QUALITATIVE MODELLING AND ANALYSIS CONCURRENT SYSTEMS WITH PETRI NETS

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OUTLINE

1.

MOTIVATION

WHY PETRI NETS ?

- a suitable intermediate representation for
  - -> *different languages*
  - -> *different stages of certainty*
- modelling power
  - -> *partial order semantics*
  - -> *applicable on any abstraction level*
  - -> *specification of limited resources possible*
- analysis power
  - -> *various complementary analysis methods*
  - -> *reliable tools*
- integration of
  qualitative and quantitative analyses
- **BUT:**
  modelling power <-> analysis power

PETRI NETS, A BIT OF HISTORY

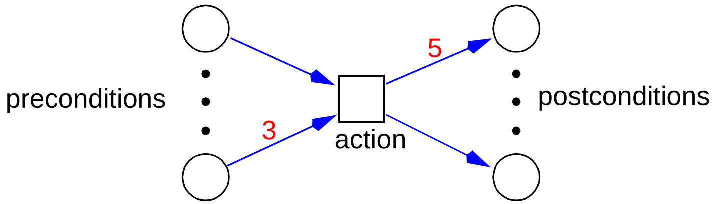
- Carl Adam Petri, 1962,
  PhD University of Technology Darmstadt
  -> *basic ideas introduced*
- early 1970's
  -> *first papers contributing to Petri net theory*
- Petri, 1976
  -> *application to chemical networks mentioned*
- early 1980's
  -> *first monographs appear*
- Reddy, 1993
  -> *first paper on bio application*
- late 1990's
  -> *increasing interest in applying Petri nets for modelling and analysis of bio networks*

2. INTRODUCTION INTO QUALITATIVE MODELLING

PETRI NETS, STRUCTURE

- two types of nodes
  - -> *places*
    "passive elements", conditions, local states, chemical compounds
  - -> *transitions*
    "active elements", events, actions, chemical reactions

- arcs

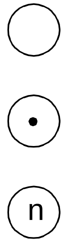


  - -> *directed*
  - -> *never arcs between nodes of the same type*
  - -> *for any node,*
    arbitrary number of pre-nodes and post-nodes

- arc inscriptions
  - -> *arc weight / multiplicity*
  - -> *amount of units of the substances involved in the basic (re-) action*

PETRI NETS, SYSTEM STATE

- tokens
  - -> *moving objects,*
    e. g. units of substances (e. g. Mole), ...



condition is not fulfilled

condition is (one times) fulfilled

condition is n times fulfilled

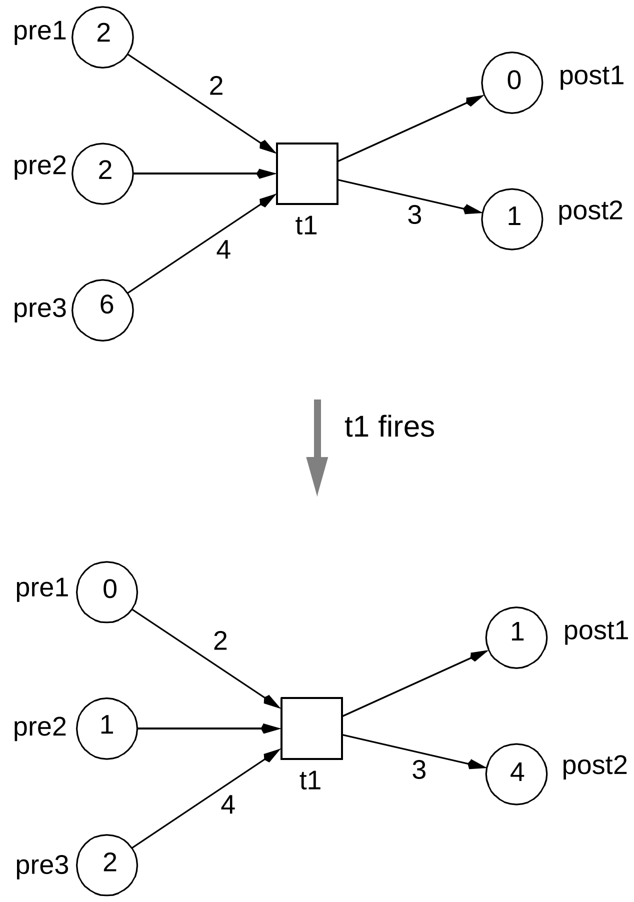
  - -> *token amount -*
    amount of available units of a given compound

