

# MODULARIZATION BY T-INVARIANTS

□ Let  $X$  denote the set of all (non-trivial) minimal T-invariants  $x$  of a given PN.

□ **dependency relation**

*Two transitions  $i, j$  depend on each other,*

*if they always appear together in all minimal T-invariants  $x$ , i.e.*

$$\forall(x \in X) : \text{supp}(x)(i) = \text{supp}(x)(j)$$

□ **equivalence relation** in the transition set, leading to a partition of T

-> *reflexive*

-> *symmetric*

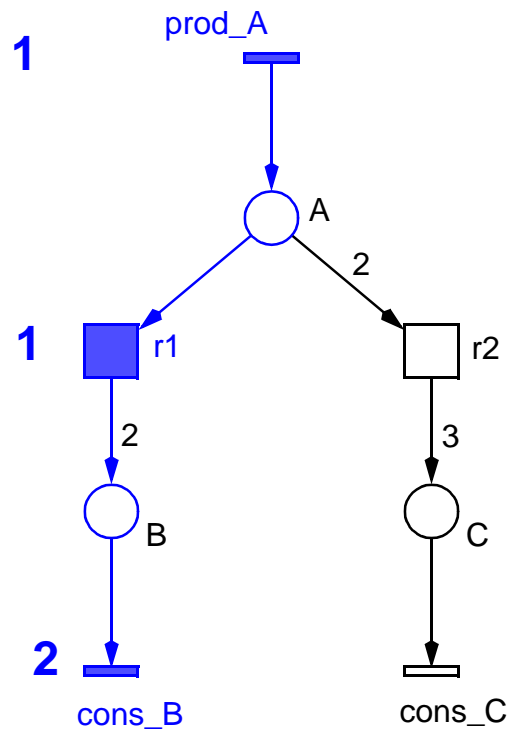
-> *transitive*

□ the **equivalence classes**  $A$  represent maximal ADT-sets

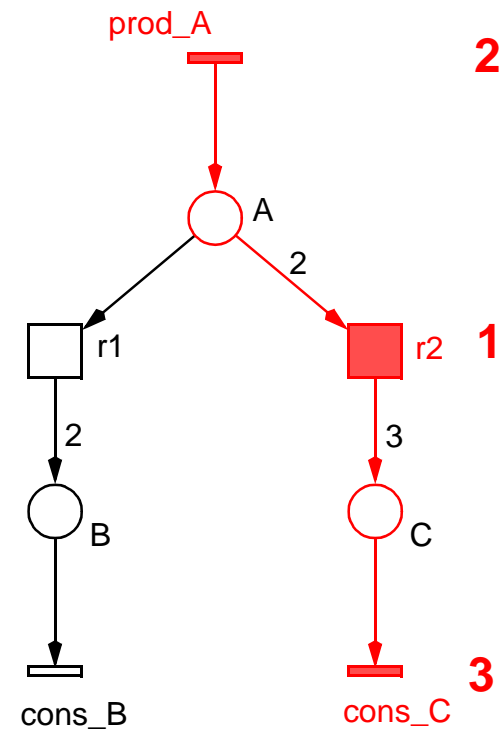
$$\forall(x \in X) : A \subseteq \text{supp}(x) \vee A \cap \text{supp}(x) = \emptyset$$

