

T- INVARIANTS

(ELEMENTARY MODES)

(EXTREME PATHWAYS)

(GENERIC PATHWAYS)

- a representation of the net structure

=> **stoichiometric matrix**

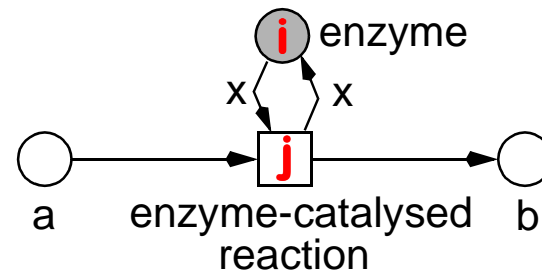
$$C =$$

| | | | | | |
|-------|----|-----|-----|-----|----|
| P \ T | t1 | ... | tj | ... | tm |
| p1 | | | | | |
| pi | | | cij | | |
| ⋮ | | | Δtj | | |
| pn | | | | | |

$$c_{ij} = (p_i, t_j) = F(t_j, p_i) - F(p_i, t_j) = \Delta t_j(p_i)$$

$$\Delta t_j = \Delta t_j^*$$

- matrix entry c_{ij} :
token change in place p_i by firing of transition t_j
- matrix column Δt_j :
vector describing the change of the whole marking by firing of t_j
- side-conditions are neglected



$$c_{ij} = 0$$

□ Lautenbach, 1973

-> Schuster, 1993

□ T-invariant x

-> *multiset of transitions*

-> integer solution of $Cx = 0, x \neq 0, x \geq 0$

□ support of a T-invariant x -> $\text{supp}(x)$

-> *set of transitions*

-> set of transitions involved, i.e. $x(i) \neq 0$

□ minimal T-invariants

-> there is no T-invariant with a smaller support

-> gcd of all entries is 1

□ any T-invariant is a non-negative linear combination of minimal ones

-> multiplication with a positive integer

-> addition

-> Division by gcd

$$kx = \sum_i a_i x_i$$

□ T-invariants = (multi-) sets of transitions = Parikh vector

- > zero effect on marking
- > reproducing a marking / system state

□ two interpretations

1. *partially ordered transition sequence* **-> behaviour understanding**
of transitions occurring one after the other
-> substance / signal flow
2. *relative transition firing rates* **-> steady state behaviour**
of transitions occurring permanently & concurrently
-> steady state behaviour

□ a minimal T-invariant defines a connected subnet

- > the T-invariant's transitions (the support),
+ all their pre- and post-places
+ the arcs in between
- > pre-set of support = post-set of support

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-> *steady state behaviour* -> **steady state behaviour**

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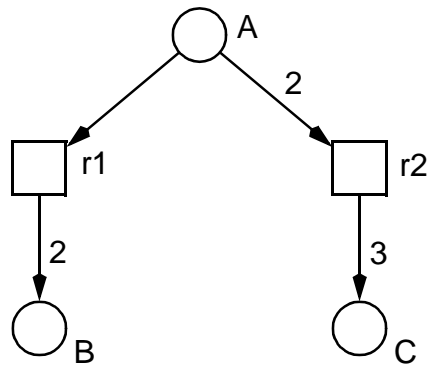
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$r2: 2 A \rightarrow 3 C$

