$= (0.4, 0.4, 0.6, 0.6) \bullet \begin{pmatrix} 1.0 & 0.8 \\ 0.0 & 0.2 \\ 0.6 & 0.0 \\ 0.4 & 1.0 \end{pmatrix}$$

$$= (\max\{0.4, 0.0, 0.36, 0.24\},\ \max\{0.32, 0.08, 0.0, 0.6\}) = (0.4, 0.6)$$

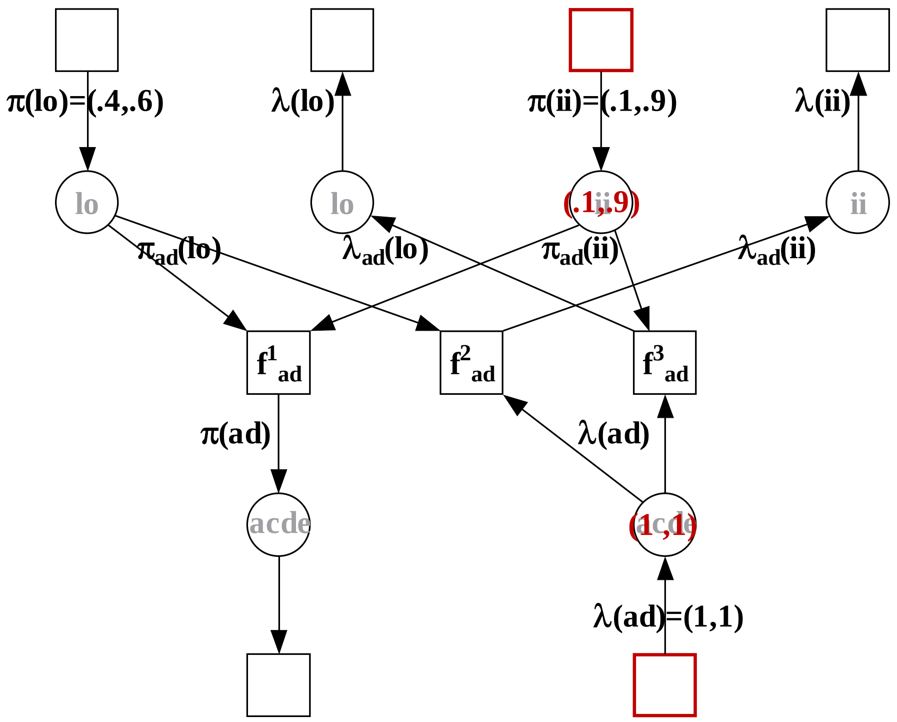
Petri Net Representation



ad	lo	ii	$f^2_{ad}(lo, ad)$
1	1	1	1.0
0	1	1	0.0
1	0	1	0.6
0	0	1	0.4
1	1	0	0.8
0	1	0	0.2
1	0	0	0.0
0	0	0	1.0

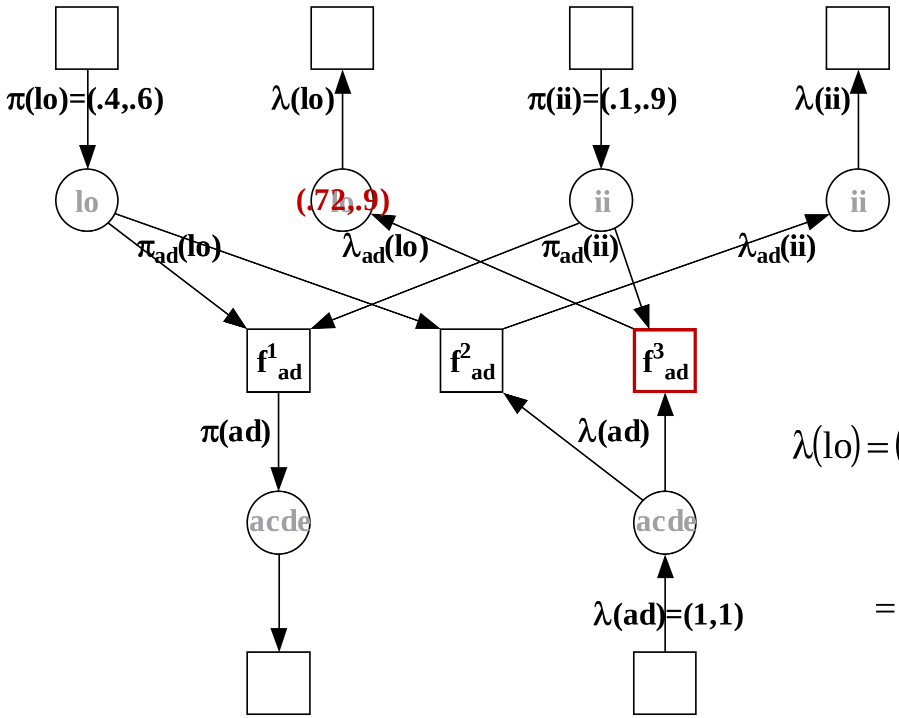
$$P^*(ii) = \pi(ii) \circ \lambda(ii) = (0.1, 0.9) \circ (0.4, 0.6) = (0.04, \boxed{0.54)}$$

Petri Net Representation



f^3_{ad}(ii, ad)			
ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	1	0	0.8
0	1	0	0.2
1	0	1	0.6
0	0	1	0.4
1	0	0	0.0
0	0	0	1.0