$r1: A \rightarrow 2 B$

$r2: 2 A \rightarrow 3 C$

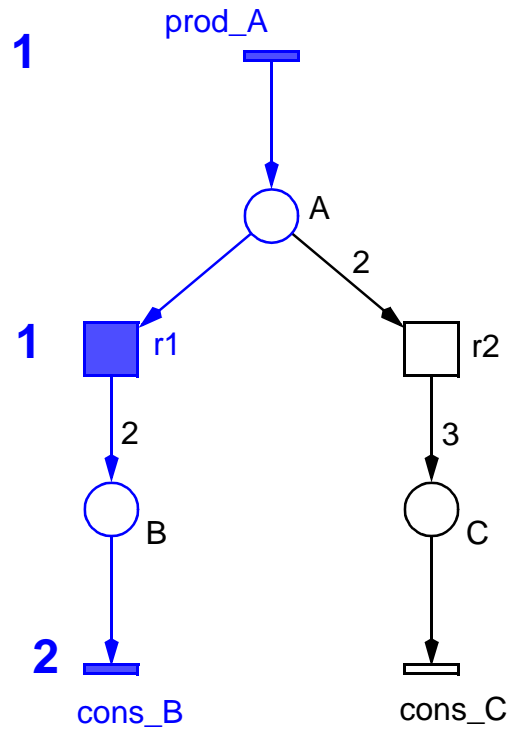


**T-INVARIANT 1**

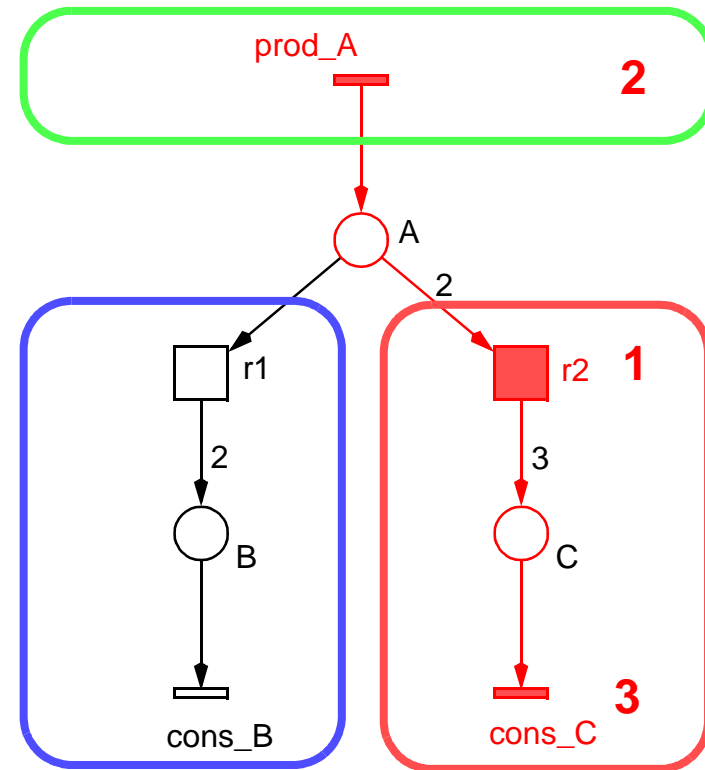


**T-INVARIANT 2**

$r1: A \rightarrow 2 B$   
 $r2: 2 A \rightarrow 3 C$



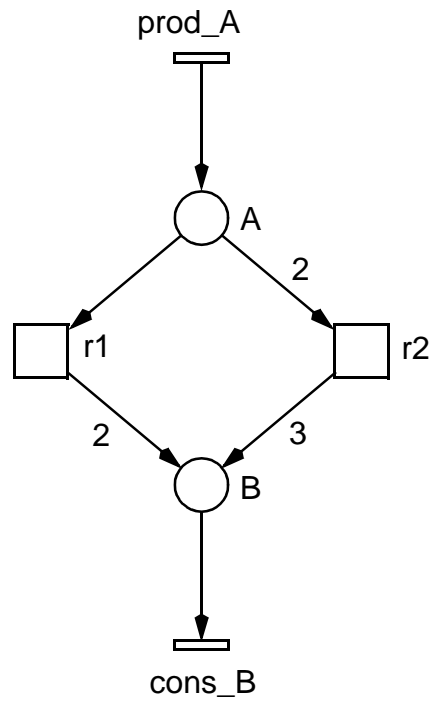
**T-INVARIANT 1**



**T-INVARIANT 2**

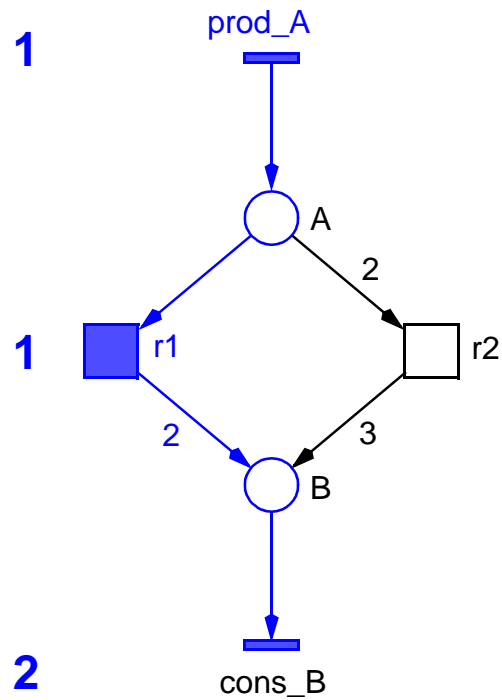
$r1: A \rightarrow 2 B$

$r2: 2 A \rightarrow 3 B$

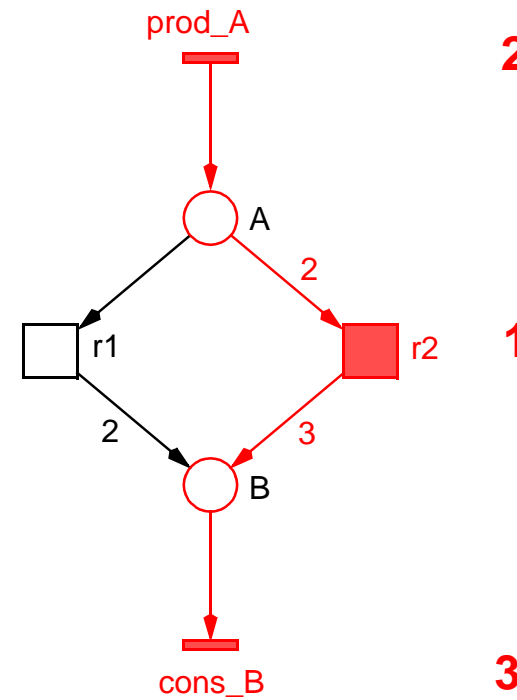


$r1: A \rightarrow 2 B$

$r2: 2 A \rightarrow 3 B$

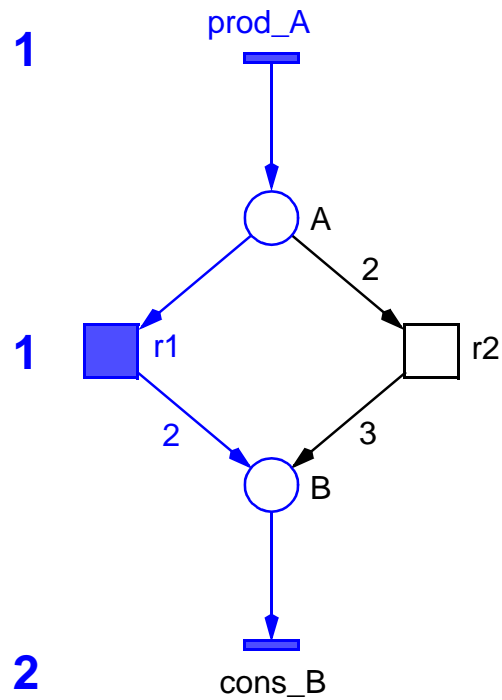


**T-INVARIANT 1**

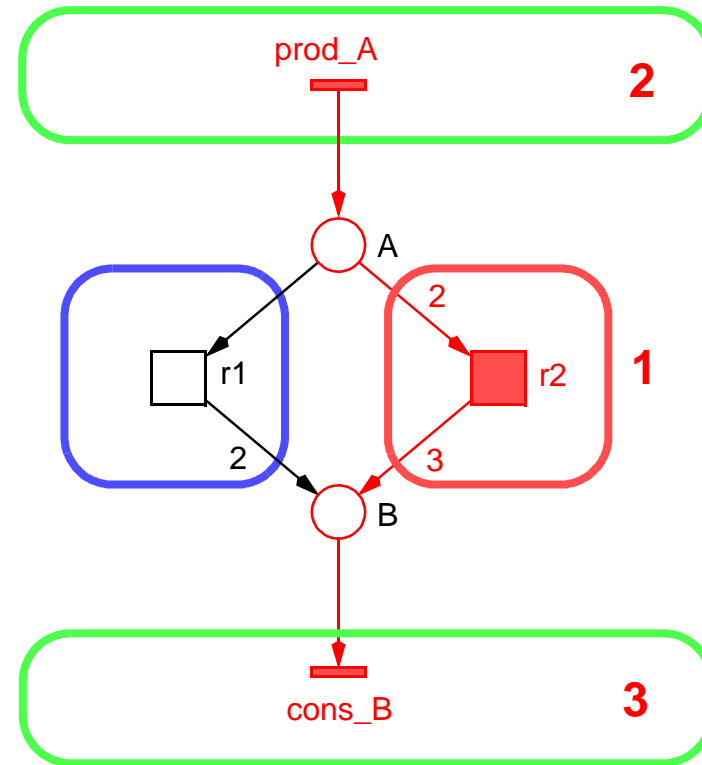


**T-INVARIANT 2**

$r1: A \rightarrow 2 B$   
 $r2: 2 A \rightarrow 3 B$



**T-INVARIANT 1**



**T-INVARIANT 2**

## ❑ maximal ADT-sets

- > *disjunctive subnets*
- > *not necessarily connected*

## minimal T-invariants

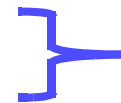
- > *overlapping subnets*
- > *connected*