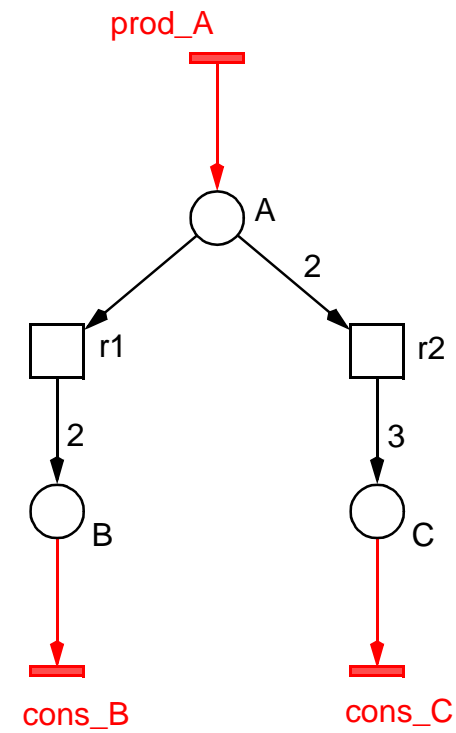
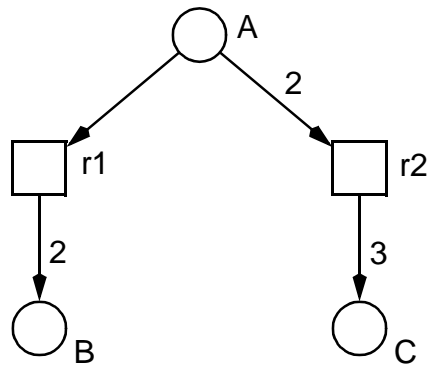
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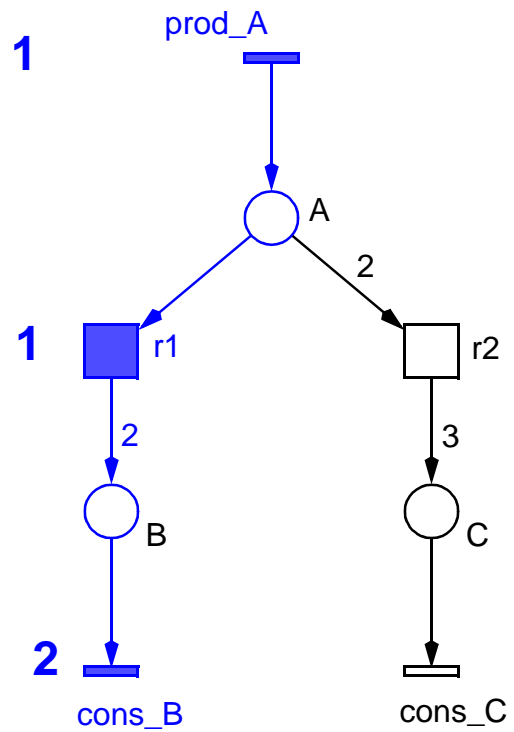
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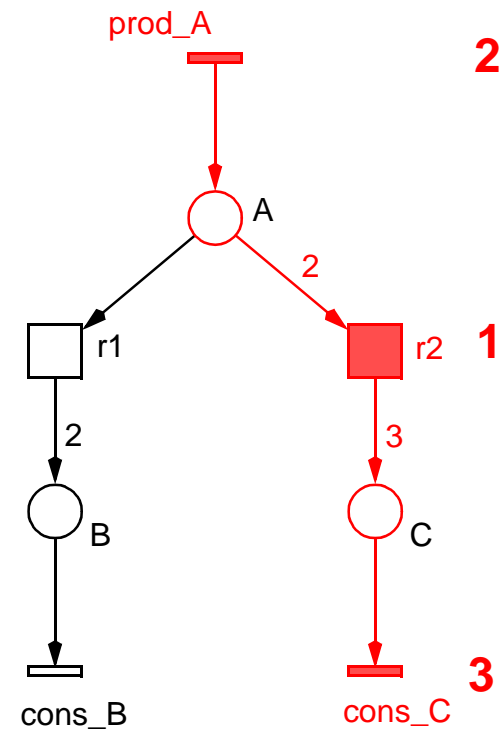


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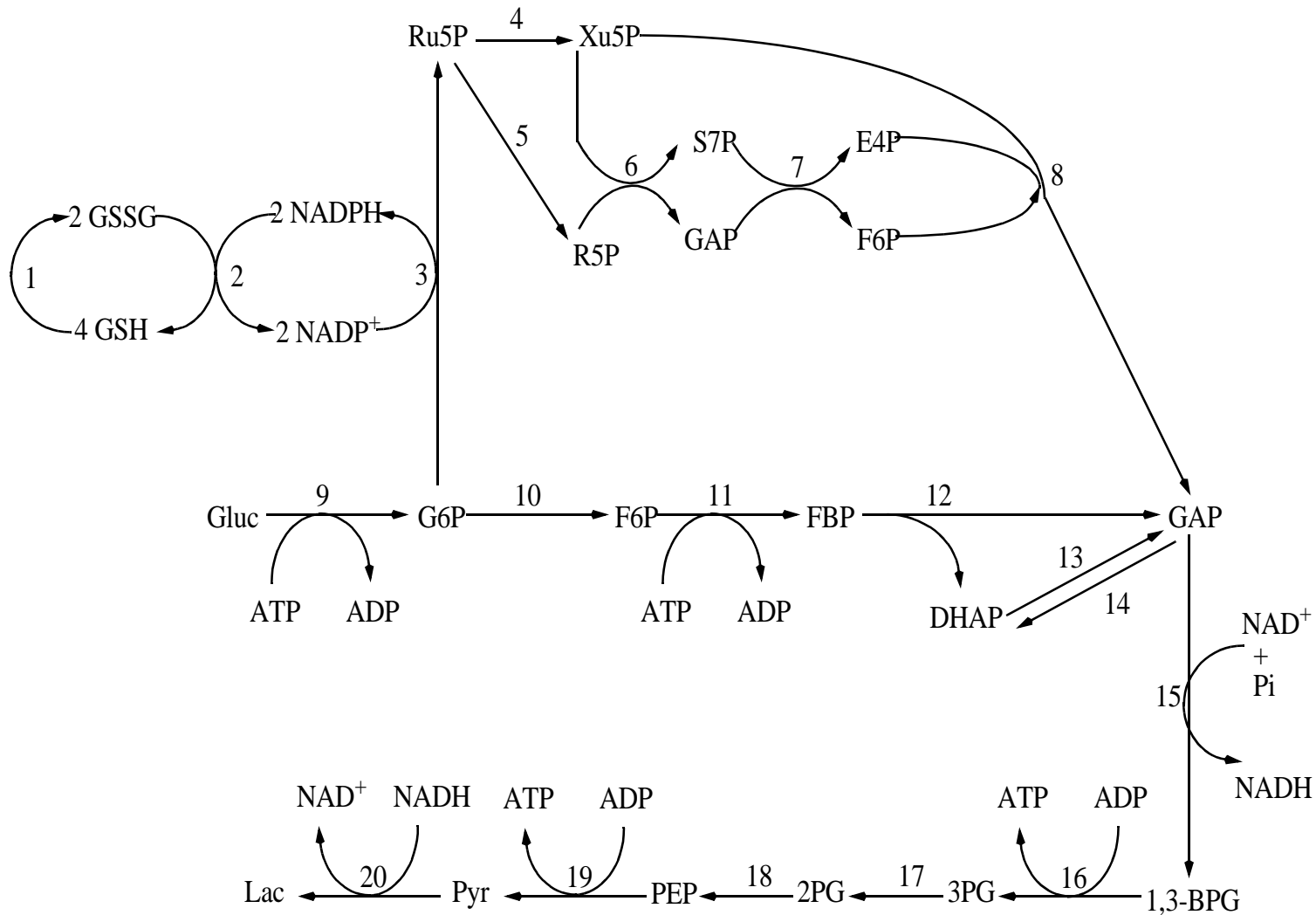
T-INVARIANT 1



