



# Colored Petri Nets to Model and Simulate Biological Systems

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# Outline

- Motivation
- Colored Petri nets-based framework
- Functionalities and features
- Constructing colored Petri net models
- Outlook

# Motivation

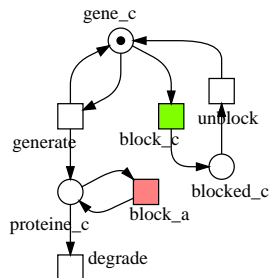
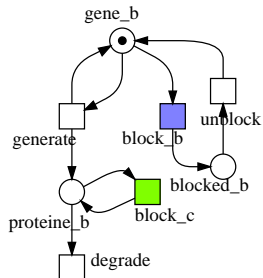
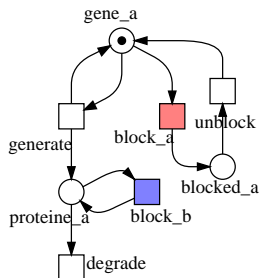
# Low-level Petri nets

- Low-level Petri nets do not scale,
- Mainly restricted to relatively small models,
- Difficult to manage and understand large-scale nets,
- Increasing risk of modeling errors for large-scale nets.

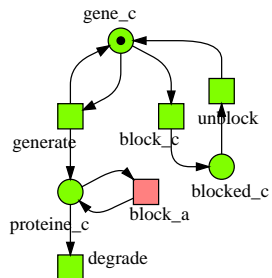
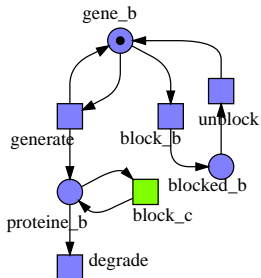
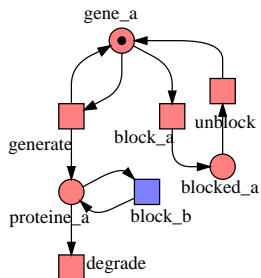
# Colored Petri nets

- Compact and readable representation,
- Increasing net size = increasing color sets,
- Analysis techniques of low-level Petri nets by automatic unfolding,
- Analysis techniques of high-level Petri nets.

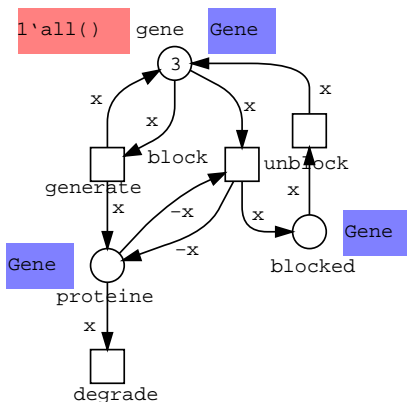
# Example: the repressilator



# Example: the repressilator



# Colored Petri net model for the repressilator



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Declarations

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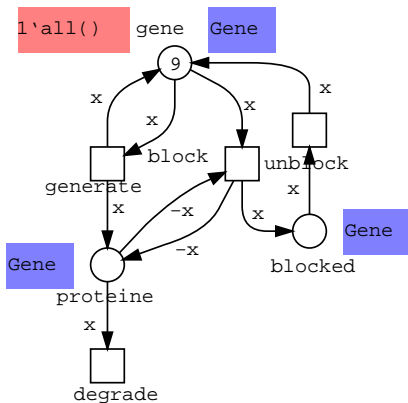
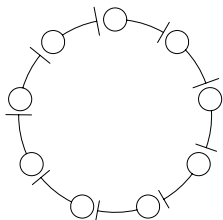
Colorset Gene = enum with a-c

Variable x: Gene

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# Colored Petri net model for the generalized repressilator