## ❑ interpretation

- > *structural decomposition into rather small subnets*
- > *smallest biologically meaningful functional units*
- > *building blocks*

## ❑ variations

- > *with / without trivial T-invariants*
- > *whole / partial set of T-invariants*



*not necessarily maximal ADT-sets*

## ❑ classification of all transitions based on the T-invariants' support



## ❑ maximal ADT-sets

-> *not necessarily connected*

## ❑ further decomposition into connected ADT-sets

-> *possibly according to primary compounds, only,  
i.e. neglecting connections by auxiliary compounds*

-> *non-maximal ADT-sets*

## ❑ **coarse network structure, definition**

-> *macro transitions*      -    *abstract from connected ADT-sets*

-> *places*                      -    *interface between functional units*

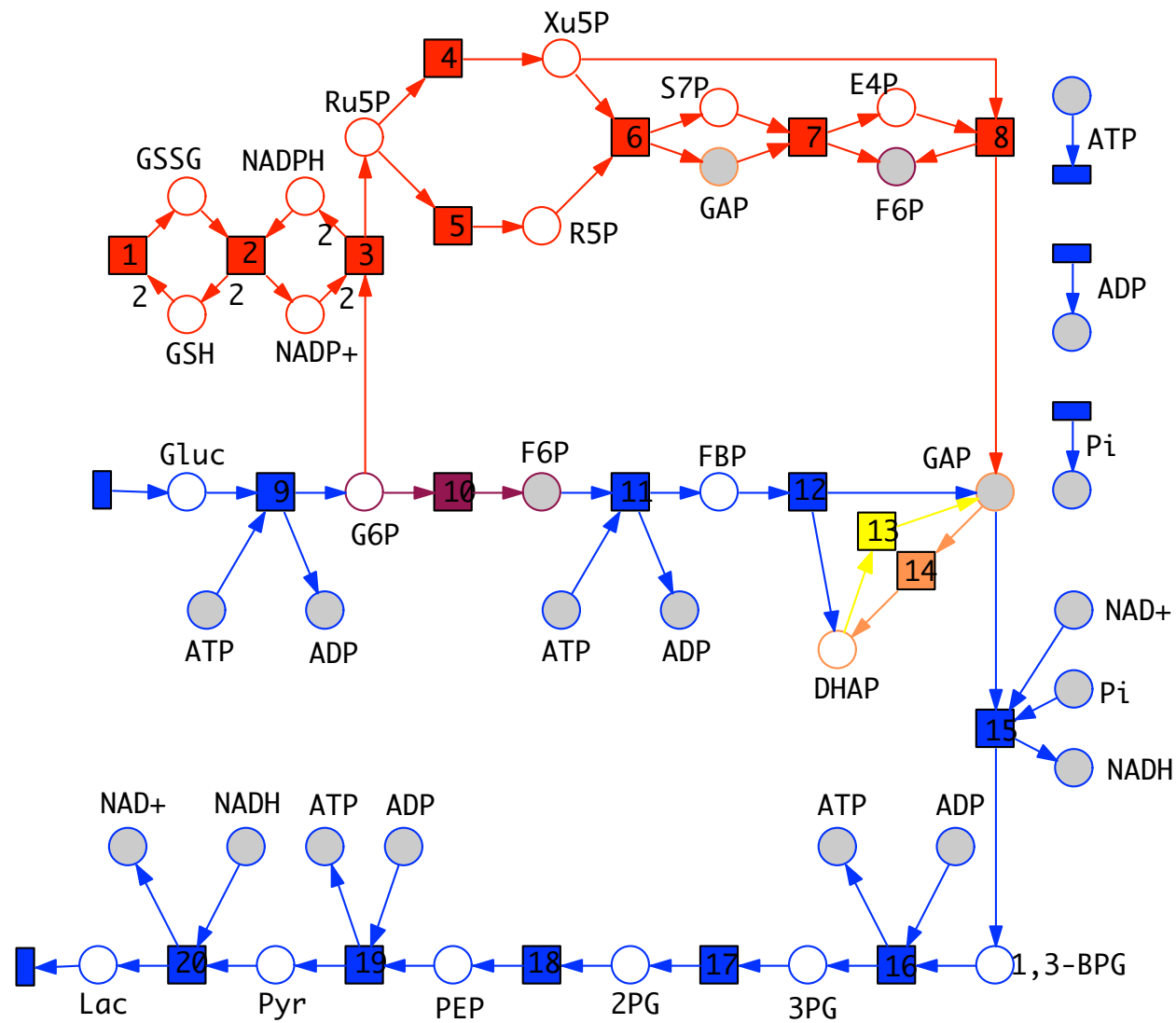
-> *(minimal) path*              -    *(minimal) T-invariant*