- marking
  - -> *How many tokens are on each place?*
  - -> *system state*
    - -> *substance distribution*
  - -> *initial marking*
    - -> *initial substance distribution*

PETRI NETS, BEHAVIOUR

- flow of tokens
  - -> *defined by firing rule*
- an action **may** happen (fire), if
  - -> *all preconditions are fulfilled (corresponding to the arc weights);*
- **if** an action happens (fires), **then**
  - -> *tokens are removed from all preconditions (corresponding to the arc weights), **and***
  - -> *tokens are added to all postconditions (corresponding to the arc weights);*
- an action happens (firing of a transition)
  - -> *atomic*
  - -> *time-less*

FIRING RULE, EXAMPLE



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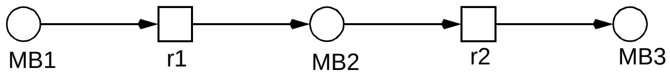
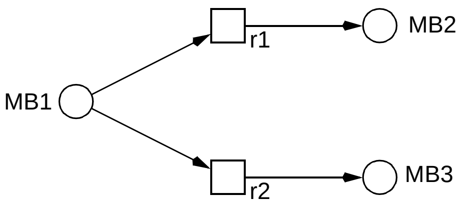
TYPICAL BASIC STRUCTURES

