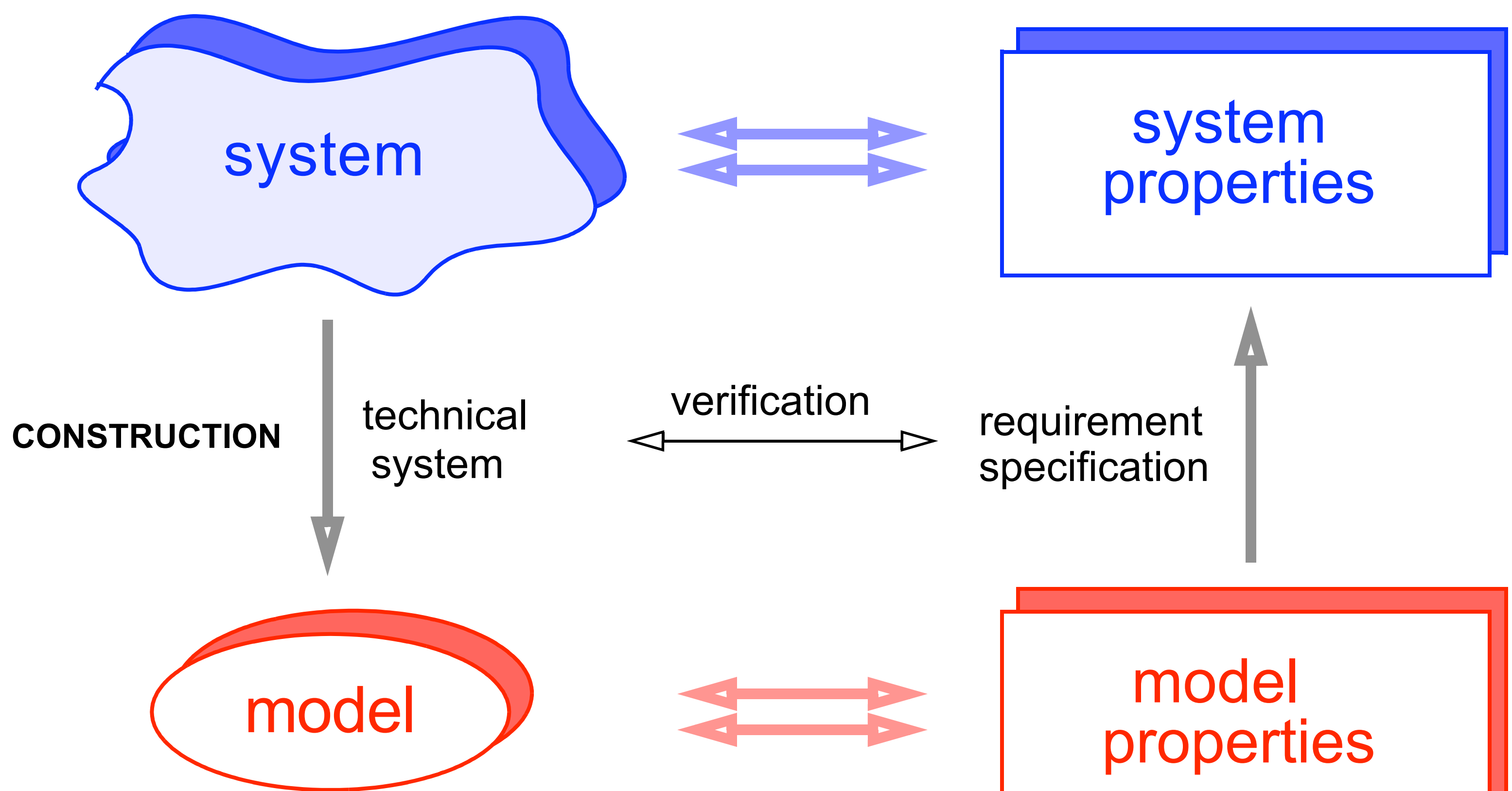
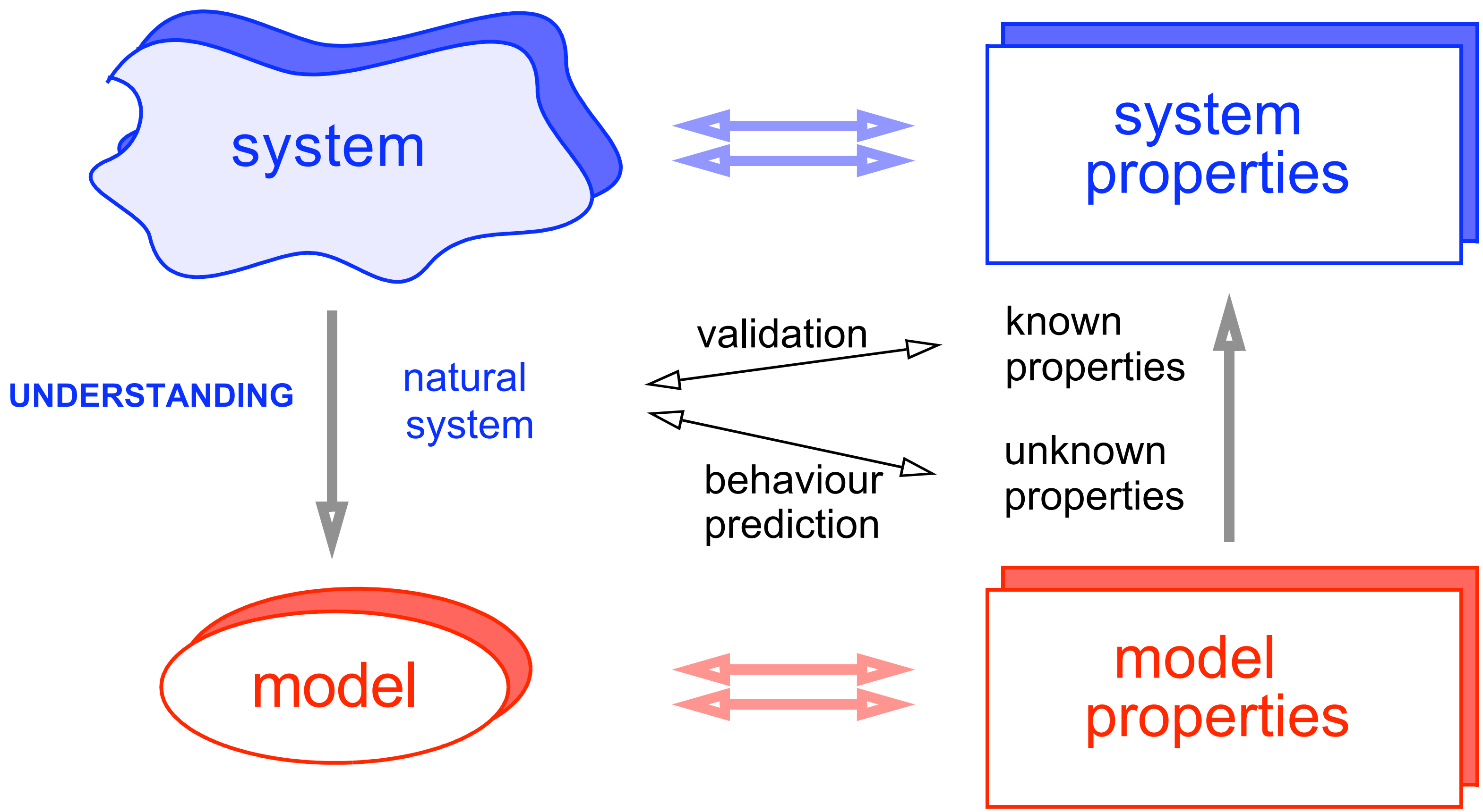
MODEL CHECKING OF CONCURRENT SYSTEMS - PART I -

