

# ON THE IMPORTANCE OF THE DEADLOCK TRAP PROPERTY FOR MONOTONIC LIVENESS

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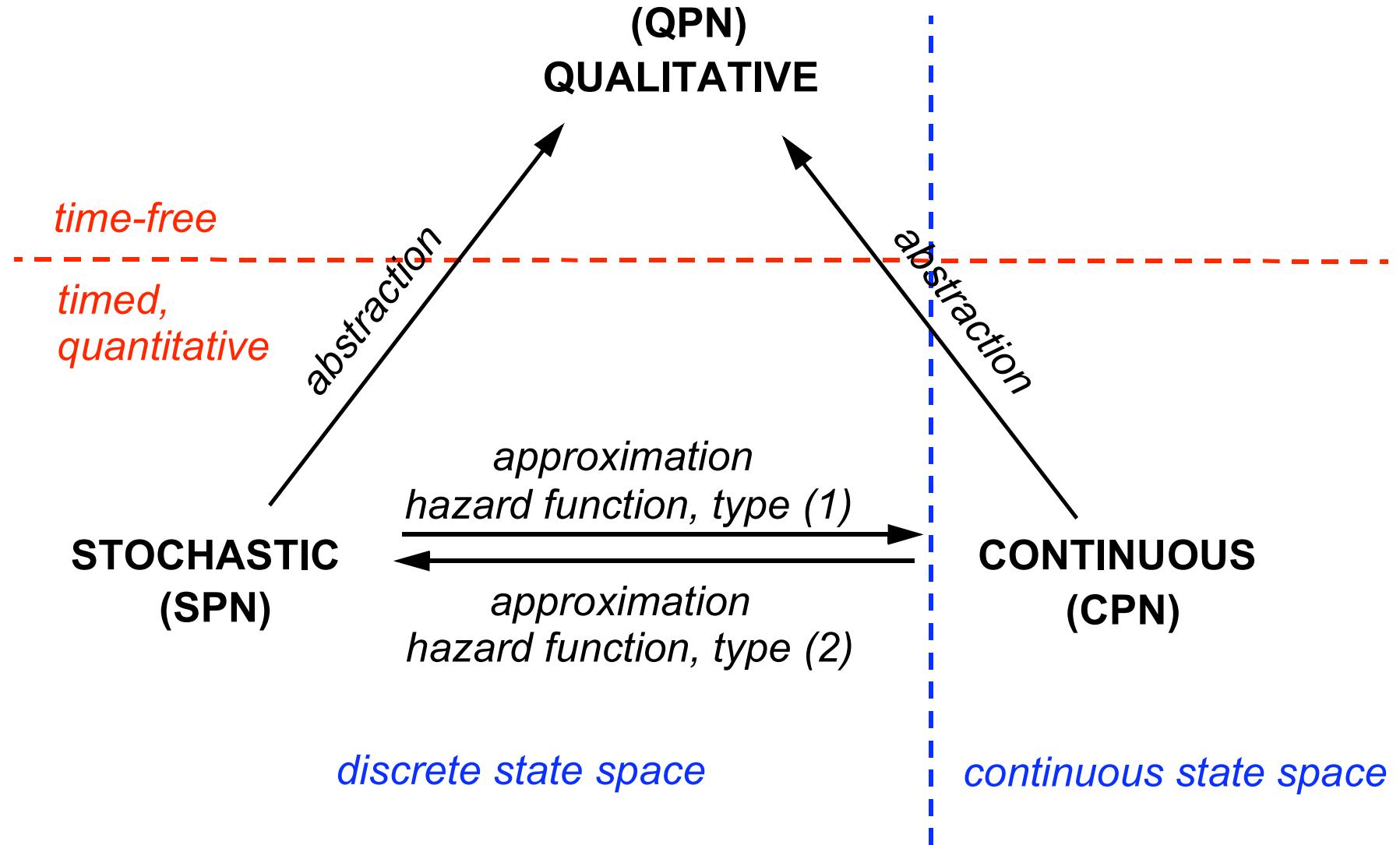
**Brandenburg University  
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**Cristian Mahulea**

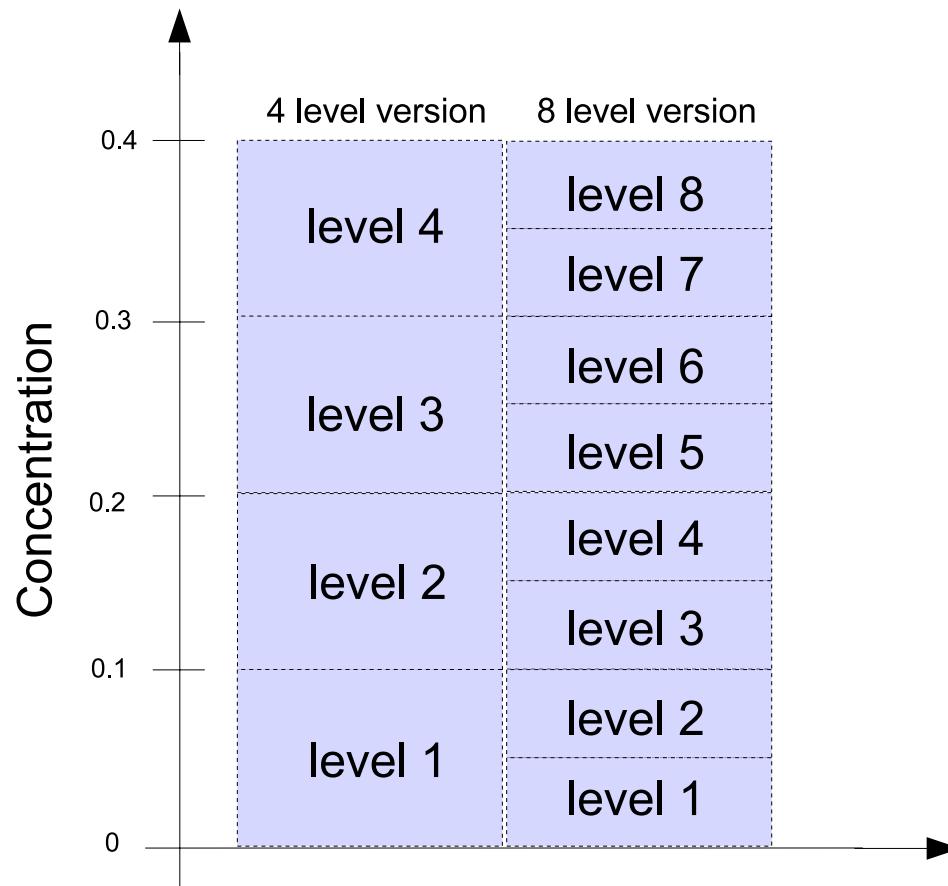
**Manuel Silva**

**Universidad de Zaragoza**



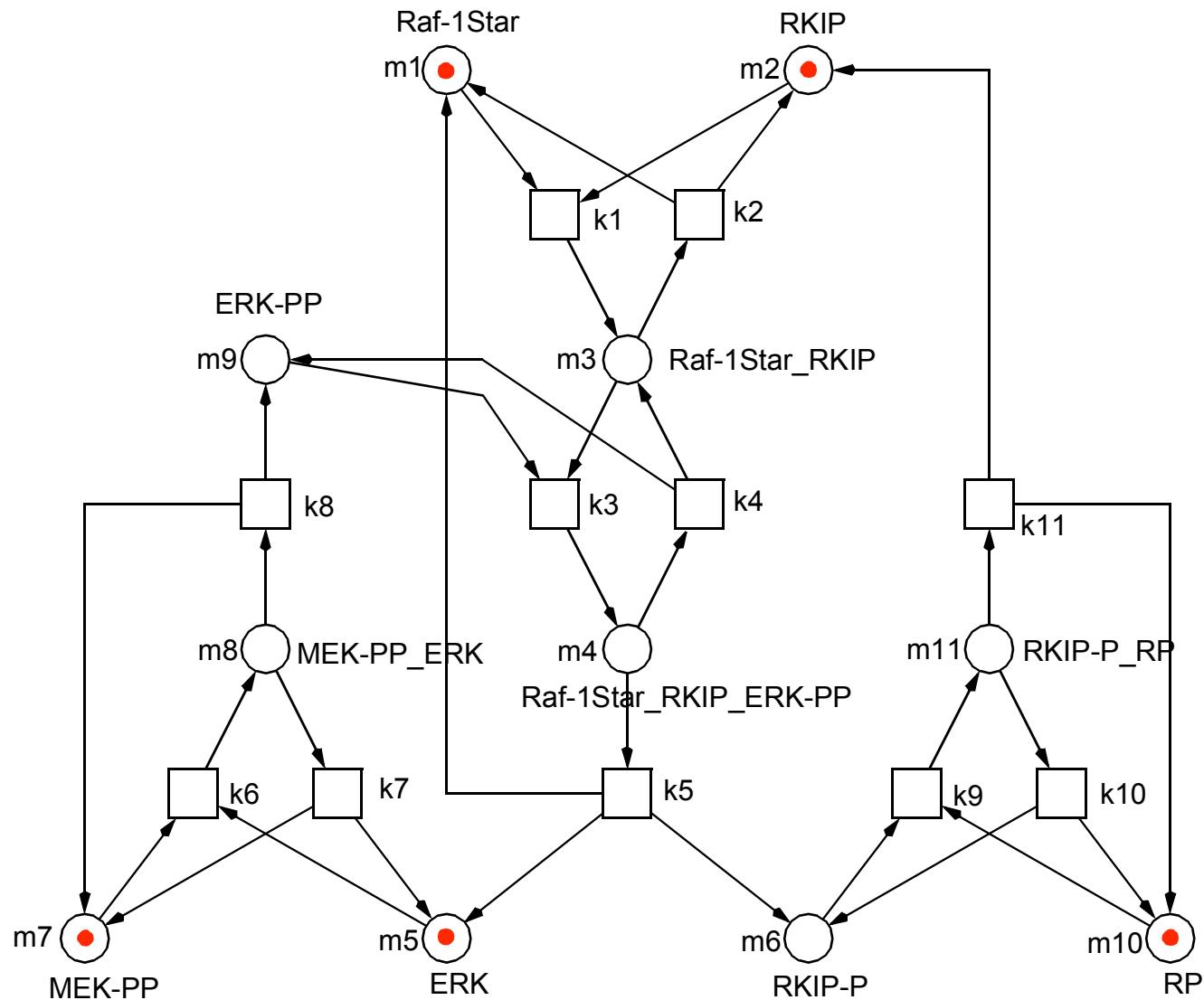


- sharing structure = sharing properties
- increasing level number = increasing accuracy