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Declarations

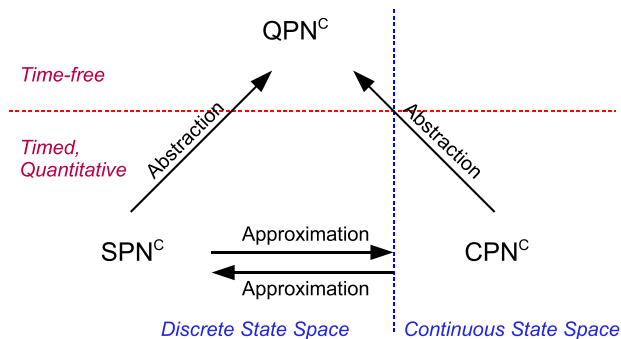
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Colorset Gene = enum with a-i

Variable x: Gene

# Colored Petri nets-based framework

# Colored Petri nets-based framework



# Colored qualitative Petri net ( $QPN^C$ )

- A colored extension of extended P/T net,  
e.g., inhibitor arc, read arc, equal arc and reset arc,
- Predefined data types for color set definition:
  - ▶ Basic types: dot, integer, string, Boolean, enumeration, index,
  - ▶ Structured types: product, union.

# Colored stochastic Petri net ( $SPN^C$ )

- A colored extension of biochemically interpreted extended stochastic Petri nets,
- Many features helpful for modeling biological systems, e.g., initial marking definition, rate function definition.

# Functionalities and features

# Functionalities

- Colored Petri net models drawn as usual.
- Automatic syntax checking of declarations and expressions.
- Qualitative animation
  - ▶ Automatic animation,
  - ▶ Single-step animation by manually choosing a binding.
- Stochastic simulation:
  - ▶ Done on an automatically unfolded Petri net,
  - ▶ Several algorithms, e.g. Gillespie stochastic simulation algorithm (SSA),
  - ▶ Results for colored or uncolored places/transitions.
- $QPN^C$  and  $SPN^C$  are exported to different net formalisms.

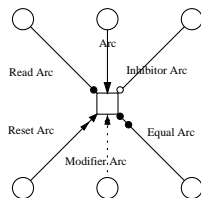
# Features for modeling biological systems


- 1 Concise specification of initial markings,
- 2 Rate function for each instance of a colored transition,
- 3 Special arc types:

- ▶ inhibitor arc,
- ▶ read arc,
- ▶ equal arc,
- ▶ reset arc,
- ▶ modifier arc,

## 4 Transition types:

- ▶ stochastic transitions with freestyle rate functions,
- ▶ immediate firing,
- ▶ deterministic firing delay,
- ▶ scheduled firing.



  
Stochastic transition

  
Immediate transition

  
Deterministic transition

  
Scheduled transition



# 1. Specification of initial markings

- Specifying colors and their corresponding tokens as usual,
- Specifying a set of colors with the same number of tokens,
- Using a predicate to choose a set of colors and then specifying the same number of tokens,
- Using the *all()* function to specify all colors with a specified number of tokens.

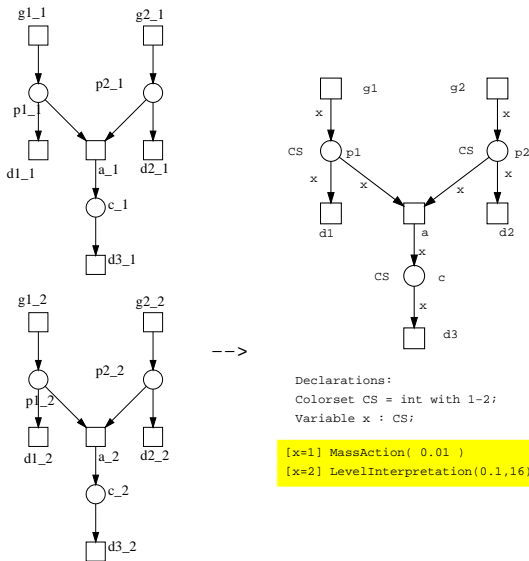
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Color/Predicate/Function	marking
1	1
4,5,7	2
$x > 10$	3
<i>all()</i>	4

