VALIDATION & VERIFICATION OF SYSTEM DESIGN SPECIFICATIONS

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OUTLINE

- **V&V needs formal semantics**

  -> *model-based system design*

- **possible formal modelling techniques**

  . . . many . . .

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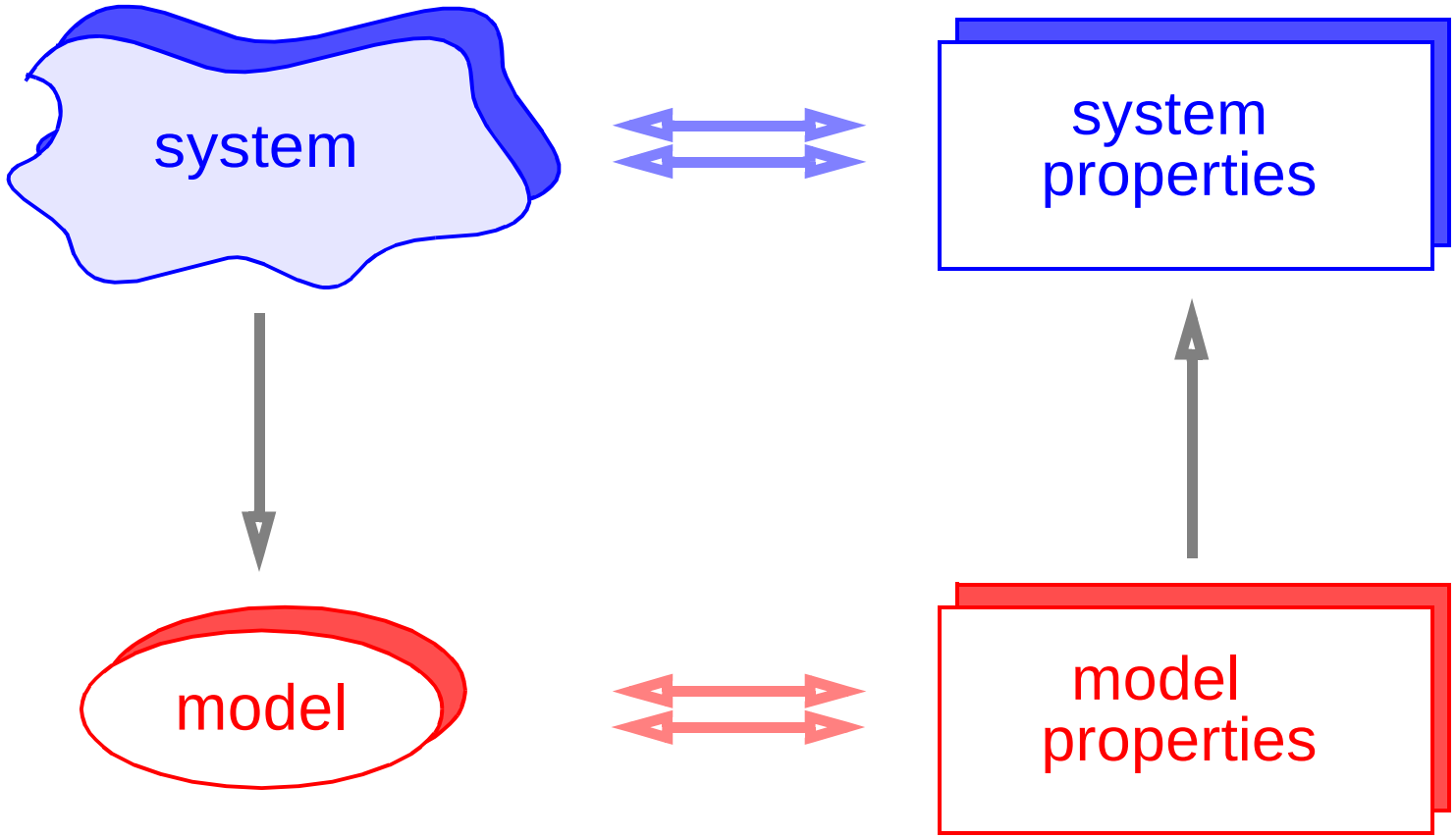
- **V&V needs formal semantics**
  -> *model-based system design*
- **possible formal modelling techniques**
  . . . many . . .
  -> *Petri nets*
- **introduction into Petri nets**
  -> *modelling and analysis*
  -> *typical Petri net properties*
  -> *typical Petri net analysis techniques*
- **applications**
  . . . many . . .
  -> *error-correcting Petri nets*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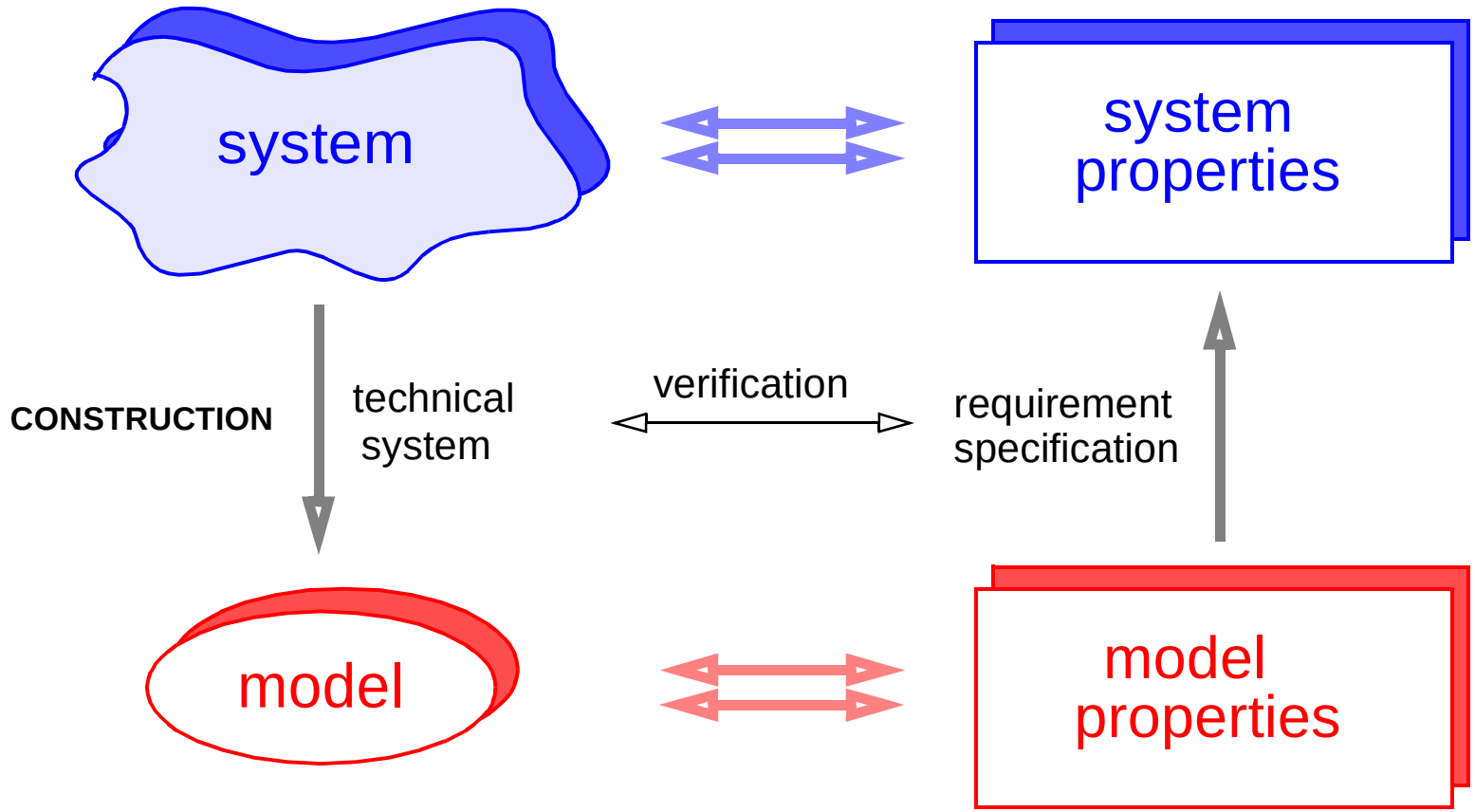
system properties
model
model properties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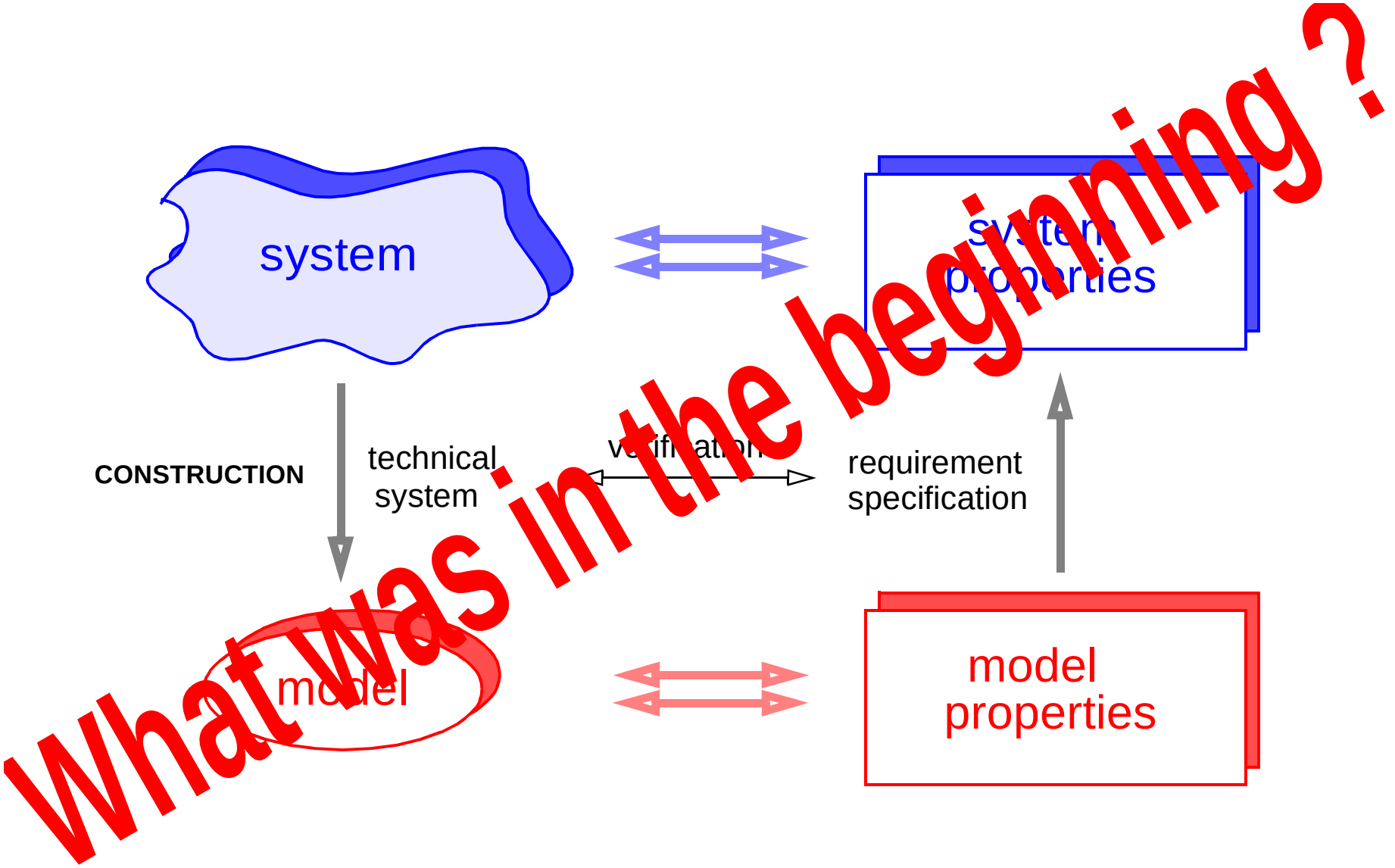
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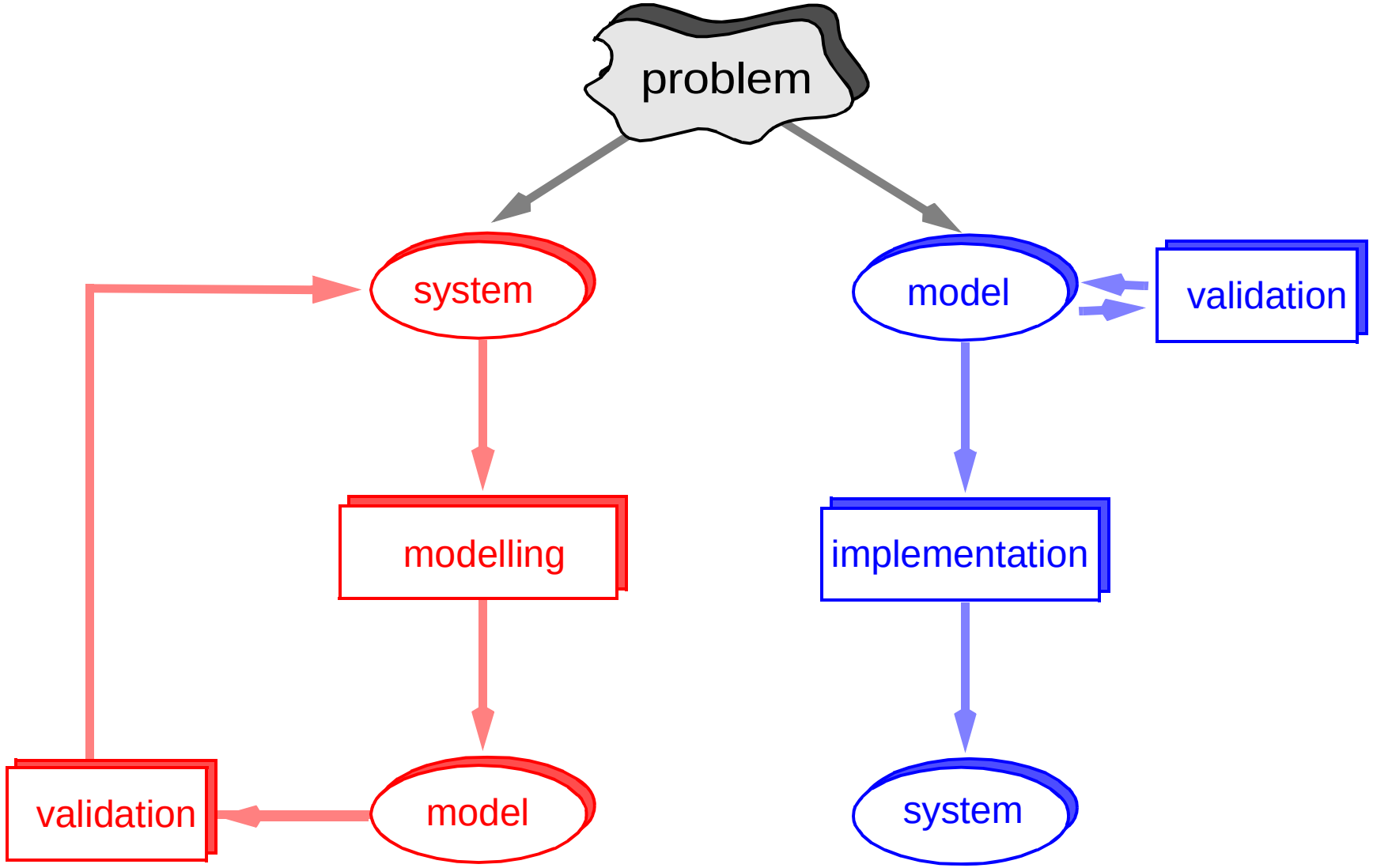
What was in the beginning ?