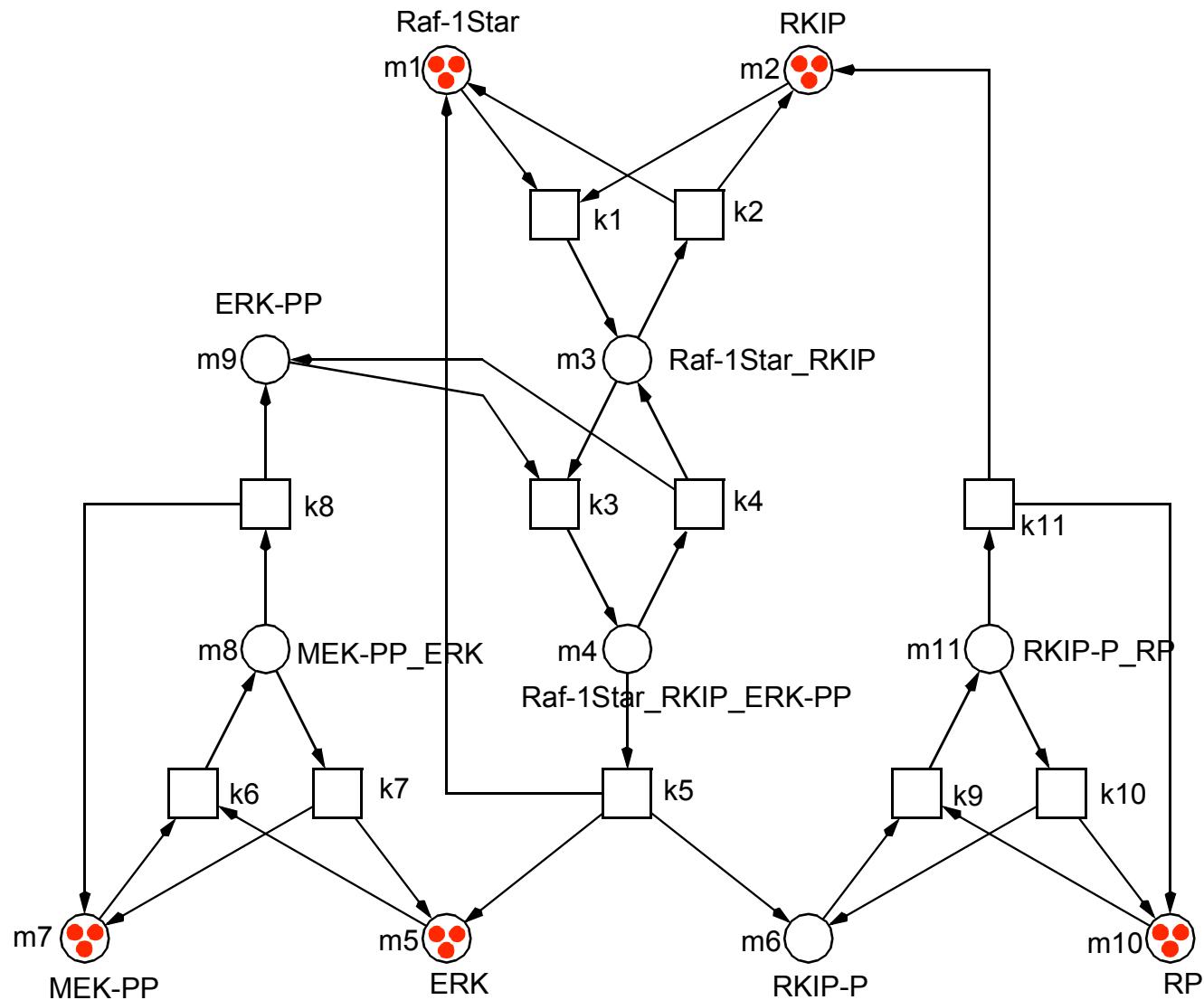
# EX1 - RKIP PATHWAY, PETRI NET

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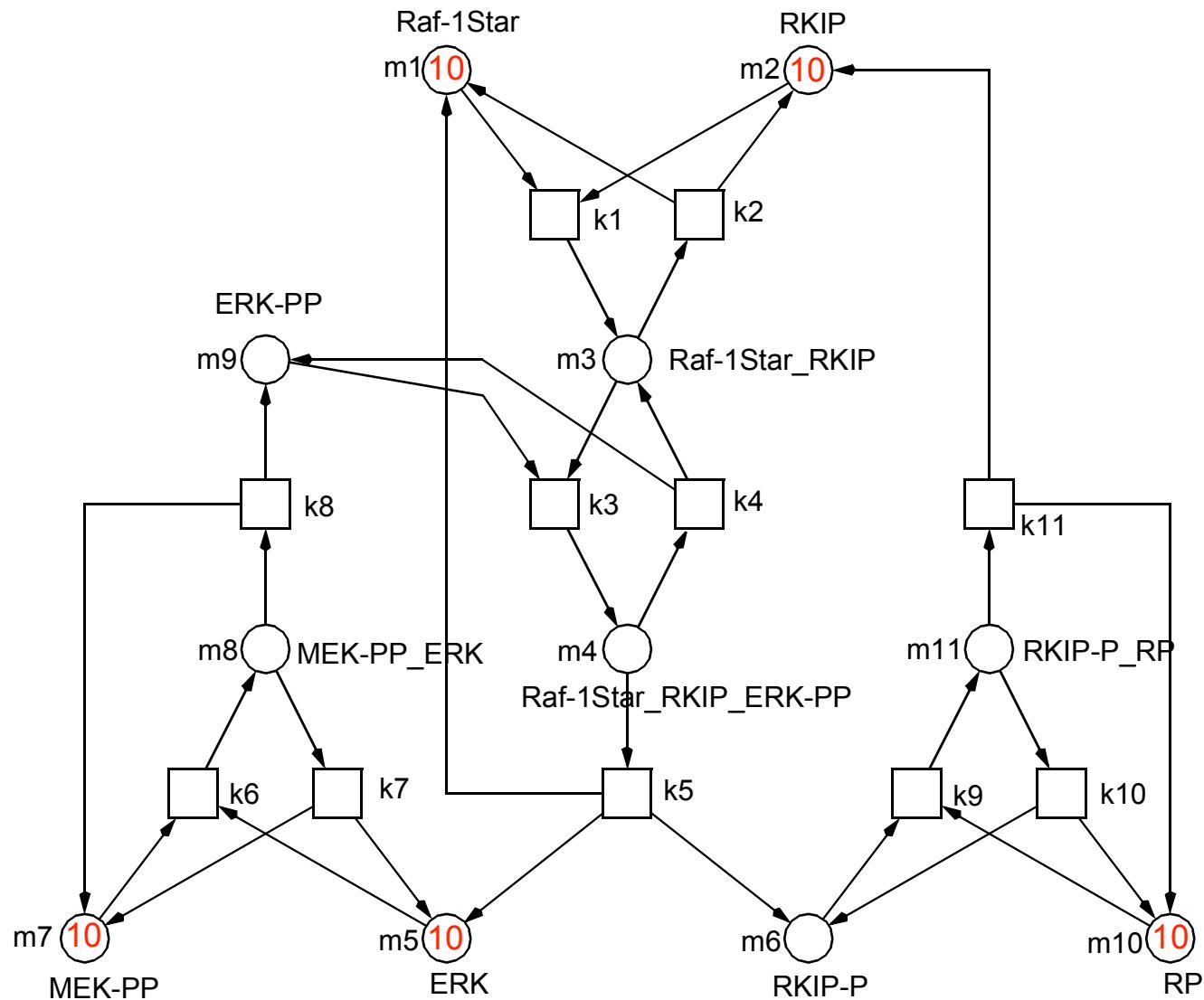
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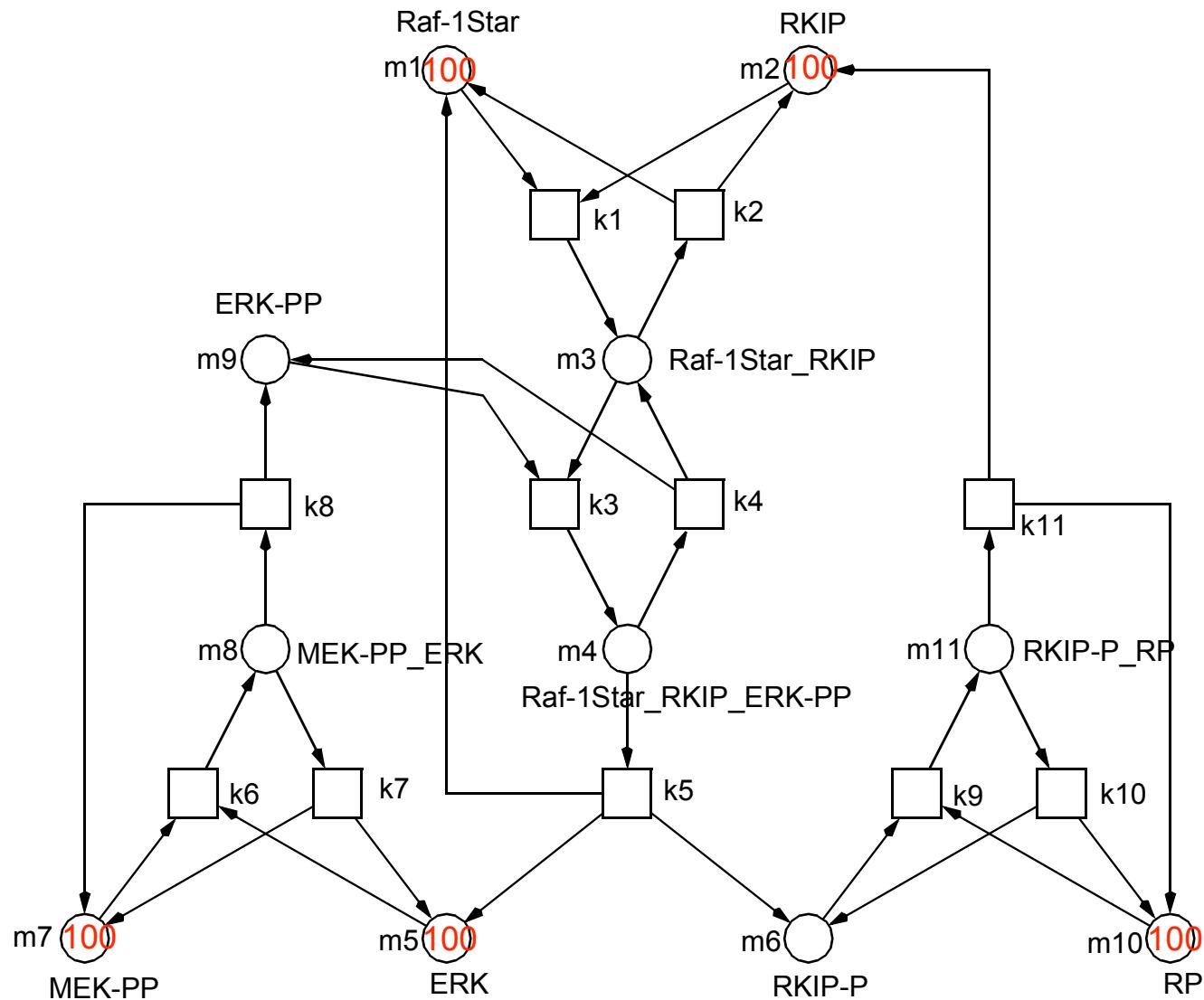
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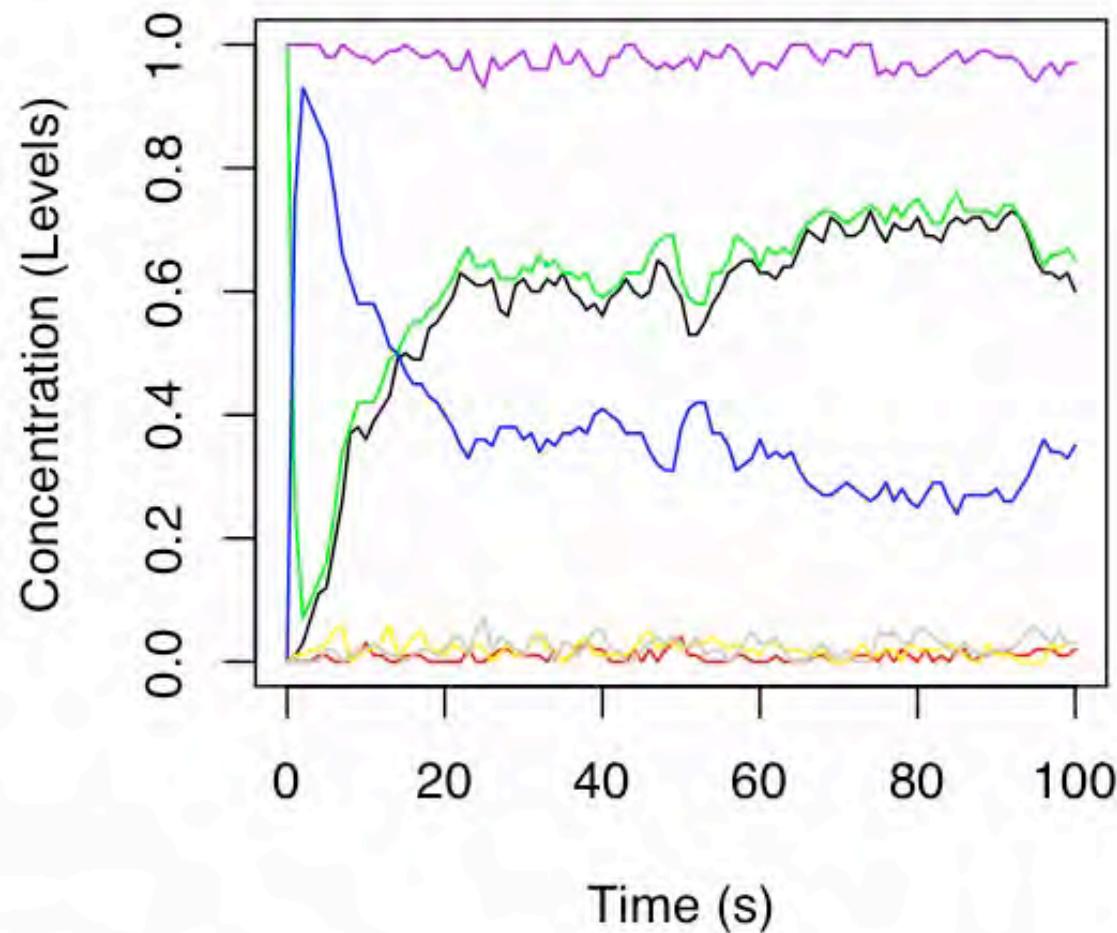


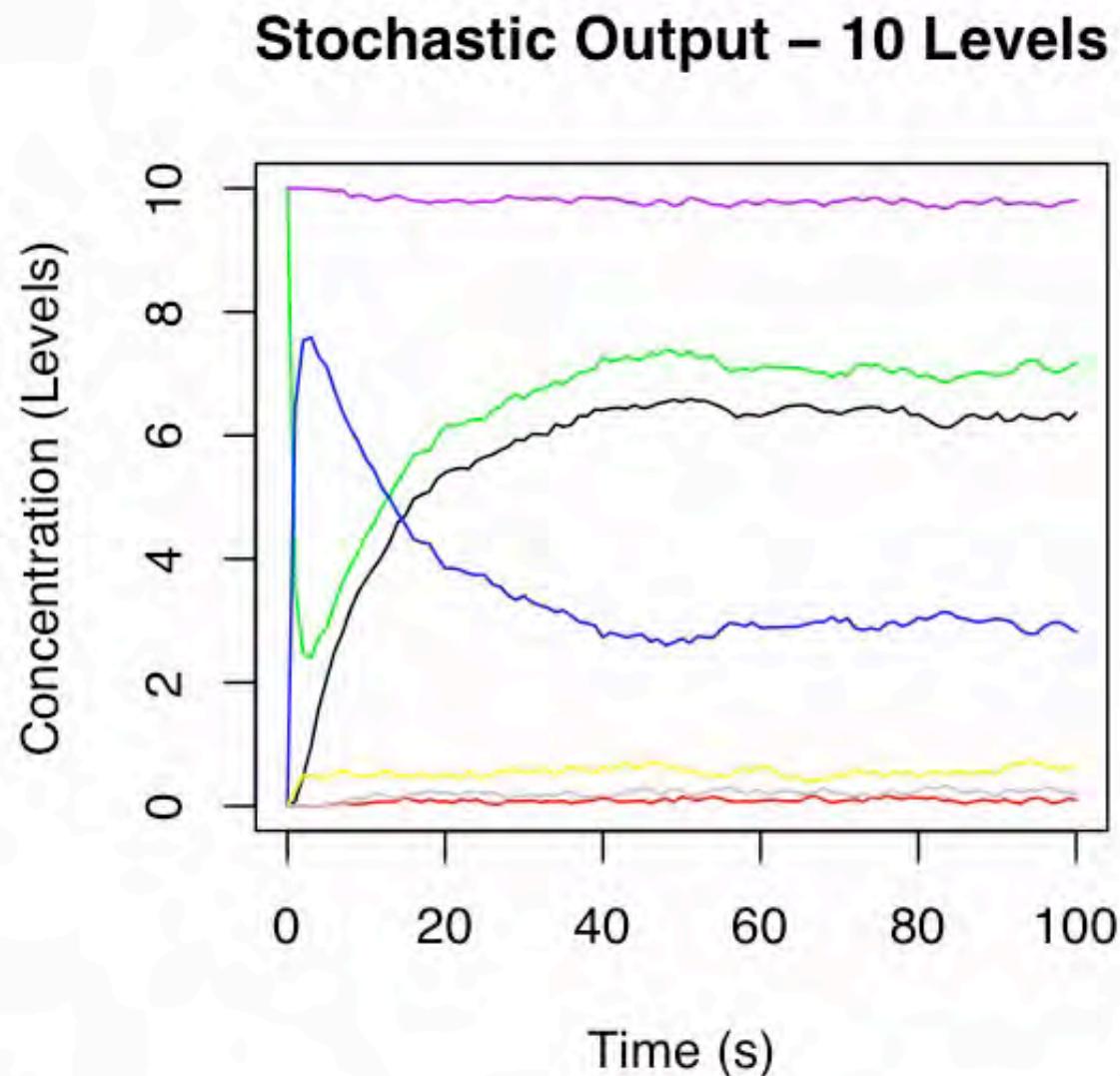
# EX1 - RKIP PATHWAY, PETRI NET

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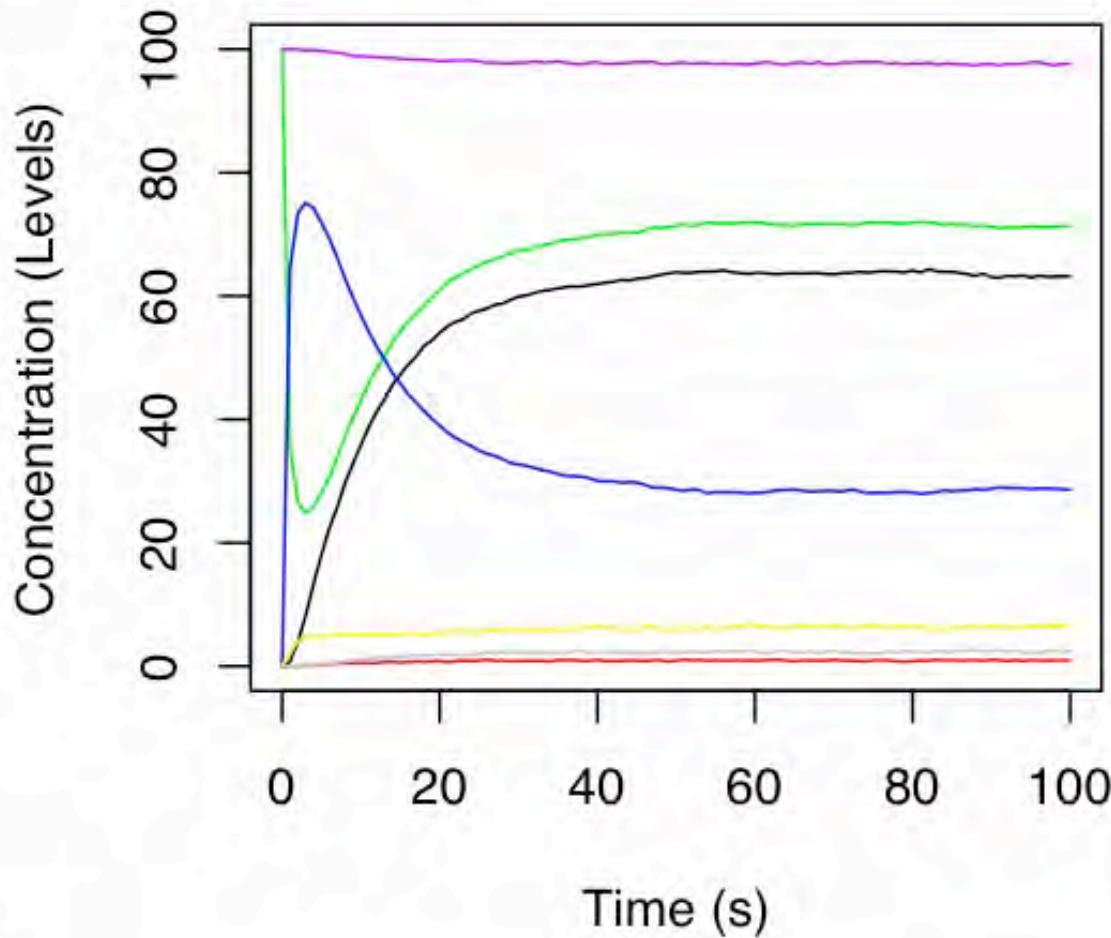