Monika Heiner

**BTU Cottbus,
Dep. of Computer Science**


system
system properties
CONSTRUCTION
technical system
verification
requirement specification
model
model properties

Model-based System Analysis



BASIC INGREDIENTS

- **a language to model the system**
  - -> *formal semantics*
  - -> *many options, e.g.*
    Petri nets

- **a language to specify model properties**
  - -> *temporal Logics,*
  - -> *several options, e.g.*
    Computational Tree Logic (CTL)

- **an analysis approach to check a model against its properties**
  - -> *model checking,*
  - -> *various approaches (algorithms + data structures), e.g.*
    using reachability graph (RG)
    = labelled state transition system (STS) = Kripke structure
    ≈ Continuous Time Markov Chain (CTMC)

The modelling language -

Petri nets,
a crash course

C. A. PETRI, NOVEMBER 2006

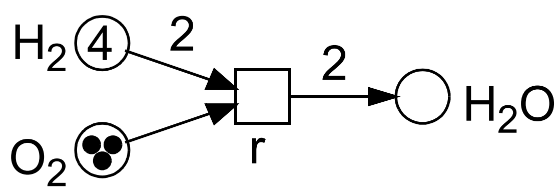
A Bit of History

PETRI NETS, BASICS - THE STRUCTURE

- **atomic actions -> Petri net transitions -> chemical reactions**

$$2\ H_2 + O_2 \rightarrow 2\ H_2O$$

input compounds



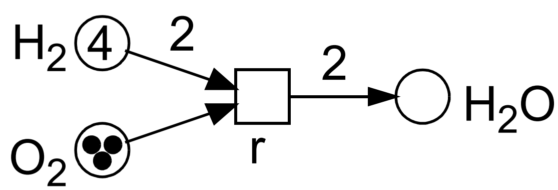
output compounds

- **local conditions -> Petri net places -> chemical compounds**
- **multiplicities -> Petri net arc weights -> stoichiometric relations**
- **condition's state -> token(s) in its place -> available amount (e.g. mol)**
- **system state -> marking -> compounds distribution**
- **$PN = (P, T, F, m_0)$, $F: (P \times T) \cup (T \times P) \rightarrow N_0$, $m_0: P \rightarrow N_0$**

- **atomic actions -> Petri net transitions -> chemical reactions**

$$2\ H_2 + O_2 \rightarrow 2\ H_2O$$

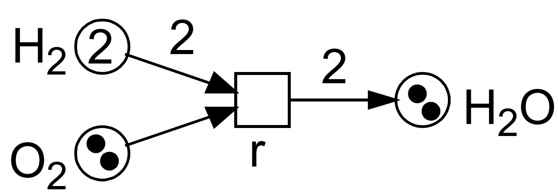
input compounds



output compounds

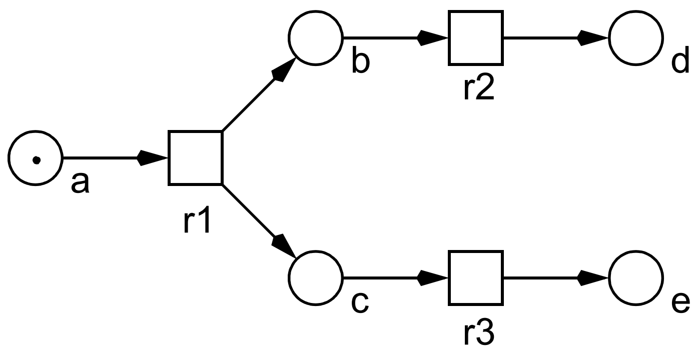
FIRING

TOKEN GAME



DYNAMIC BEHAVIOUR (substance/signal flow)

PARTIAL ORDER VERSUS INTERLEAVING SEMANTICS



- **possible interleaving runs**
  - -> *r1 - r2 - r3*
  - -> *r1 - r3 - r2*

- **totally ordered runs**

-> **INTERLEAVING SEMANTICS**

all totally ordered runs

- **order between r1 - r2 and r1 - r3**
  - -> *causality* *x < y* [x-y]
  - -> *dependency*

- **no order between r2 , r3**
  - -> *concurrency* *x || y*
  - -> *independency*

- **partial order run**

  r1 < *r2*, *r3*

-> **PARTIAL ORDER SEMANTICS**

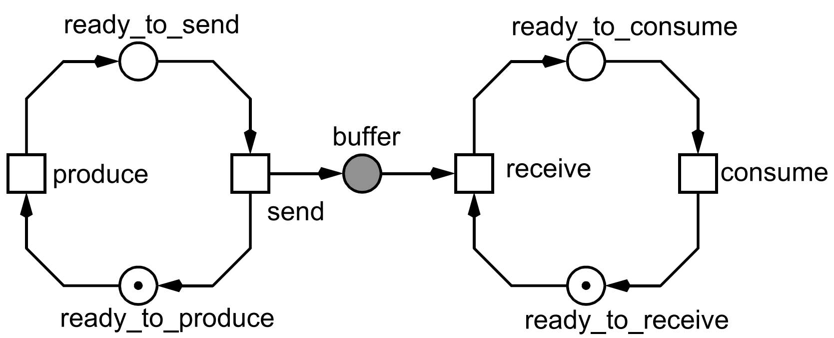
“true concurrency semantics”