Petri Net Representation



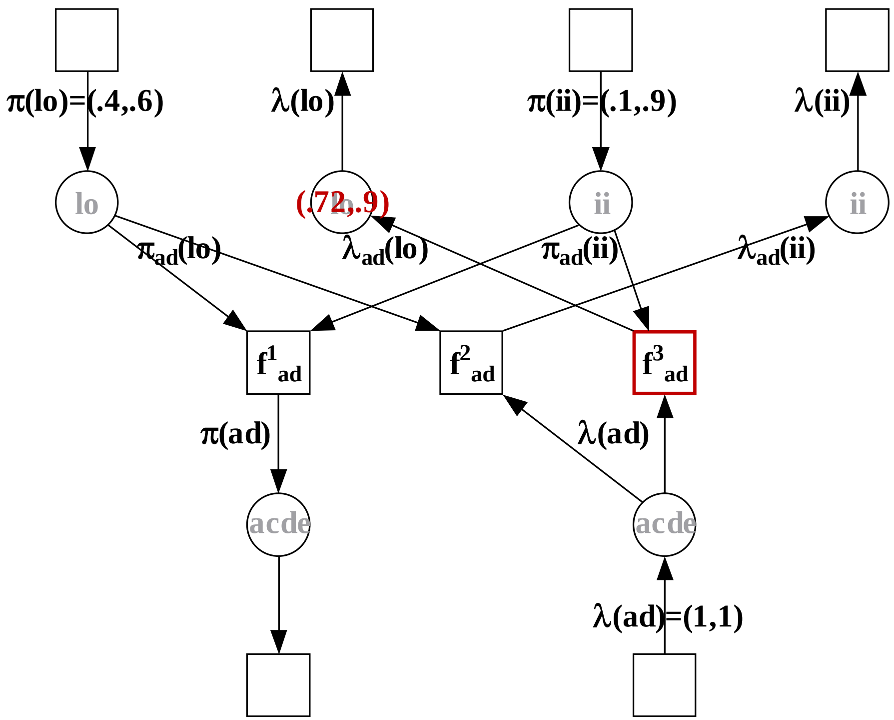
$f^3_{ad}(ii, ad)$			
ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	1	0	0.8
0	1	0	0.2
1	0	1	0.6
0	0	1	0.4
1	0	0	0.0
0	0	0	1.0

$$\lambda(lo) = (\pi_{ad}(ii)) \times (\lambda(ad)) \bullet f^3_{ad} = ((0.1, 0.9) \times (1.0, 1.0)) \bullet f^3_{ad}$$

$$= (0.1, 0.1, 0.9, 0.9) \bullet \begin{pmatrix} 1.0 & 0.6 \\ 0.0 & 0.4 \\ 0.8 & 0.0 \\ 0.2 & 1.0 \end{pmatrix}$$

$$= (\max\{0.1, 0.0, 0.72, 0.18\},\ \max\{0.06, 0.04, 0.0, 0.9\}) = (0.72, 0.9)$$

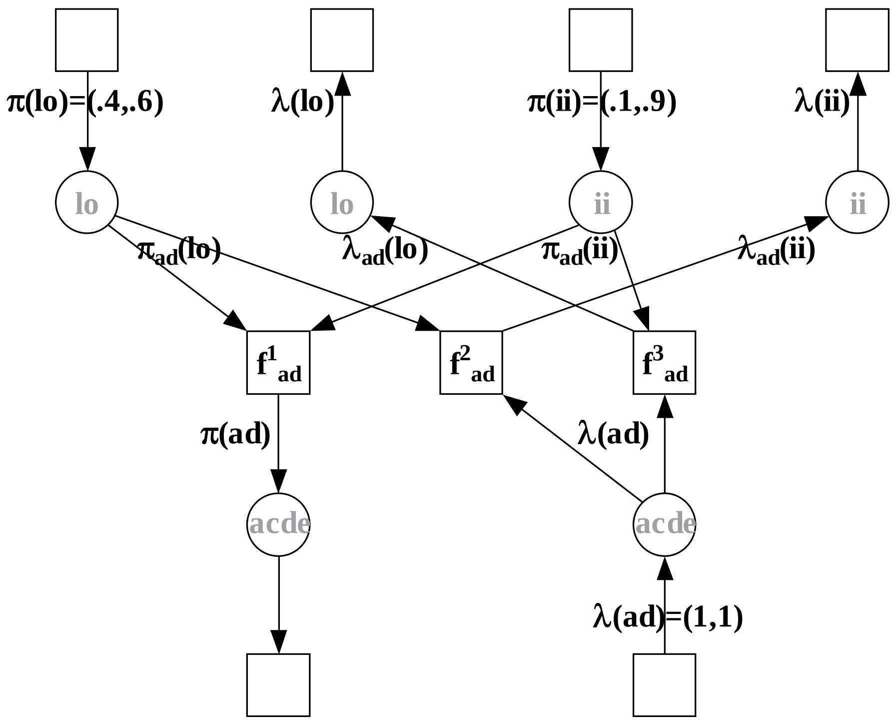
Petri Net Representation



$f^3_{ad}(ii, ad)$			
ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	1	0	0.8
0	1	0	0.2
1	0	1	0.6
0	0	1	0.4
1	0	0	0.0
0	0	0	1.0

$$P^*(lo) = \pi(lo) \circ \lambda(lo) = (0.4, 0.6) \circ (0.72, 0.9) = (0.288, 0.54)$$

Petri Net Representation



$P^*(lo) = (0.288, \boxed{0.54})$

$P^*(ii) = (0.04, \boxed{0.54})$

$P^*(ad) = (0.288, \boxed{0.54})$

(not normalized beliefs)

that means that
without any evidence
(initialization!)