T-INVARIANT 2

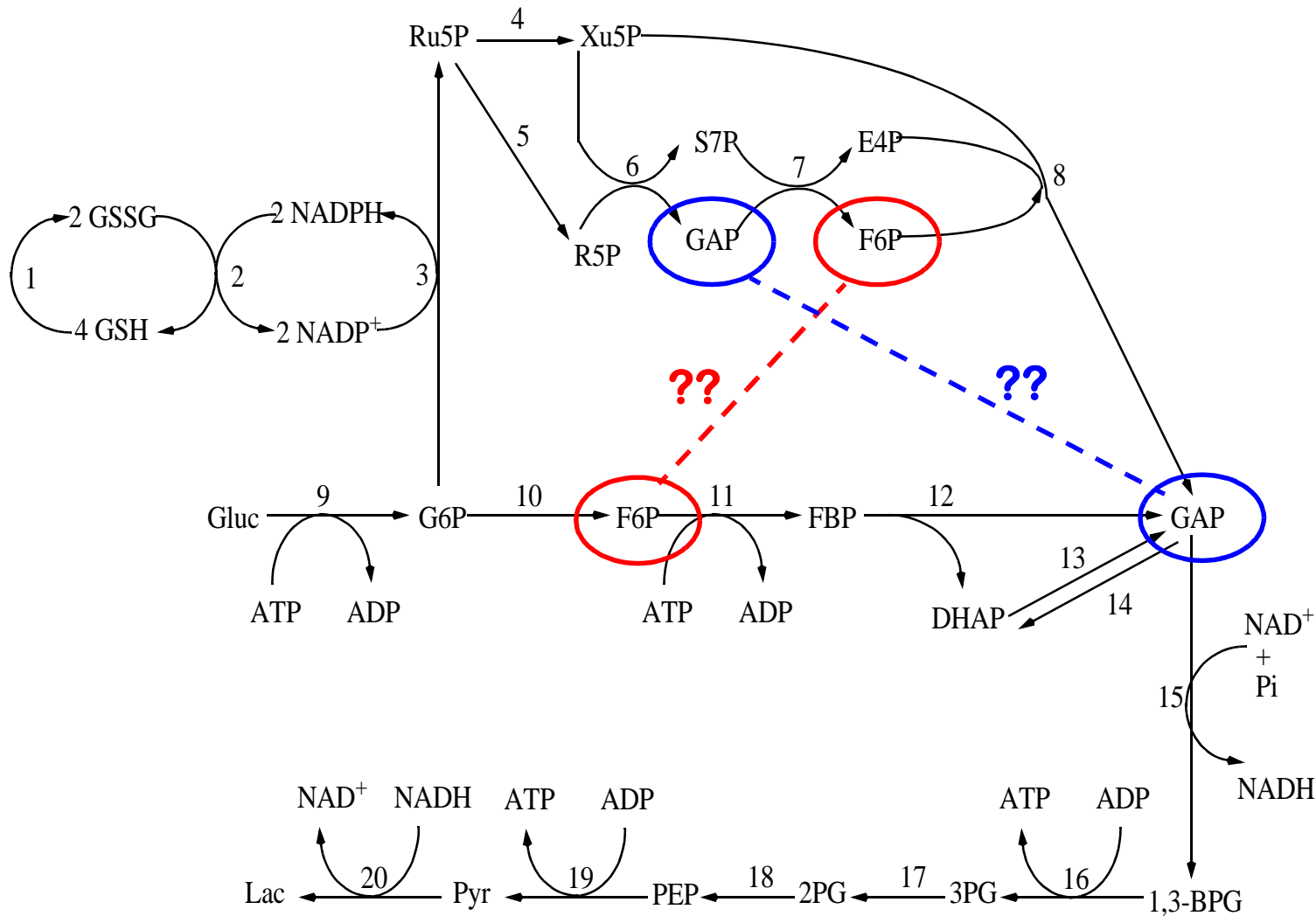
Ex1 - Glycolysis and Pentose Phosphate Pathway

[Reddy 1993]