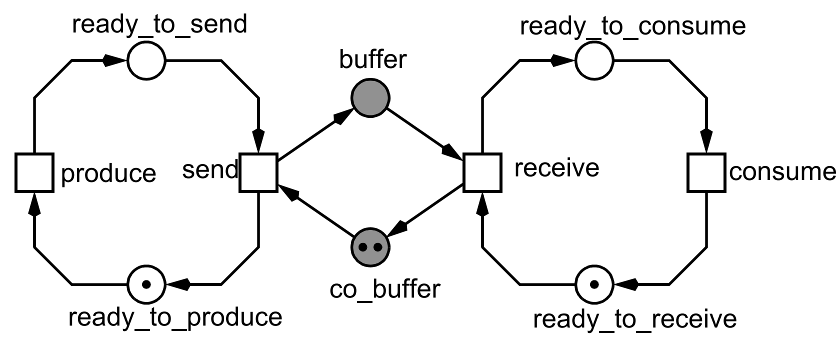
all partially ordered runs

Some examples

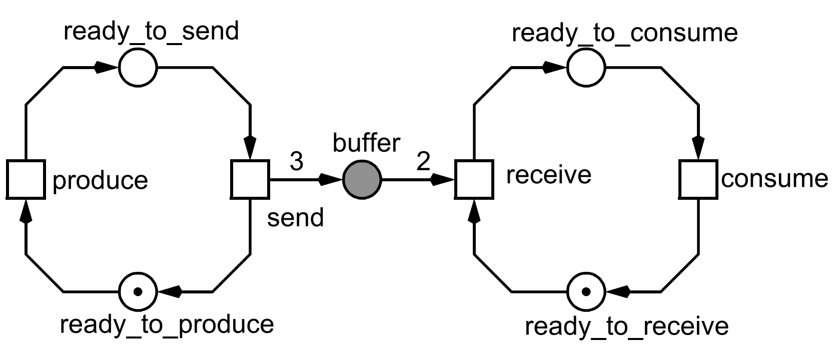
EXAMPLE 1 - PRODUCER/CONSUMER SYSTEM IN FOUR VERSIONS

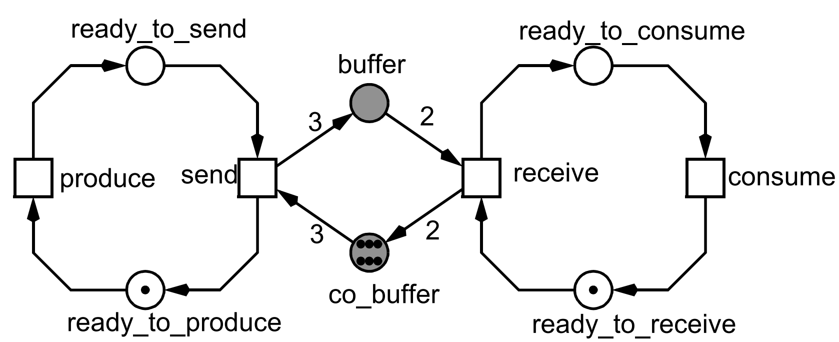
❑ SYSTEMS WITHOUT ARC WEIGHTS