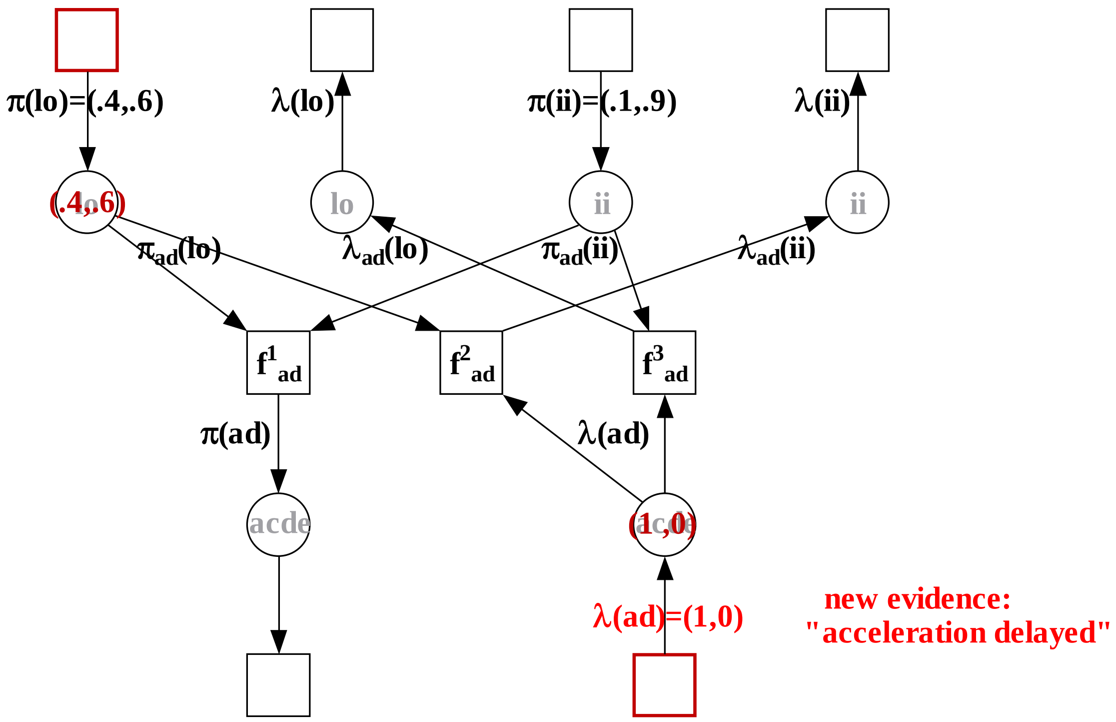
"no lack of oil"

"ignition normal"

"acceleration not delayed"

is most probable:
so, the **explanation set is $\{lo_0, ii_0, ad_0\}$**

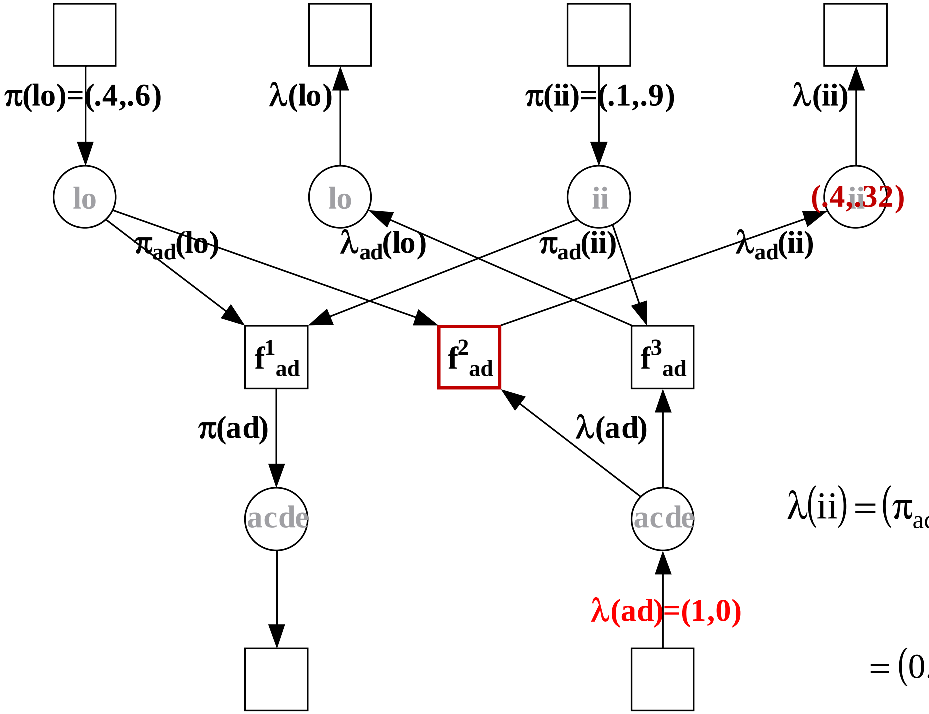
Petri Net Representation



			f^2_{ad} (lo, ad)
ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	0	1	0.6
0	0	1	0.4
1	1	0	0.8
0	1	0	0.2
1	0	0	0.0
0	0	0	1.0