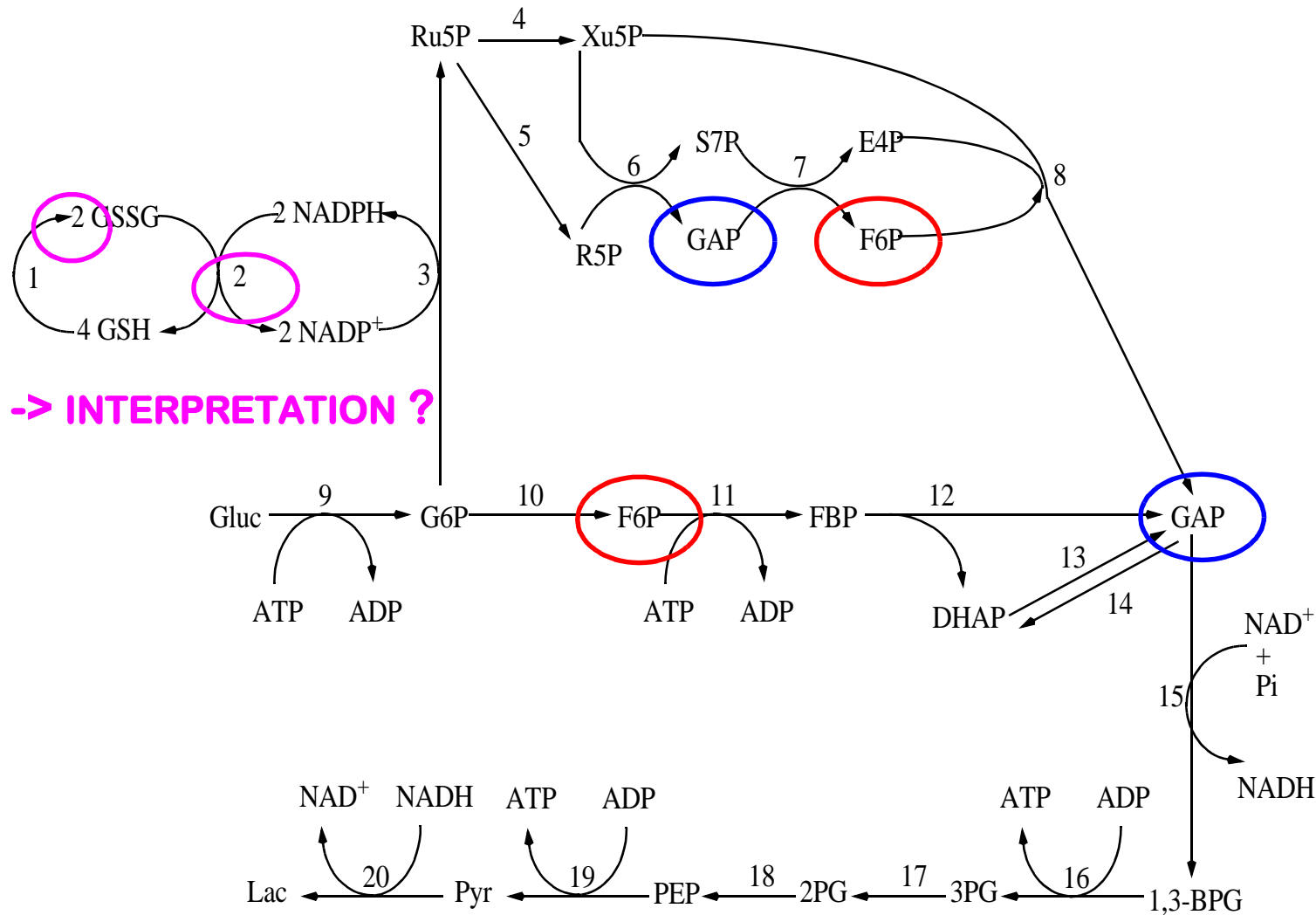
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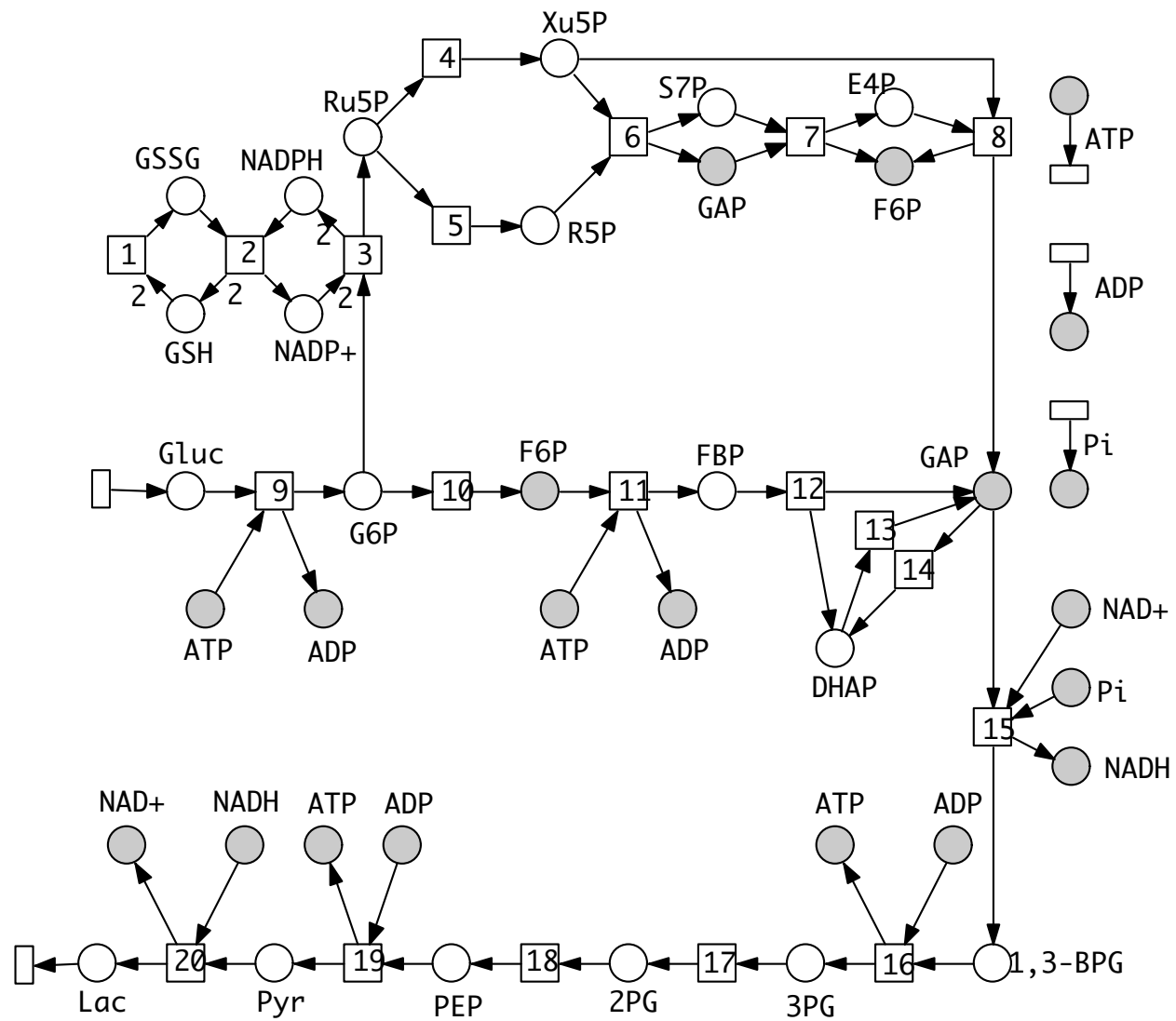
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