## Stochastic Output – 1 Level

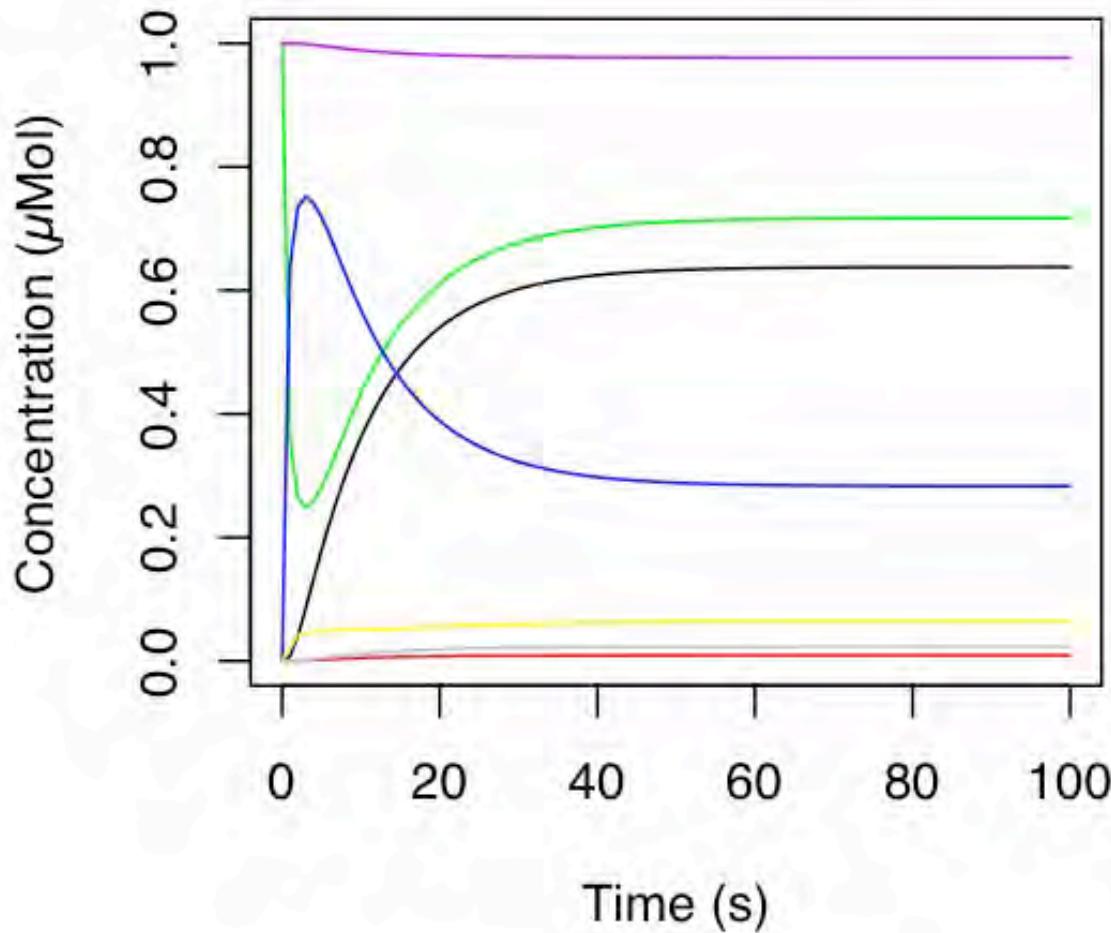




## Stochastic Output – 100 Levels



## Deterministic Output



... HOWEVER ...

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*PN & Systems Biology*

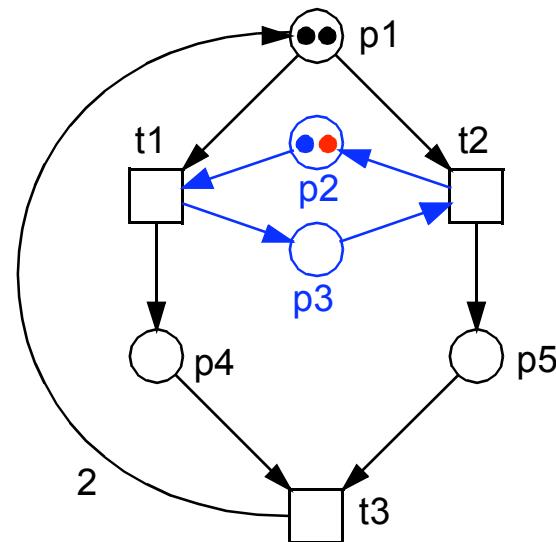
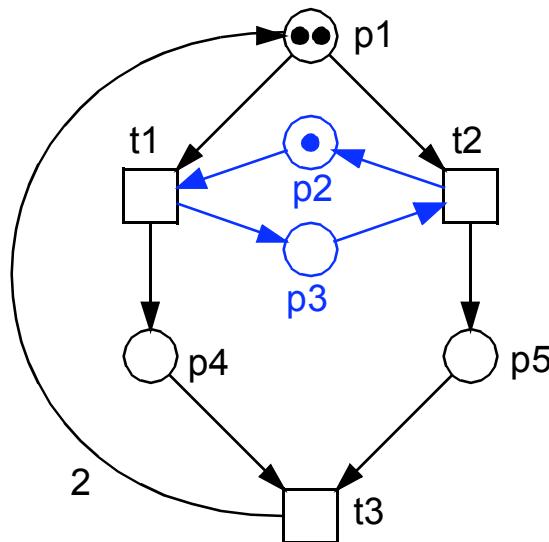




*... behavioural properties  
are generally not preserved under marking increase*

- holds generally for sub-structures (EFC) only !

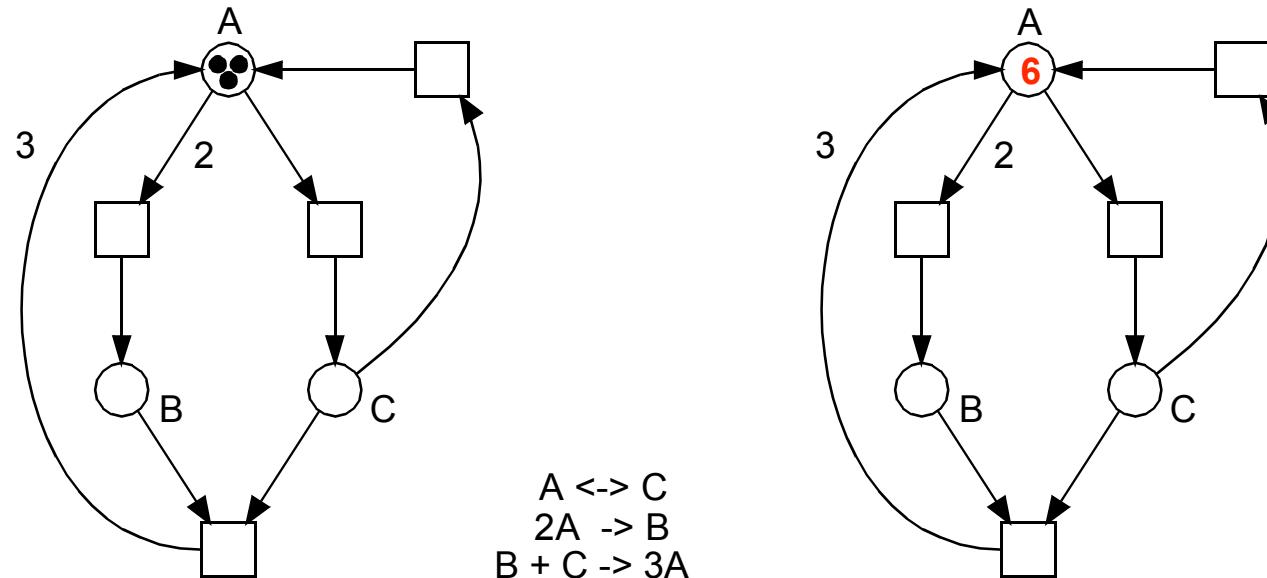
[STARKE 1990]



ORD	PUR	HOM	NBM	CSV	SCF	CON	SC	FT0	TF0	FPO	PF0	NC
N	Y	Y	Y	Y	N	Y	Y	N	N	N	N	<b>ES</b>
<b>DTP</b>	CPI	CTI	SCTI	SB	k-b	1-b	DCF	DSt	DTr	LIV	REV	
<b>N</b>	Y	Y	Y	Y	Y	N	Y	0	N	Y	Y	

- liveness may also be lost by multiple markings

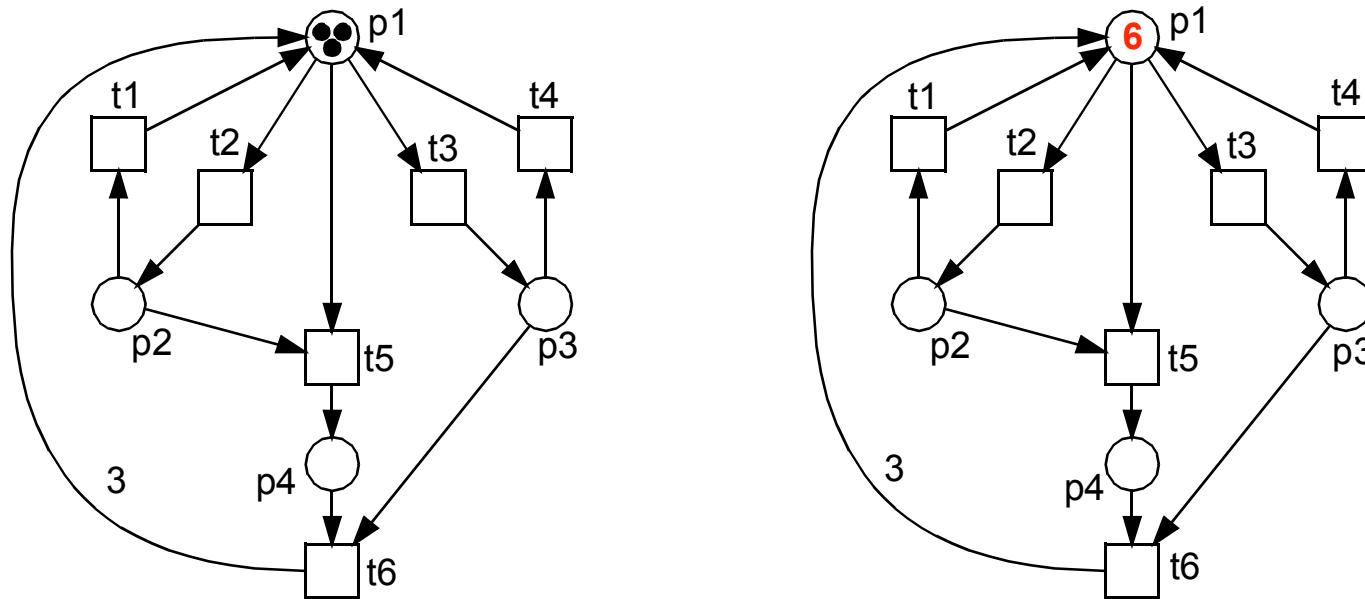
[BALBO 2009]



ORD	PUR	HOM	NBM	CSV	SCF	CON	SC	FT0	TF0	FPO	PFO	NC	
N	Y	N	N	N	N	Y	Y	N	N	N	N	<b>ES</b>	
DTP	CPI	CTI	SCTI	SB	k-b	1-b	DCF	DSt	DTr	LIV	REV		
N	Y	Y	Y	Y	Y	N	N	0	N	Y	Y		
										1	Y	N	N

- liveness may also be lost in homogenous/ordinary nets

[POPOVA 2010]



ORD	PUR	HOM	NBM	CSV	SCF	CON	SC	FT0	TF0	FPO	PF0	NC	
N	Y	Y	Y	N	N	Y	Y	N	N	N	N	nES	
DTP	CPI	CTI	SCTI	SB	k-b	1-b	DCF	DSt	DTr	LIV	REV		
N	Y	Y	Y	Y	Y	N	N	0	N	Y	Y		
										1	Y	N	N

- **homothetic liveness**

- > *liveness preservation while multiplying the initial marking by k*

- > *precondition for reasonable continuization (fluidization)*

- **monothonic liveness**

- > *liveness preservation for arbitrary marking increase*

- **monothonic liveness -> homothetic liveness**

- ... but not vice versa*

*-> necessary / sufficient criteria ?*

# **DEADLOCK TRAP PROPERTY (DTP)**

**STRUCTURAL DEADLOCK**

~~**DEADLOCK TRAP PROPERTY**~~

**(DTP)**

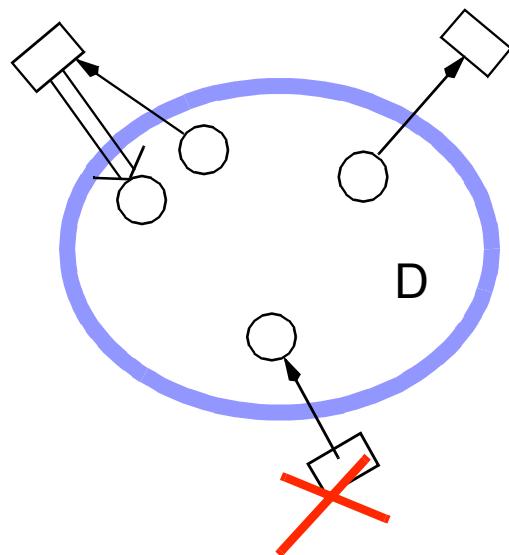
**SIPHON**



# DEADLOCK TRAP PROPERTY (DTP)

**Siphon D**

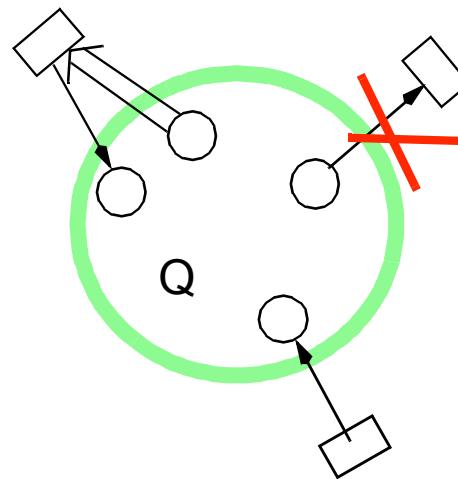
$$FD \subseteq DF$$



any transition putting token into the set  
also takes token from it:  
an empty siphon will never get marked

**Trap Q**

$$QF \subseteq FQ$$

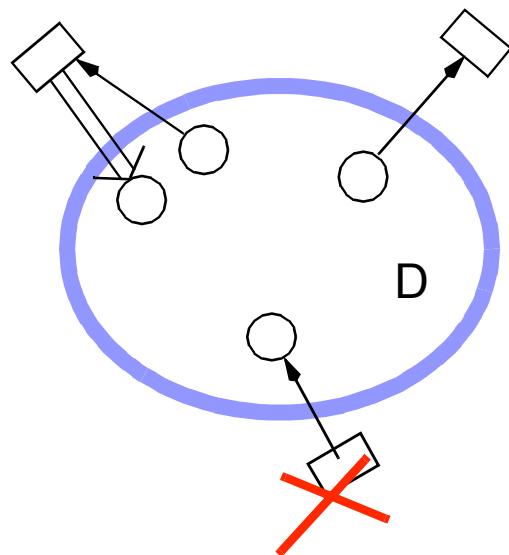


any transition taking tokens from the set  
also puts token into it:  
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# DEADLOCK TRAP PROPERTY (DTP)

**Siphon D**

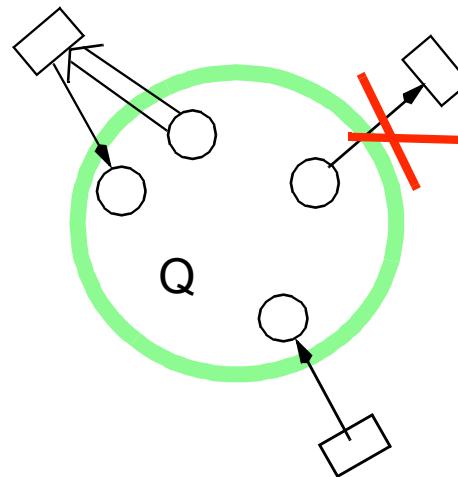
$$FD \subseteq DF$$



any transition putting token into the set  
also takes token from it:  
an empty siphon will never get marked