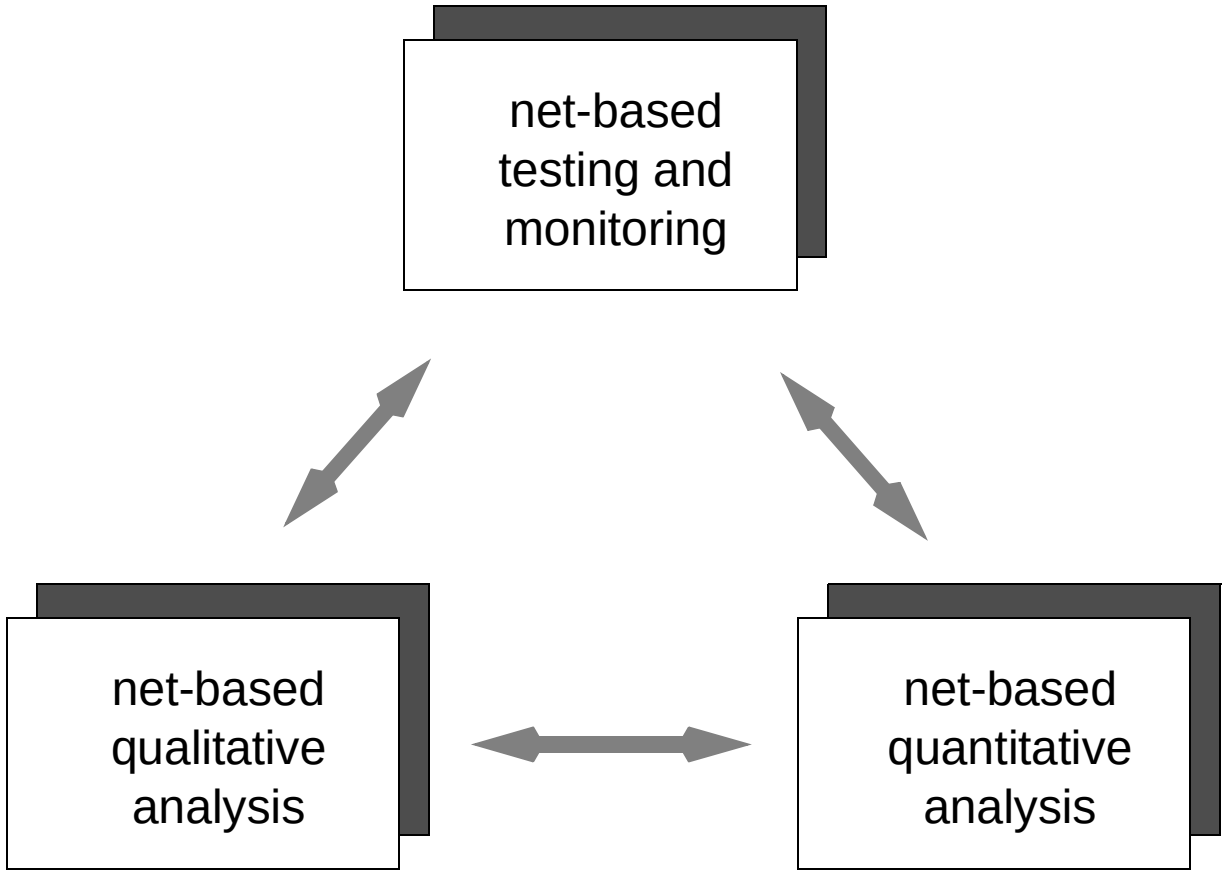
MODEL-BASED V&V, TWO APPROACHES



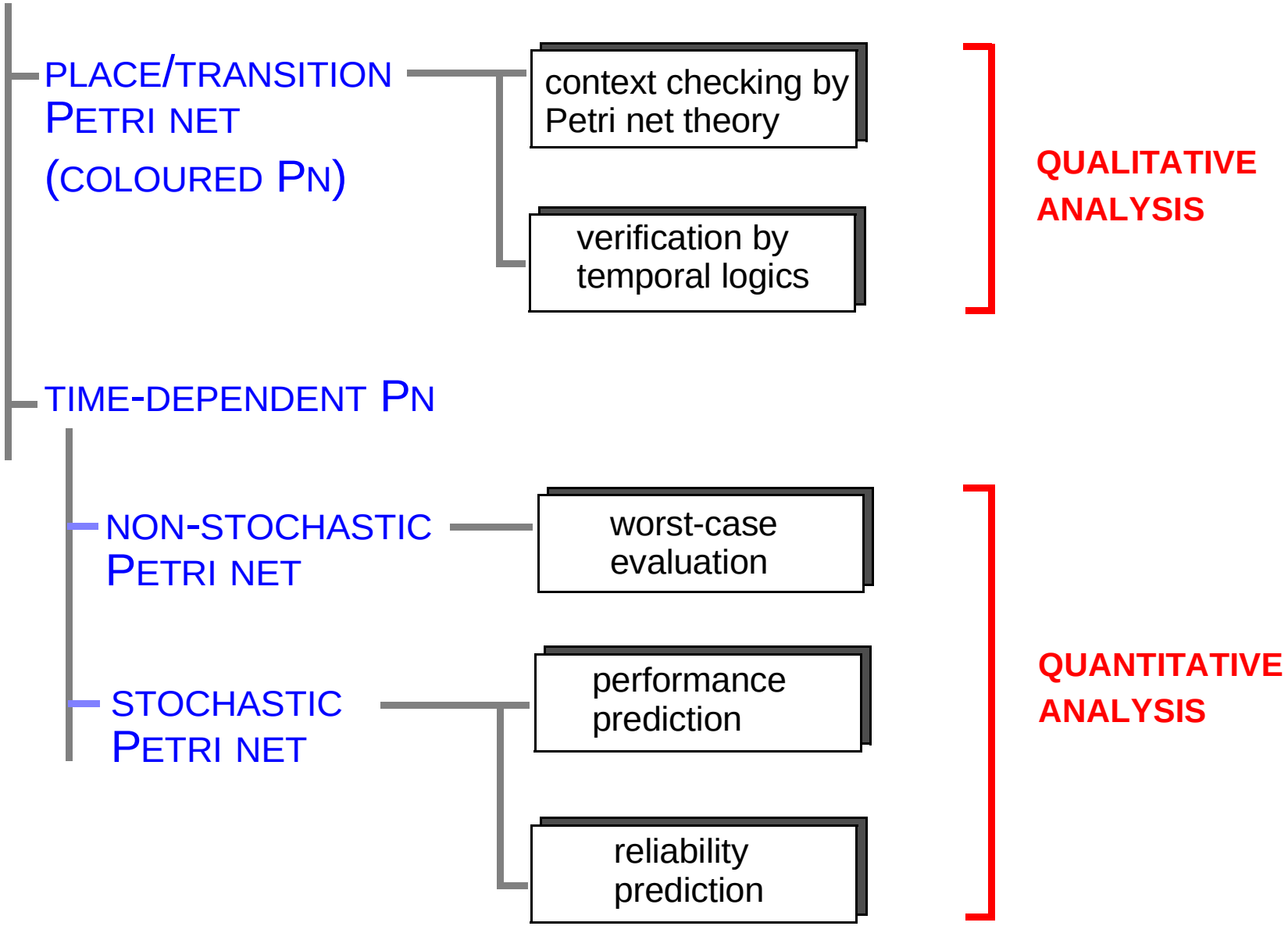
WHY PETRI NETS?