Petri Net Representation

			$f^2_{ad}(lo, ad)$
ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	0	1	0.6
0	0	1	0.4
1	1	0	0.8
0	1	0	0.2
1	0	0	0.0
0	0	0	1.0

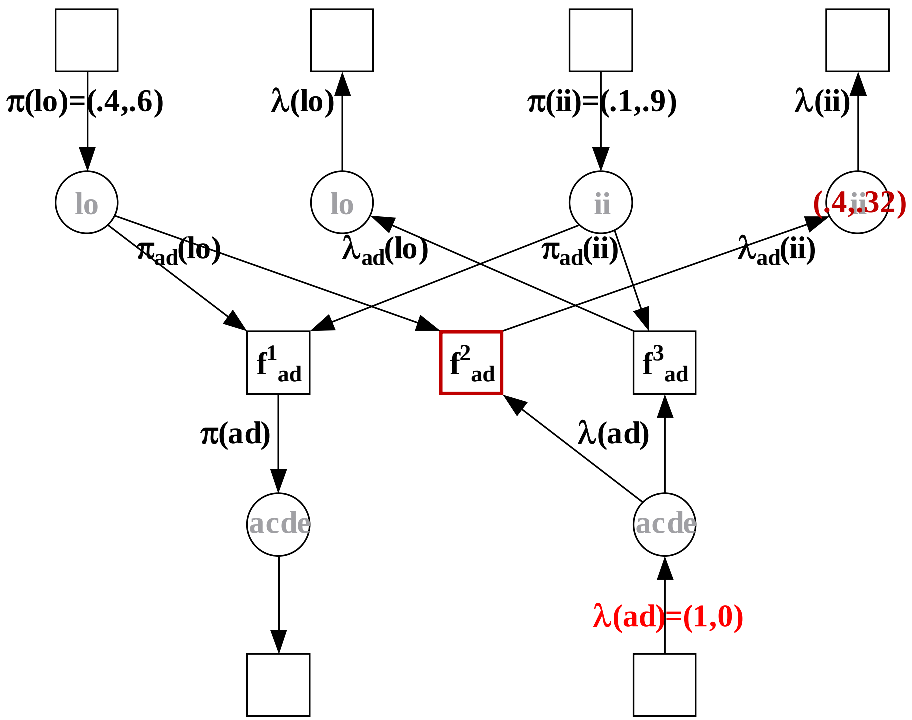


$$\lambda(ii) = (\pi_{ad}(lo)) \times (\lambda(ad)) \bullet f^2_{ad} = ((0.4, 0.6) \times (1.0, 0.0)) \bullet f^2_{ad}$$

$$= (0.4, 0.0, 0.6, 0.0) \bullet \begin{pmatrix} 1.0 & 0.8 \\ 0.0 & 0.2 \\ 0.6 & 0.0 \\ 0.4 & 1.0 \end{pmatrix}$$

$$= (\max\{0.4, 0.0, 0.36, 0.0\},\ \max\{0.32, 0.0, 0.0, 0.\}) = (0.4, 0.32)$$

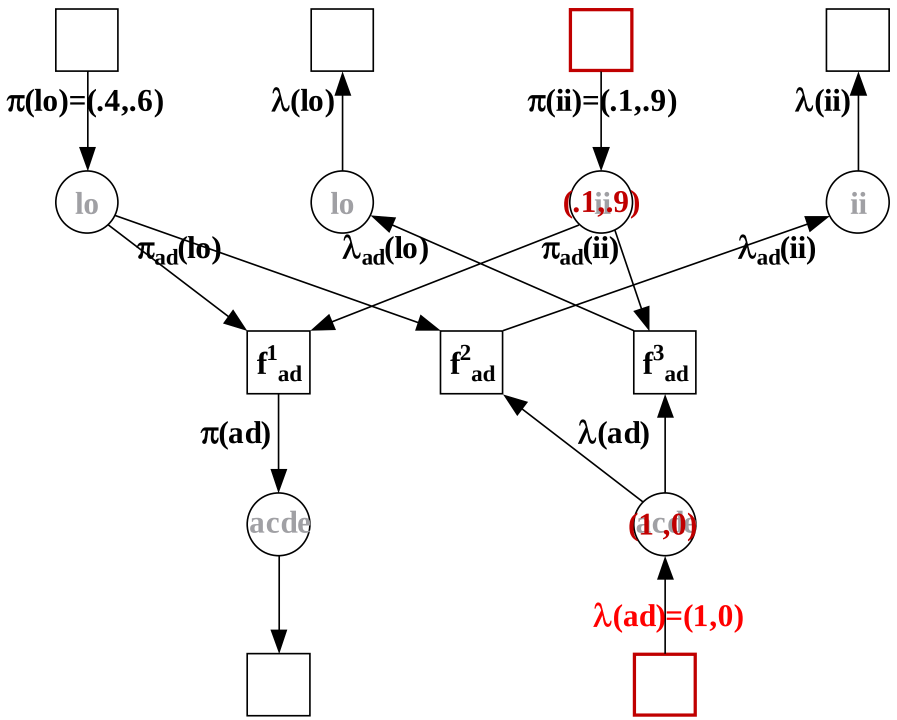
Petri Net Representation



ad	lo	ii	$f^2_{ad}(lo, ad)$
1	1	1	1.0
0	1	1	0.0
1	0	1	0.6
0	0	1	0.4
1	1	0	0.8
0	1	0	0.2
1	0	0	0.0
0	0	0	1.0