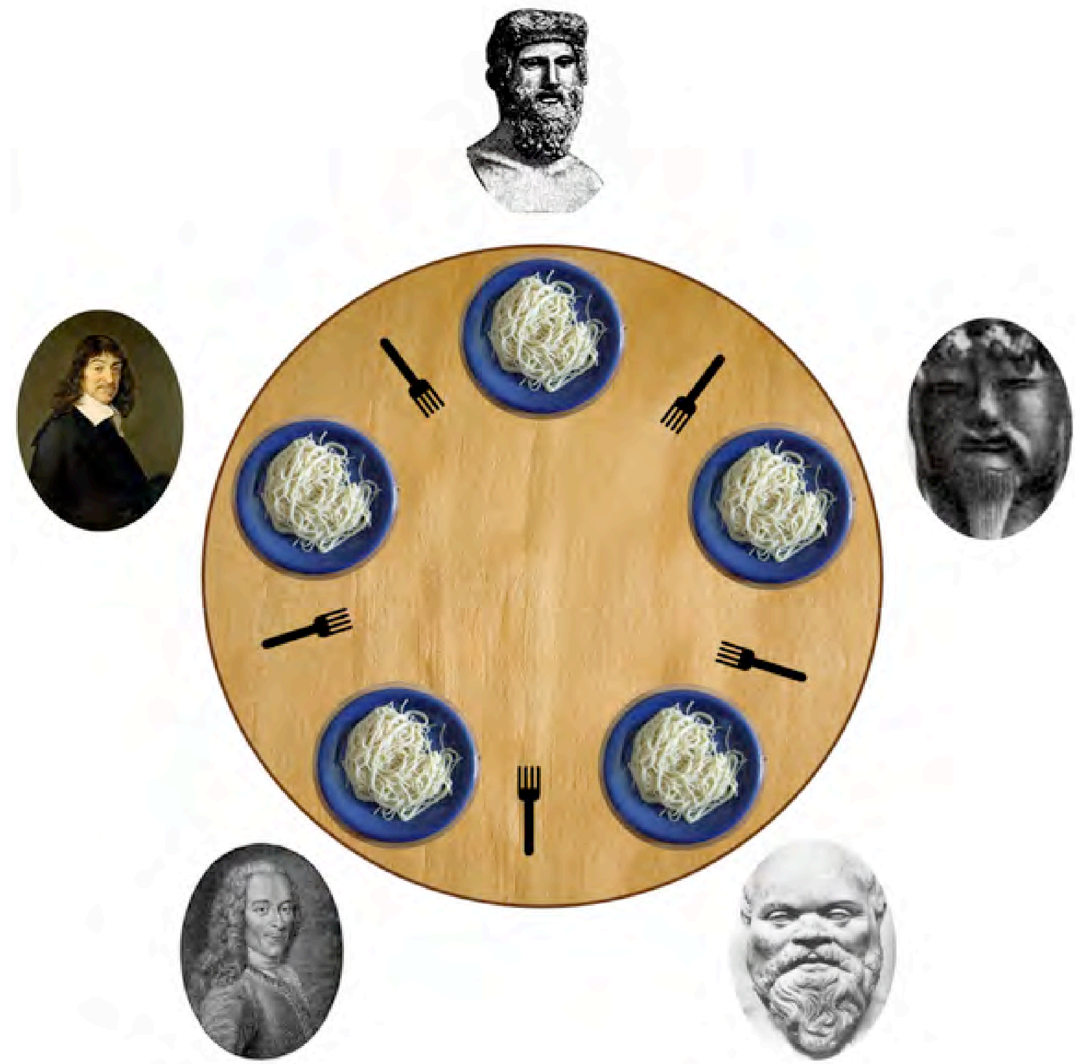


❑ SYSTEMS WITH ARC WEIGHTS



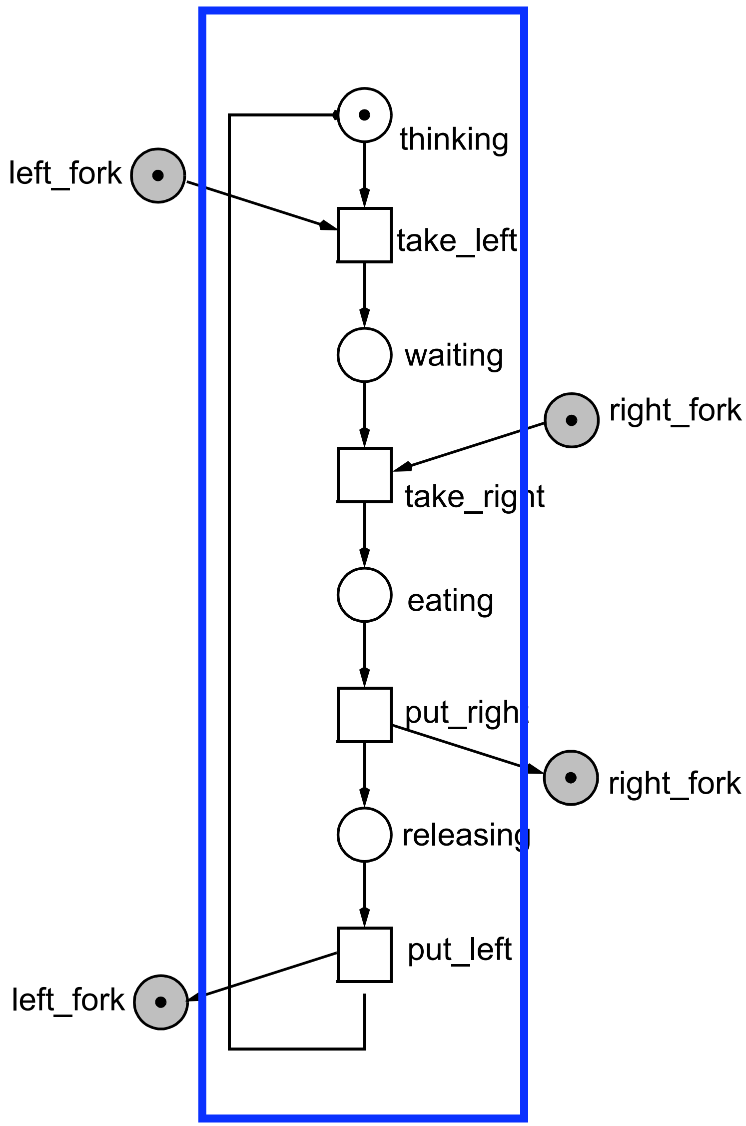


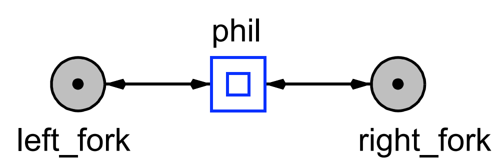
EXAMPLE 2 - DINING PHILOSOPHERS



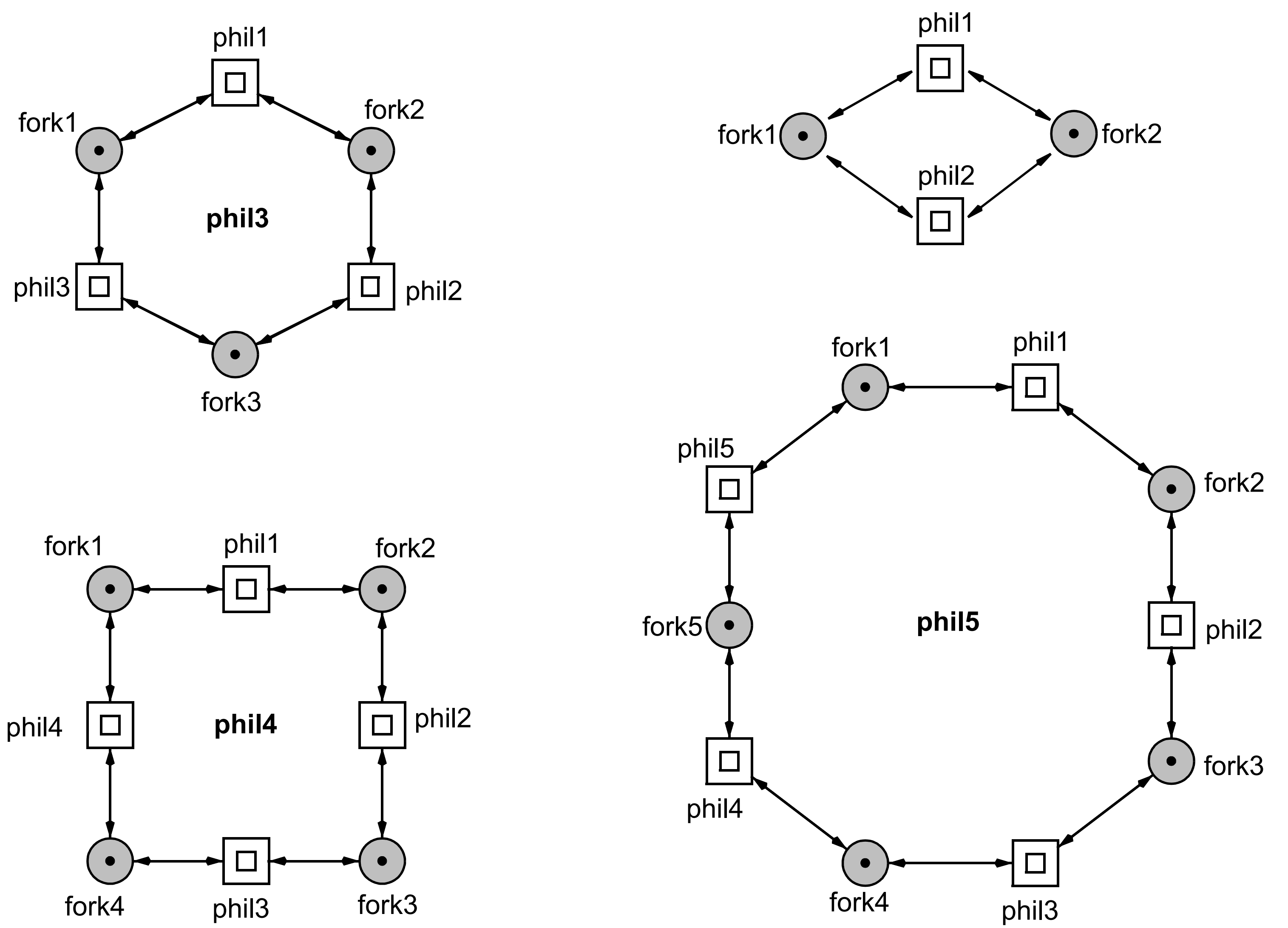
http://en.wikipedia.org/wiki/Dining_philosophers_problem