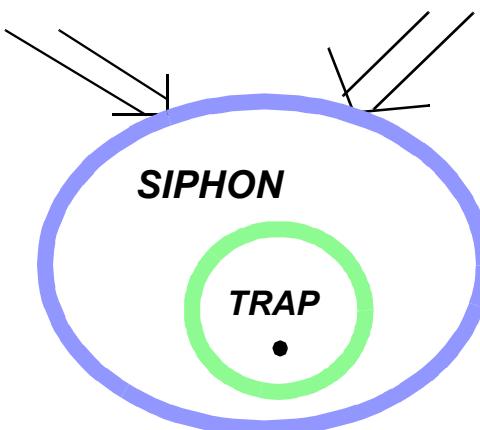
**Trap Q**

$$QF \subseteq FQ$$



any transition taking tokens from the set  
also puts token into it:  
a marked trap will never get empty

**DTP:** each siphon contains a  
marked trap (at  $m_0$ )



- allows to decide liveness, sometimes

Prop.1: ORD & no siphons  $\rightarrow$  live

Prop.2: ORD & DTP  $\rightarrow$  no DSt

Prop.3: ES & DTP  $\rightarrow$  live

Prop.4: EFC  $\rightarrow$  ( DTP  $\leftrightarrow$  live )

- allows to decide liveness, sometimes

Prop.1: ORD & no siphons  $\rightarrow$  live

Prop.2: ORD & DTP  $\rightarrow$  no DSt

Prop.3: ES & DTP  $\rightarrow$  live

Prop.4: EFC  $\rightarrow$  ( DTP  $\leftrightarrow$  live )

- Monotonic DTP -> sufficient conditions

Prop.1m: ORD & no siphons  $\rightarrow$  monotonically live

Prop.2m: ORD & DTP  $\rightarrow$  monotonically no DSt

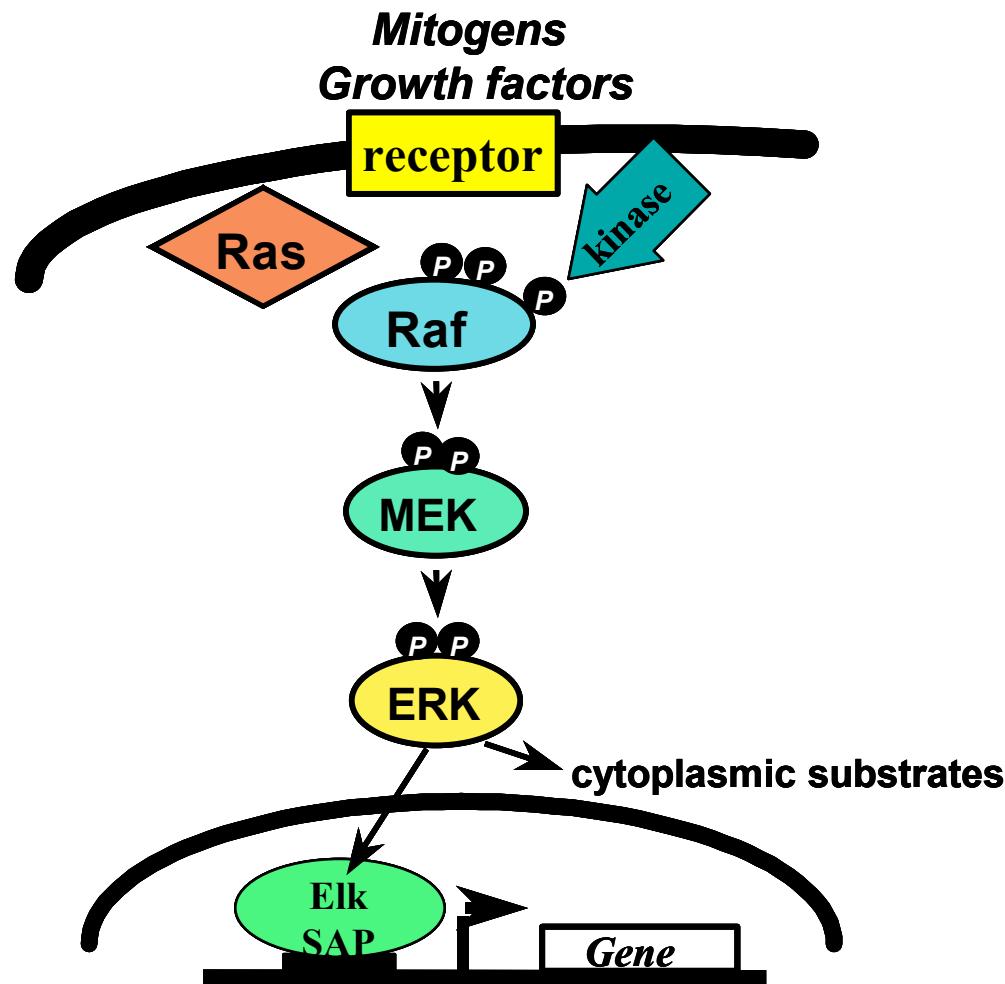
Prop.3m: ES & DTP  $\rightarrow$  monotonically live

Prop.4m: EFC  $\rightarrow$  ( DTP  $\leftrightarrow$  monotonically live )

# SOME BIO PETRI NETS, WHERE IT HELPS

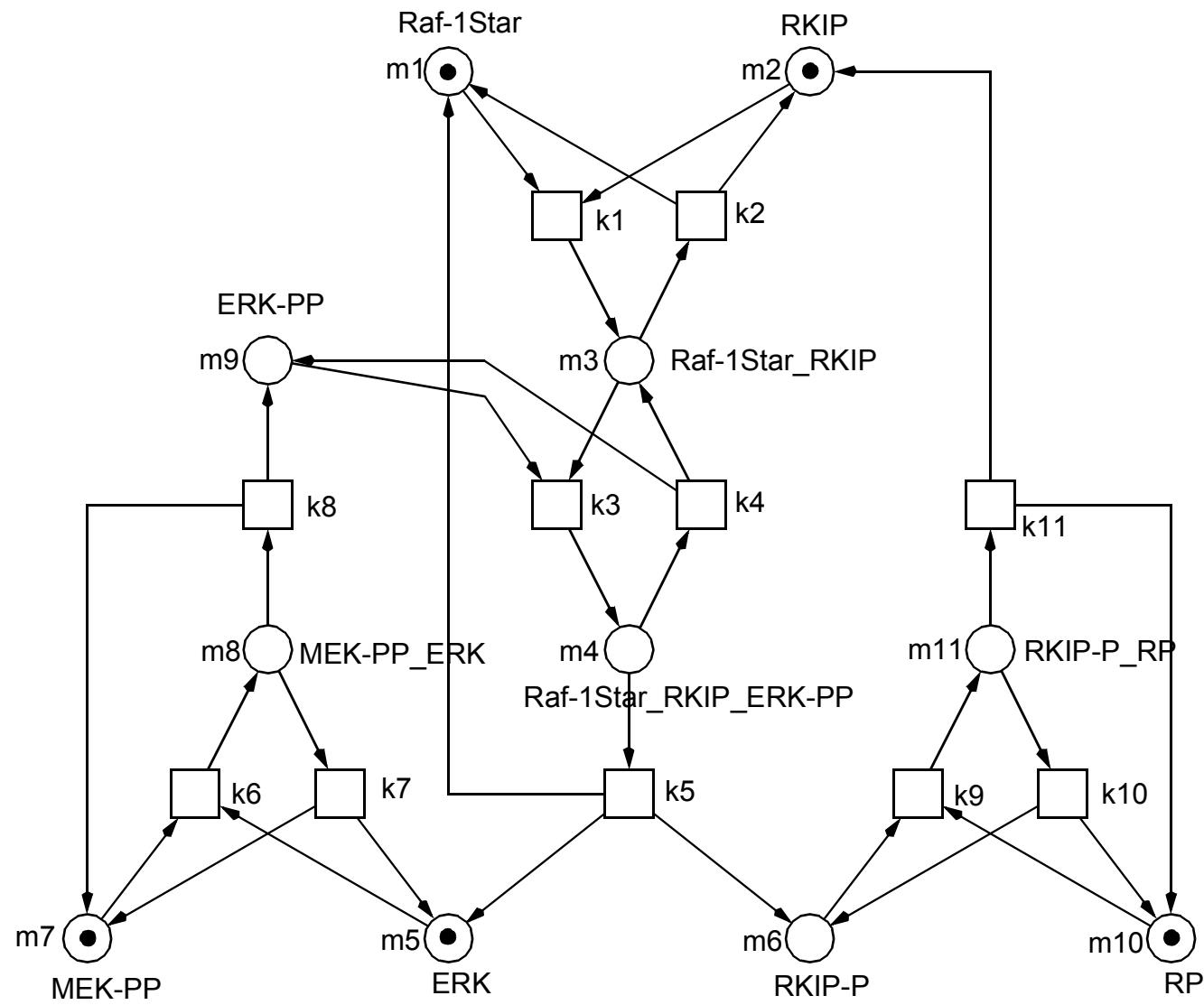
*(ordinary, DTP, live, consistent)*

...one pathway...



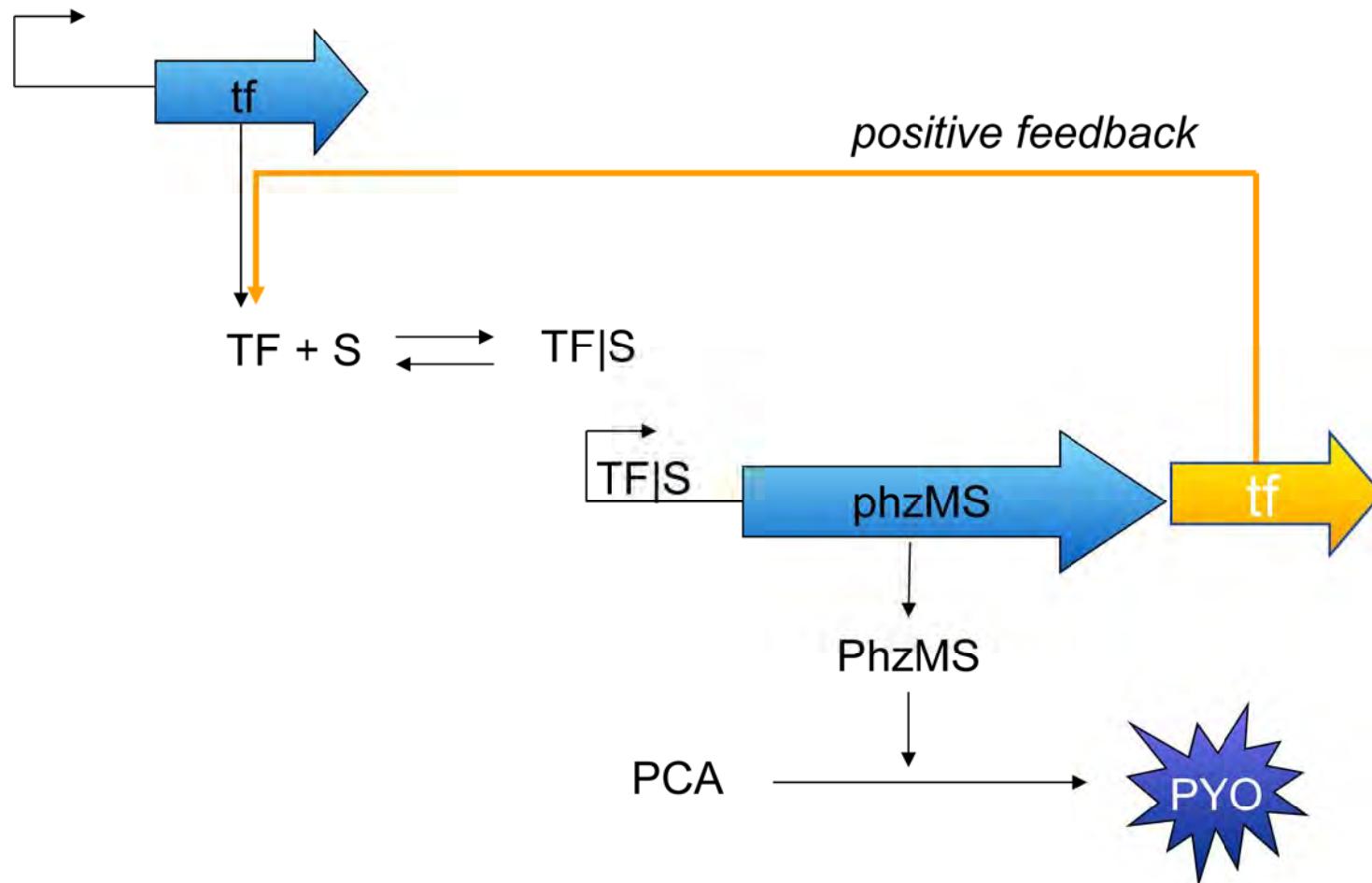
# EX1 - RKIP PATHWAY, PETRI NET (PROP.3)

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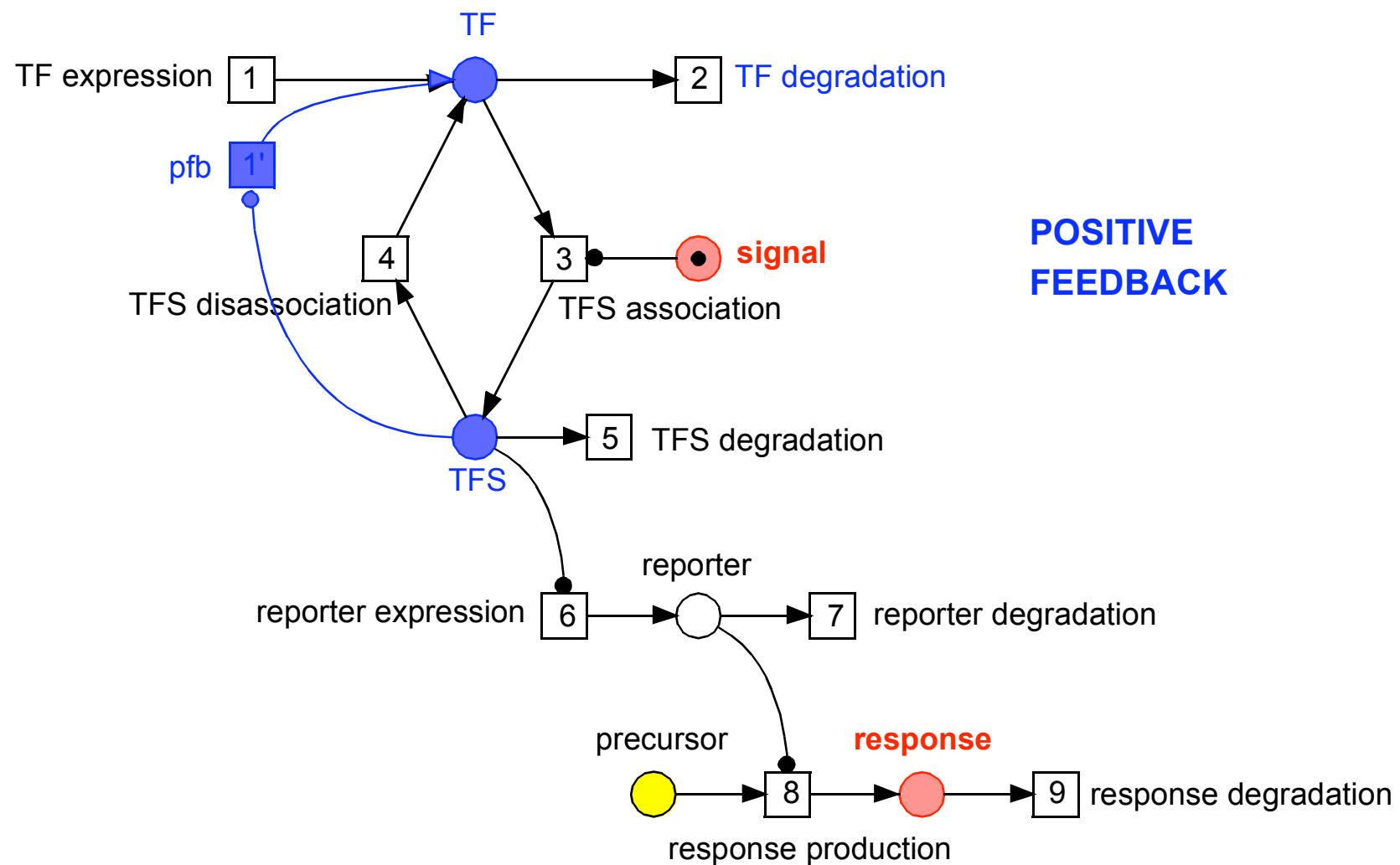
## Ex2 - BIOSENSOR