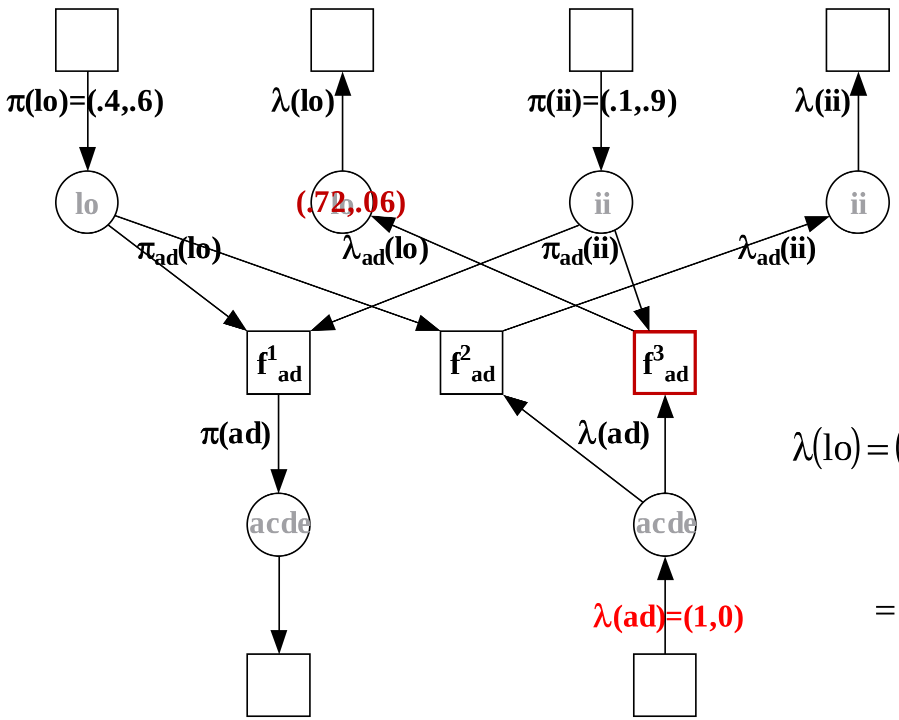
$$P^*(ii) = \pi(ii) \circ \lambda(ii) = (0.1, 0.9) \circ (0.4, 0.32) = (0.04, \boxed{0.288})$$

Petri Net Representation



$f^3_{ad}(ii, ad)$			
ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	1	0	0.8
0	1	0	0.2
1	0	1	0.6
0	0	1	0.4
1	0	0	0.0
0	0	0	1.0

Petri Net Representation



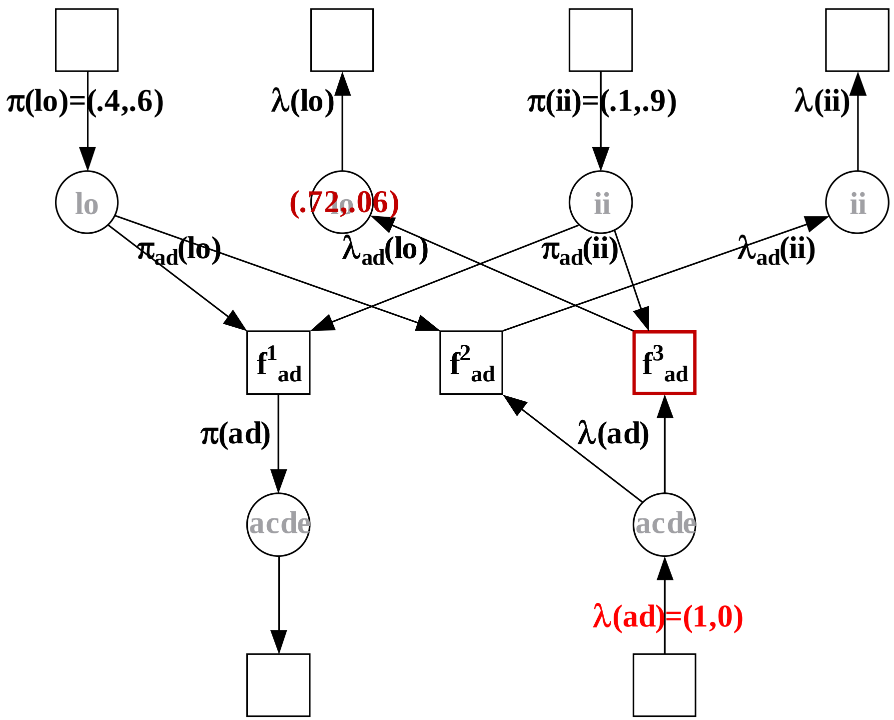
$f^3_{ad}(ii, ad)$			
ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	1	0	0.8
0	1	0	0.2
1	0	1	0.6
0	0	1	0.4
1	0	0	0.0
0	0	0	1.0

$$\lambda(lo) = (\pi_{ad}(ii)) \times (\lambda(ad)) \bullet f^3_{ad} = ((0.1, 0.9) \times (1.0, 0.0)) \bullet f^3_{ad}$$

$$= (0.1, 0.0, 0.9, 0.0) \bullet \begin{pmatrix} 1.0 & 0.6 \\ 0.0 & 0.4 \\ 0.8 & 0.0 \\ 0.2 & 1.0 \end{pmatrix}$$

$$= (\max\{0.1, 0.0, 0.72, 0.0\}, \ \max\{0.06, 0.0, 0.0, 0.0\}) = (0.72, 0.06)$$