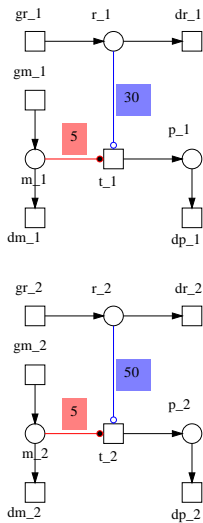
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Colorset CS = int with 1-100;

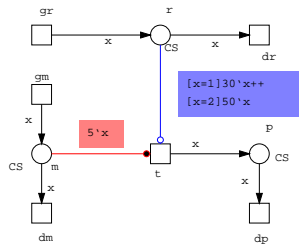
## 2. Rate function for each instance of a colored transition



### 3. Special arc types



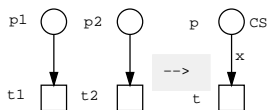
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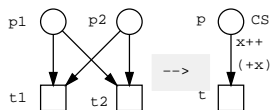
Declarations:  
 Colorset CS=int with 1,2;  
 Variable x:CS;

# Constructing colored Petri net models

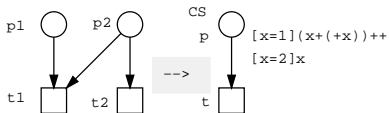
# Basic colored Petri net components



(a)



(b)



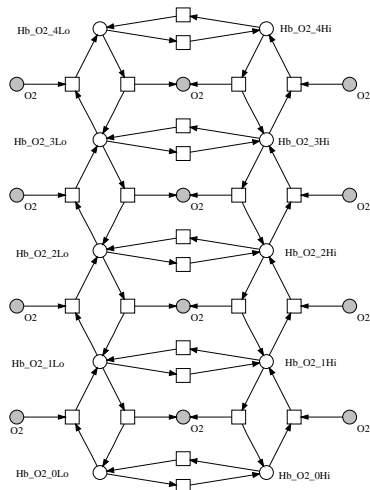
(c)

Declarations:  
 colorset CS = int with 1,2;  
 variable x : CS ;

(d)

# Cooperative binding of oxygen to hemoglobin

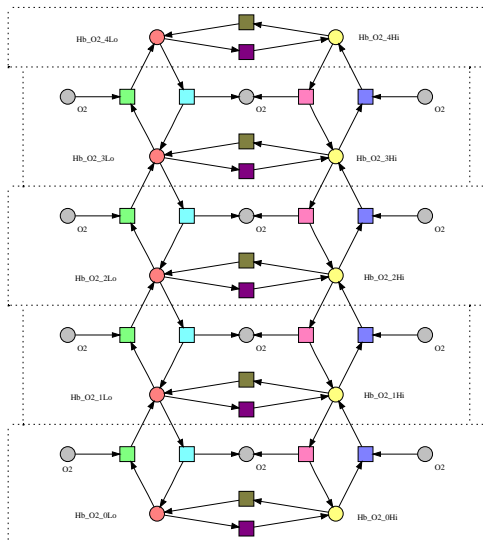
- Binding of oxygen to the four subunits of a hemoglobin heterotetramer,
- Oxygen (O<sub>2</sub>) is represented in the form of multiple copies of one logical place



Reference: W. Marwan, A. Wagler, R. Weismantel: Petri Nets as a Framework for the Reconstruction and Analysis of Signal Transduction Pathways and Regulatory Networks. Natural Computing. 9 (2009)

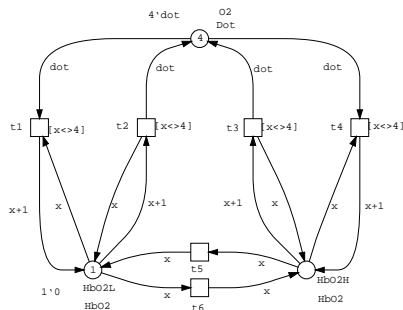
# Cooperative binding of oxygen to hemoglobin

- Subnet partition,
- Node set partition.