- **a suitable intermediate representation for**
  - -> *different (specification/programming) languages,*
  - -> *different phases of software development cycle,*
  - -> *different validation methods;*
- **modelling power**
  - -> *partial order (true concurrency) semantics*
  - -> *applicable on any abstraction level*
  - -> *specification of limited resources possible*
- **analyzing power**
  - -> *combination of static and dynamic analysis techniques*
  - -> *rich choice of methods, algorithms, tools*
- **BUT: modelling power <-> analyzing power**

INTEGRATION OF QUALITATIVE AND QUANTITATIVE ANALYSES



PETRI NETS, MODEL CLASSES

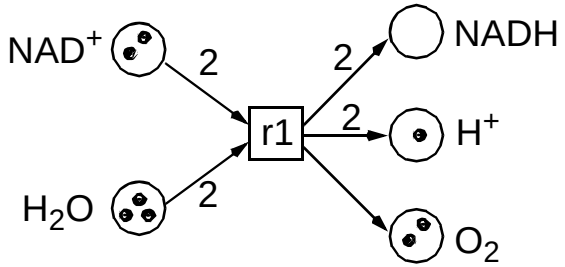


Petri nets,
a crash course

- **atomic actions -> Petri net transitions -> chemical reactions**

$$2\ NAD^+ + 2\ H_2O \rightarrow 2\ NADH + 2\ H^+ + O_2$$

input compounds



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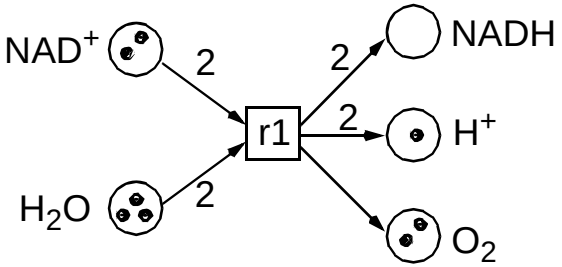

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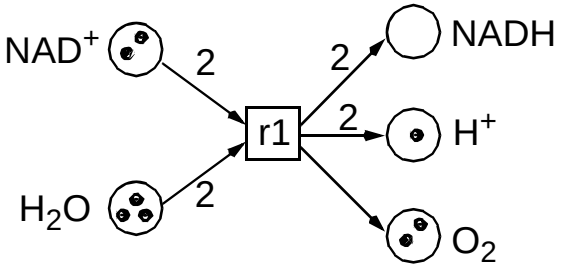

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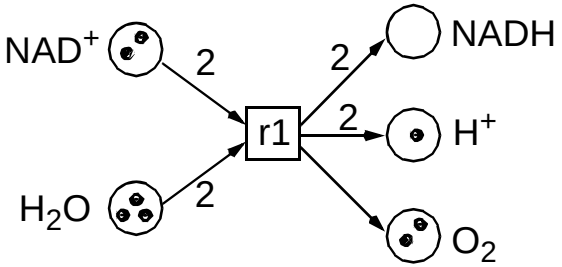

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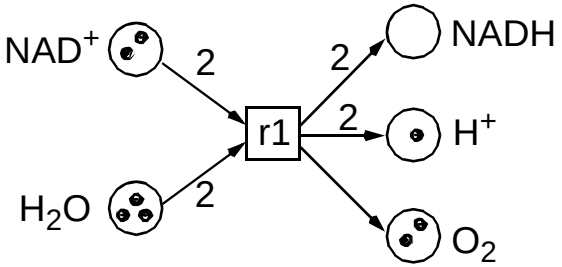

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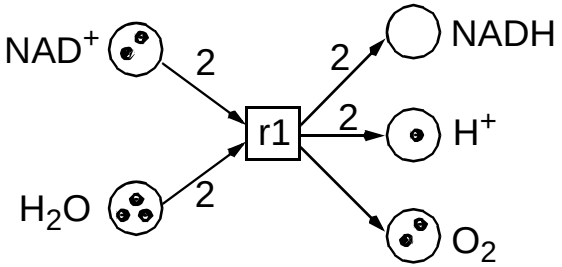

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- **PN = (P, T, F, m_0), F: (P x T) U (T x P) -> N_0, m_0: P -> N_0**

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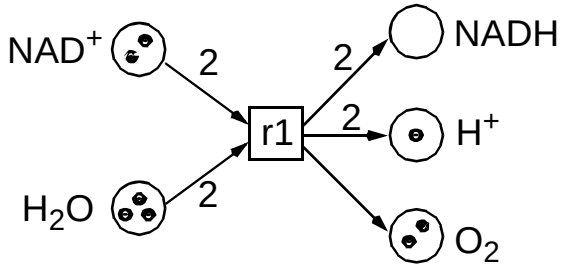

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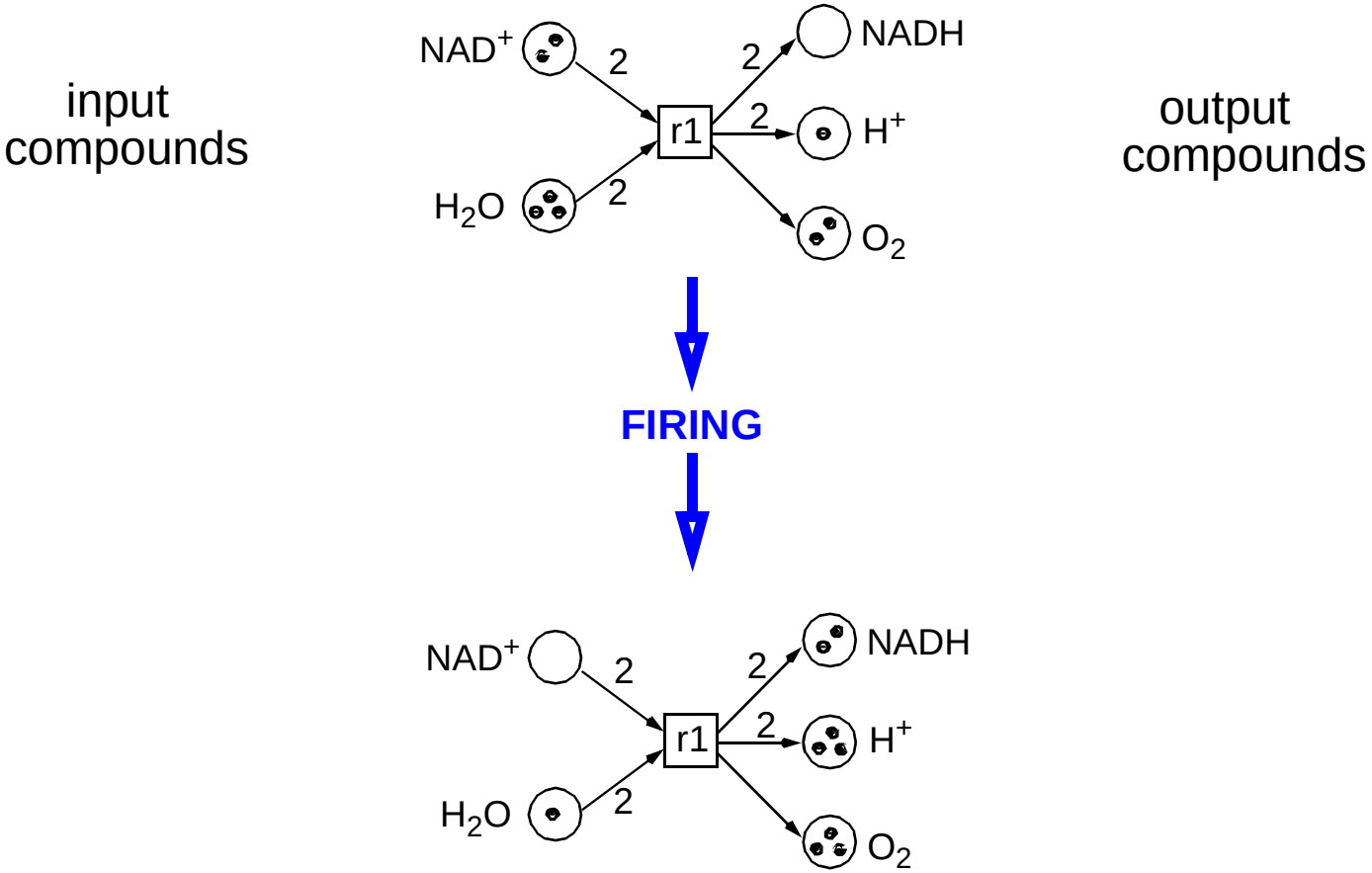

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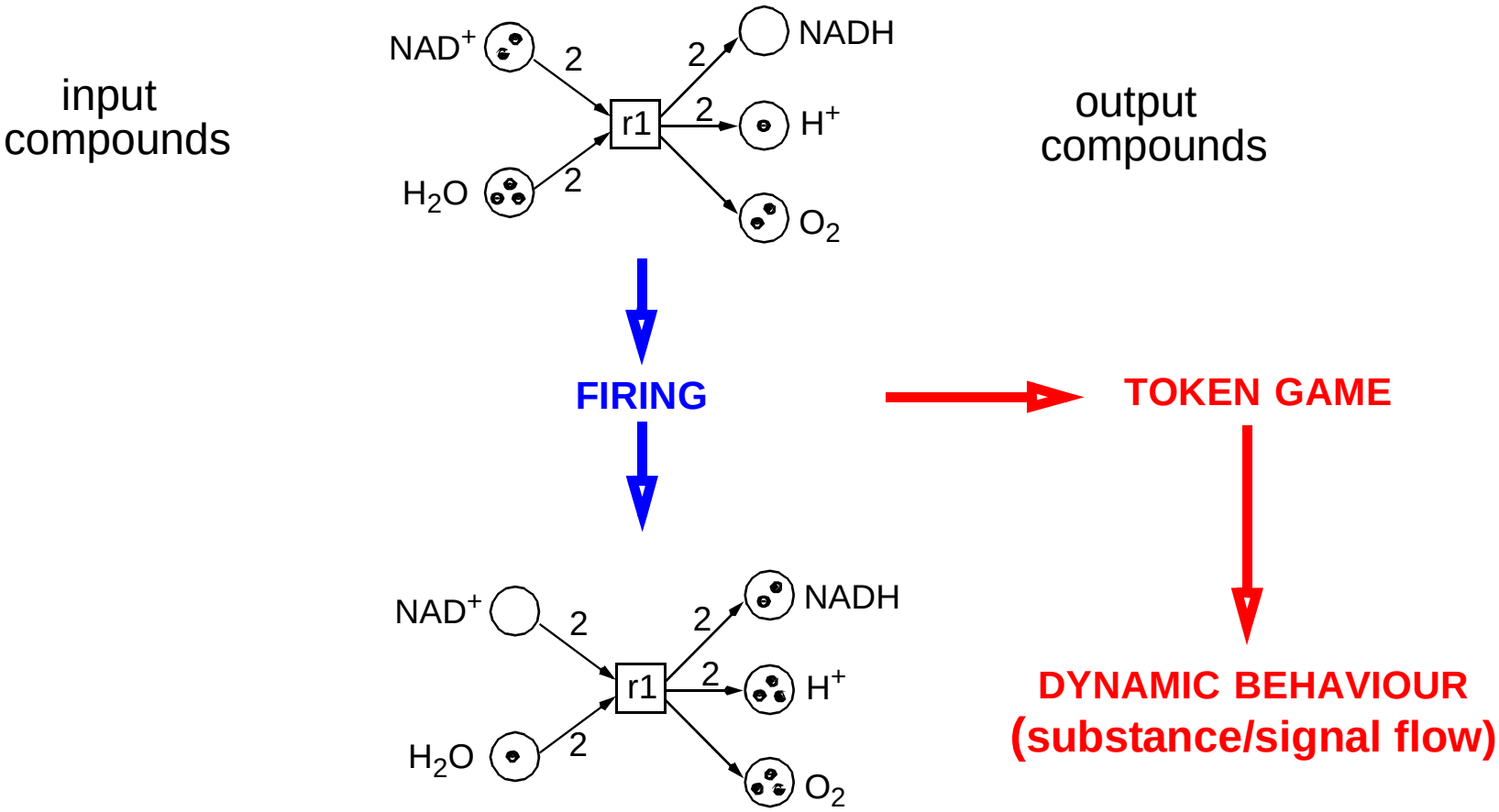
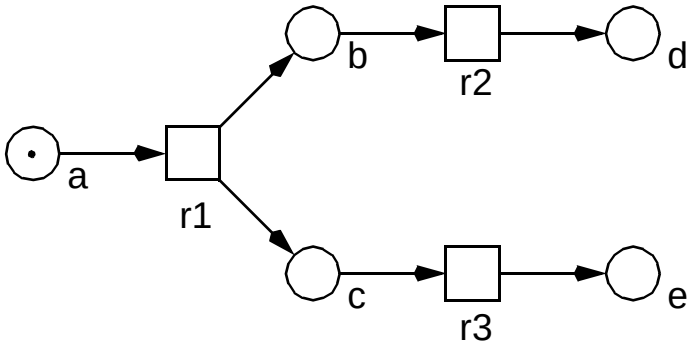