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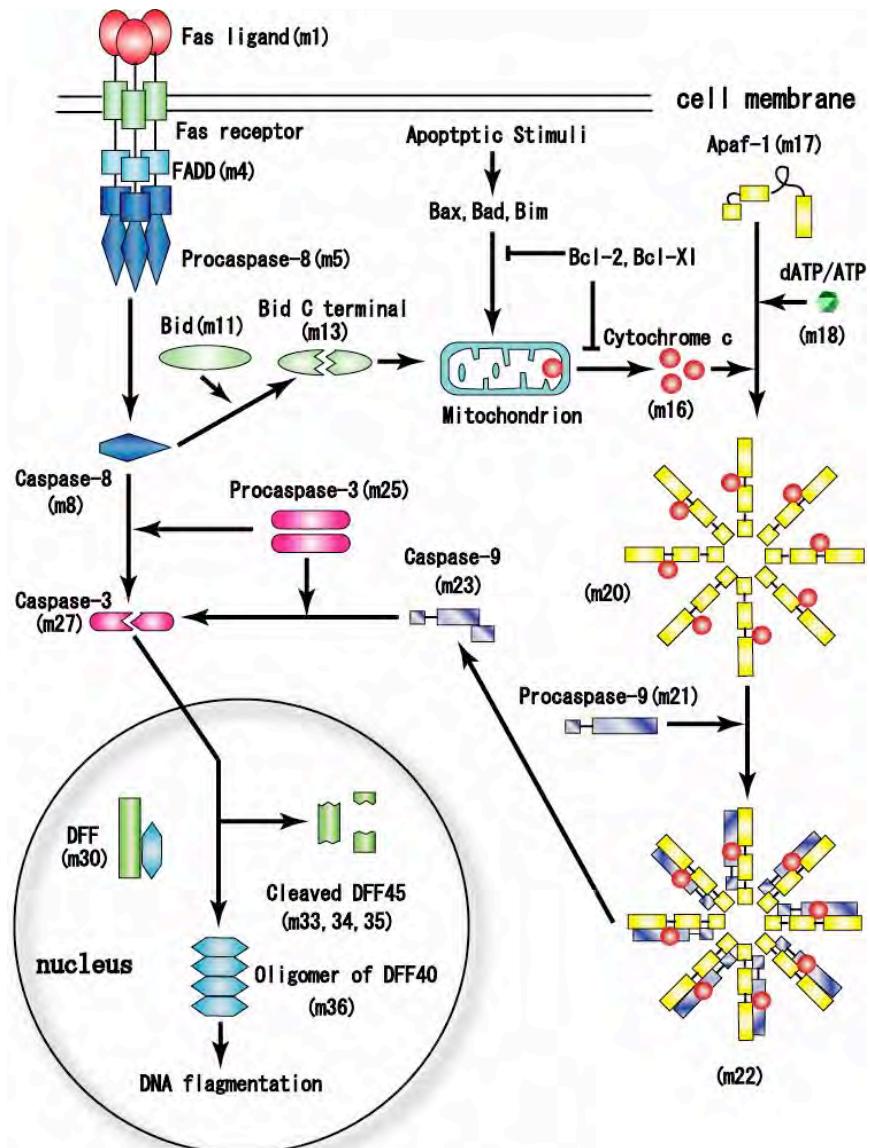
## Ex2 - BIOSENSOR, PETRI NET (PROP.3)

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# Ex3 - APOPTOSIS IN MAMMALIAN CELLS

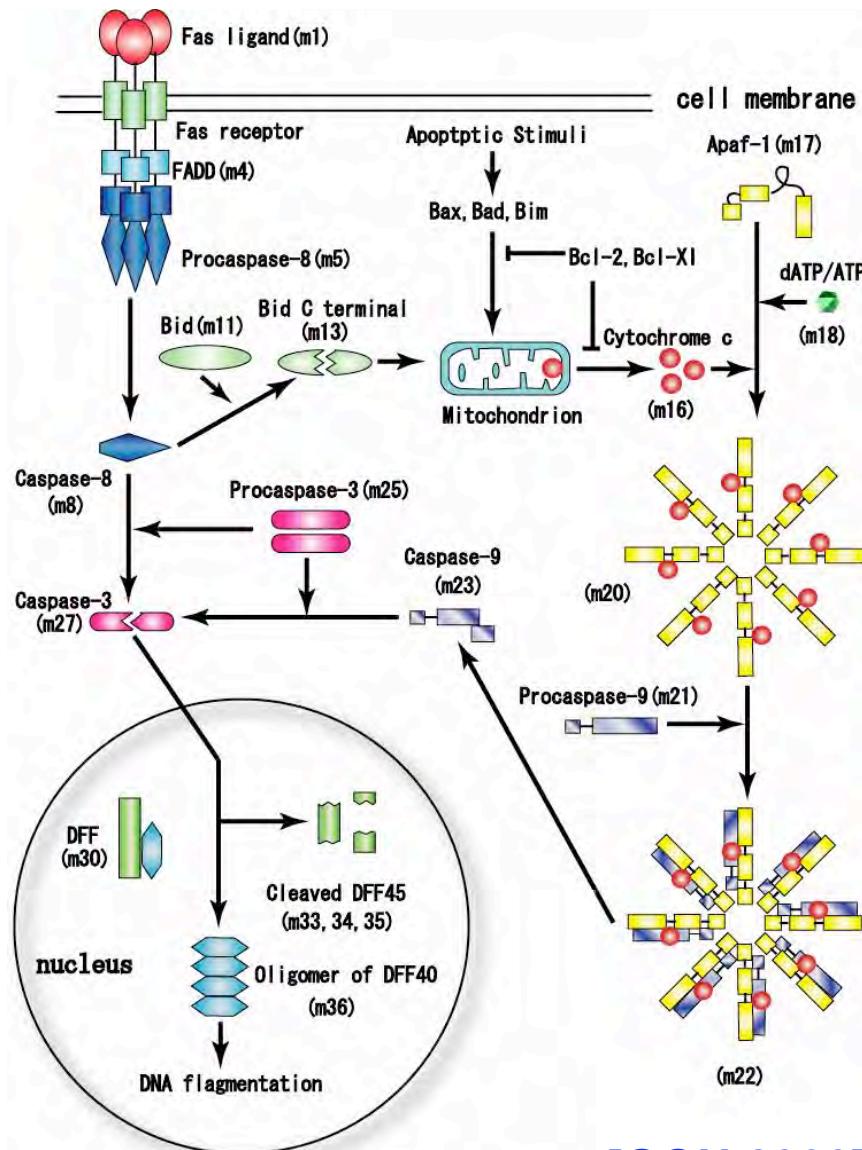
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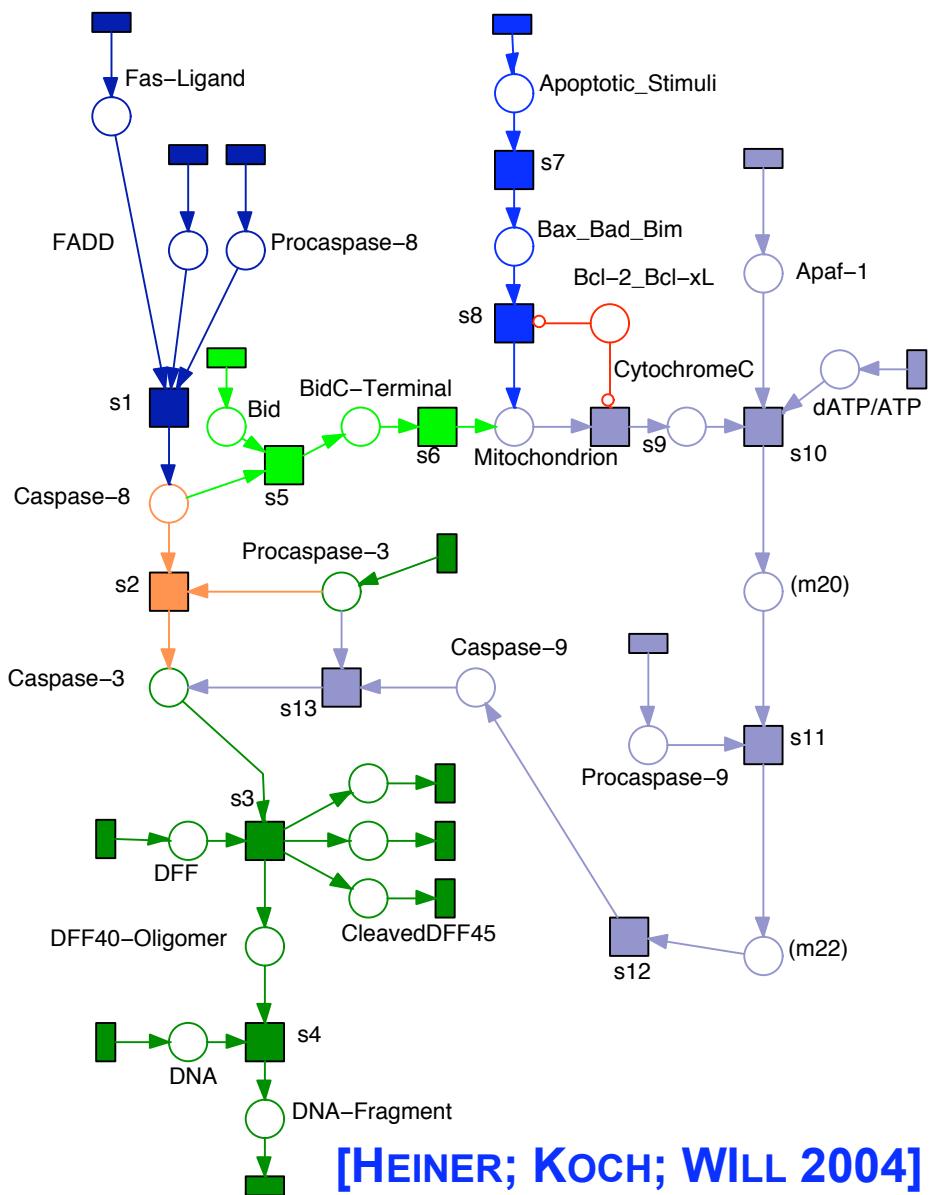
[GON 2003]

# Ex3 - APOPTOSIS IN MAMMALIAN CELLS (PROP.1)

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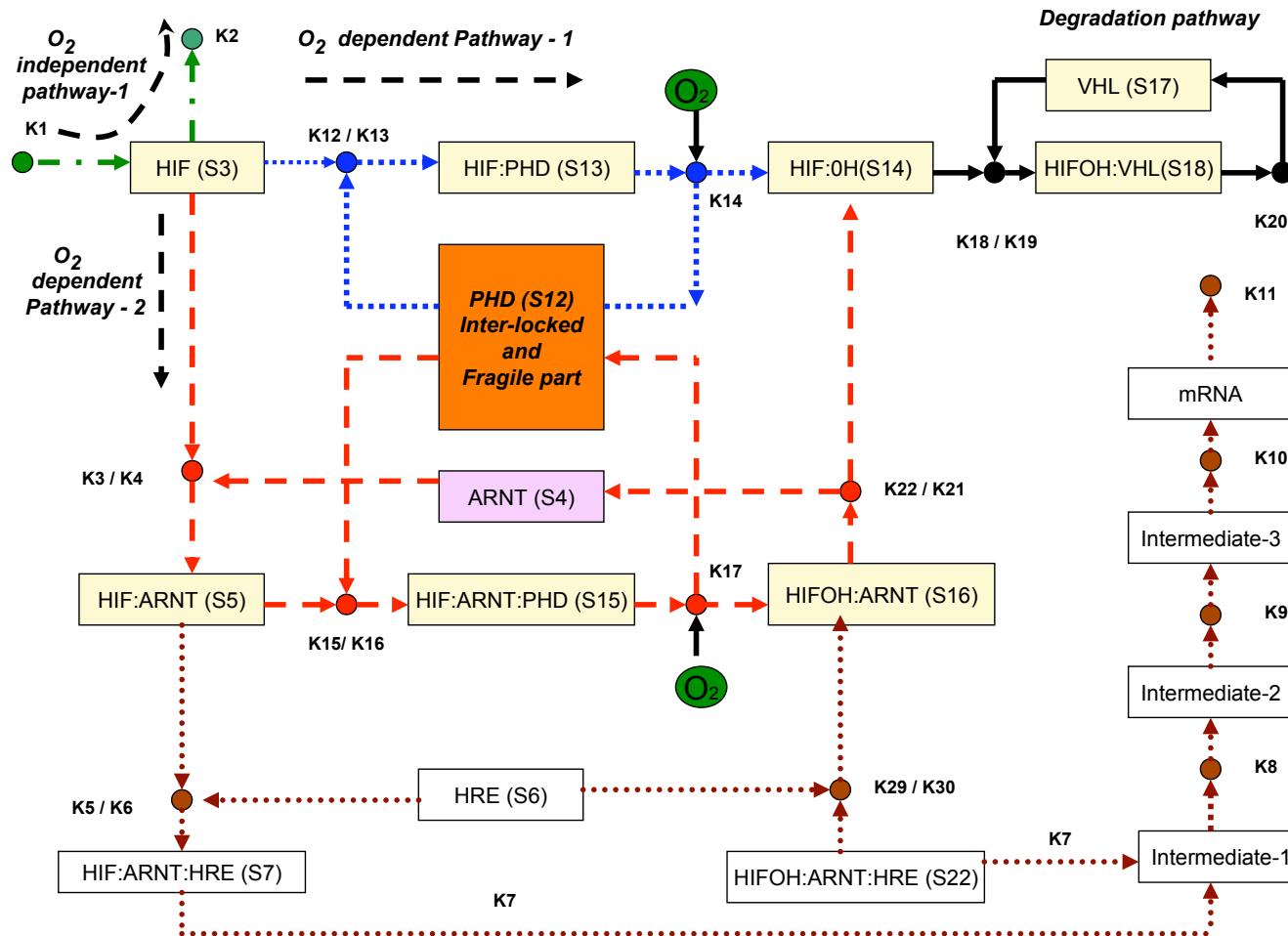


[GON 2003]



[HEINER; KOCH; WILL 2004]