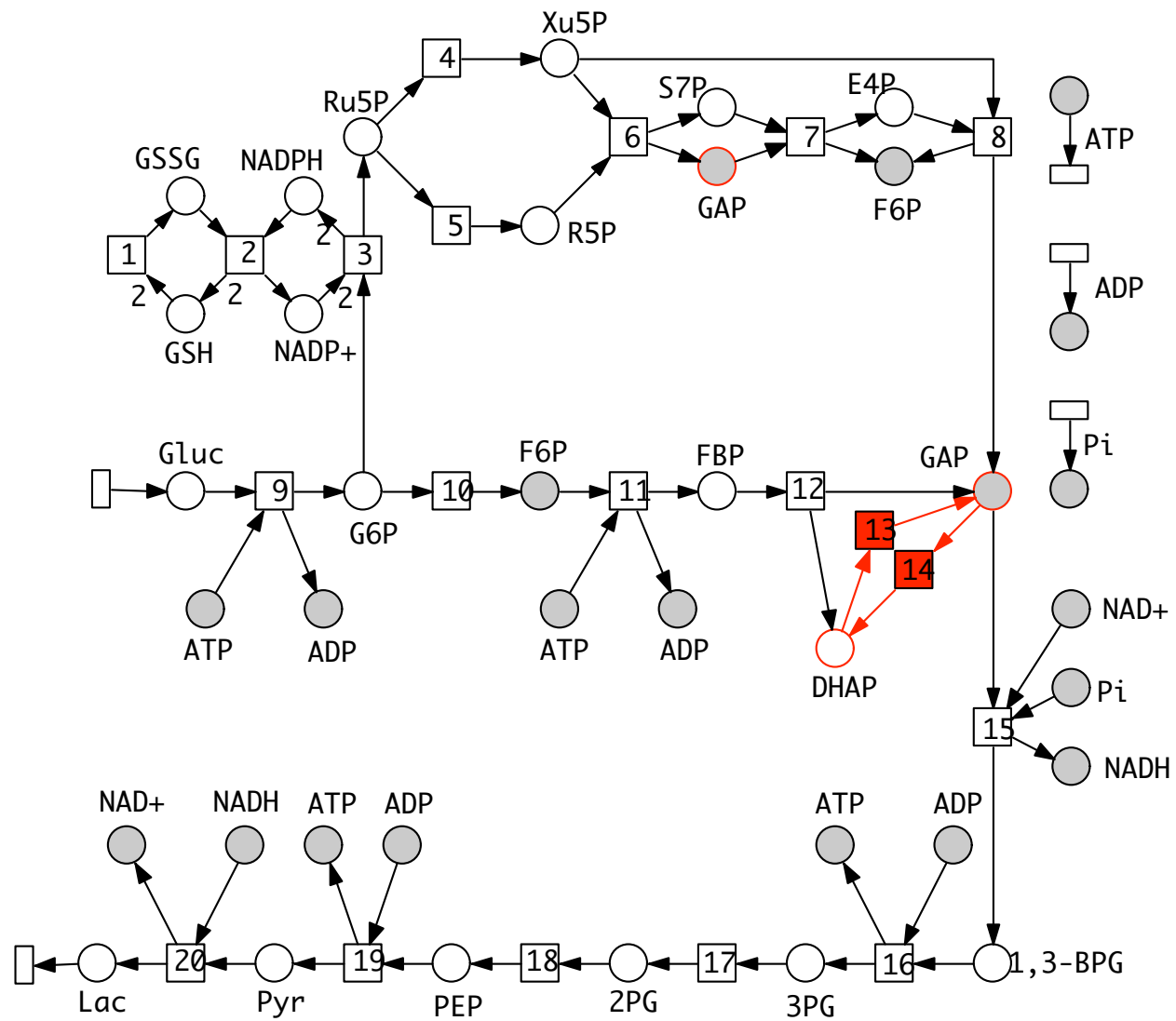
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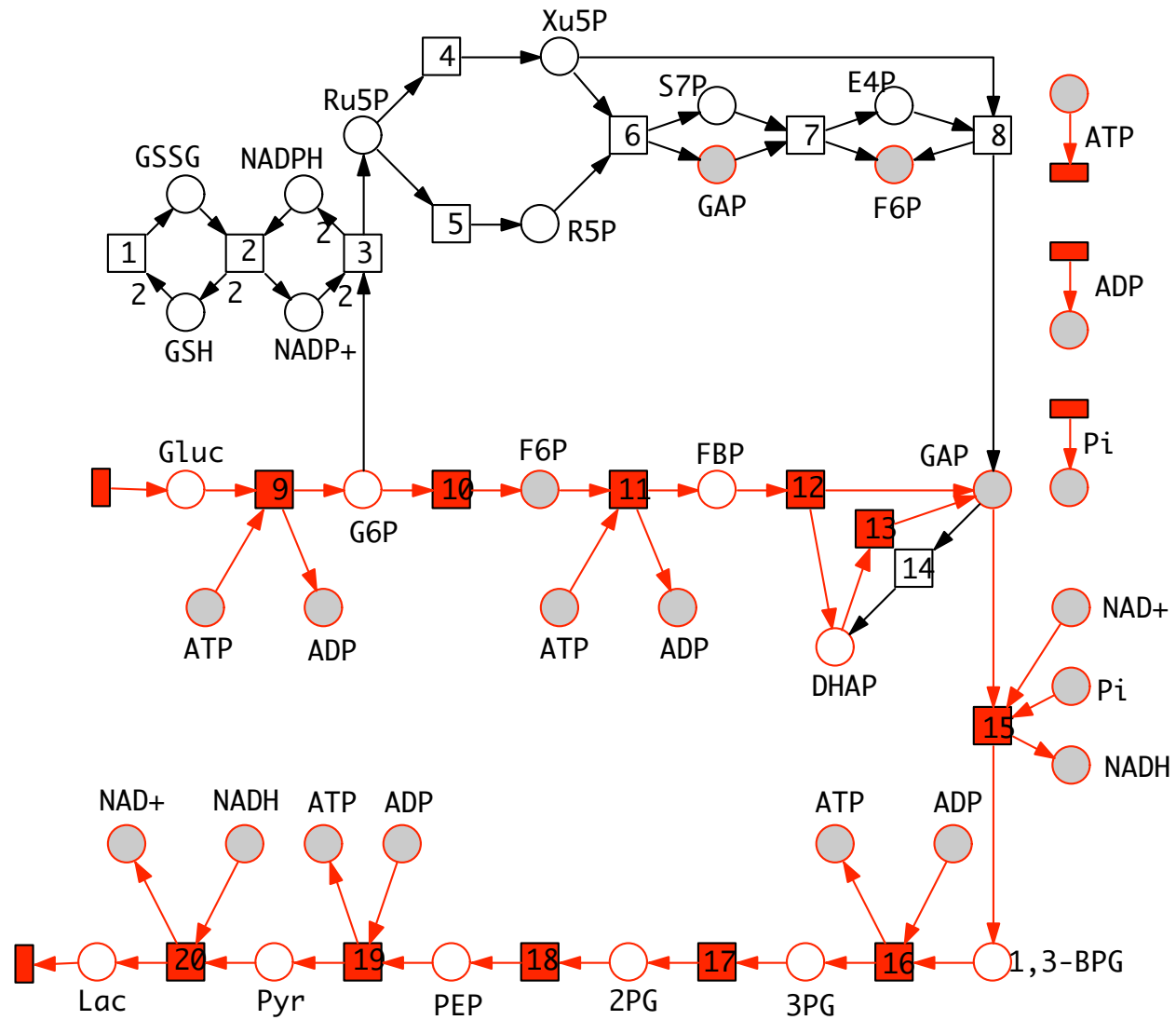