Example 2 - Dining Philosophers, One Philosopher

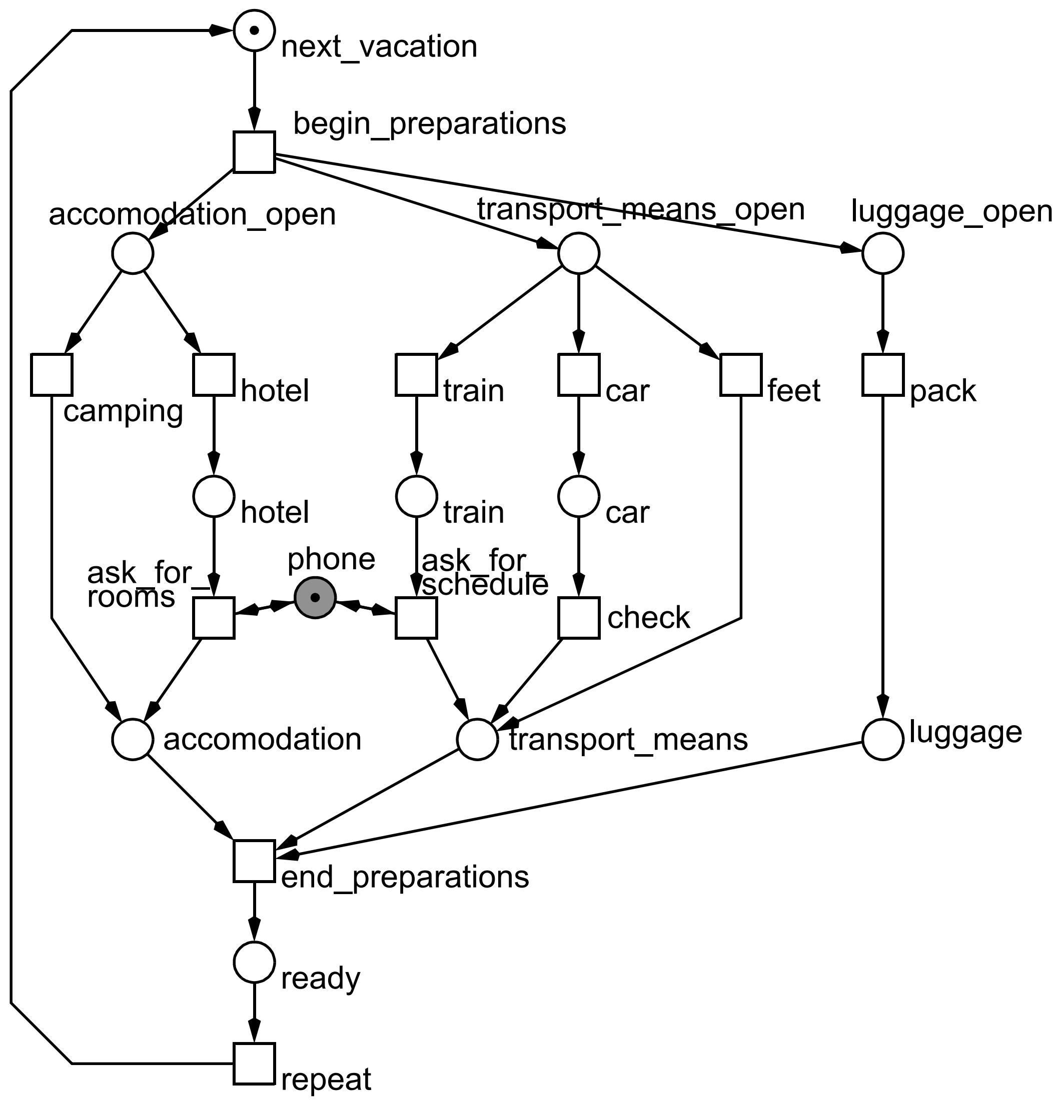




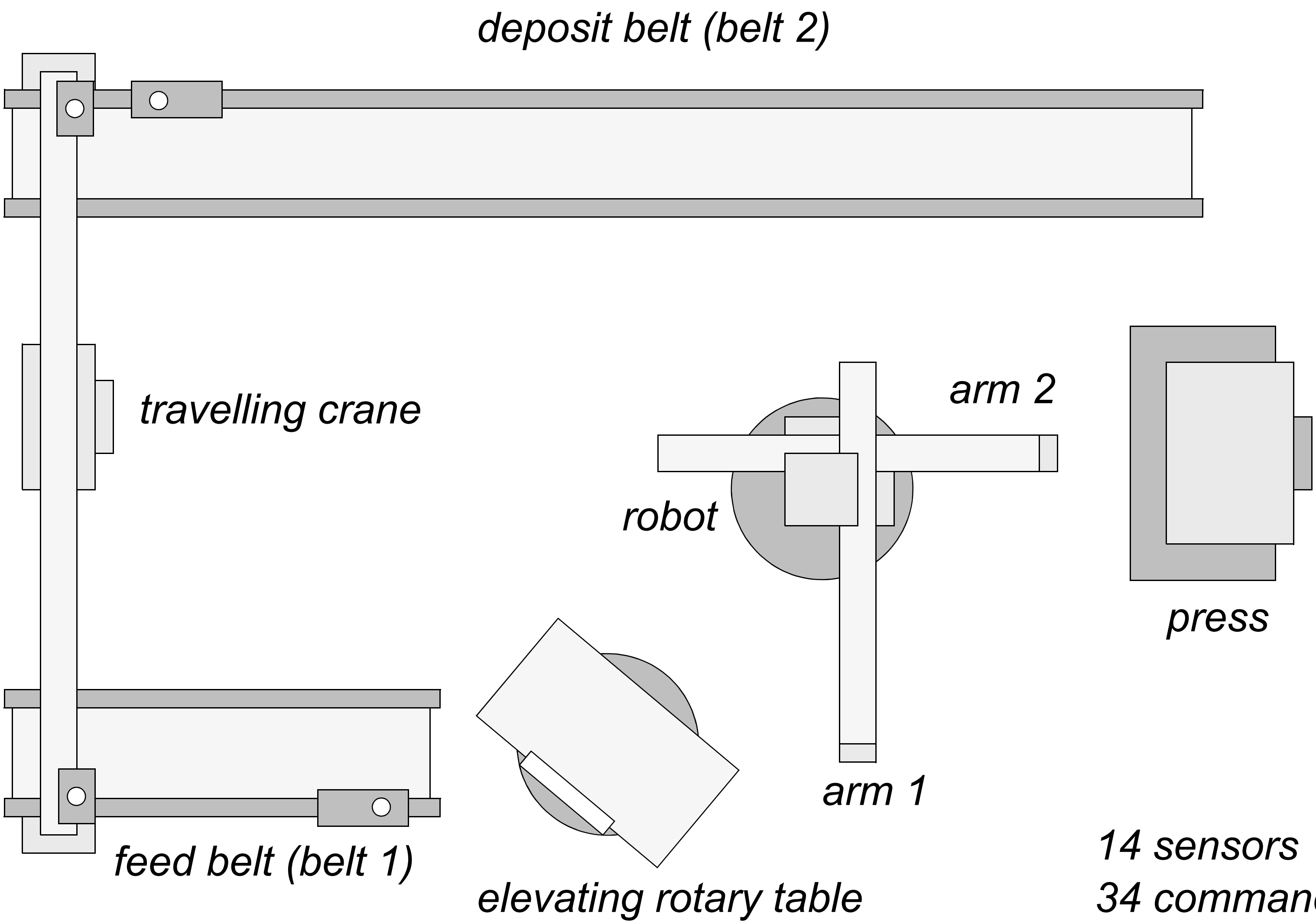
Example 2 - System of N Philosophers



Example 3 - Travel Planing

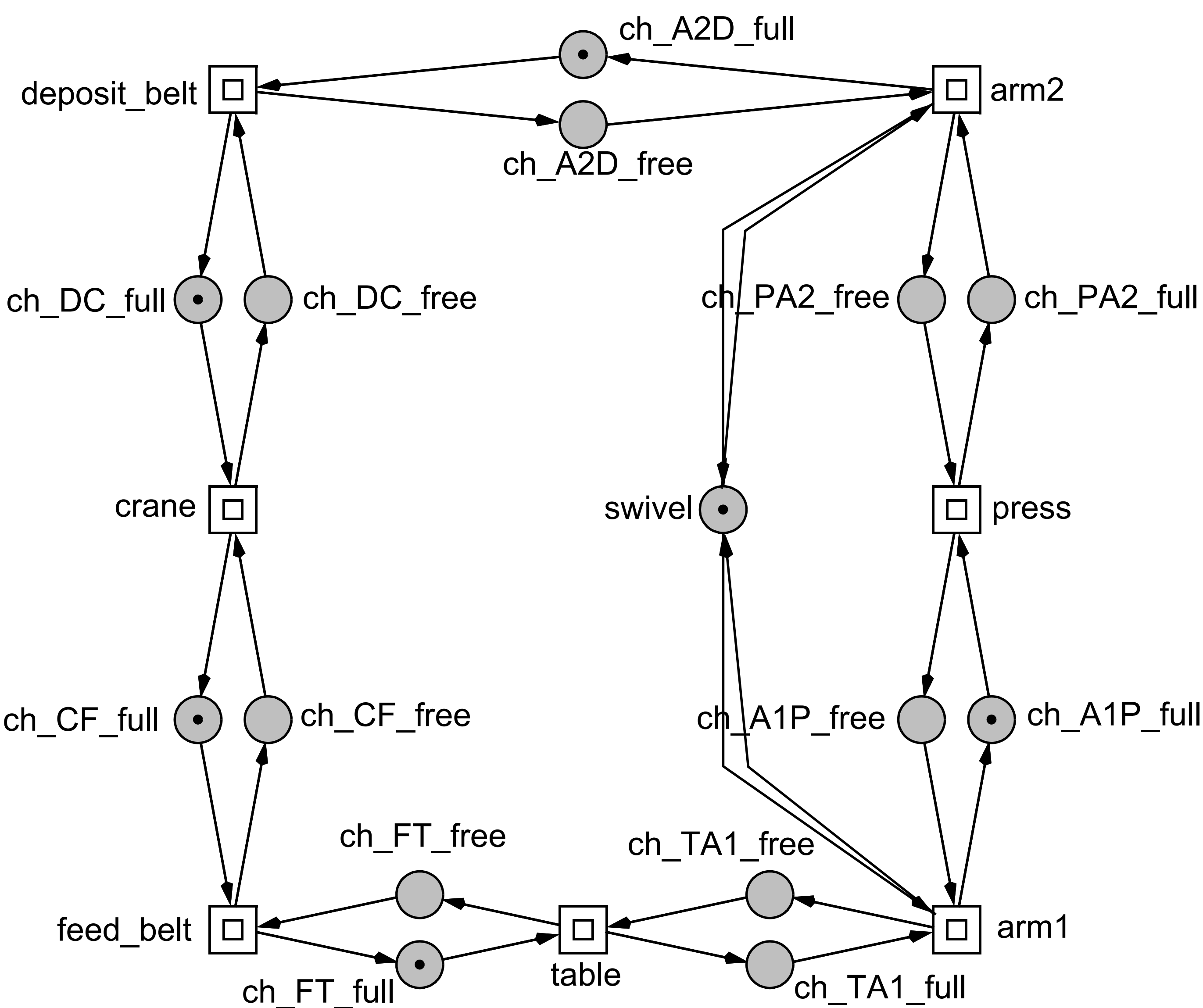


EXAMPLE 4 - PRODUCTION CELL



14 sensors
34 commands

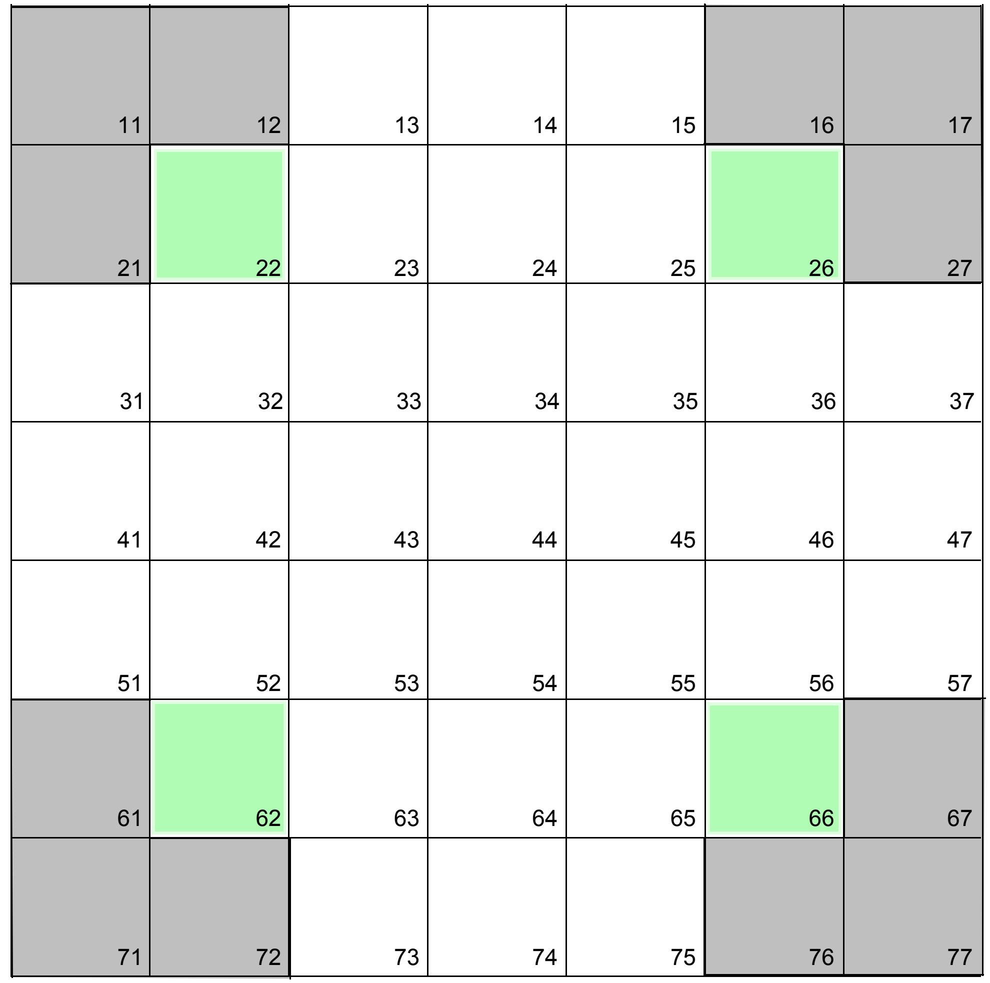