PARTIAL ORDER VERSUS INTERLEAVING SEMANTICS



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  - -> *dependency*
- **no order between r2 , r3**
  - -> *concurrency* $x \,||\, y$
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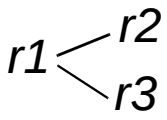
- **partial order run**



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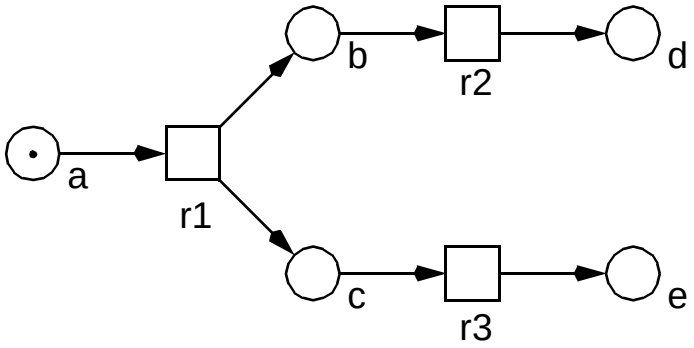

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$$r1 \begin{cases} r2 \\ r3 \end{cases}$$

-> **PARTIAL ORDER SEMANTICS**
"true concurrency semantics"

all partially ordered runs

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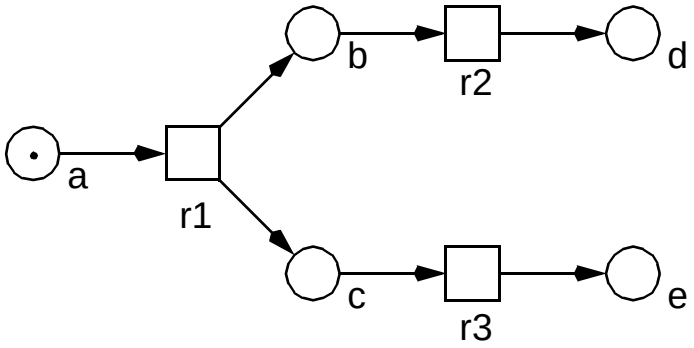

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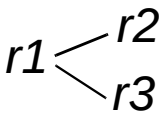

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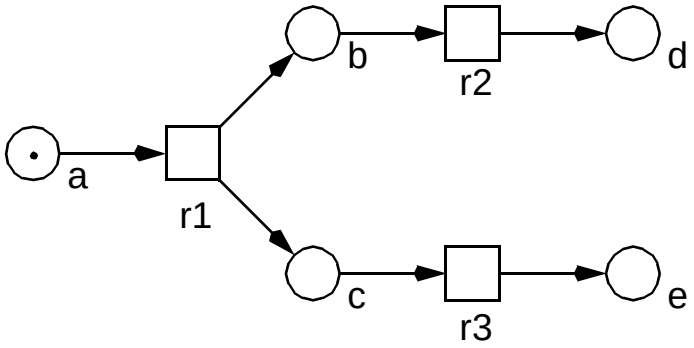

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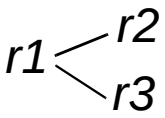

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PETRI NET, EXAMPLE 1

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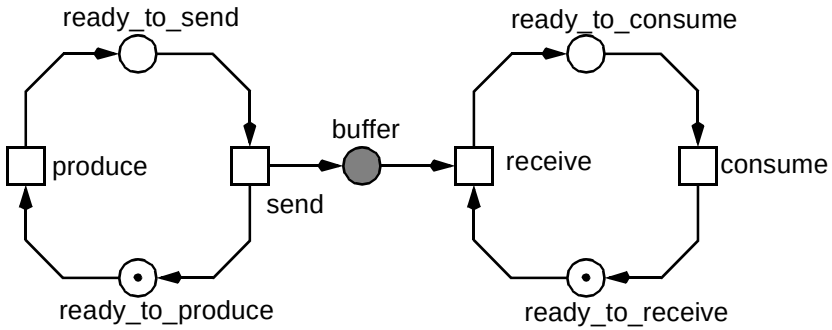

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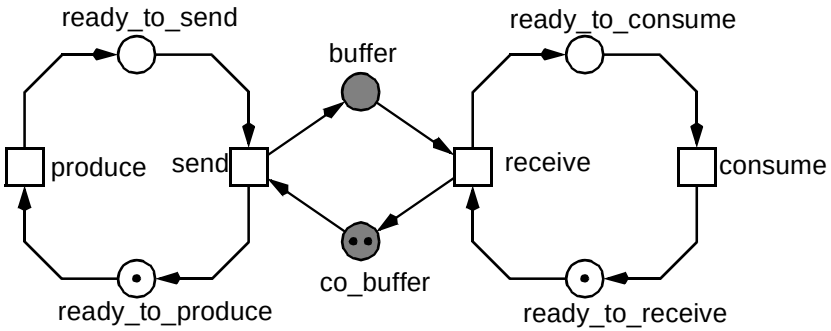

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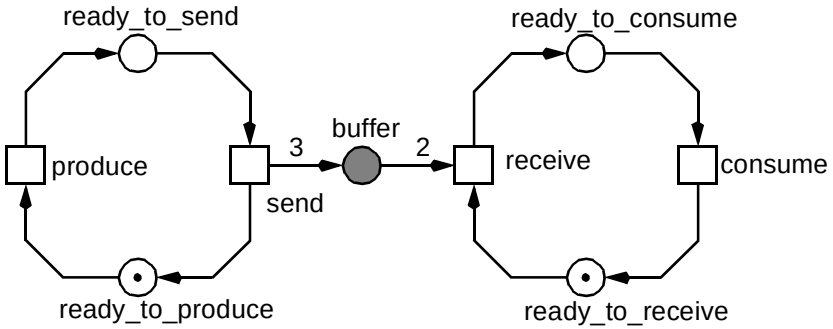

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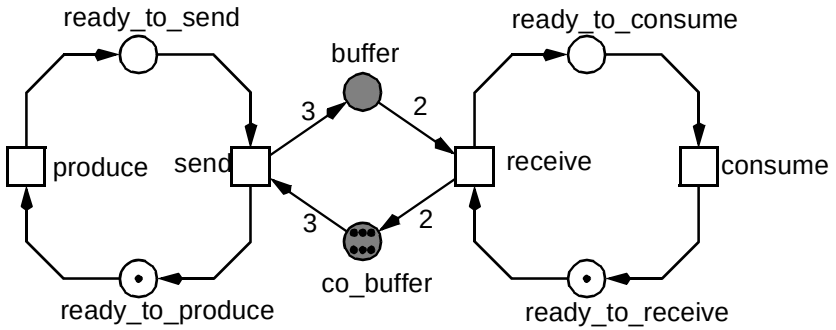
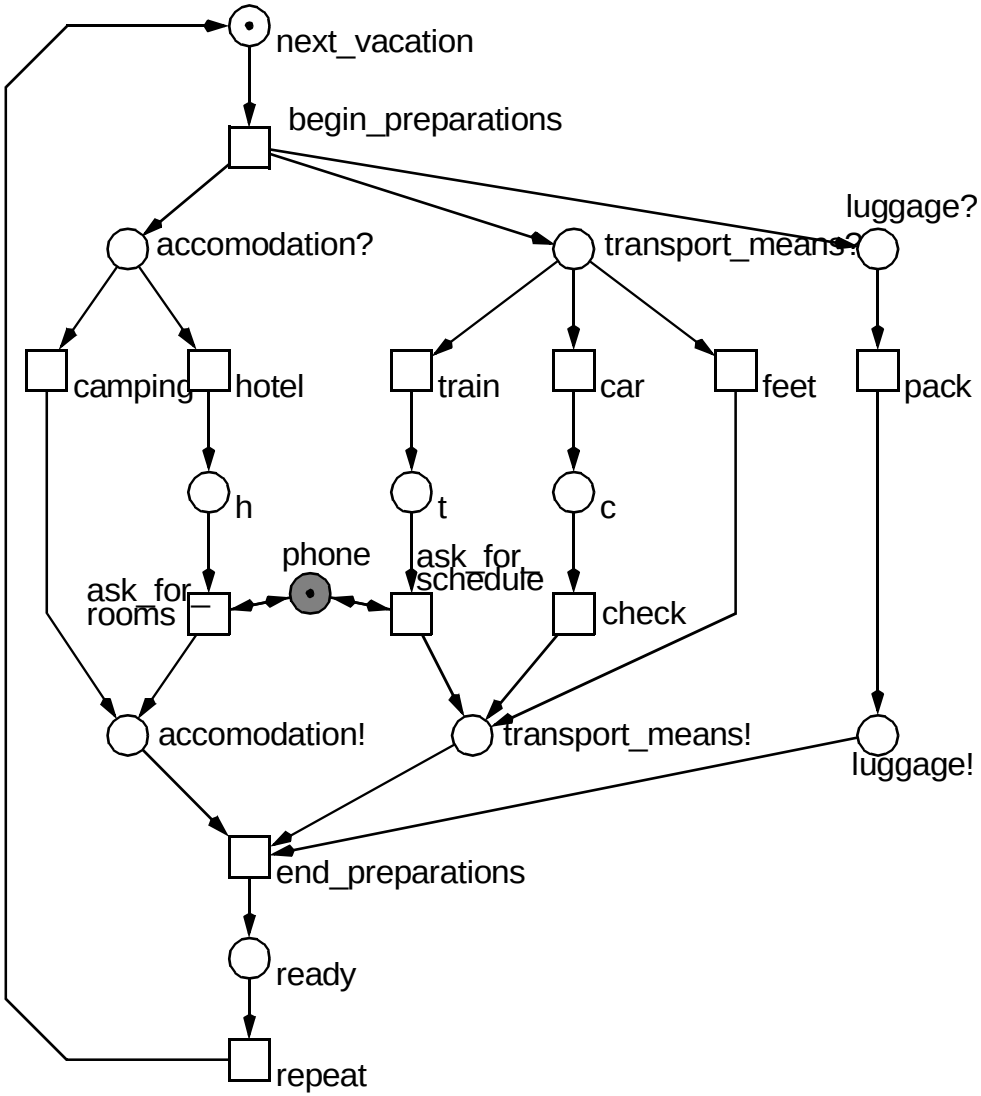


PETRI NET, EXAMPLE 2



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GENERAL BEHAVIOURAL PROPERTIES

SPECIAL BEHAVIOURAL PROPERTIES

Petri nets,
typical analysis techniques

Petri nets, typical analysis techniques

MODEL ANIMATION (?)