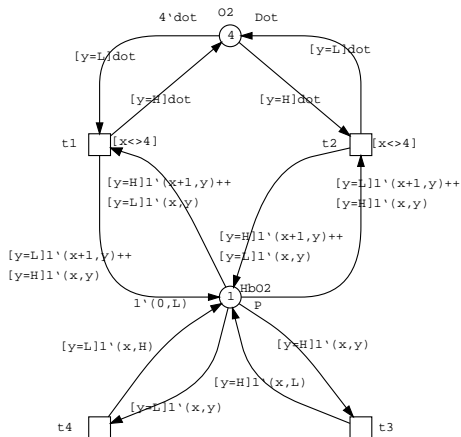
# Cooperative binding of oxygen to hemoglobin

- Define color sets and variables,
- Assign color sets to places,
- Define guards/rate functions for transitions
- Write arc expressions.





# Cooperative binding of oxygen to hemoglobin



# Cooperative binding of oxygen to hemoglobin: declarations for the colored models

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## Declarations

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Colorset Dot = dot;

Colorset HbO2 = int with 0-4;

Colorset Level = enum with H,L;

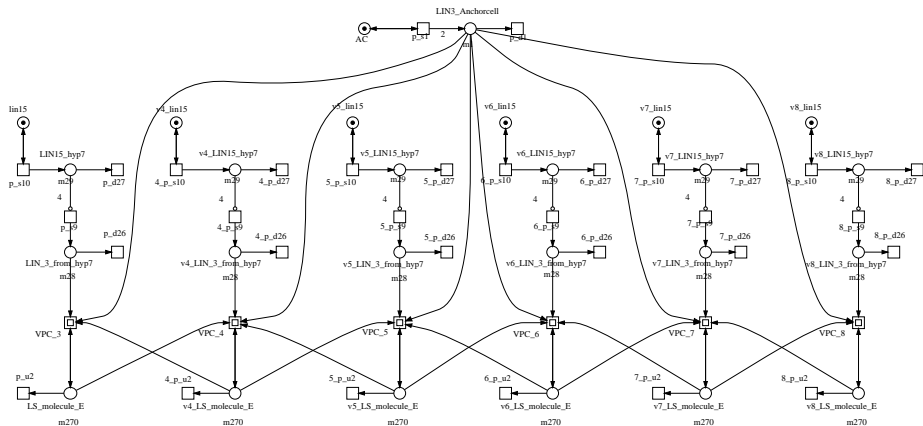
Colorset P = product with HbO2  $\times$  Level;

Variable x: HbO2;

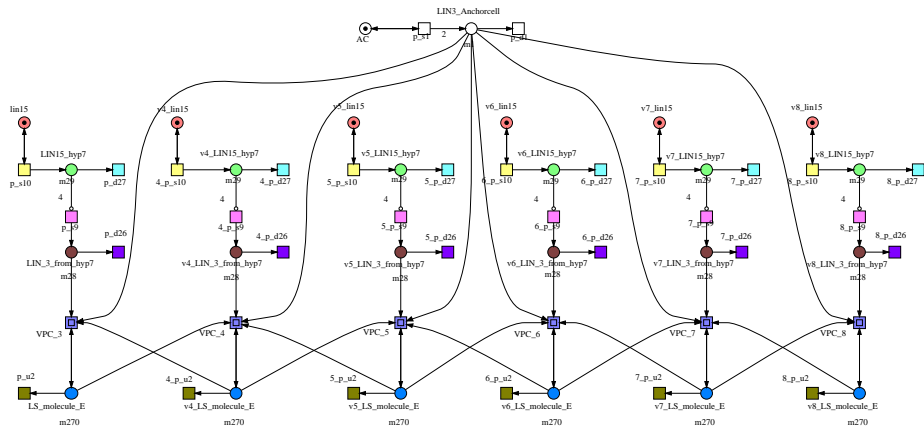
Variable y: Level;