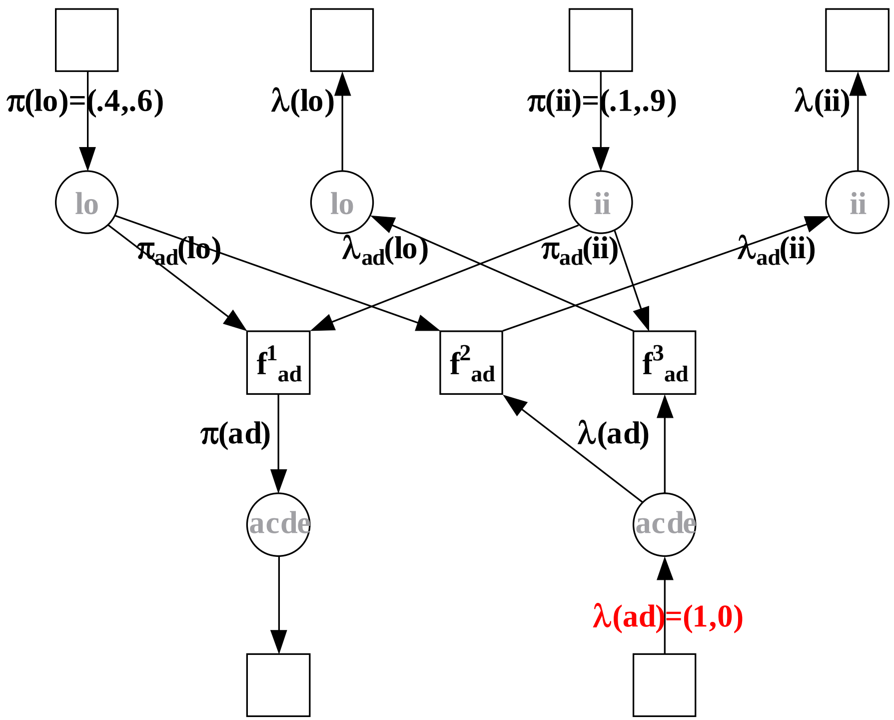
Petri Net Representation



$f^3_{ad}(ii, ad)$			
ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	1	0	0.8
0	1	0	0.2
1	0	1	0.6
0	0	1	0.4
1	0	0	0.0
0	0	0	1.0

$$P^*(lo) = \pi(lo) \circ \lambda(lo) = (0.4, 0.6) \circ (0.72, 0.06) = (\boxed{0.288,} 0.036)$$

Petri Net Representation



$P^*(lo) = (0.288, 0.036)$ change

$P^*(ii) = (0.04, 0.288)$ smaller

that means that
with evidence

"acceleration delayed"

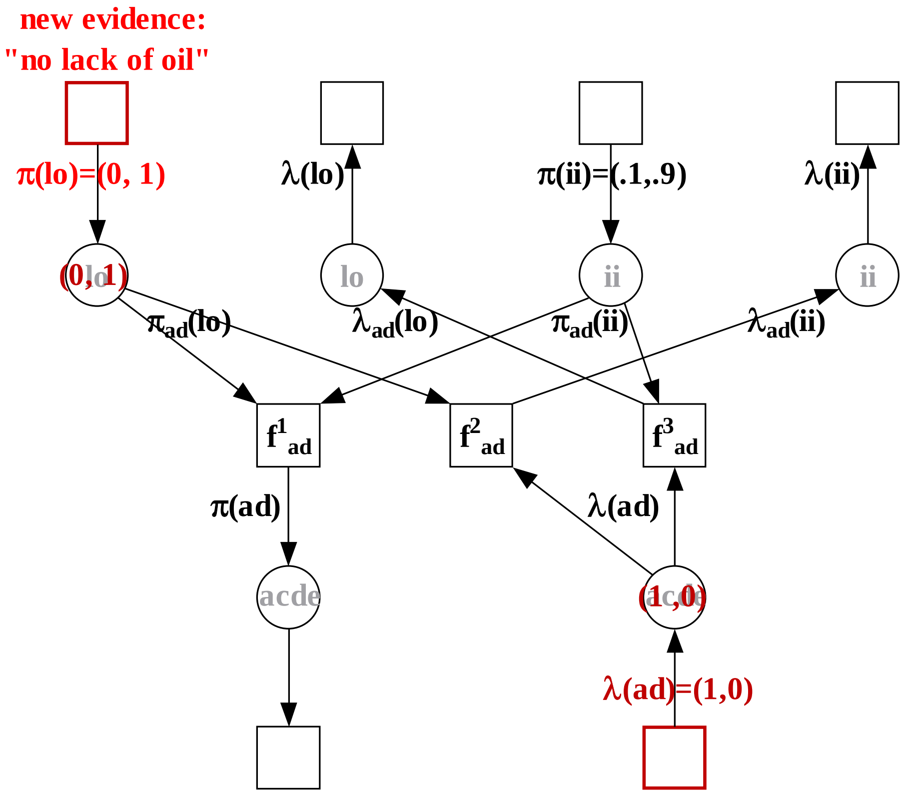
we get the former result that

"lack of oil"