Dynamic Analyses

- **reachability / occurrence graph,**
  - -> *(labelled) state transition graph*
  - -> *Kripke structure, CTMC, . . .*
- **nodes**
  - -> *system states / markings*
- **arcs**
  - -> *the (single) firing transition*
  - -> *single step firing*
- **interleaving semantics**
  - -> *(sequential) finite automaton*
  - -> *concurrency == enumerating all interleaving sequences*
- **reachability graph construction - simple algorithm**

- **boundedness**
  -> *finite graph*
- **reversibility**
  -> *one Strongly Connected Component (SCC)*
- **liveness**
  -> *every transition contained in all terminal SCC*
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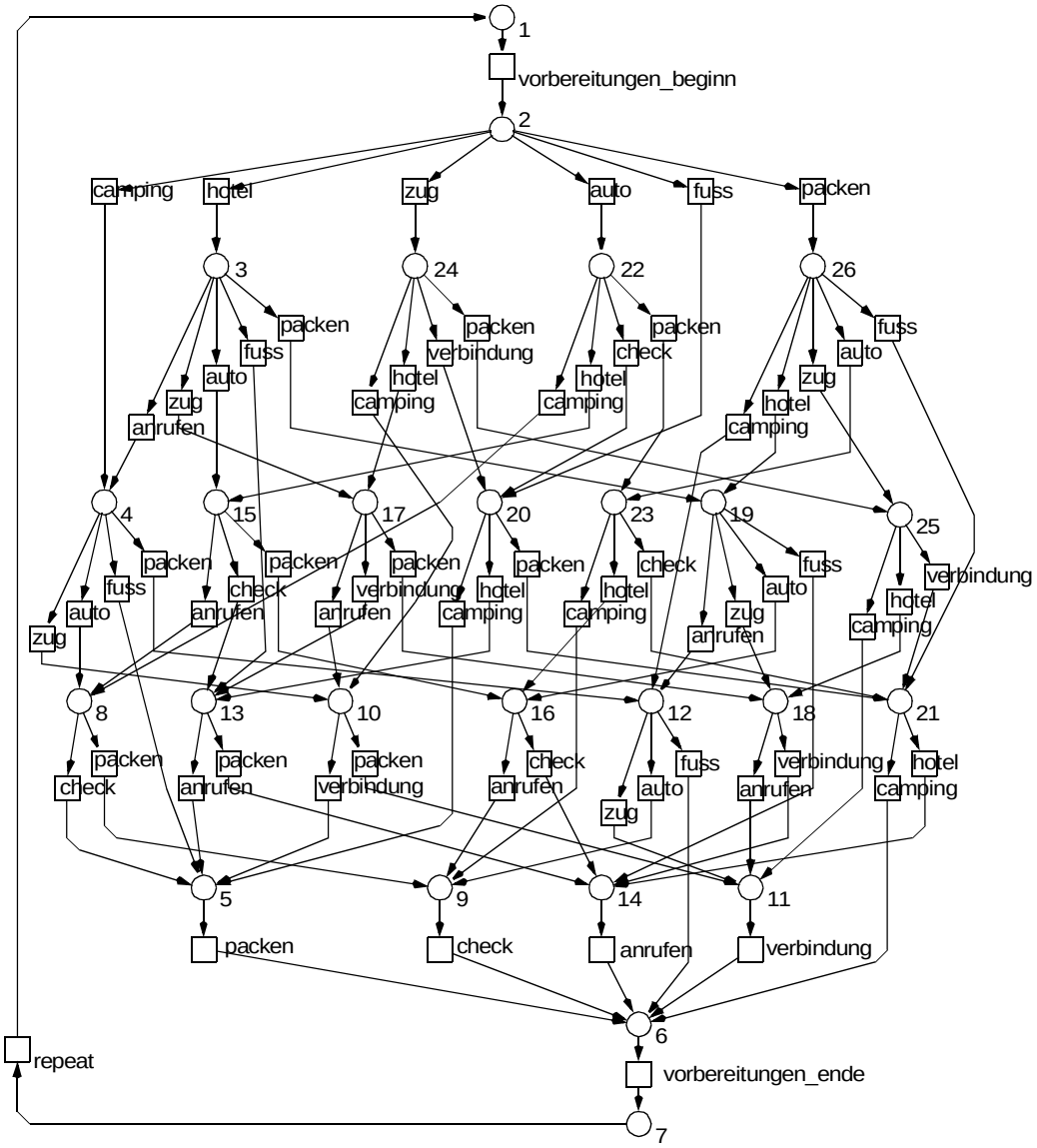
-> reachability graphs tend to be huge <-

REACHABILITY GRAPH, Example 2



- infinite for unbounded nets

REACHABILITY GRAPH, STATE SPACE COMPLEXITY

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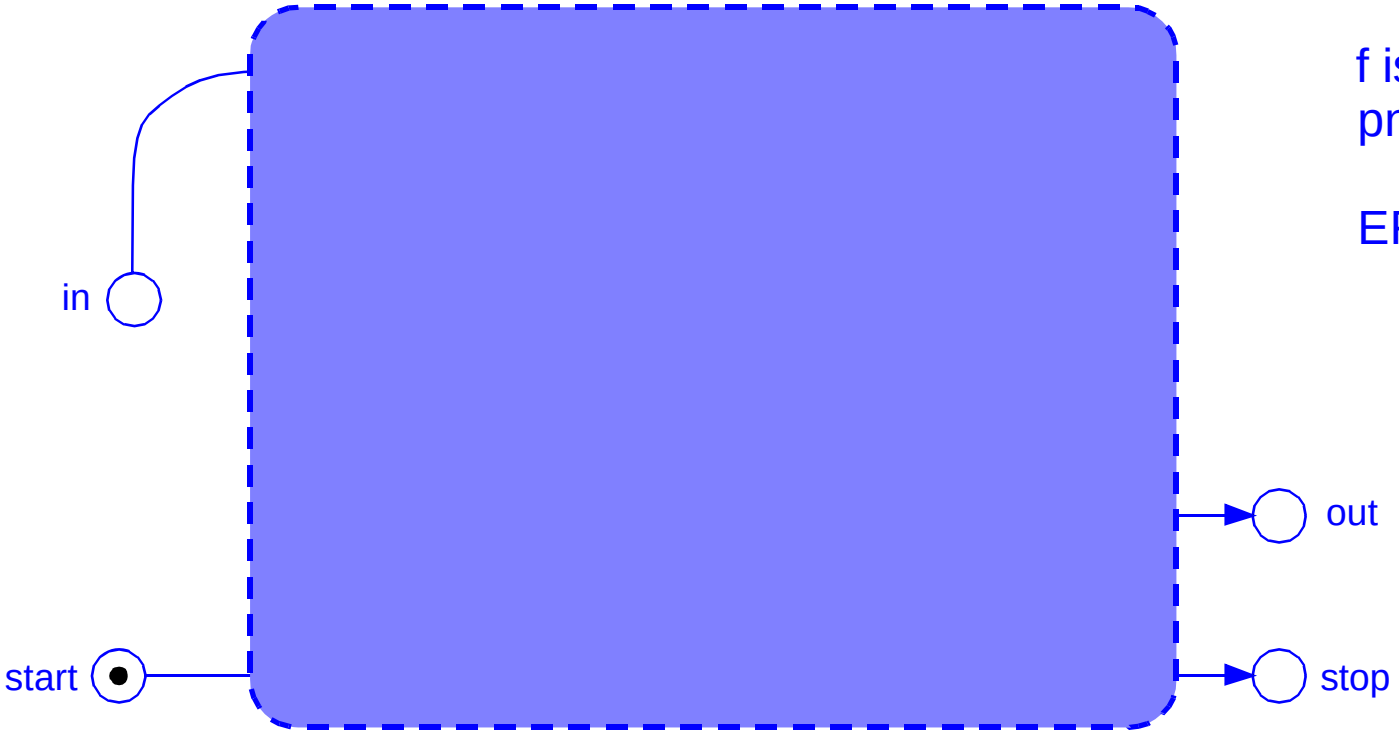

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EF(out = f(in) &
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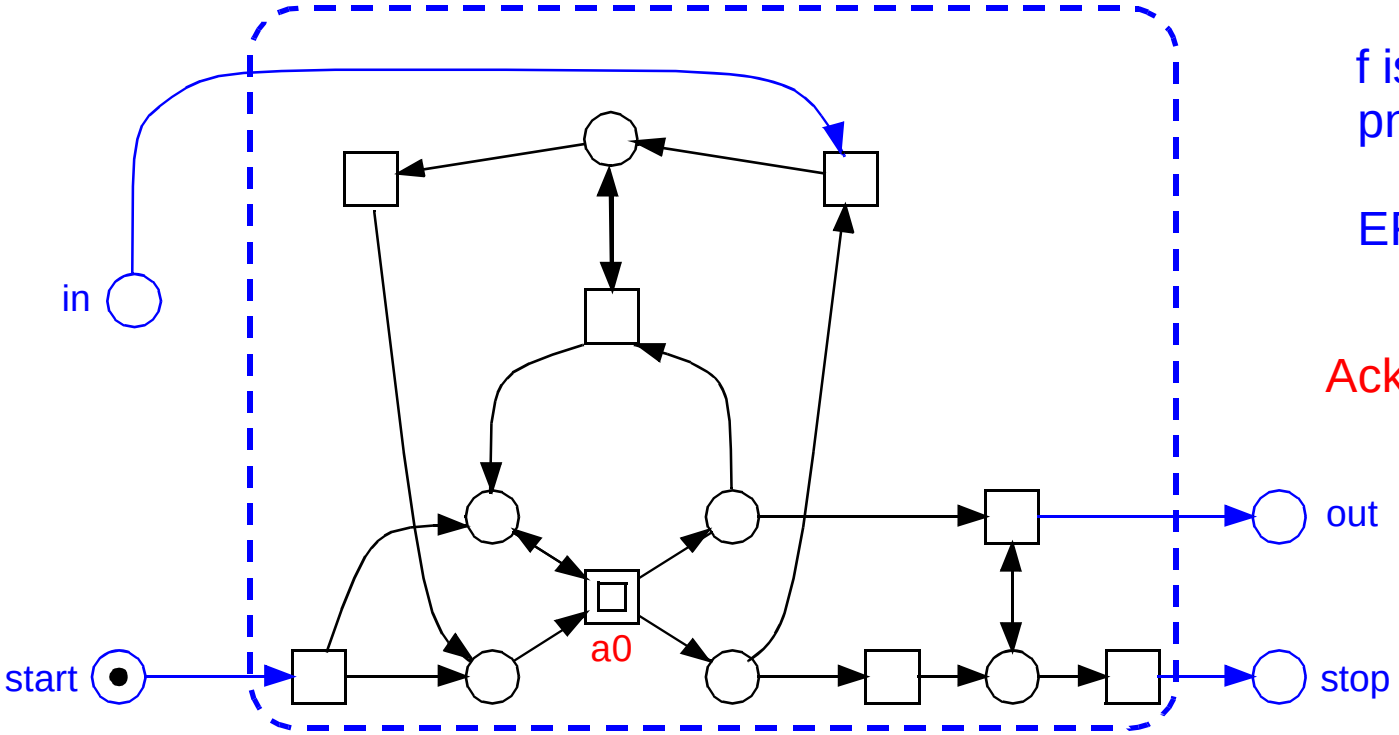