## ❑ **coarse network structure, what for?**

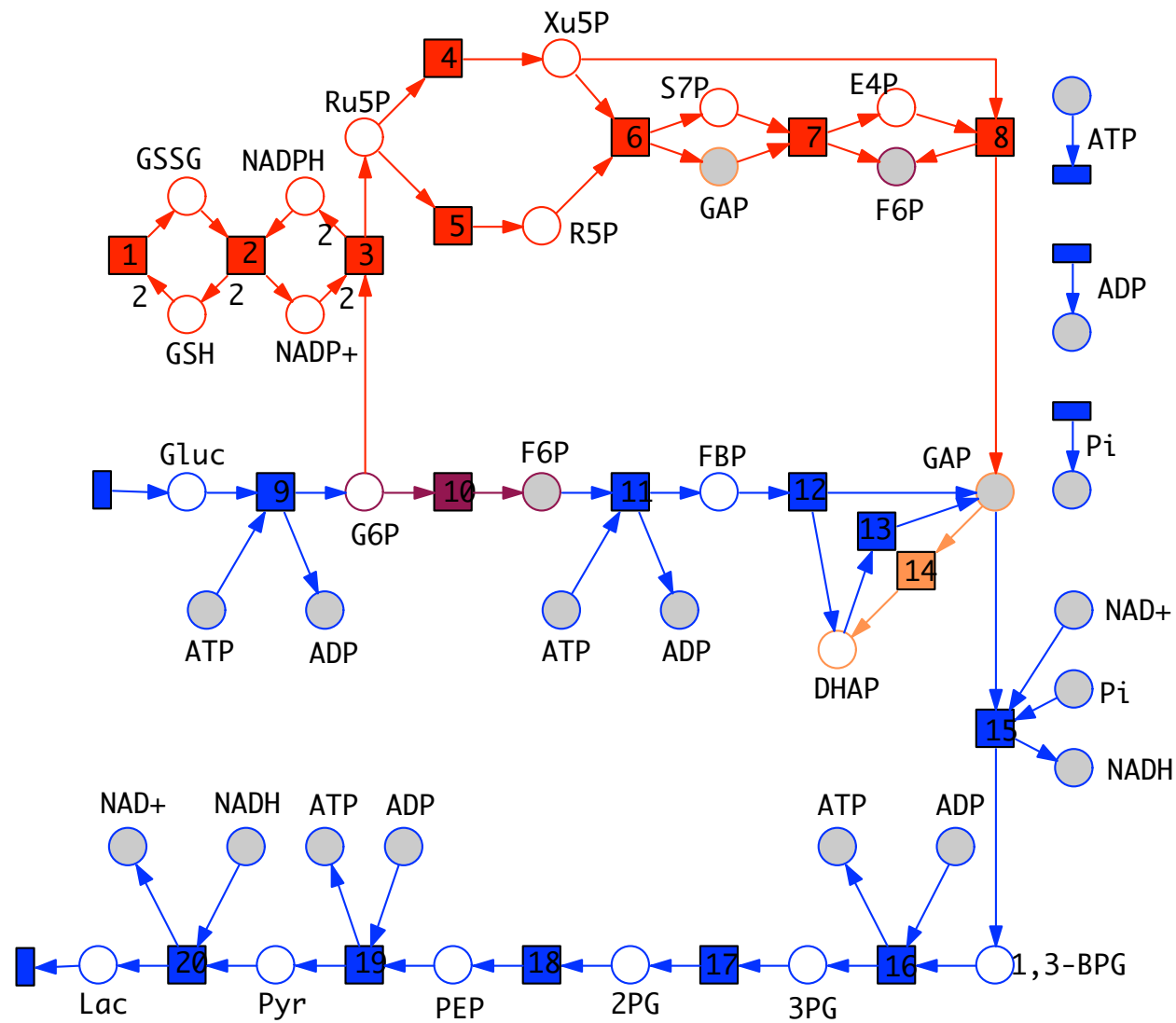
-> *set of T-invariants gets structured*