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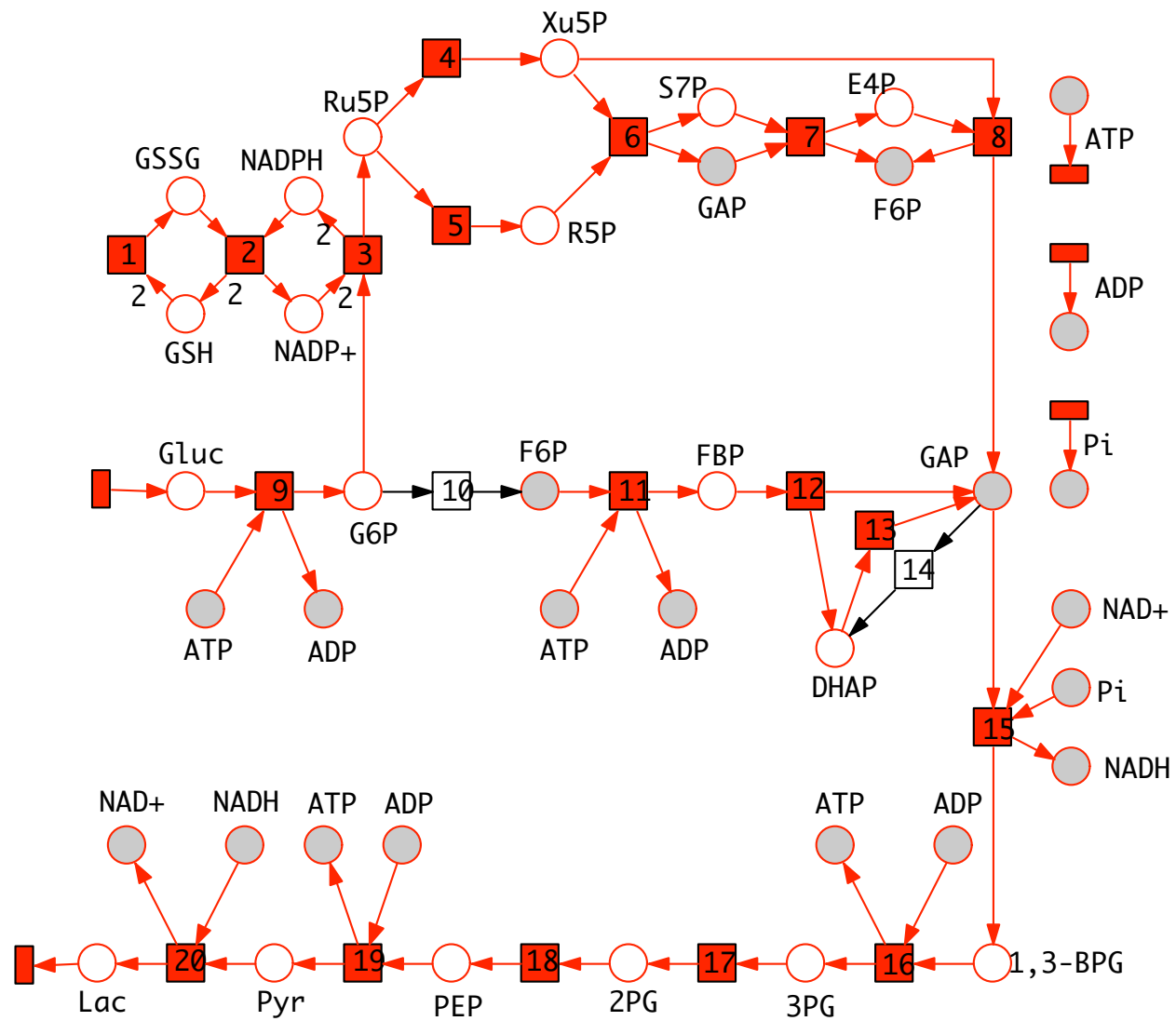
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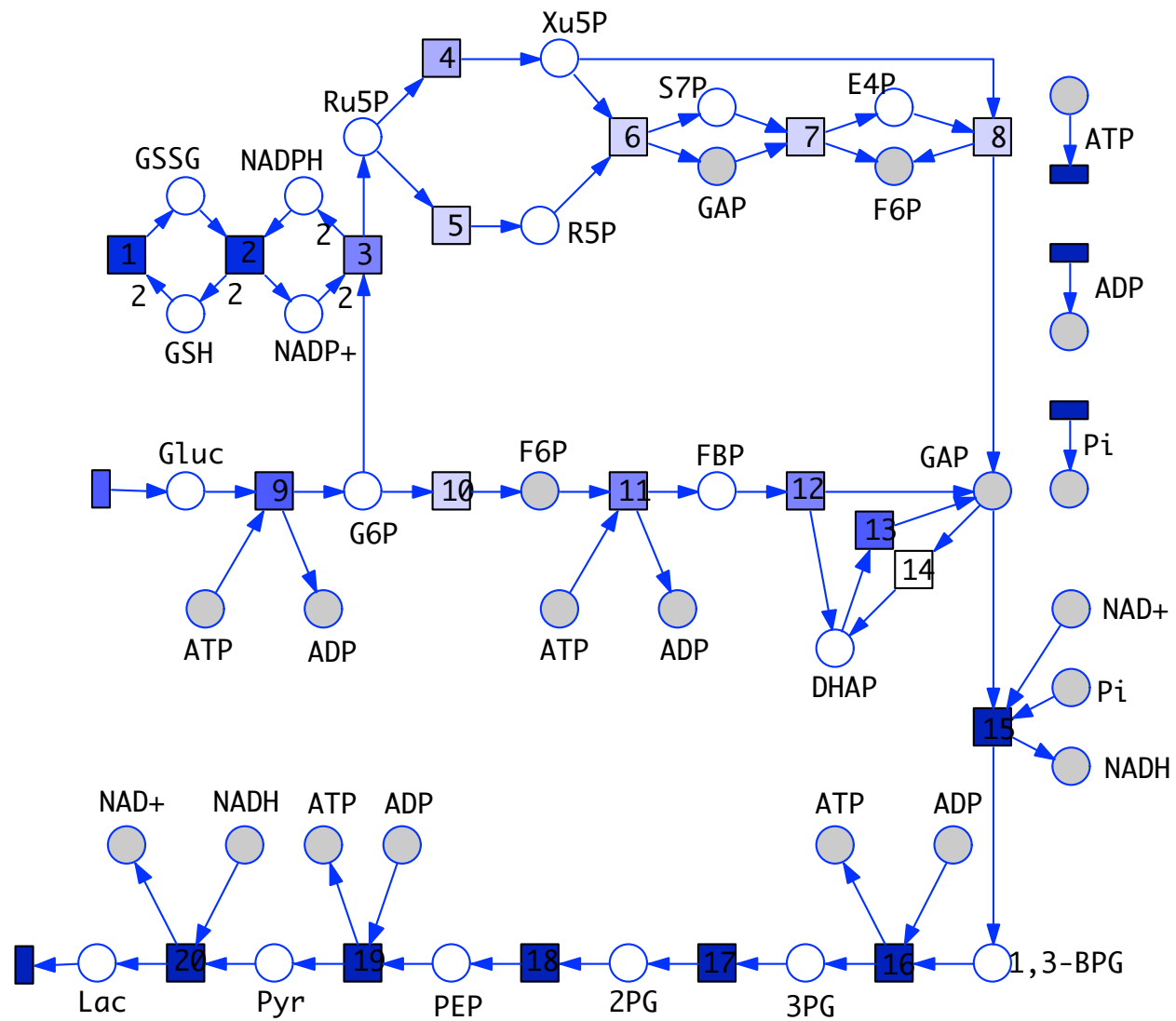