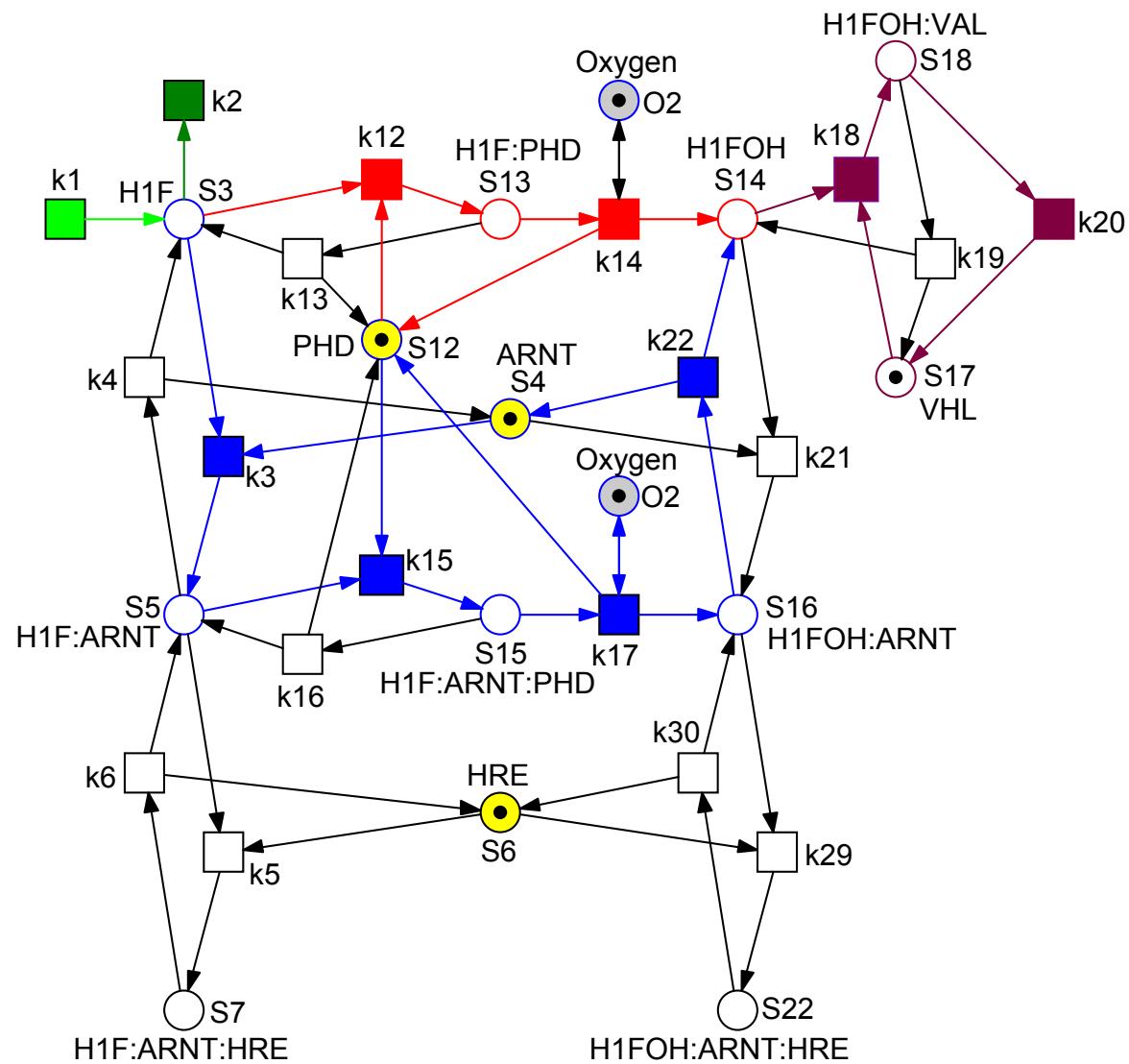
[YU ET AL. 2007]



## Ex4 - HYPOXIA, PETRI NET (PROP.2)

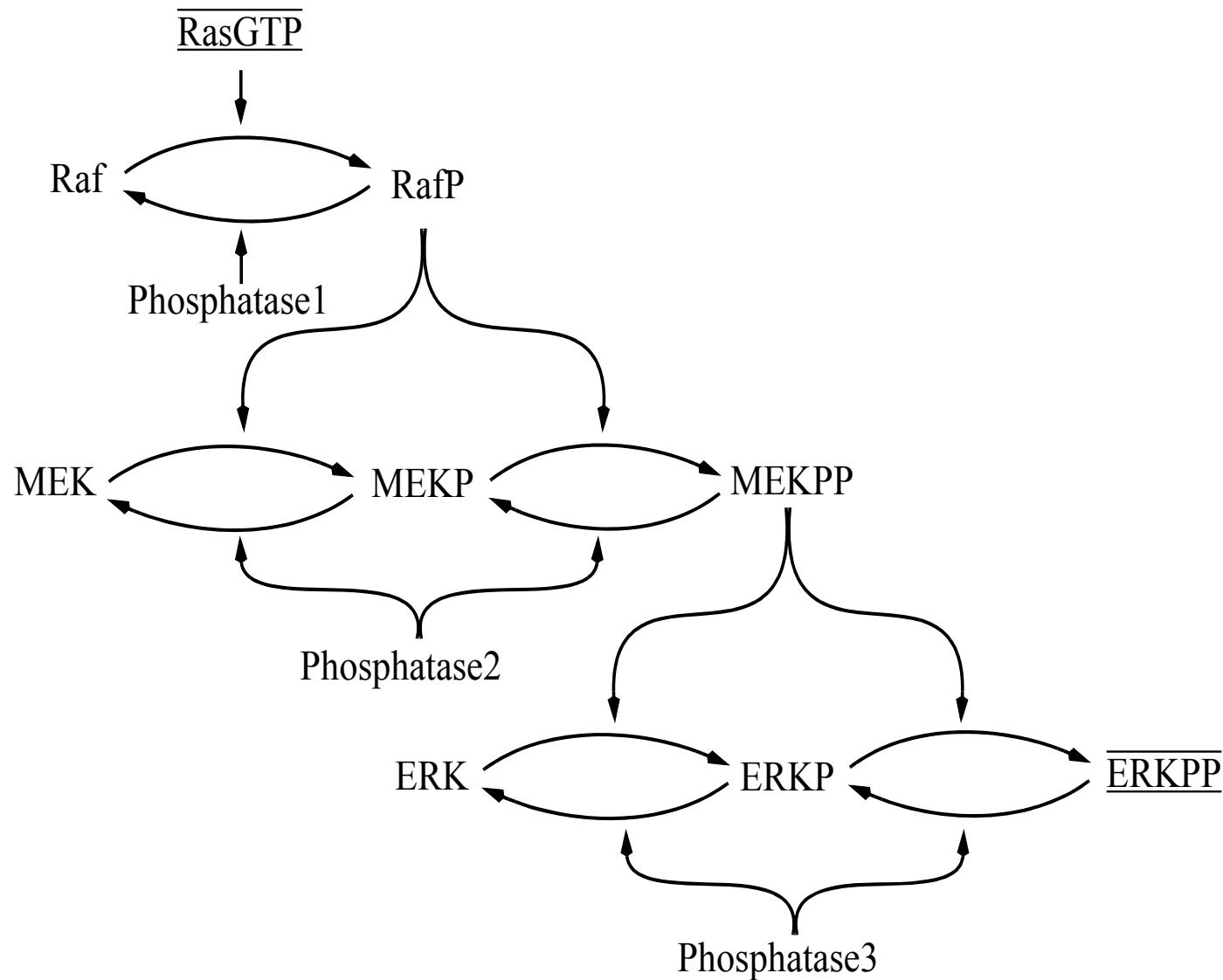
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LIVENESS SHOWN  
BY REDUCTION



## EX5 - MAPK SIGNALLING CASCADE

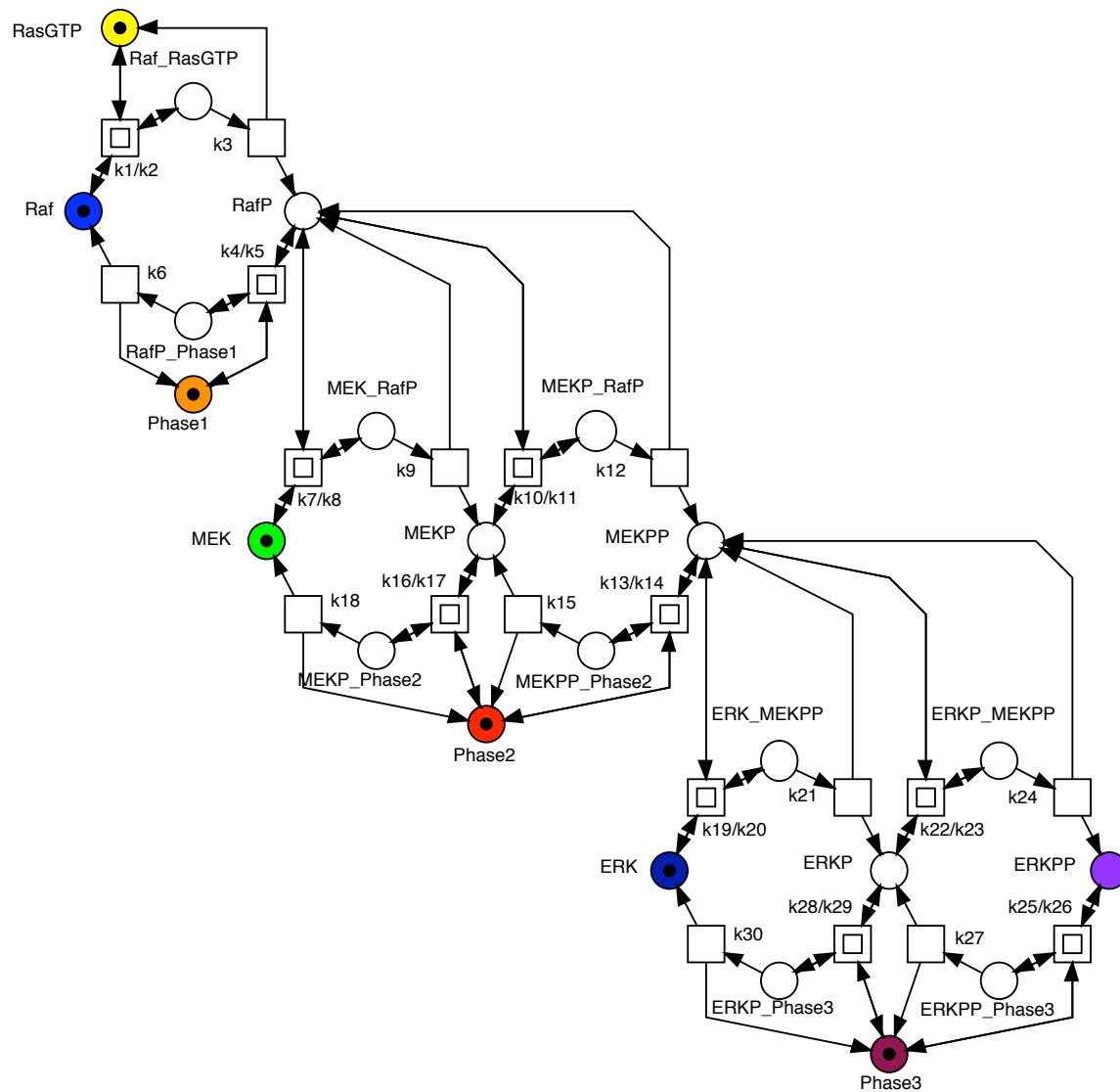
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# EX5 - MAPK SIGNALLING CASCADE, PETRI NET (PROP.2)

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## LIVENESS SHOWN BY DYNAMIC ANALYSIS (RG)



# **A NECESSARY CONDITION FOR MONOTONIC LIVENESS OF ORDINARY NETS**



- **two steps**

- > *structural check*

- > *marking check*

- **possible outcomes of structural check**

- (1) *there are no siphons:*

- > *DTP holds, monotonically live (Prop. 1m)*

- (2) *there are bad siphons - siphons, not containing a trap*

- > *there does not exist a marking such that the DTP holds*

- (3) *each siphon contains a trap*

- > *there does exist a marking such that the DTP holds*

## CASE: BAD SIPHONS

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- do not prevent liveness

- BUT,  
a bad siphon D  
can always be emptied  
by increasing the  
initial (live) marking

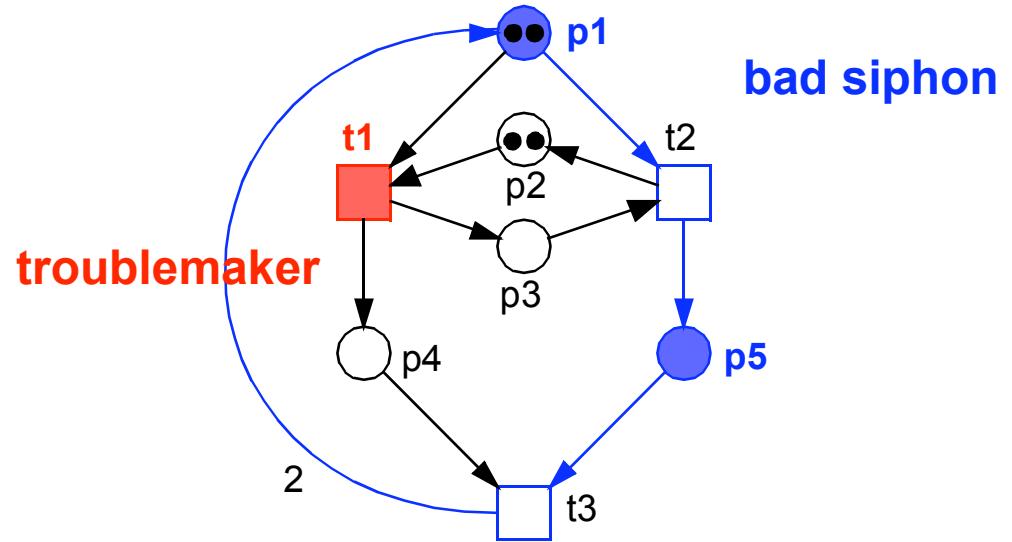
- troublemakers  
 $t \in DF - FD$

- Lemma

-> necessary condition

ORD -> (monotonically live -> no bad siphons)

- all our non-monotonically live examples have bad siphons



### proposition

-> *Each siphon of a live system is initially marked.*

### question

-> *In a live net, can this token be in the siphon, but not in the trap?*

-> *the DTP wouldn't hold;*

### observation

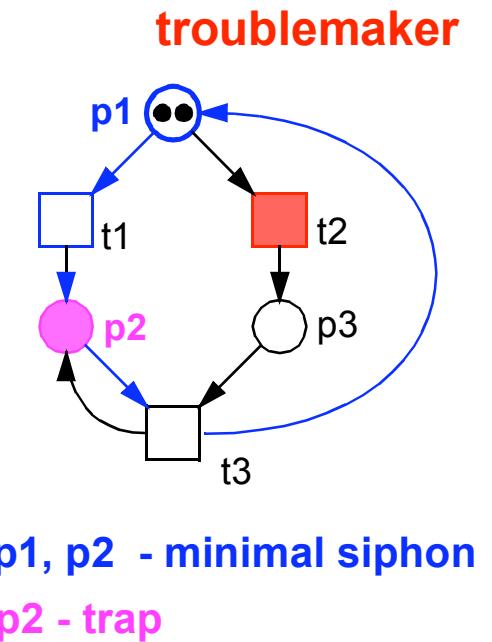
-> *A minimal siphon can properly contain a maximal trap.*

-> *The initial token could be in the siphon, but not in the trap.*

### However

-> *troublemaking transitions could empty the siphon, as in the case "Bad Siphon".*

-> *The system can not be monotonically live.*



□ **Theorem**

-> *necessary condition*

*ORD* -> (*monotonically live* -> *DTP*)

□ **Theorem**

-> *necessary condition*

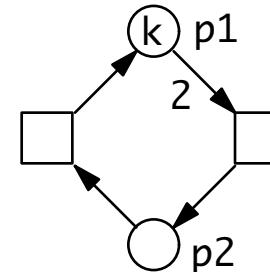
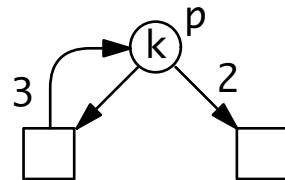
*ORD* -> (*monotonically live* -> *DTP*)

□ **Corollary**

-> *necessary and sufficient condition*

*ES* -> (*monotonically live* <-> *DTP*)