-> *better understanding of the net behaviour*

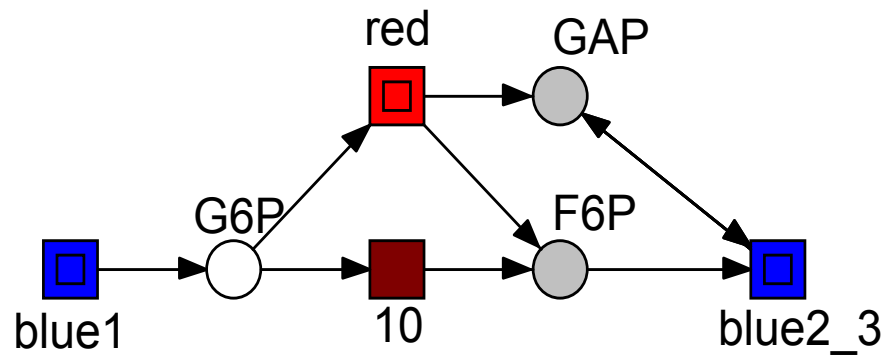
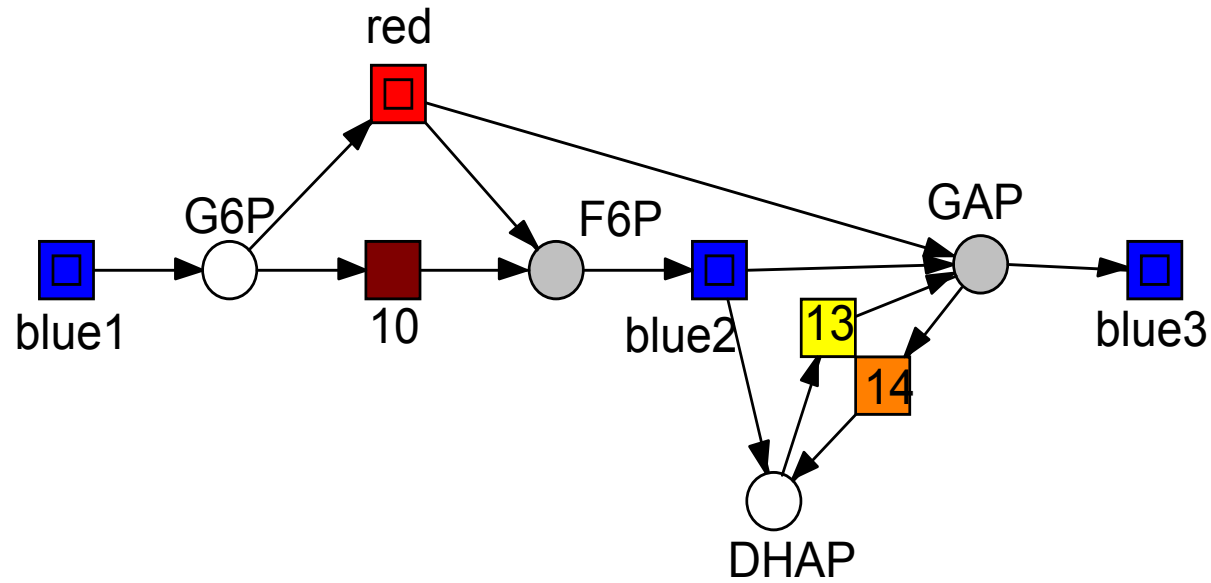
# Ex1 - Glycolysis and Pentose Phosphate Pathway



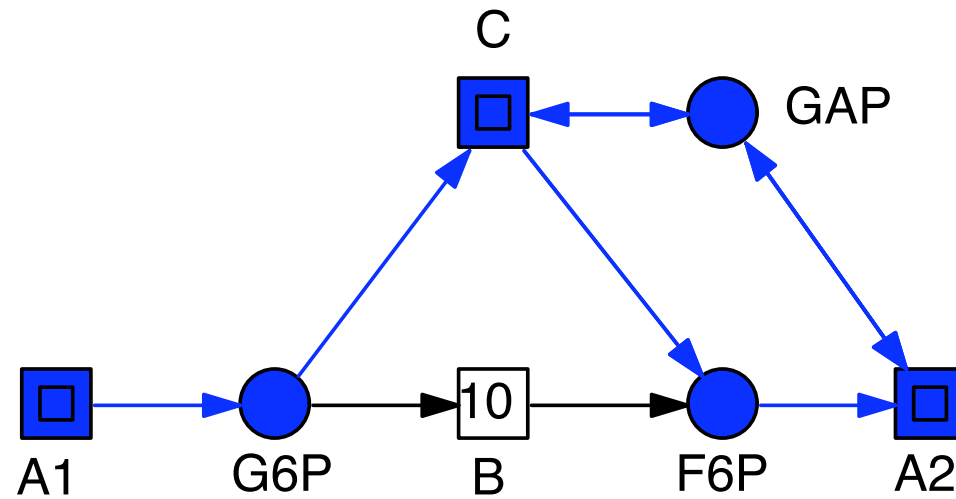
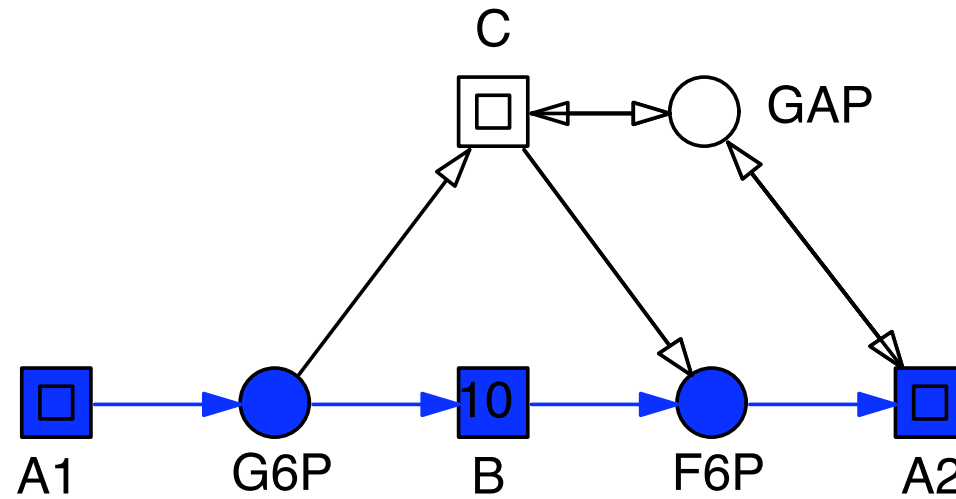
# Ex1 - Glycolysis and Pentose Phosphate Pathway