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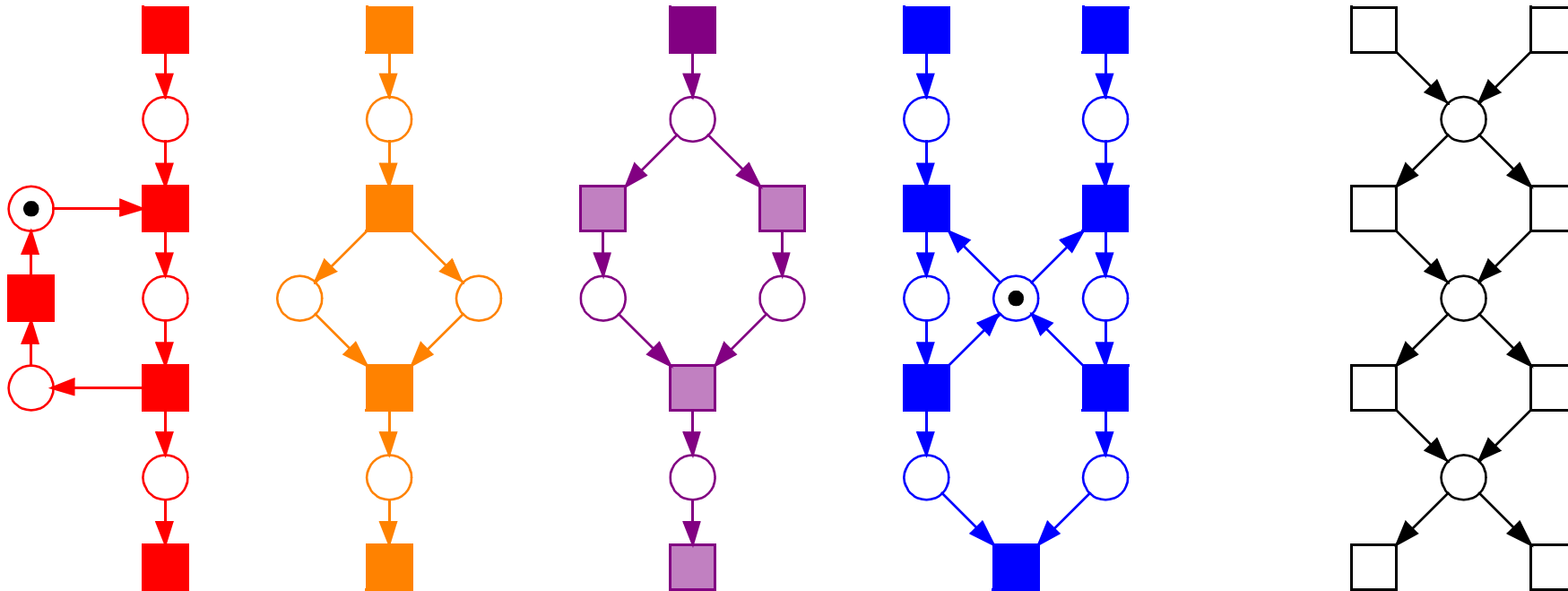


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- T-invariants may contain any structure



- minimal T-invariants generally overlap

-> combinatorial effect brings explosion in the number of min. T-invariants (2^4)