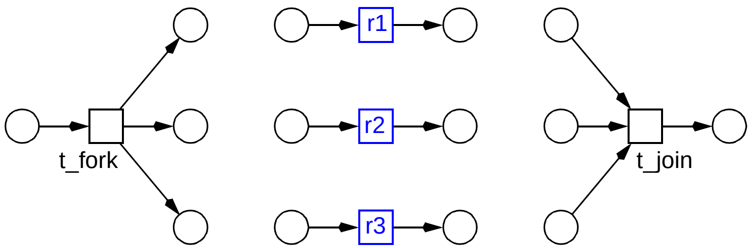
- CHAIN OF REACTIONS



- (FREE-CHOICE) BRANCHING / CONFLICT



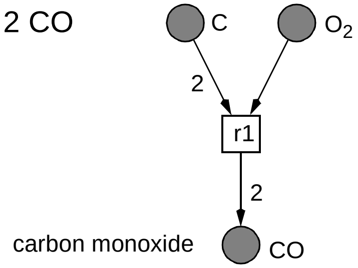
- CONCURRENCY



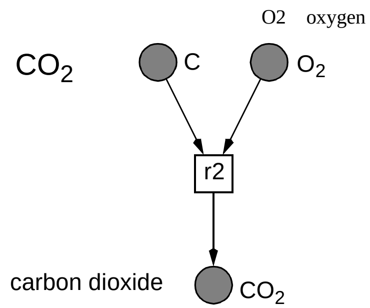
r1, r2, r3 are concurrent = independent

EXAMPLE, CARBON OXIDATION, BASIC REACTIONS

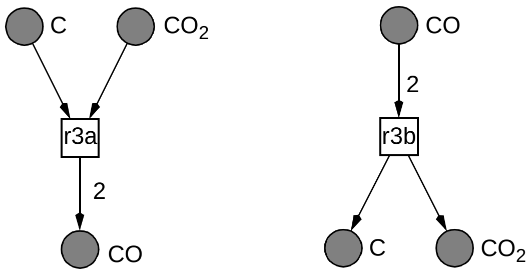
r1) $2\ C + O_2$ --> $2\ CO$



r2) $C + O_2$ --> CO_2

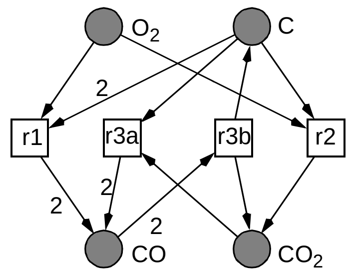


r3) $C + CO_2$ <--> $2\ CO$



EXAMPLE, COMPOSITION

BASIC MODEL



SYSTEM'S TOTAL EQUATION

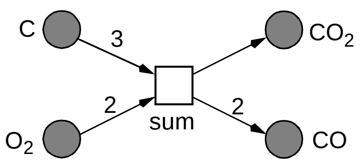
1) $2\,C + O_2 \rightarrow 2\,CO$

2) $C + O_2 \rightarrow CO_2$

[3) $C + CO_2 \leftrightarrow 2\,CO$]

$3\,C + 2\,O_2 \rightarrow 2\,CO + CO_2$

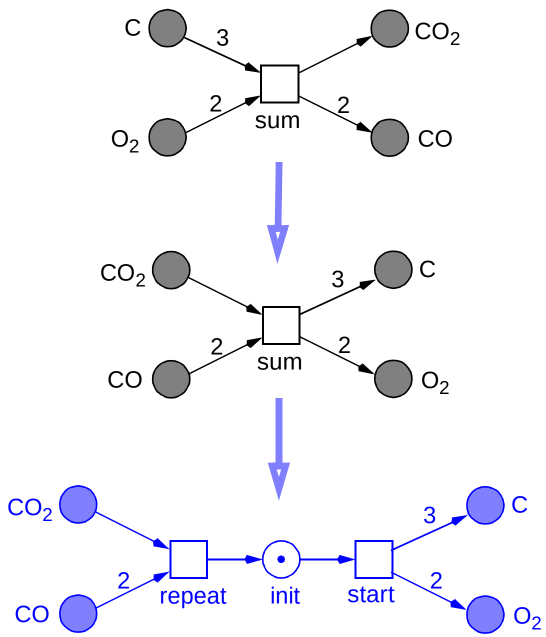
MODEL OF THE SYSTEM'S TOTAL EQUATION



ENVIRONMENT BEHAVIOUR

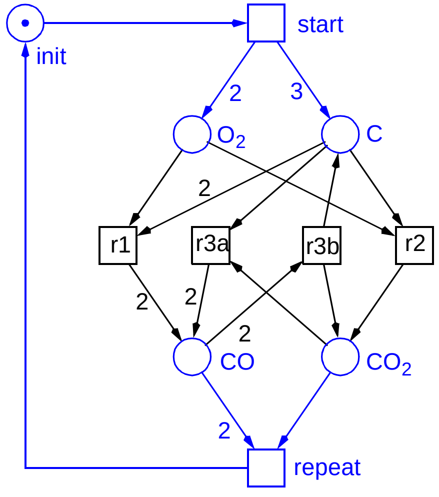
- strong assumptions about quantitative relations of input / output compounds

- 'inverse' total equation



- there are no boundary nodes

CARBON OXIDATION, SYSTEM MODEL,



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3. INTRODUCTION INTO QUALITATIVE ANALYSIS

Petri Net Properties, Overview / INA

1. Simple Structural Properties

ORD	ordinary *(1-multiplicity of all arcs)*
HOM	homogeneous *(all output arcs of a given place have the same multiplicity)*
NBM	non-blocking multiplicity *(for each place applies: MIN multiplicity of input arcs >= MAX multiplicity of output arcs)*
PUR	pure *(no side conditions)*
CSV	conservative *(any firing preserves token amount)*
SCF	static conflict free
CON	connected
SC	strongly connected
Ft0	there is a transition without pre-place
tF0	there is a transition without post-place
Fp0	there is a place without pre-transition
pF0	there is a place without post-transition
MG	marked graph *(synchronization graph)*
SM	state machine
FC	free choice net
EFC	extended free choice net
ES	extended simple net

Petri Net Properties, Overview / INA

2. More Expensive Structural Properties

DTP	deadlock trap property
SMC	state machine coverable *(covered with SM components)*
SMD	state machine decomposable *(covered with SCSM components)*
SMA	state machine allocatable
CPI	covered with place invariants
CTI	covered with transition invariants
SB	structurally bounded