"ignition normal"

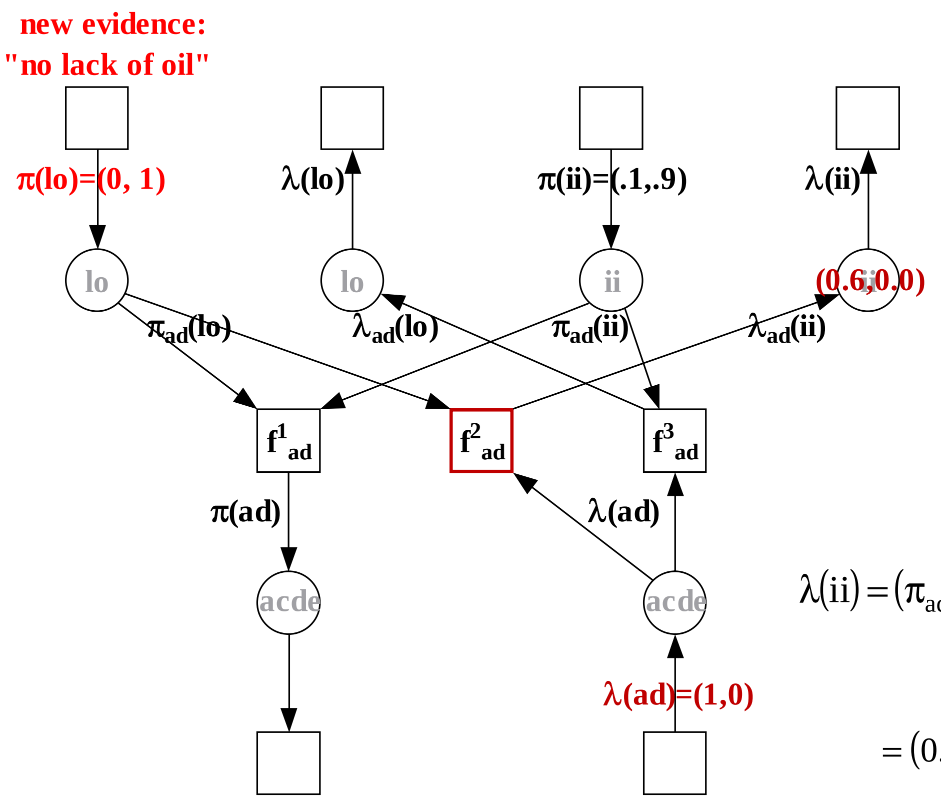
is most probable:
so, the **explanation set is $\{lo_1, ii_0\}$**

Petri Net Representation



f^2_{ad} (lo, ad)			
ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	0	1	0.6
0	0	1	0.4
1	1	0	0.8
0	1	0	0.2
1	0	0	0.0
0	0	0	1.0

Petri Net Representation



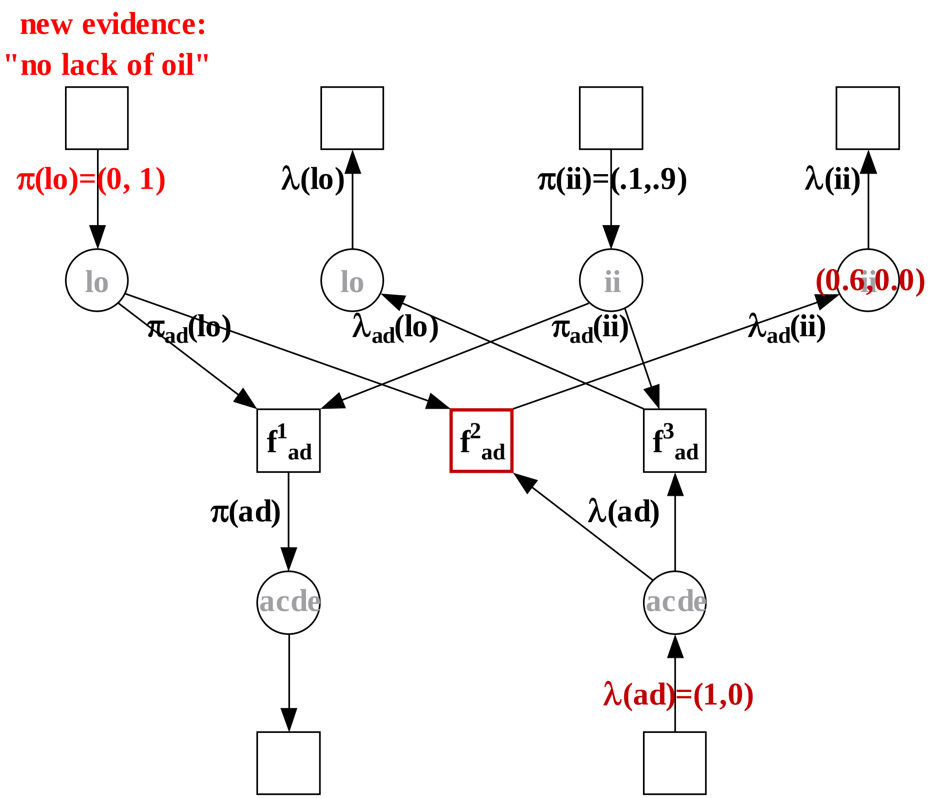
$f^2_{ad}(lo, ad)$			
ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	0	1	0.6
0	0	1	0.4
1	1	0	0.8
0	1	0	0.2
1	0	0	0.0
0	0	0	1.0