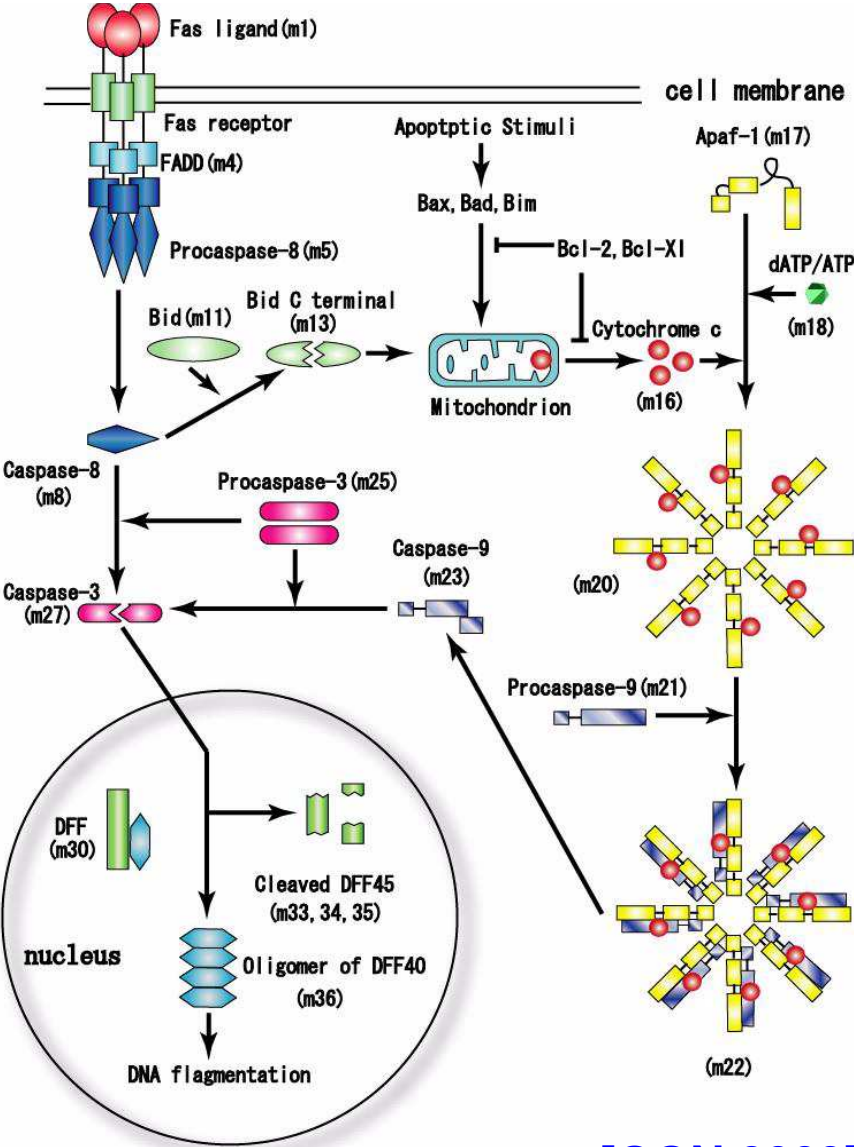
# Ex1 - Glycolysis and Pentose Phosphate Pathway