Example 4 - Closed System, Coarse Structure



231 P,
202 T,
65 pages

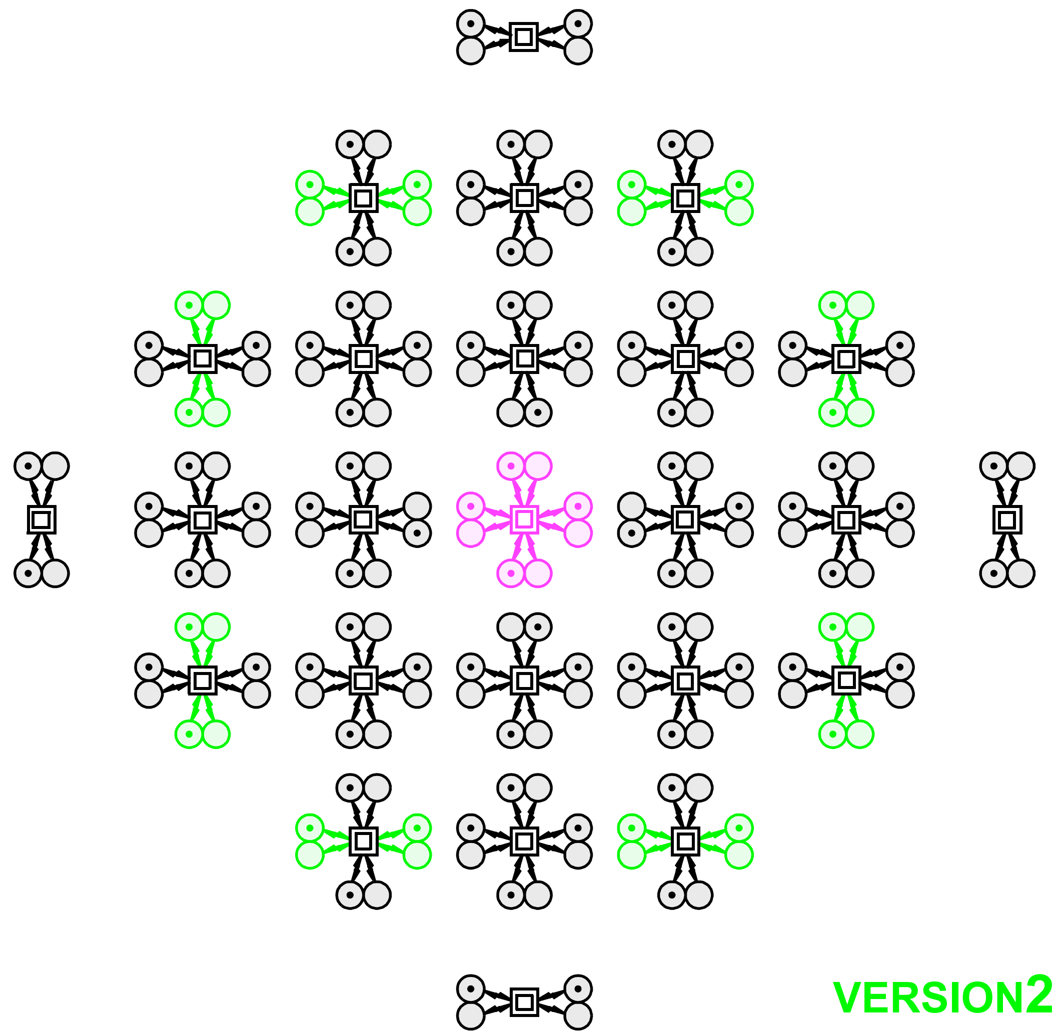
Example 5 - SOLITAIRE GAME

- two versions,
  green squares Y/N
- all but one squares
  carry tokens
- remove tokens
  by jumbing over them
- goal of the game:
  only one token left
- questions:
  is there a solution ?
- always ?