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Ackermann function a1

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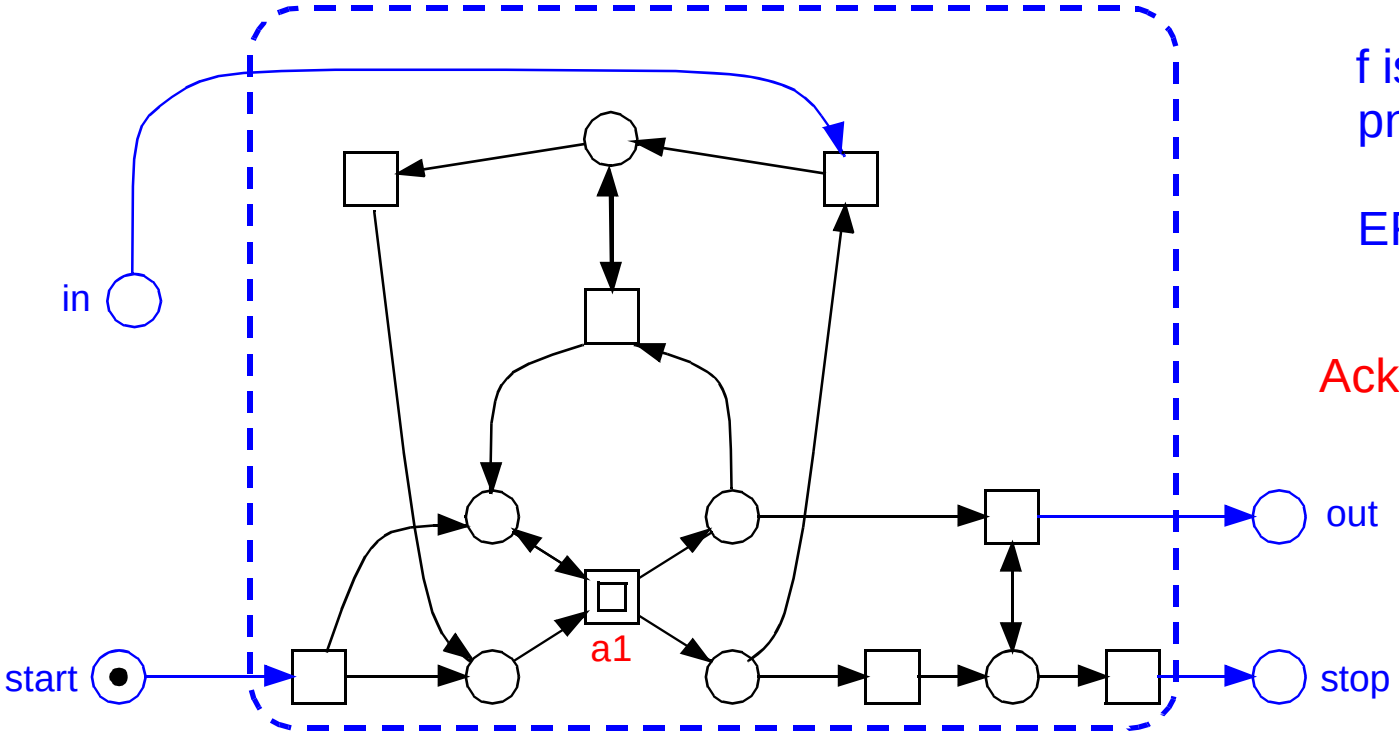

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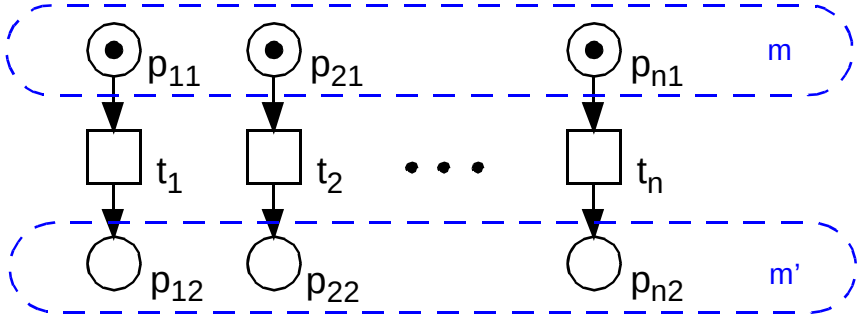
Ackermann function a2

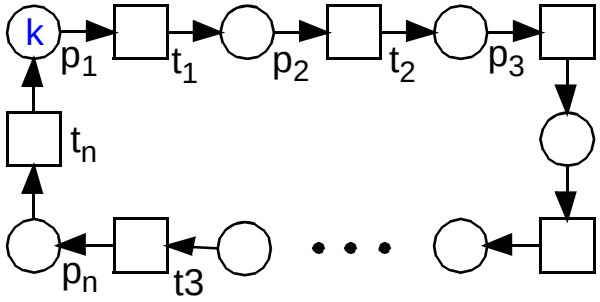
STATE SPACE COMPLEXITY, CAUSES



n! interleaving sequences

m -> m'

2^n - 2 intermediate states



$$\frac{(n + k - 1)!}{(n - 1)!\, k!}$$ states

(combination with repetition)

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  -> *analysis of general behavioural system properties,*
  i.e. boundedness, liveness, reversibility

  -> *model checking of special behavioural system properties,*
  e.g. reachability of a given (sub-) system state (with constraints),
  reproducability of a given (sub-) system state (with constraints)

  => expressed in temporal logics (CTL / LTL),
  as very flexible & powerful query language

Static Analyses

STRUCTURAL ANALYSIS, SOME EXAMPLES

- **boundary nodes**
  - -> *input transitions -> not BND*
  - -> *input places -> not LIVE*
  - -> *LIVE & BND -> no boundary nodes*
- **conservative -> BND**
- . . .
- **Deadlock-Trap Property (DTP)**
  - -> *no structural deadlock -> live*
  - -> *ORD & DTP -> no dead states*
  - -> *ORD & ES & DTP -> LIVE*
  - -> *ORD & EFC & DTP <-> LIVE*

INCIDENCE MATRIX C

- **a representation of the net structure**

$C =$

P \ T	t1 . . .	tj	. . . tm
p1			
pi		cij	
: pn		Δtj	

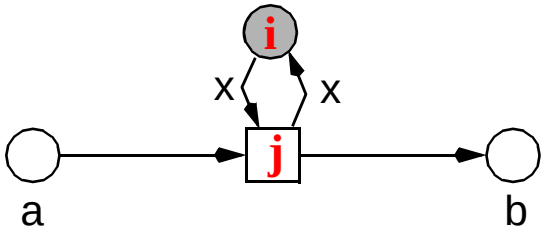
$cij = (pi, tj) = F(tj,pi) - F(pi, tj) = \Delta\, tj(pi)$

$\Delta tj = \Delta\, tj(*)$

- **matrix entry cij:**
  token change in place pi by firing of transition tj

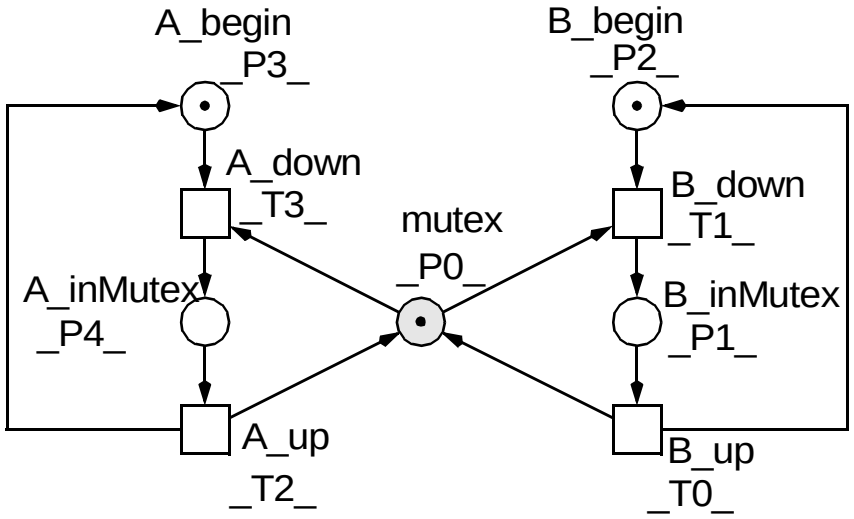
- **matrix column Δtj:**
  vector describing the change of the whole marking by firing of tj

- **side-conditions are neglected**

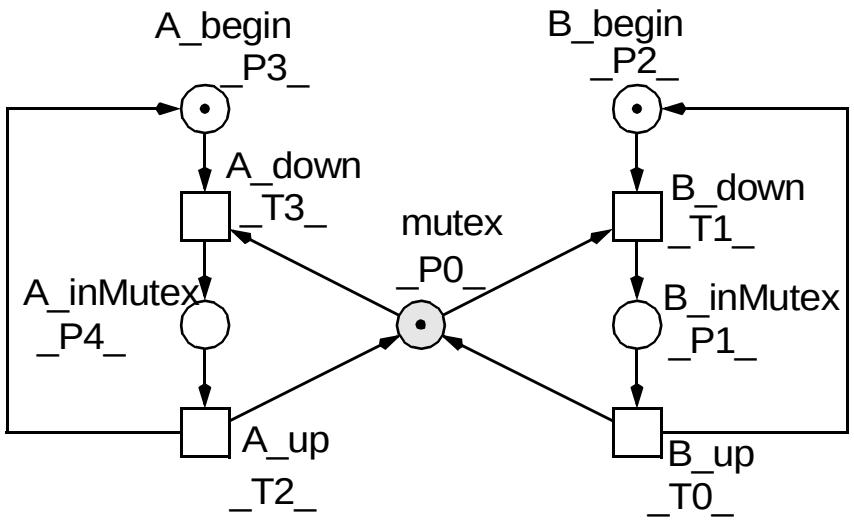


cij = 0

INCIDENCE MATRIX, EXAMPLE

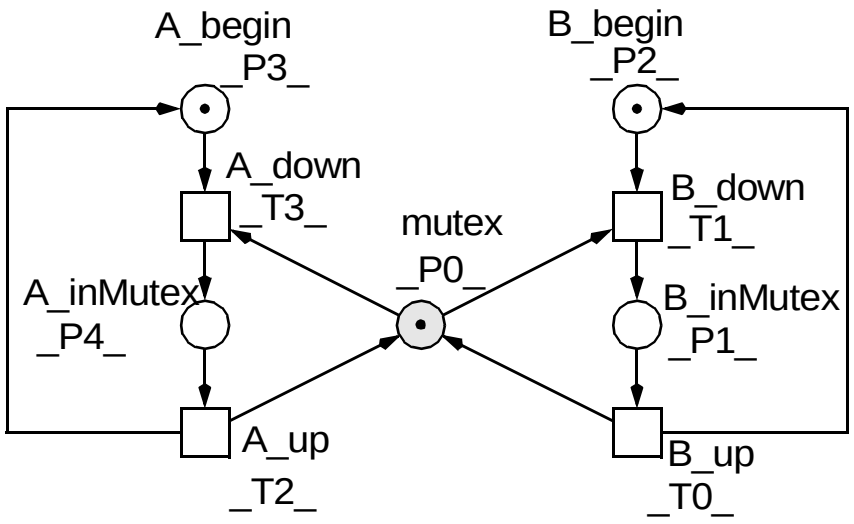


INCIDENCE MATRIX, EXAMPLE



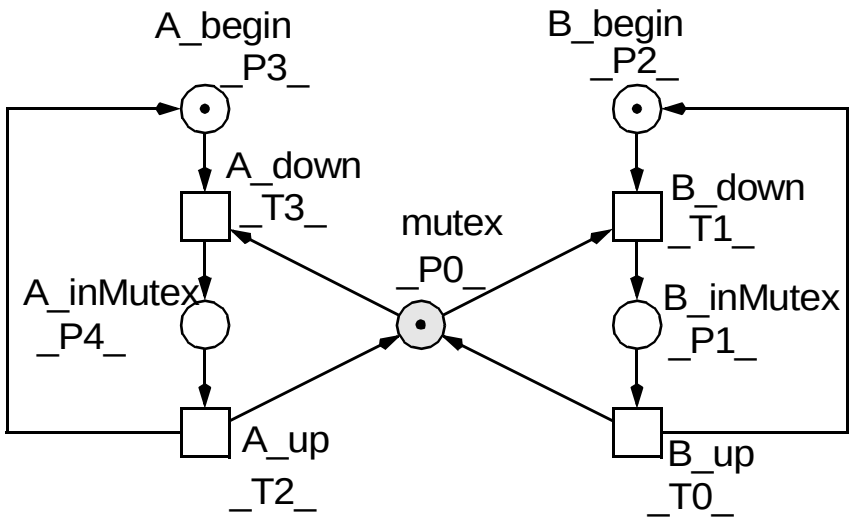
P \ T	B_up	B_down	A_up	A_down
mutex	+1	-1	+1	-1
B_inMutex	-1	+1	0	0
B_begin	+1	-1	0	0
A_begin	0	0	+1	-1
A_inMutex	0	0	-1	+1