3. Behavioural Properties

B	bounded
REV	reversible *(the initial state m_0 can be reached again from all reachable states: home state)*
DSt	dead states *(a state where no transition is enabled)*
BSt	bad states *(a state where a fact is enabled)*
DTr	dead transitions *(at the initial state)*
DCF	dynamically conflict free
L	live
LV	live, excepted transitions dead at the initial marking
L&S	live & safe *(1-bounded)*

BEHAVIOURAL NET PROPERTIES, OVERVIEW

general semantic properties:

- MARKABILITY of places
  - markable *(place liveness)*
  - k-bounded *(1-bounded / safe)*
  - unbounded
- LIVENESS of transitions
  - zero times firing *(m_0-dead)*
  - finite times firing *(dead, non-live)*
  - infinite times firing, probably *(live)*
  - infinite times firing, definitely *(livelock free)*
- REACHABILITY of states
  - dead states
  - reproducibility
  - reversibility *(m_0 - home state)*

special semantic properties:

- REACHABILITY of states (continued)
  - bad states *(facts)*
  - user-specified states
- NET INVARIANTS
  - transition invariants
  - place invariants
- temporal relationship of logic formulae
  - safety properties
  - progress properties

QUALITATIVE ANALYSIS METHODS, OVERVIEW

static analysis:

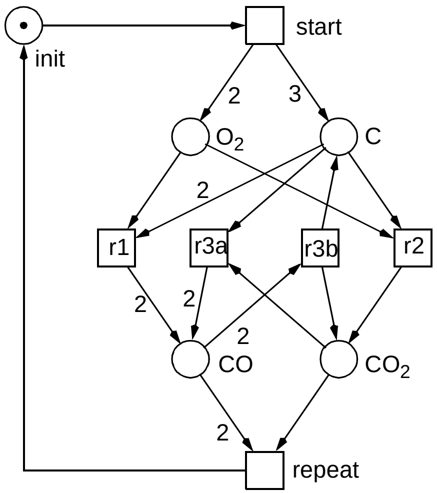
- NET REDUCTION
- STRUCTURAL PROPERTIES
- LINEAR PROGRAMMING
  - place / transition invariants
  - state equation
  - trap equation

dynamic analysis *(model checking)*:

- STATE SPACE ANALYSIS
  - (complete) reachability graph
  - compressed state spaces
    - BDDs, NDDs, ..., xDDs
    - Kronecker products
  - reduced state spaces
    - coverability graph
    - symmetry
    - stubborn sets
  - branching process

REACHABILITY GRAPH (RG)

- nodes - system states
- arcs - the (single) firing transition
- example - carbon oxidation, environment style 3



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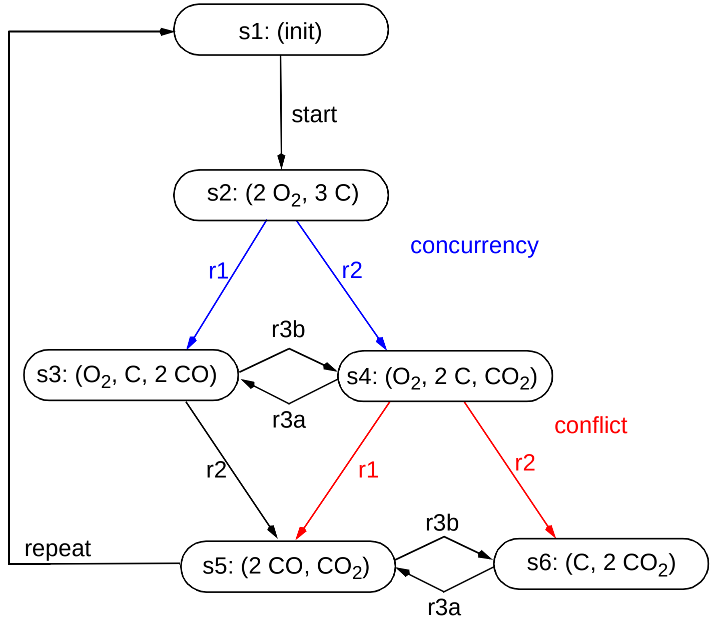
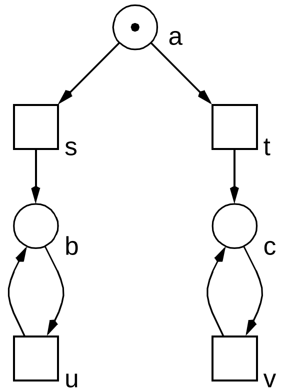
RG (CARBON OXIDATION)