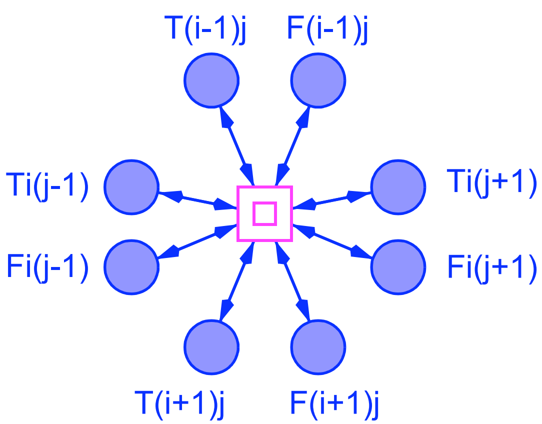
Example 5 - SOLITAIRE GAME

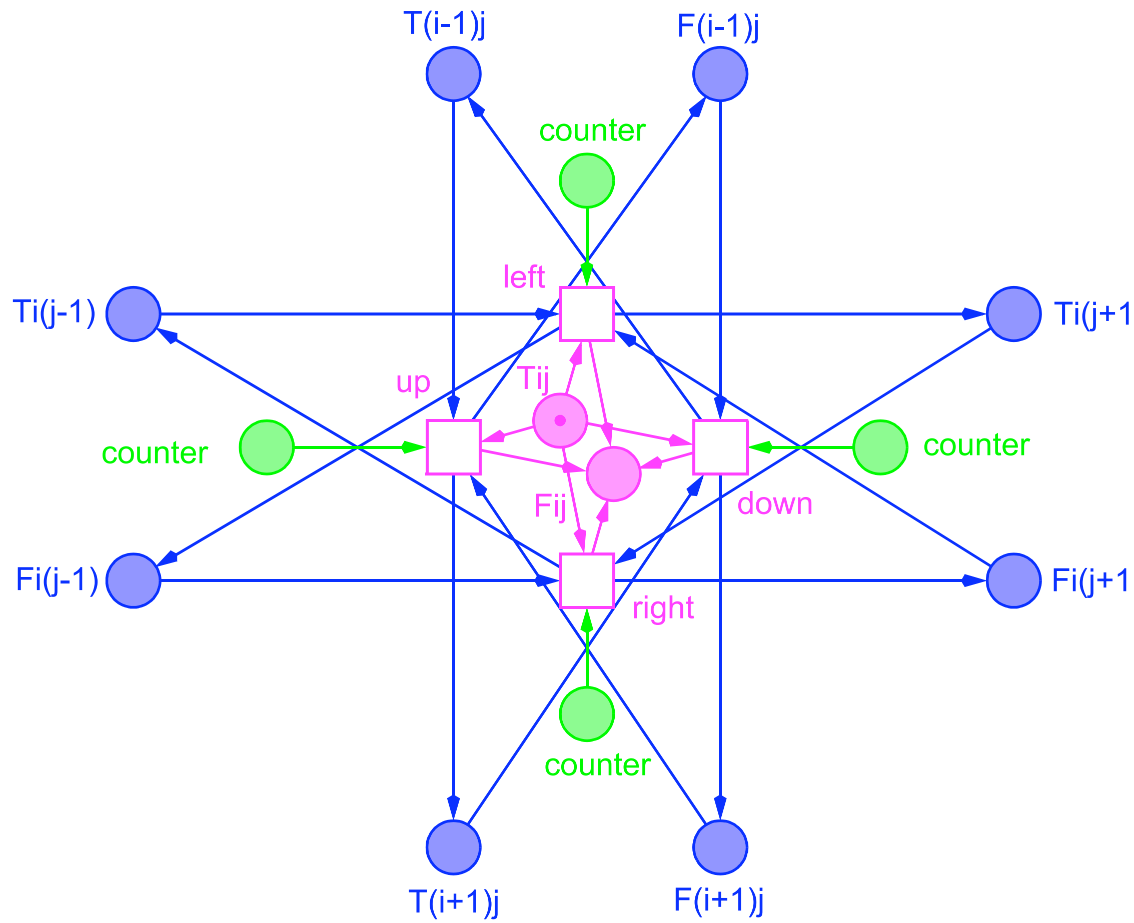
- two-level hierarchical pn
- only one square net component
- two states for each square i: T(i), F(i)
- goal of the game: dead state(s) with Σ T(i) = 1
- reachable ?
- for any initial marking ?