INCIDENCE MATRIX, EXAMPLE



P \ T	B_up	B_down	A_up	A_down	m0
mutex	+1	-1	+1	-1	1
B_inMutex	-1	+1	0	0	0
B_begin	+1	-1	0	0	1
A_begin	0	0	+1	-1	1
A_inMutex	0	0	-1	+1	0

INCIDENCE MATRIX, EXAMPLE



P \ T	B_up	B_down	A_up	A_down
mutex	+1	-1	+1	-1
B_inMutex	-1	+1	0	0
B_begin	+1	-1	0	0
A_begin	0	0	+1	-1
A_inMutex	0	0	-1	+1

A_down

m0 →

1

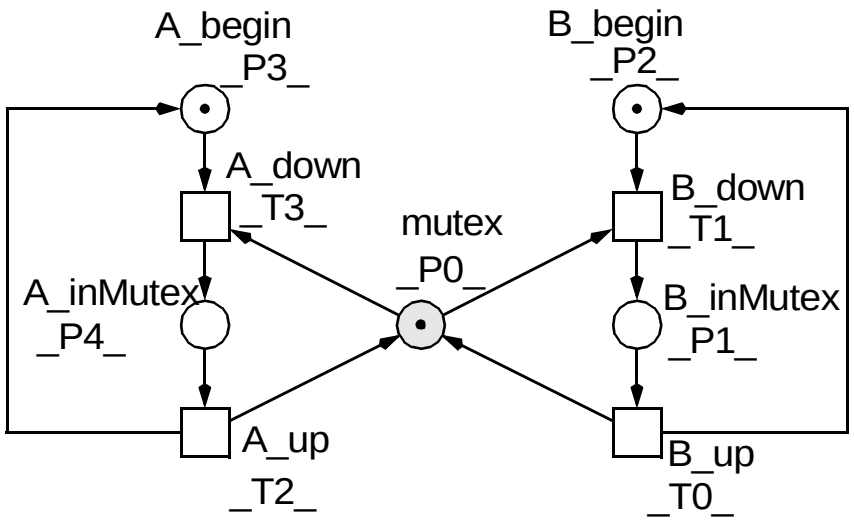
0

1

1

0

INCIDENCE MATRIX, EXAMPLE



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mutex	+1	-1	+1	-1
B_inMutex	-1	+1	0	0
B_begin	+1	-1	0	0
A_begin	0	0	+1	-1
A_inMutex	0	0	-1	+1

A_down

m0 →

1

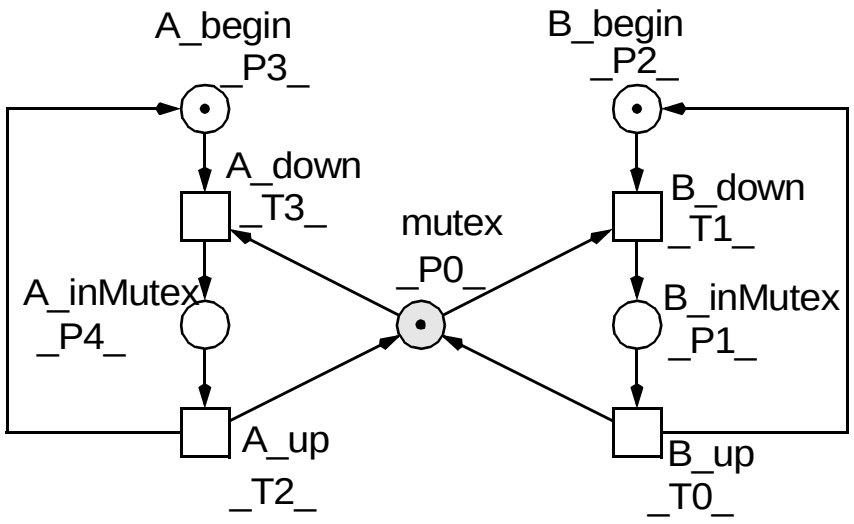
0

1

1

0

INCIDENCE MATRIX, EXAMPLE

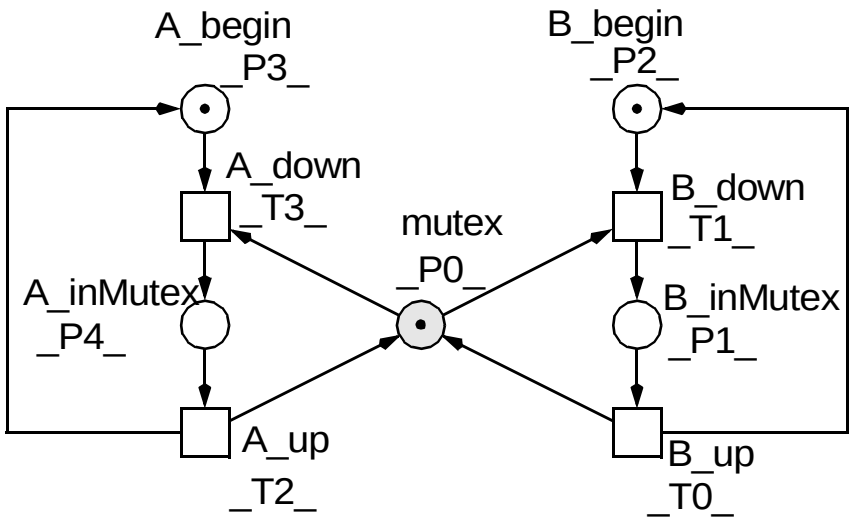


P \ T	B_up	B_down	A_up	A_down
mutex	+1	-1	+1	-1
B_inMutex	-1	+1	0	0
B_begin	+1	-1	0	0
A_begin	0	0	+1	-1
A_inMutex	0	0	-1	+1

A_down

m0	→	m1
1		0
0		0
1		1
1		0
0		1

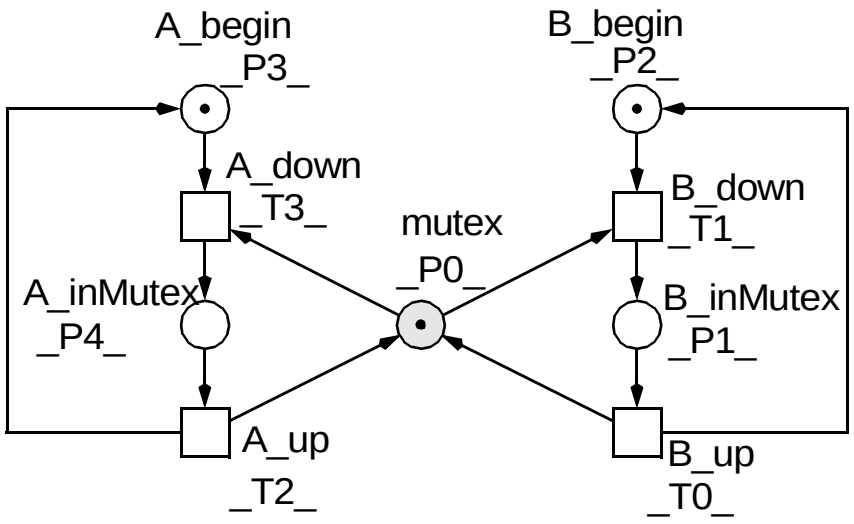
INCIDENCE MATRIX, EXAMPLE



P \ T	B_up	B_down	A_up	A_down
mutex	+1	-1	+1	-1
B_inMutex	-1	+1	0	0
B_begin	+1	-1	0	0
A_begin	0	0	+1	-1
A_inMutex	0	0	-1	+1

m0	A_down →	m1	A_up →
1		0	
0		0	
1		1	
1		0	
0		1	

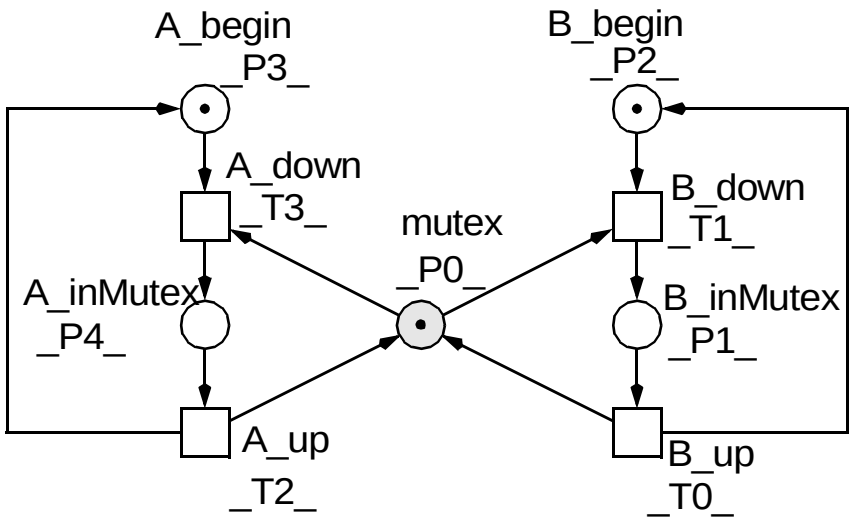
INCIDENCE MATRIX, EXAMPLE



P \ T	B_up	B_down	A_up	A_down
mutex	+1	-1	+1	-1
B_inMutex	-1	+1	0	0
B_begin	+1	-1	0	0
A_begin	0	0	+1	-1
A_inMutex	0	0	-1	+1

A_down		A_up	
m0	→	m1	→ m0
1		0	1
0		0	0
1		1	1
1		0	1
0		1	0

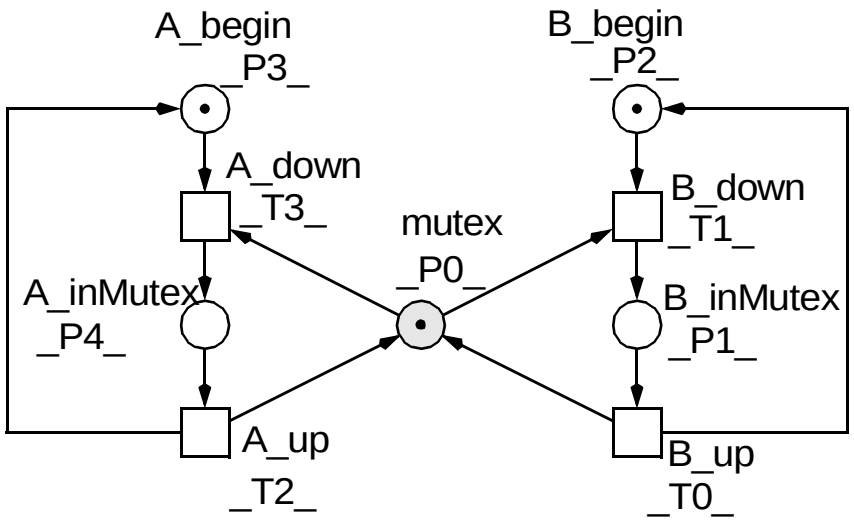
INCIDENCE MATRIX, EXAMPLE



P \ T	B_up	B_down	A_up	A_down
mutex	+1	-1	+1	-1
B_inMutex	-1	+1	0	0
B_begin	+1	-1	0	0
A_begin	0	0	+1	-1
A_inMutex	0	0	-1	+1