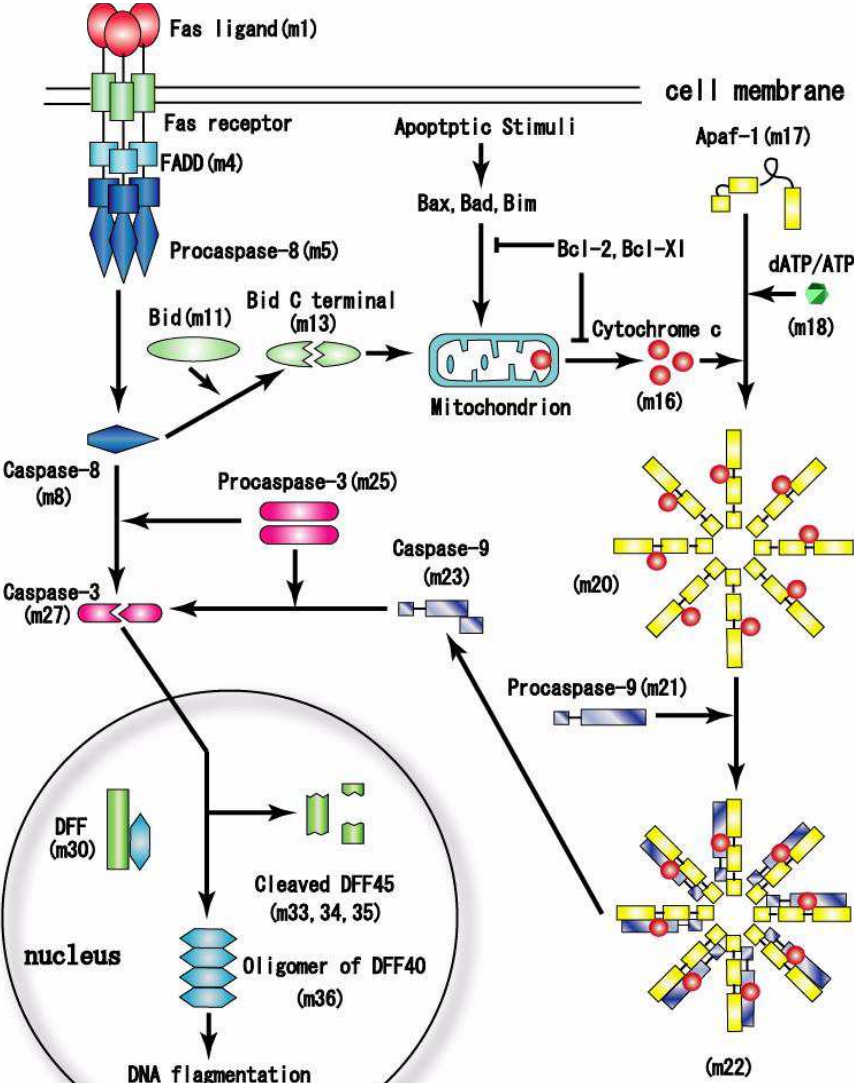
# Ex1 - Glycolysis and Pentose Phosphate Pathway