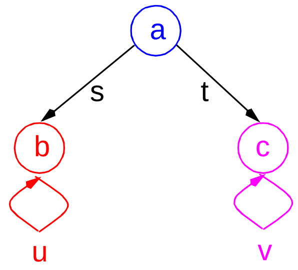


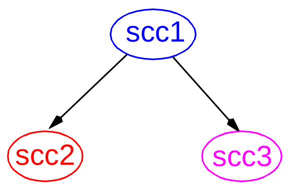
-> *interleaving description of the whole system behaviour*

EXAMPLE: RG AND THREE BASIC PN PROPERTIES

- no concurrency
  -> rg(pn) == pn
- rg - finite
  -> bounded pn
- rg - not sc
  -> pn not reversible
- no dead states,
  but liveness?
- condensed rg
  node - sc component (scc)
  scc:
  maximal set of sc nodes;
  a terminal scc
  -> *possible terminal system behaviour*
  -> *must contain all transitions in a live pn*
- not all terminal scc contain all transitions
  -> the pn is not live



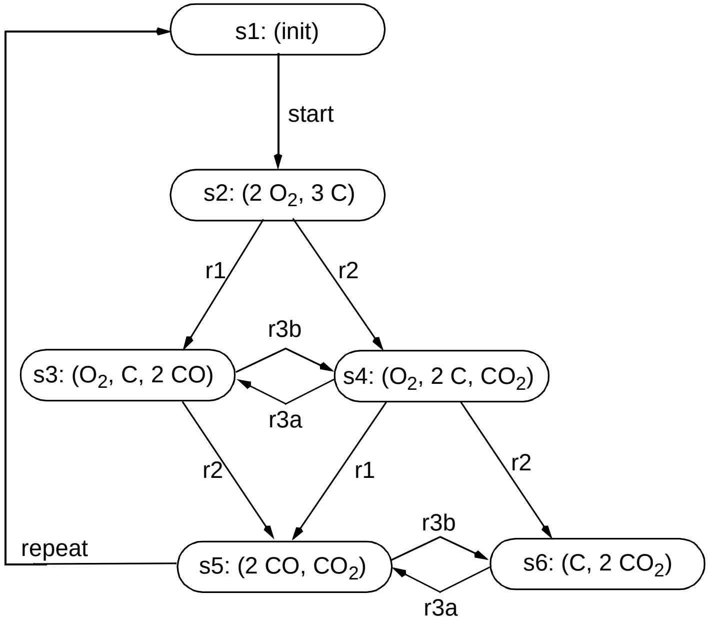




BASIC PROPERTIES & RG, SUMMARY

- How many tokens may reside at most in a given place . . .
  -> (0, 1, k, oo) ?
  - -> boundedness
  - -> rg is finite
- How often may a transition fire . . .
  -> (0-times, n-times, oo-times) ?
  - -> liveness
  - -> every terminal scc contains all transitions
- Is the initial system state . . .
  -> always reachable again ?
  - -> reversibility
  - -> rg is sc (consists of one scc)

RG(CARBON OXIDATION), EVALUATION



- ❑ RG is finite
  -> *BND*
- ❑ 1 Strongly Connected Component (SCC)
  -> *REV*
- ❑ the only SCC contains all transitions
  -> *LIVE*