0
0
0
1

INCIDENCE MATRIX, EXAMPLE

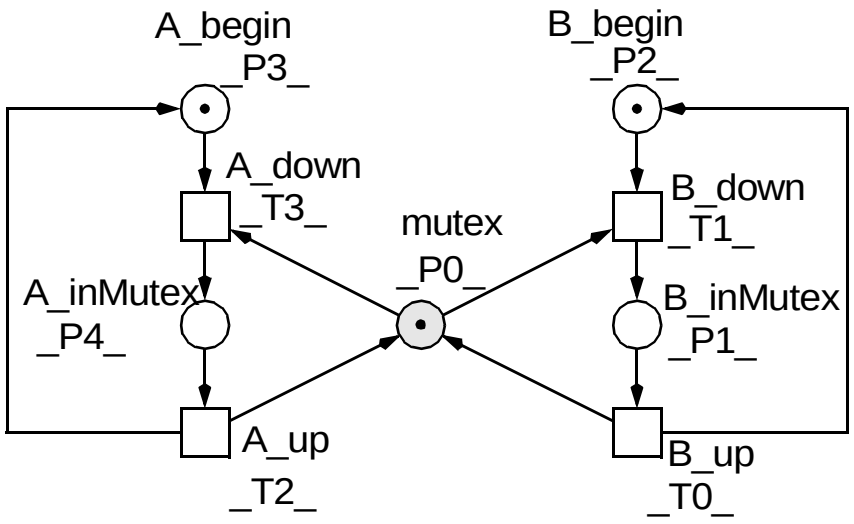


P \ T	B_up	B_down	A_up	A_down
mutex	+1	-1	+1	-1
B_inMutex	-1	+1	0	0
B_begin	+1	-1	0	0
A_begin	0	0	+1	-1
A_inMutex	0	0	-1	+1

0	
0	
0	
1	

-1	
0	
0	
-1	
1	

INCIDENCE MATRIX, EXAMPLE



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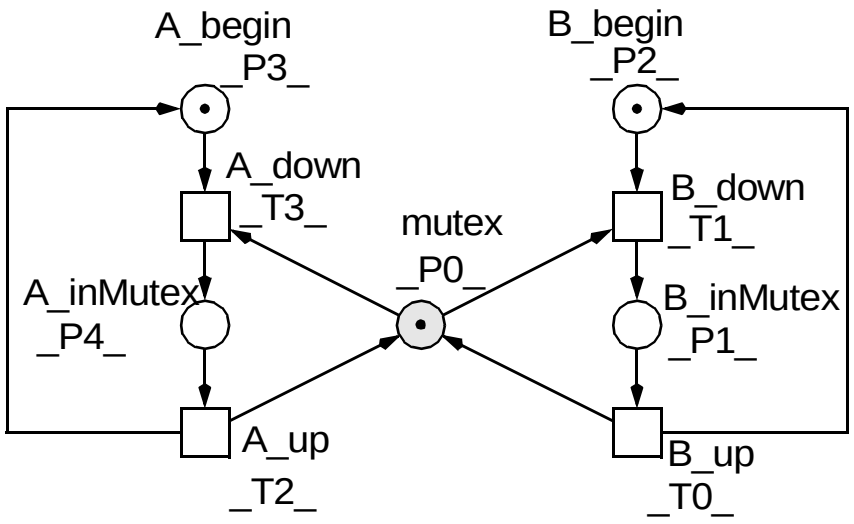
mutex	+1	-1	+1	-1
B_inMutex	-1	+1	0	0
B_begin	+1	-1	0	0
A_begin	0	0	+1	-1
A_inMutex	0	0	-1	+1

			m0 →	m1
0				
0				
0				
1				
-1		1		0
0		0		0
0	+	1	=	1
-1		1		0
1		0		1

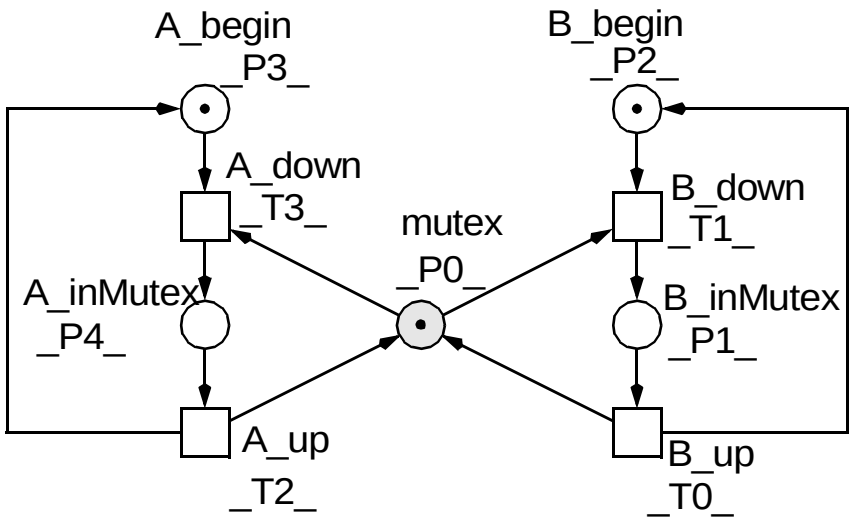
INCIDENCE MATRIX, EXAMPLE



P \ T	B_up	B_down	A_up	A_down
mutex	+1	-1	+1	-1
B_inMutex	-1	+1	0	0
B_begin	+1	-1	0	0
A_begin	0	0	+1	-1
A_inMutex	0	0	-1	+1

			m1 →	m0
0				
0				
1				
0				
1		0		1
0		0		0
0	+	1	=	1
1		0		1
-1		1		0

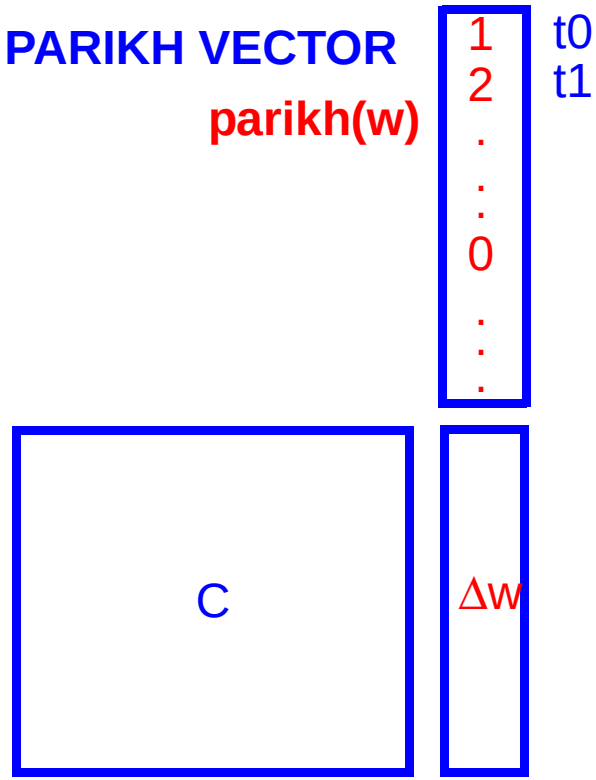
INCIDENCE MATRIX, EXAMPLE



P \ T	B_up	B_down	A_up	A_down
mutex	+1	-1	+1	-1
B_inMutex	-1	+1	0	0
B_begin	+1	-1	0	0
A_begin	0	0	+1	-1
A_inMutex	0	0	-1	+1

				m0 → m0
0		1		1
0		0		0
0	+	1	=	1
0		1		1
0		0		0

STATE EQUATION



Let the word w = t1-t0-t1-. . .
be a sequence of firing transitions.

The change of the marking

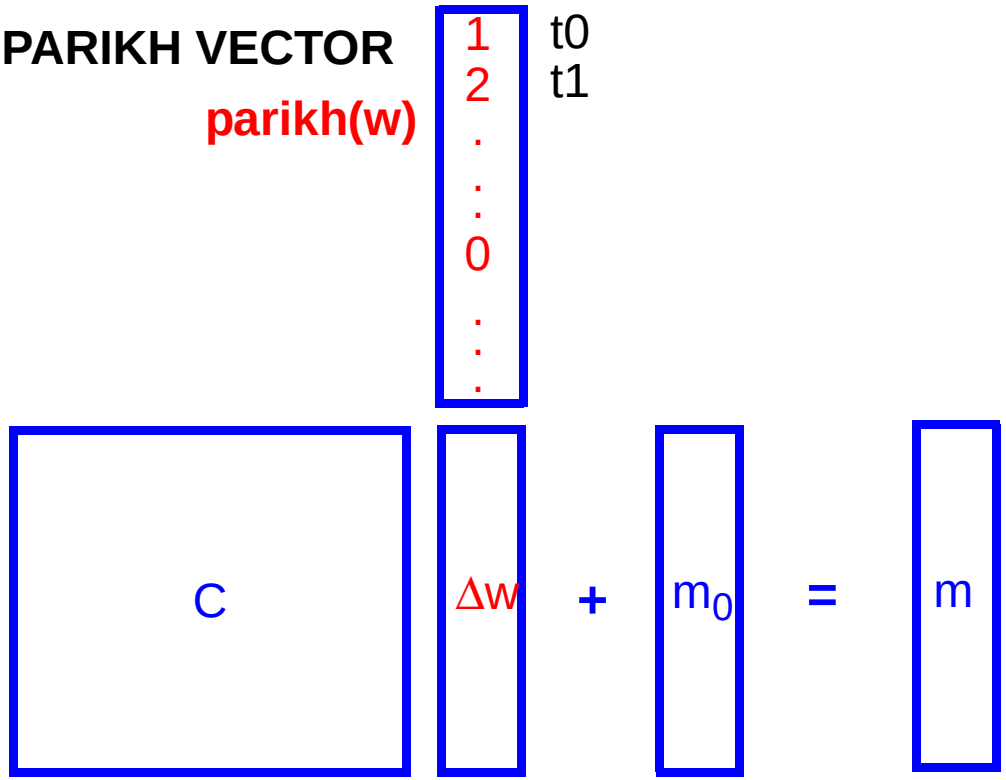
Δw

by firing that sequence
can be computed
by multiplying
the incidence matrix C
with the
Parikh vector parikh(w)
of that transition sequence.

STATE EQUATION

PARIKH VECTOR

parikh(w)



STATE EQUATION

- **linear programming problem**

$$m = m_0 + C\,x,$$
$$x >= 0$$

x **- T-vector**

- **There exists an integer solution for every reachable marking m (the Parikh vector of the transition sequence going to m).**

- **the integer solvability is a necessary condition for the reachability of a marking**

- **NON-REACHABILITY CHECK**

  -> *if there is no integer solution, then the marking is not reachable*

- **each reachable marking is a linear combination of**

  -> *initial marking*

  -> *columns of incidence matrix*

P-INVARIANTS, BASICS

- **Lautenbach, 1973**
- **P-invariants** -> *multisets of places*
  - -> *integer solutions y of* $yC = 0, y \neq 0, y \geq 0$
- **minimal P-invariants**
  - -> *there is no P-invariant with a smaller support* -> *sets of places*
  - -> *gcD of all entries is 1*
- **any P-invariant is a non-negative linear combination of minimal ones**
  - -> *multiplication with a positive integer*
  - -> *addition*
  - -> *Division by gcD*

$$ky = \sum_i a_i y_i$$

- **Covered by P-Invariants (CPI)**
  - -> *each place belongs to a P-invariant*
  - -> *CPI => BND (sufficient condition)*

T-INVARIANTS, BASICS

- **Lautenbach, 1973** -> **Schuster, 1993**
- **T-invariants** -> *multisets of transitions*
  - -> *integer solutions x of* $Cx = 0, x \neq 0, x \geq 0$ -> *Parikh vector*
- **minimal T-invariants**
  - -> *there is no T-invariant with a smaller support* -> *sets of transitions*
  - -> *gcD of all entries is 1*
- **any T-invariant is a non-negative linear combination of minimal ones**
  - -> *multiplication with a positive integer*
  - -> *addition*
  - -> *Division by gcD*

$$kx = \sum_i a_i x_i$$

- **Covered by T-Invariants (CTI)**
  - -> *each transition belongs to a T-invariant*
  - -> *BND & LIVE => CTI (necessary condition)*

Petri nets,
an application

-> error-correcting Petri nets