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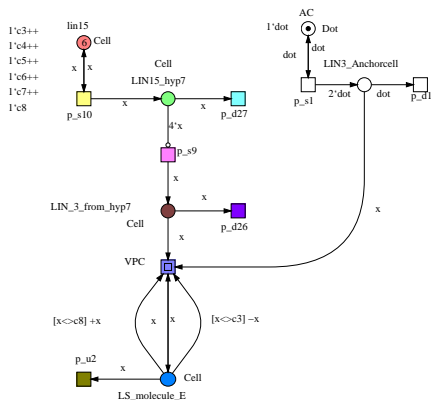
# C. elegans vulval development model



# C. elegans vulval development model



## C. elegans vulval development model

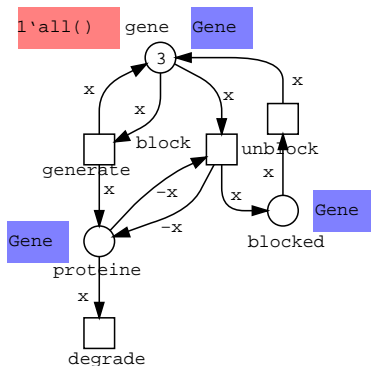


Colorset Dot = dot;

Colorset Cell = enum with c3,c4,c5,c6,c7,c8;

Variable x: Cell;

# Colored stochastic Petri net model for the repressilator



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Declarations

---

Colorset Gene = enum with a-c

Variable x: Gene

---

# Colored stochastic Petri net model for the repressilator

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## Transition Rate function

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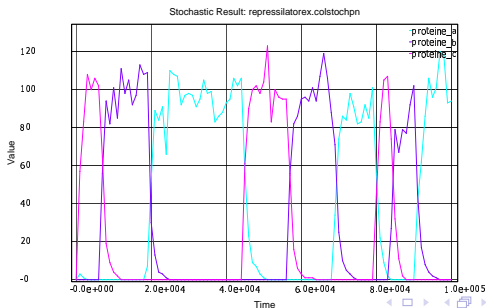
generate  $0.1 * gene$

block  $1.0 * proteine$

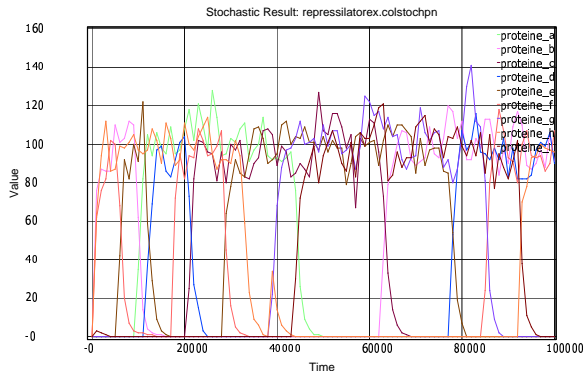
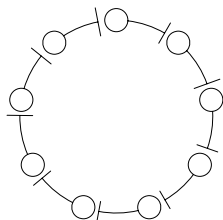
unblock  $0.0001 * blocked$

degrade  $0.001 * proteine$

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# Colored stochastic Petri net model for the repressilator



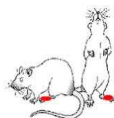


# Outlook

# Outlook

- Improving  $QPN^C$  and  $SPN^C$  prototypes,
- Developing analysis tools for  $SPN^C$ ,
- Developing the  $CPN^C$  prototype,
- Considering how to automatically create colored Petri nets from non-coloured Petri nets (automatic folding),
- working on a  $SPN^C$  model for *C. elegans* vulval development,
- Working on a case study, the underlying uncolored model: about 110,000 places and 135,000 transitions.

# Acknowledgments



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and Research

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- Thank Wolfgang Marwan, Christian Rohr, and Martin Schwarick for their assistance in model construction and software development,
- Thank the anonymous referees for many constructive comments.

# Thanks for your attention !

Begin to demonstrate  $QPN^C/SPN^C$