# **MONOTONIC LIVENESS OF NON-ORDINARY NETS**



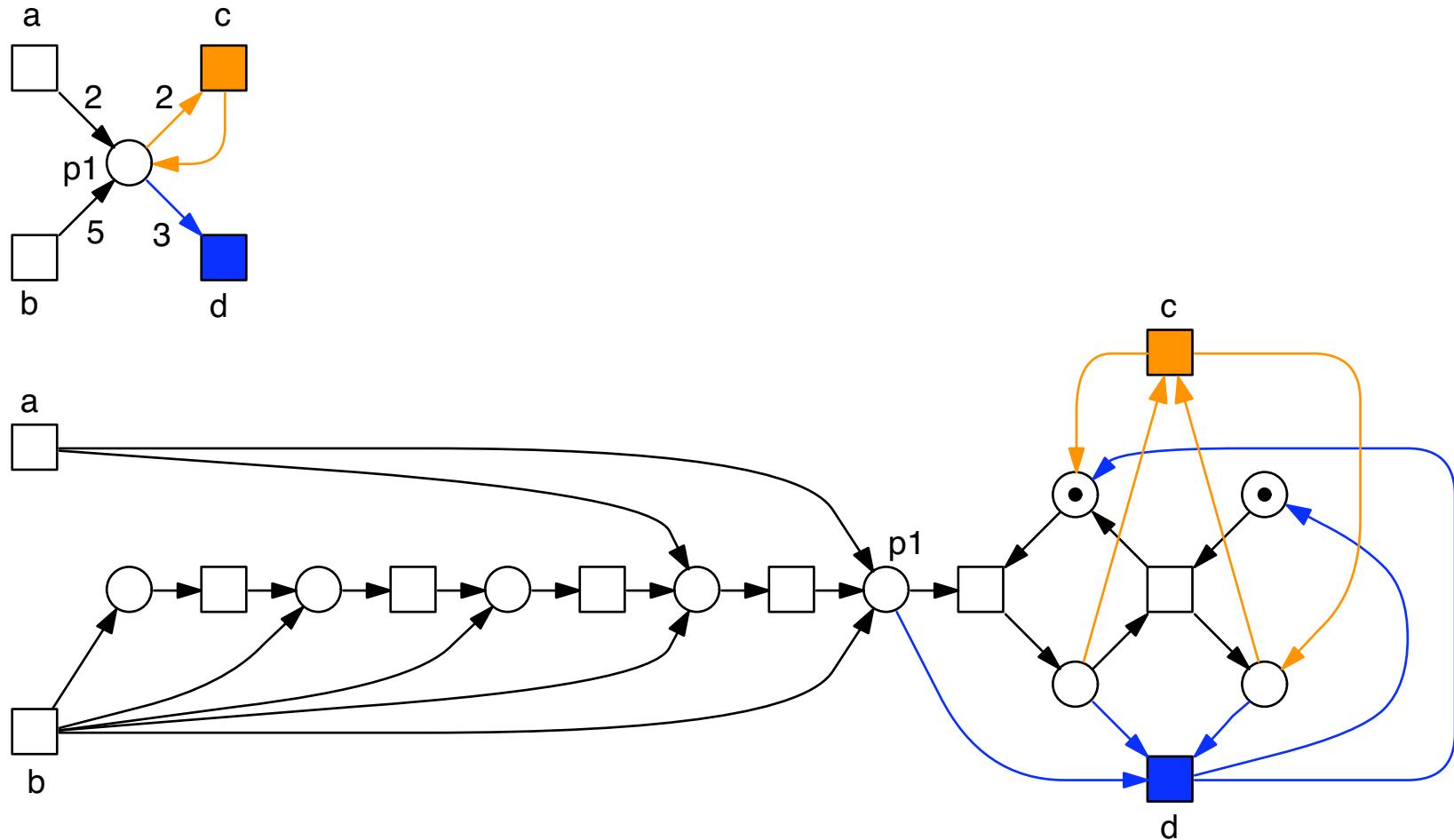
*FC net*

- p is a bad siphon**  
-> *DTP is never true*
- for k odd, the system is live**
- for k even, the system is non-live**

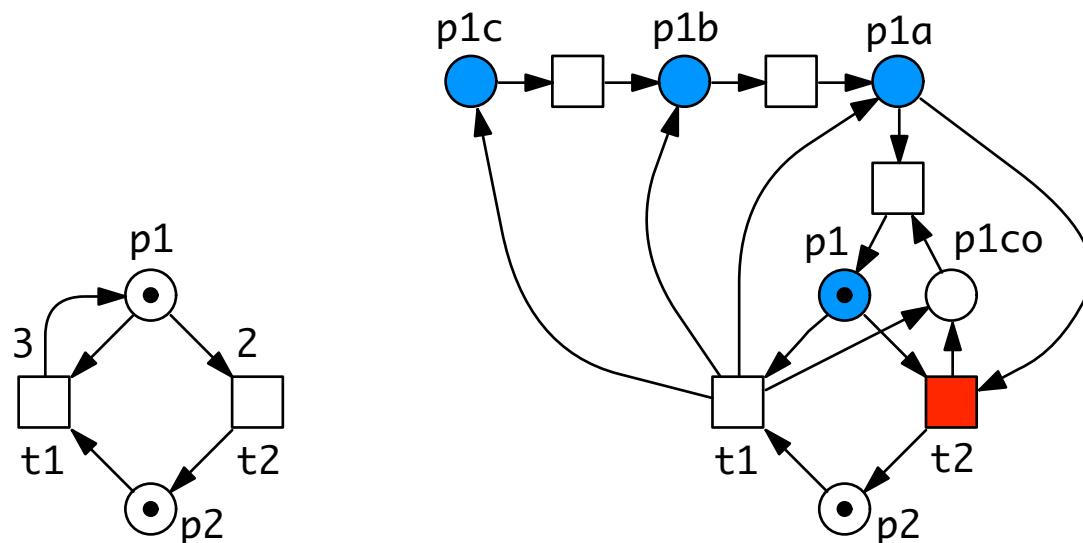
- p1, p2 is siphon and trap**
- for  $k \geq 1$ , the DTP is true,  
but the system has a dead state**
- for any  $k > 0$ , the system  
will unavoidably reach a dead state**

**THE DTP IS NEITHER NECESSARY NOR SUFFICIENT FOR  
NON-EXISTANCE OF DEAD STATES IN NON-ORDINARY NETS.**

- non-ordinary nets can be simulated by ordinary ones  
-> simulation preserves firing language under interleaving semantics



- the non-ordinary net is monotonically live iff its simulation by an ordinary net is monotonically live.

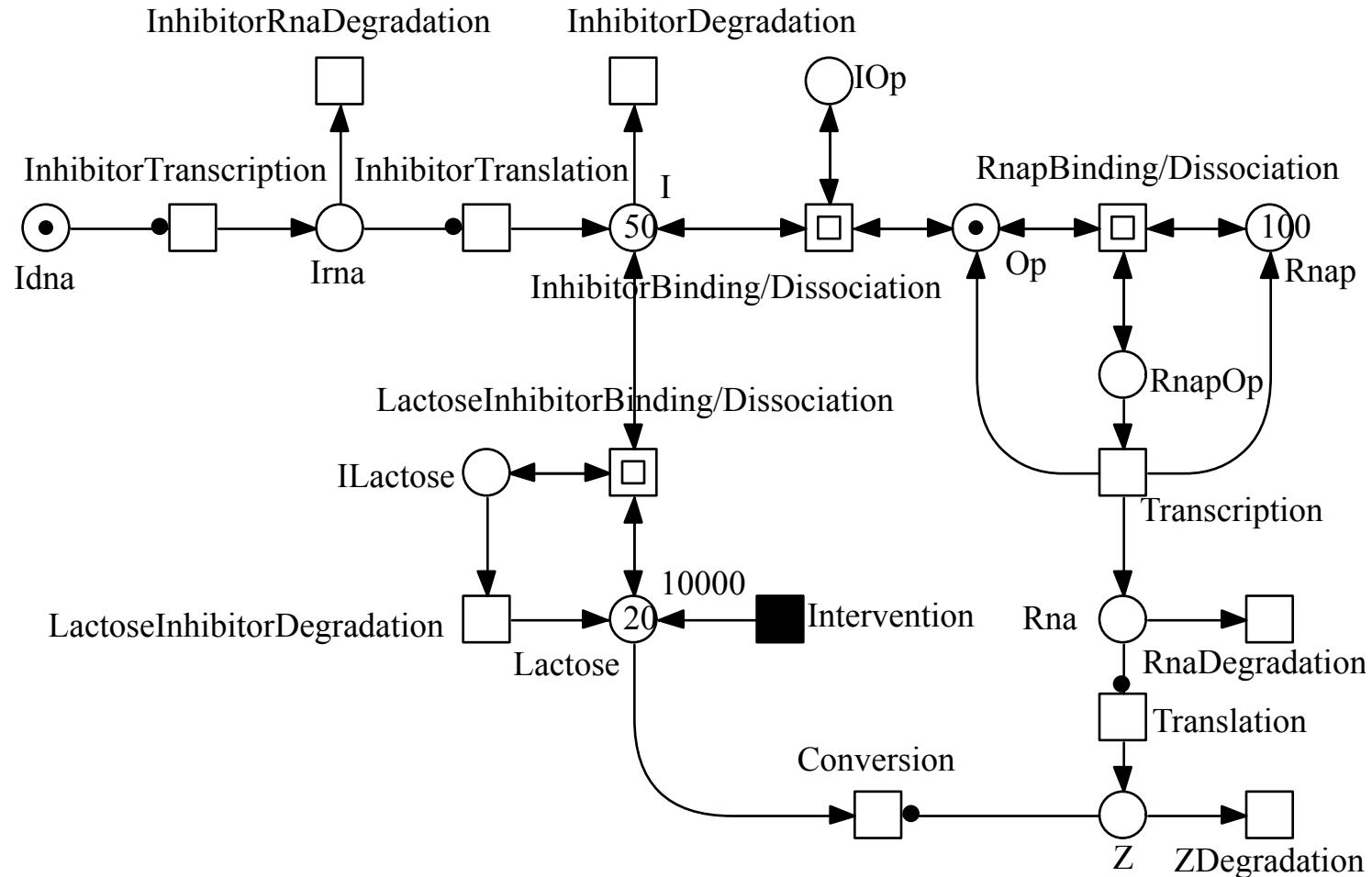


# BIO PETRI NETS - SOME MORE EXAMPLES

*(non-ordinary, DTP, live, consistent)*

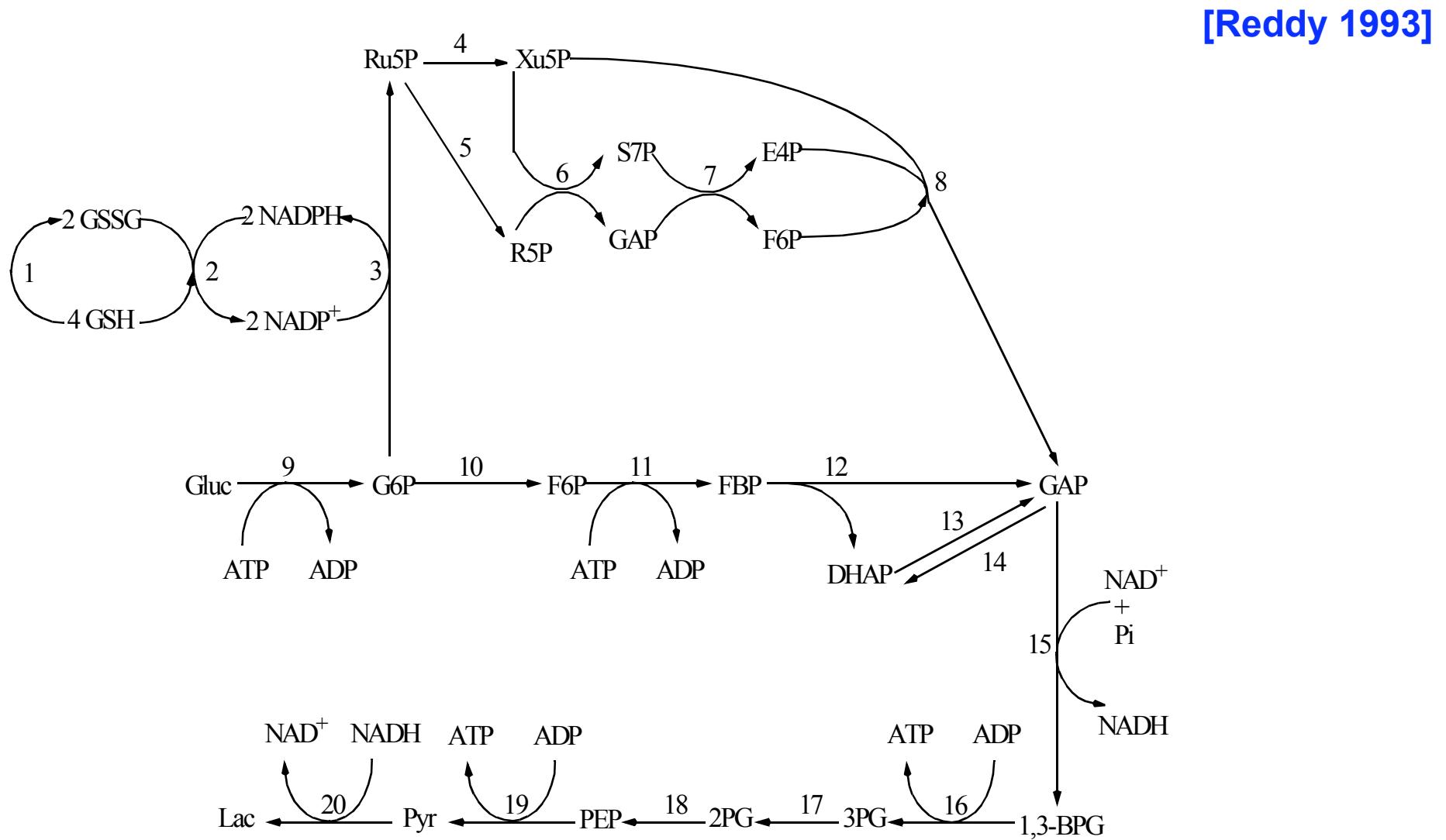
## Ex6 - LAC OPERON (WILKINSON 2006)

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# Ex7 - Glycolysis/Pentose Phosphate Pathway

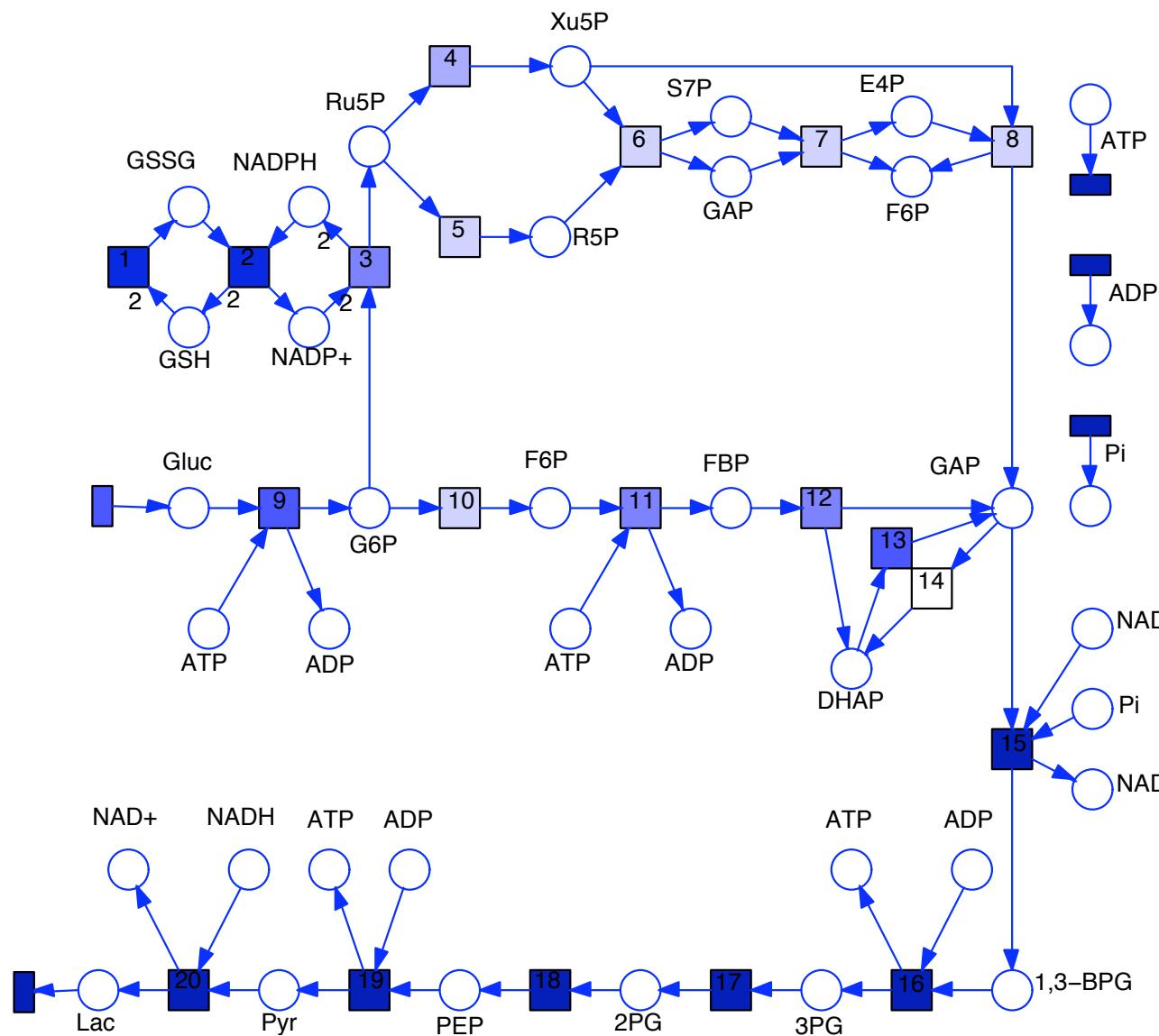
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# Ex7 - Glycolysis/Pentose Phosphate Pathway (HOM)