$$\lambda(ii) = (\pi_{ad}(lo)) \times (\lambda(ad)) \bullet f^2_{ad} = ((0.0, 1.0) \times (1.0, 0.0)) \bullet f^2_{ad}$$

$$= (0.0, 0.0, 1.0, 0.0) \bullet \begin{pmatrix} 1.0 & 0.8 \\ 0.0 & 0.2 \\ 0.6 & 0.0 \\ 0.4 & 1.0 \end{pmatrix}$$

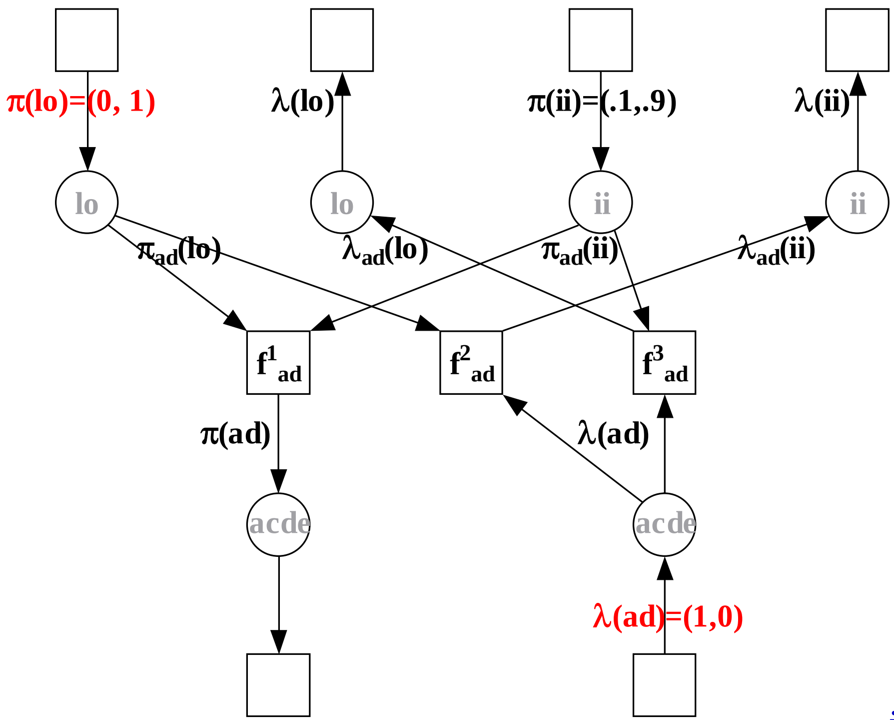
$$= (\max\{0.0, 0.0, 0.6, 0.0\},\ \max\{0.0, 0.0, 0.0, 0.\}) = (0.6, 0.0)$$

Petri Net Representation



$f^2_{ad}(lo, ad)$			
ad	lo	ii	
1	1	1	1.0
0	1	1	0.0
1	0	1	0.6
0	0	1	0.4
1	1	0	0.8
0	1	0	0.2
1	0	0	0.0
0	0	0	1.0

$$P^*(ii) = \pi(ii) \circ \lambda(ii) = (0.1, 0.9) \circ (0.6, 0.0) = (0.06, 0.0)$$



$P^*(ii) = (0.06, 0.0)$ change

that means that
with evidence

"acceleration delayed"

"no lack of oil"

we get the result that

"ignition irregular"

is most probable:
so, the **explanation set is { ii_1}**

Petri Net Representation

$\pi(lo)=(.4,.6)$ $P^*(lo)=(0.288, 0.54)$ $BEL(lo)=(0.348, 0.652)$ explanation set

$\pi(ii)=(.1,.9)$ $P^*(ii)=(0.04, 0.54)$ $BEL(ii)=(0.069, 0.931)$ = $\{lo_0, ii_0, ad_0\}$

$\lambda(ad)=(1,1)$ $P^*(ad)=(0.288, 0.54)$ $BEL(ad)=(0.348, 0.652)$

$\pi(lo)=(.4,.6)$ $P^*(lo)=(0.288, 0.036)$ $BEL(lo)=(0.889, 0.111)$ explanation set

$\pi(ii)=(.1,.9)$ $P^*(ii)=(0.04, 0.288)$ $BEL(ii)=(0.122, 0.878)$ = $\{lo_1, ii_0\}$

smaller

$\lambda(ad)=(1,0)$

= former result

{lo, igno}

$\pi(lo)=(0, 1)$

$\pi(ii)=(.1,.9)$ $P^*(ii)=(0.06, 0.0)$ $BEL(ii)=(1.0, 0.0)$ explanation set

$\lambda(ad)=(1,0)$ = $\{ii_1\}$