REACHABILITY GRAPH, CONSTRUCTION ALGORITHM

PROCEDURE rg (**IN** Net pn, **IN** Marking $m0$,
OUT MSet $nodes$, **OUT** ArcSet $arcs$);

MSet $U = \{m0\}$, // unprocessed markings
$N = \varnothing$; // rg nodes
ArcSet $E = \varnothing$; // rg arcs (pre, post, t)
Marking m'; // successor marking
Transition t;

WHILE $U \neq \varnothing$ **DO**
choose one $m \in U$;
$U = U - \{m\}$; $N = N \cup \{m\}$;

FOR ALL t enabled at m **DO**
$m' = m + \Delta t$;
IF $m' \notin N \cup U$ // new marking
THEN $U = U \cup \{m'\}$
ENDIF;
$E = E \cup \{(m, m', t)\}$
ENDFOR

ENDWHILE;

$nodes = N$; $arcs = E$;

ENDPROC rg.

REACHABILITY GRAPH, OBSERVATIONS

- **unbounded** Petri net
  -> *the RG is* ***infinite***
  bounded Petri net
  -> *the RG is* ***finite***

- simple construction algorithm
  -> *single step firing rule*

- concurrency
  -> *enumeration of all interleaving sequences*

- branching arcs in the RG
  -> *conflict* ***OR***
  -> *concurrency*

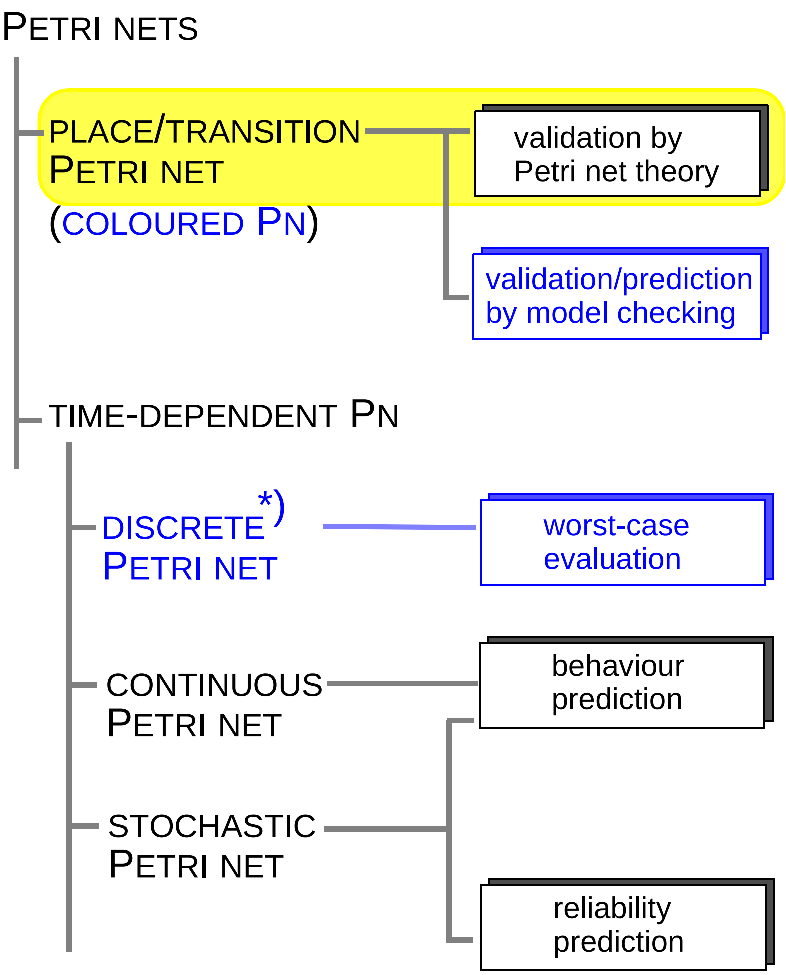
- RG tend to be very large
  -> *automatic evaluation necessary*

- worst case: over-exponential growth
  -> *alternative analyses techniques ?*

5.

SUMMARY AND OUTLOOK

MODEL CLASSES



*) DISCRETELY TREATABLE

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