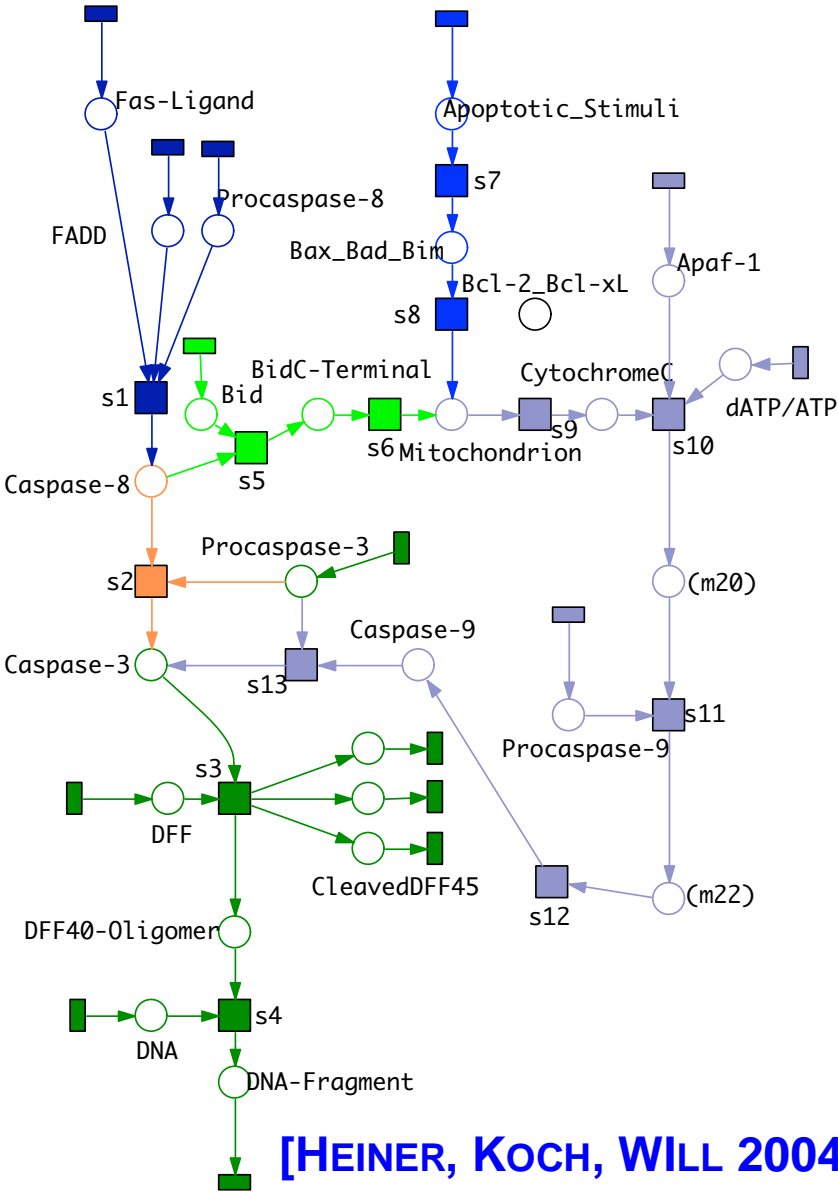
# Ex2 - APOPTOSIS IN MAMMALIAN CELLS



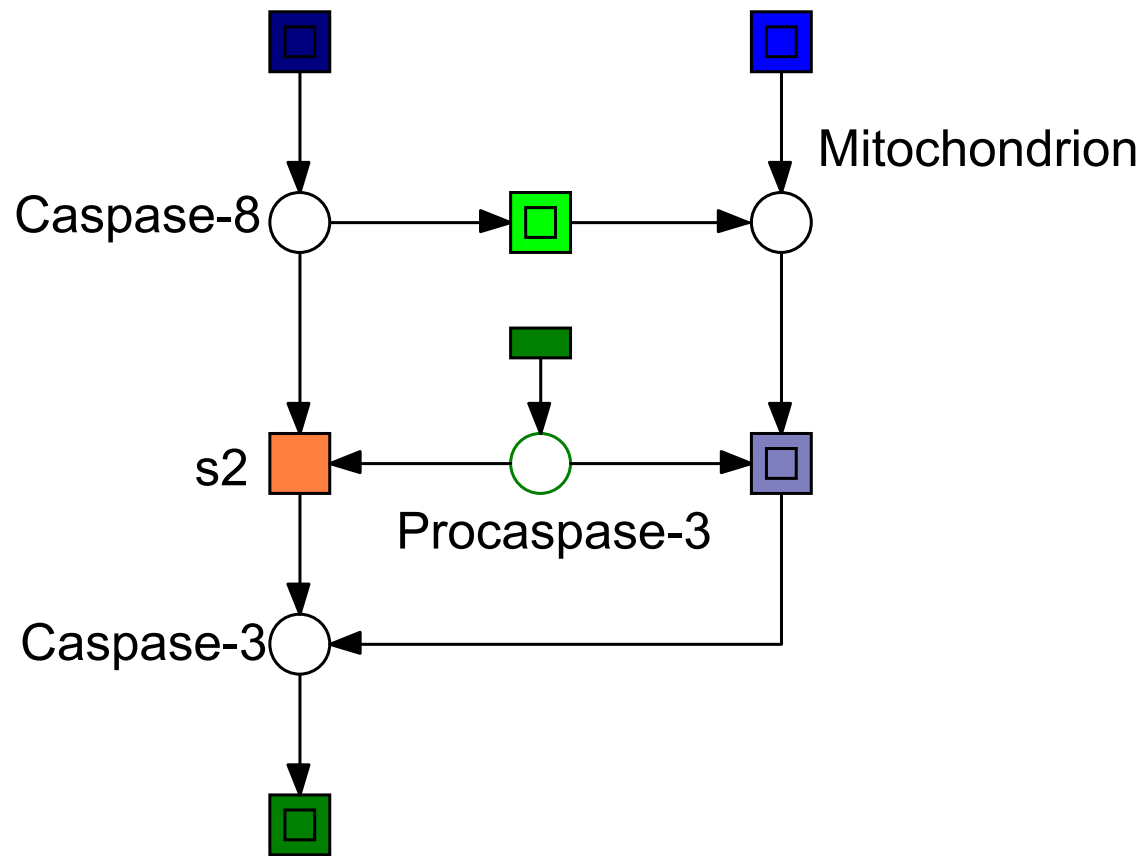
# Ex2 - APOPTOSIS IN MAMMALIAN CELLS



[GON 2003]

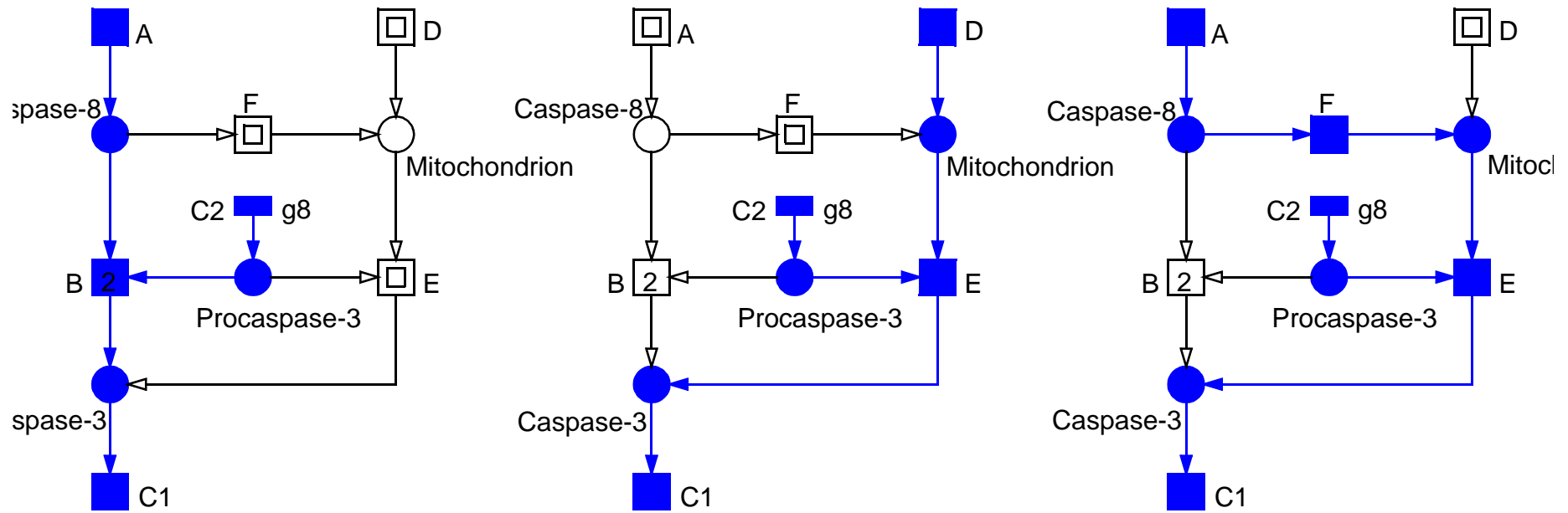


[HEINER, KOCH, WILL 2004]





# Ex2: APOPTOSIS IN MAMMALIAN CELLS





[KOCH, JUNKER, HEINER 2005]

# Ex3 - Carbon Metabolism in Potato Tuber



[KOCH, JUNKER, HEINER 2005]

*ADT-sets without trivial T-invariants*