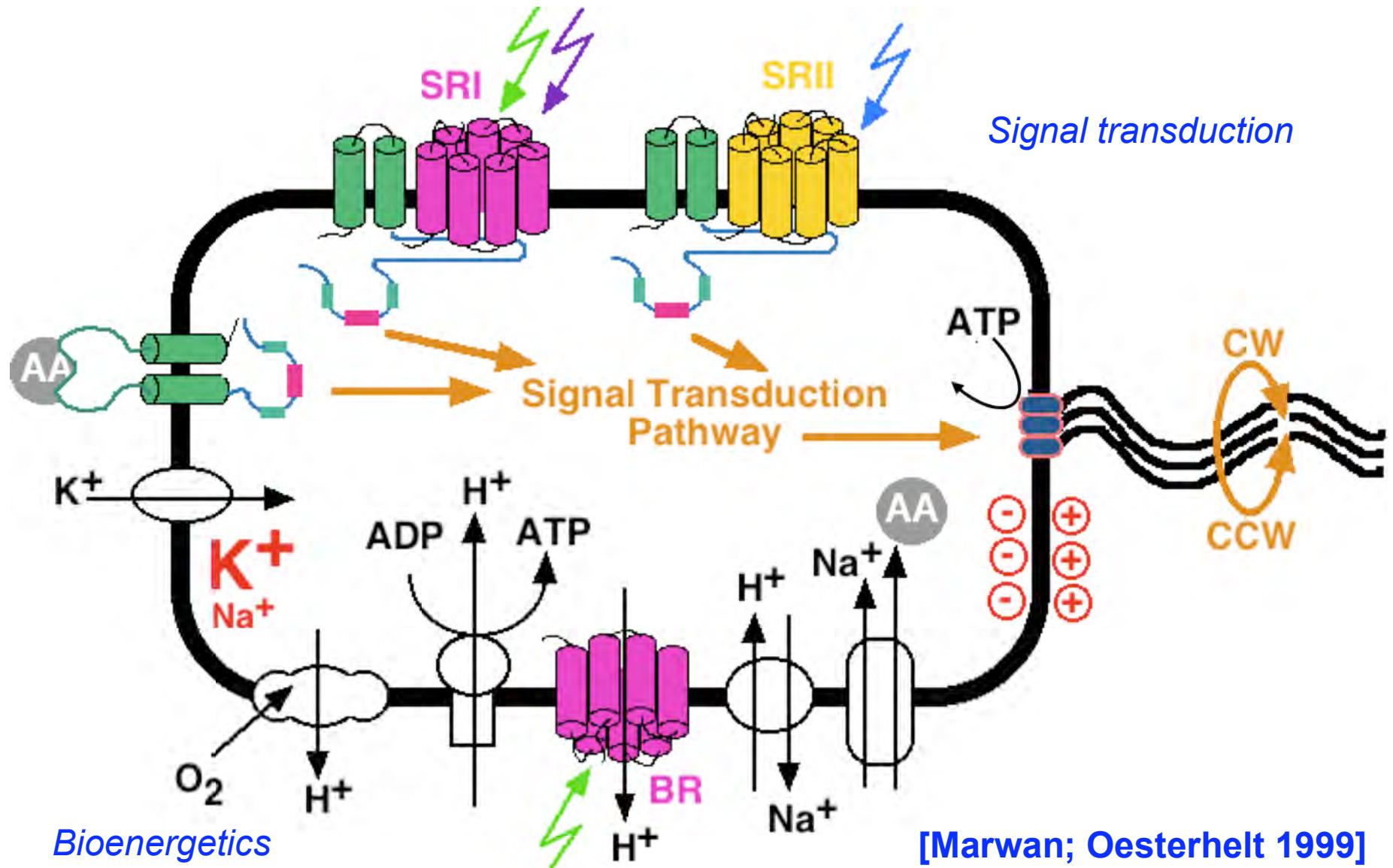
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[Reddy 1993]



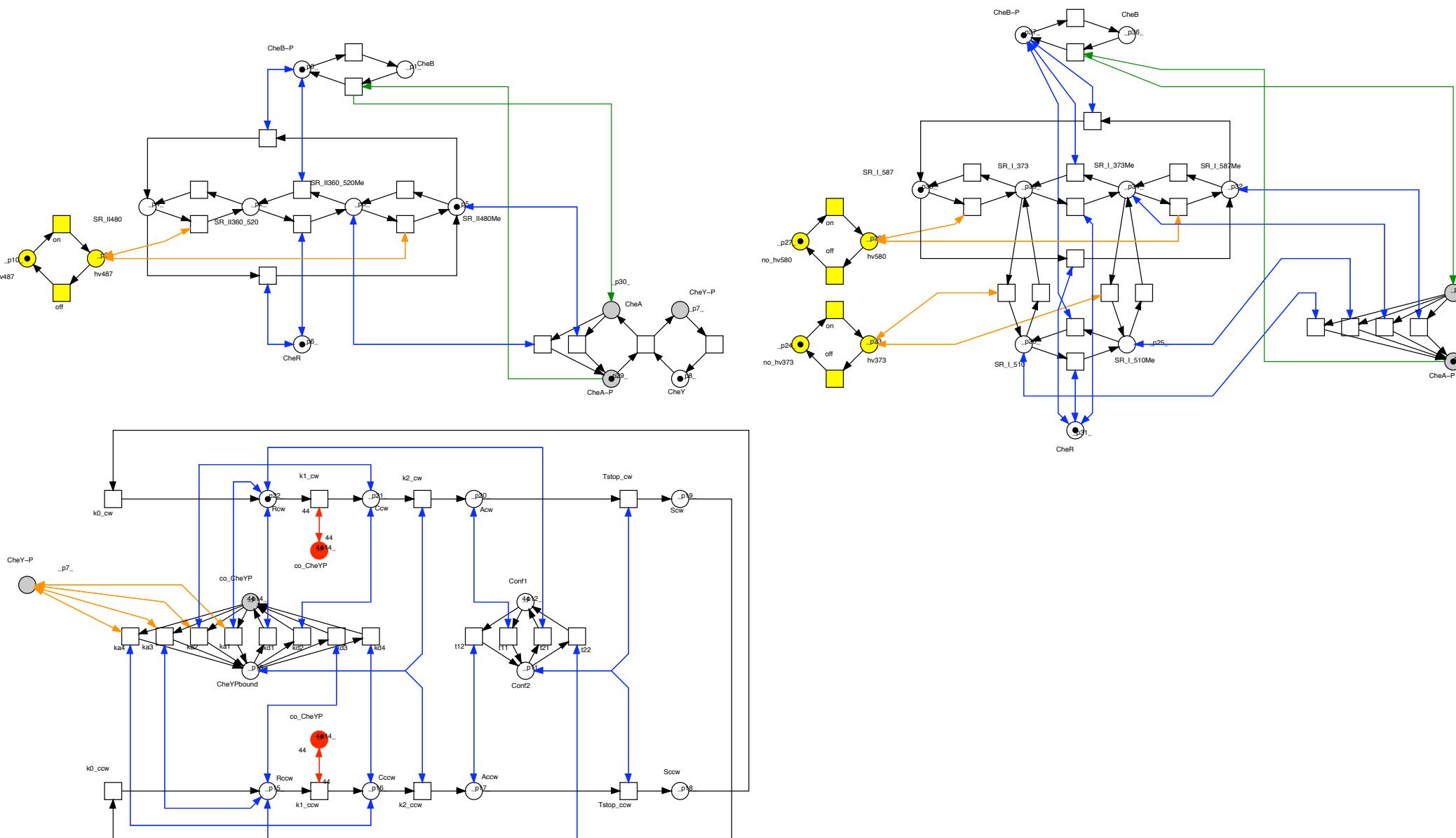
## Ex8 - SWITCH CYCLE HALOBACTERIUM SALINARUM

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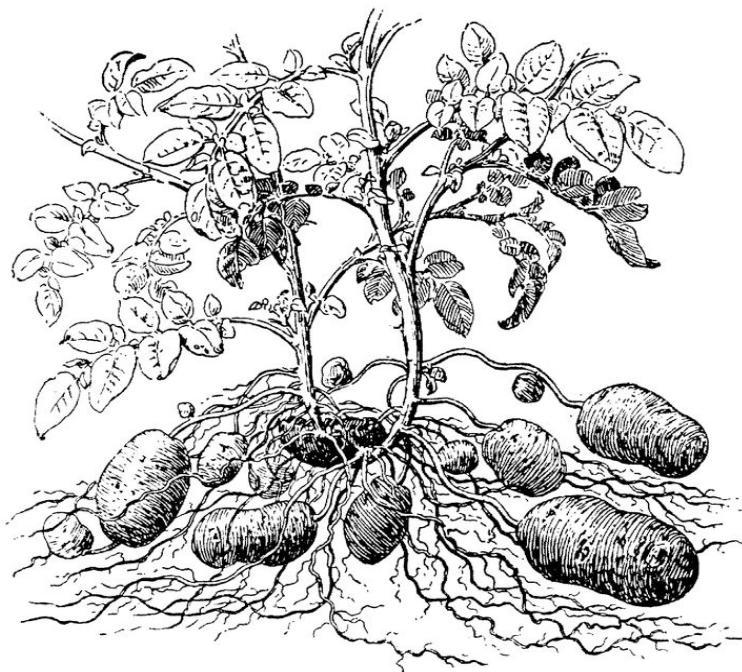
# Ex8 - SWITCH CYCLE HALOBACTERIUM SALINARUM (BND)

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# Ex9 - Carbon Metabolism in Potato Tuber

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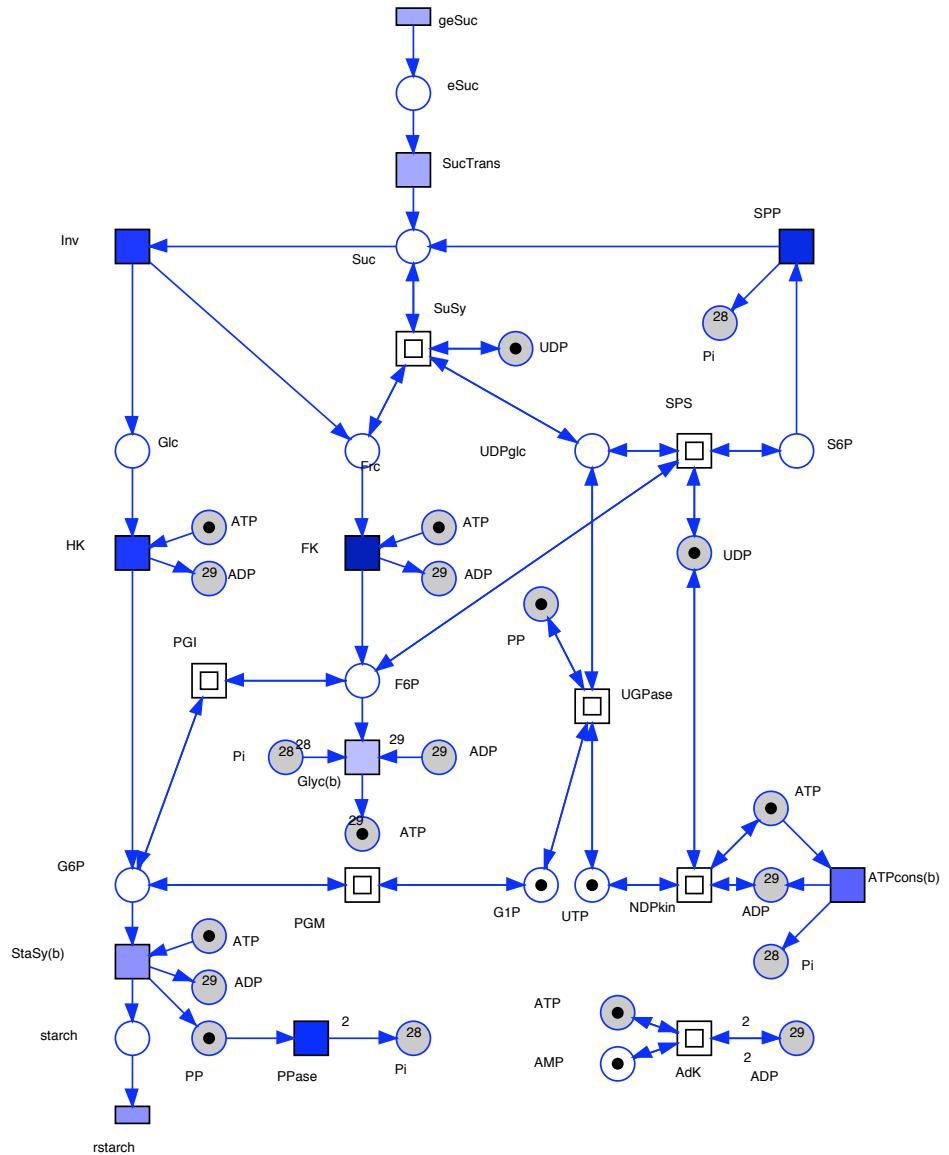
[Koch; JUNKER; HEINER 2005]

# Ex9 - Carbon Metabolism in Potato Tuber

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[Koch; JUNKER; HEINER 2005]



# **SUMMARY & OUTLOOK**

- **necessary and sufficient criteria for monotonic liveness**

- > *ordinary nets*

- > *non-ordinary nets*

- **generalisations**

- > *DTP & HOM & NBM (Starke 1990)*

- **other net structures ?**

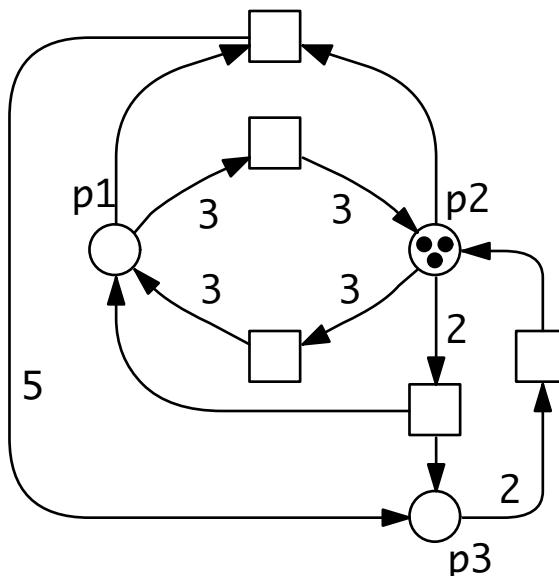
- > *mono-T-semiflow nets, see paper  
liveness and deadlock freeness coincide;  
thus, DTP ensures monotonic liveness, too*

- > *Freely Related T-semiflow nets and extensions?*

- > . . .

## □ what next?

- > homothetic liveness
- > monotonic/homothetic boundedness/reversibility ?



$m_0 = (0, 3, 0)$   
live & bounded

$m_0 = (0, 4, 0)$   
live & unbounded

[RECALDE 1998]



**THANKS !**