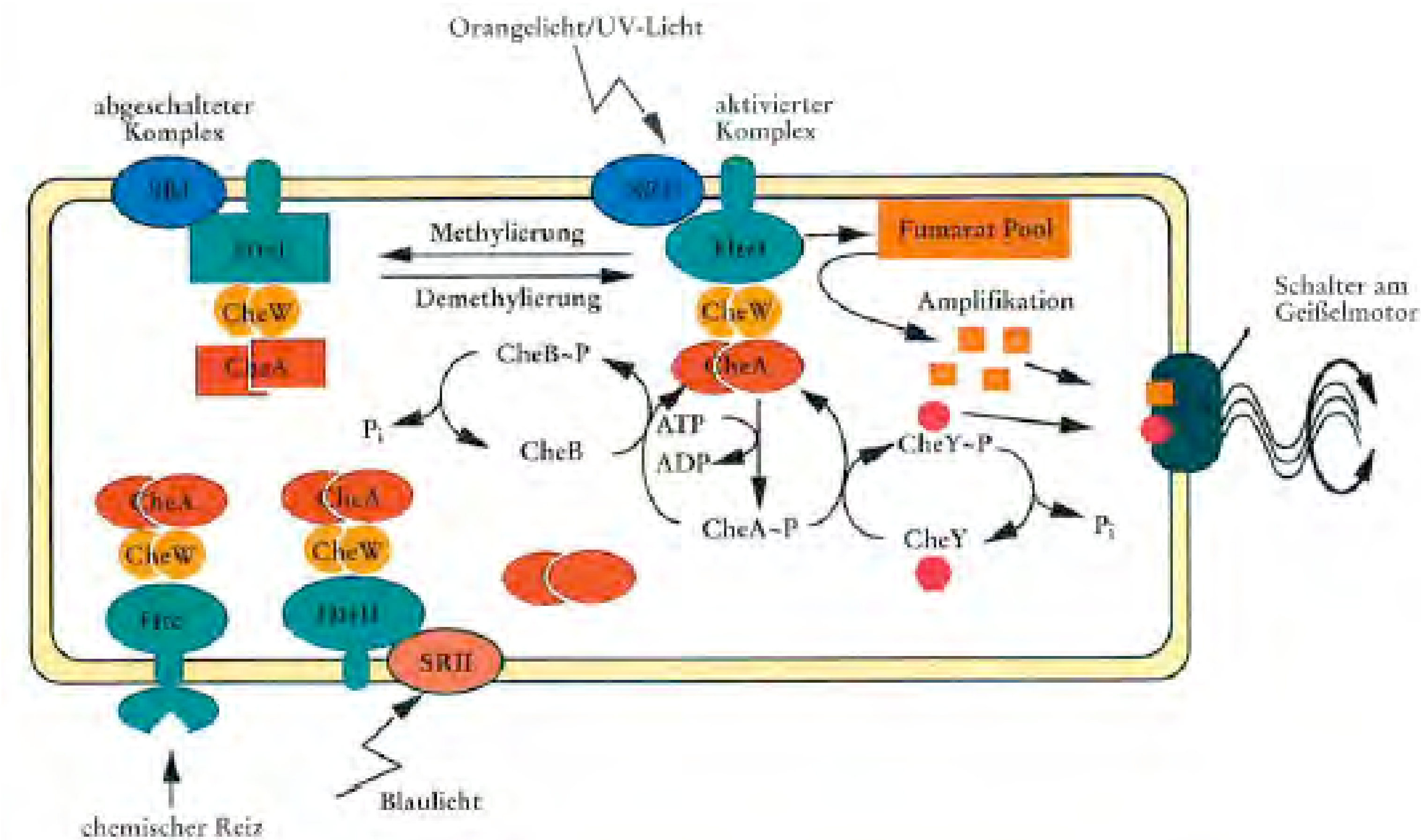
Example 5 - SOLITAIRE GAME

- square component
- counter facilitates reachbility question, but hinders analysis



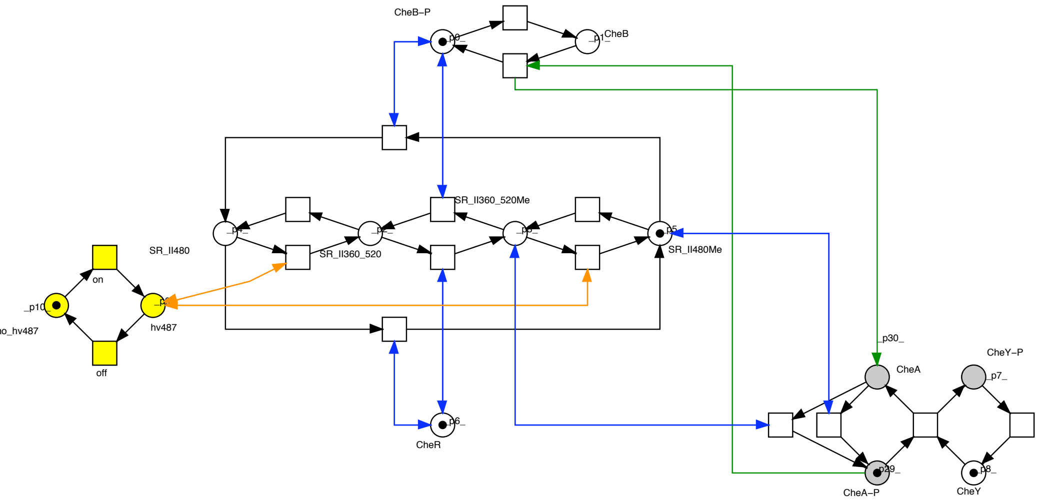


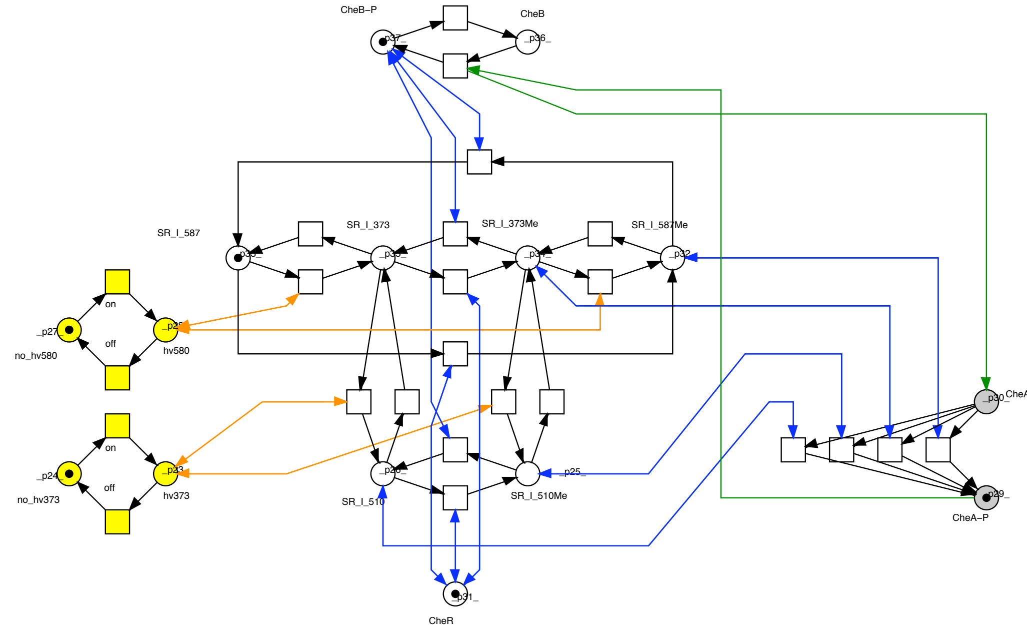
EXAMPLE 6 - HALOBACTERIUM SALINARUM

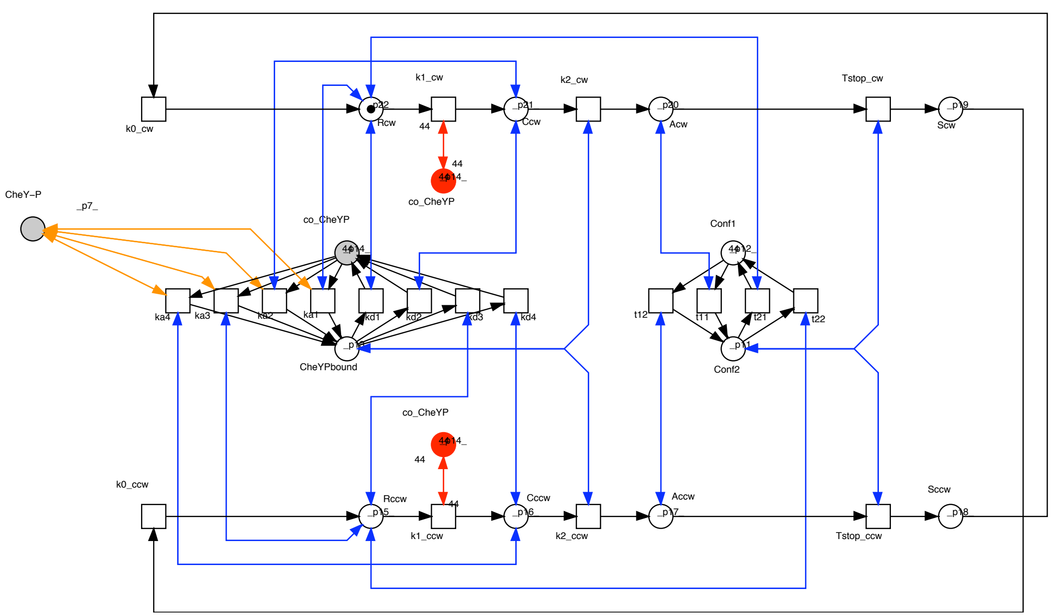


[Marwan; Oesterhelt 1999]

EXAMPLE 6 - HALOBACTERIUM SALINARUM





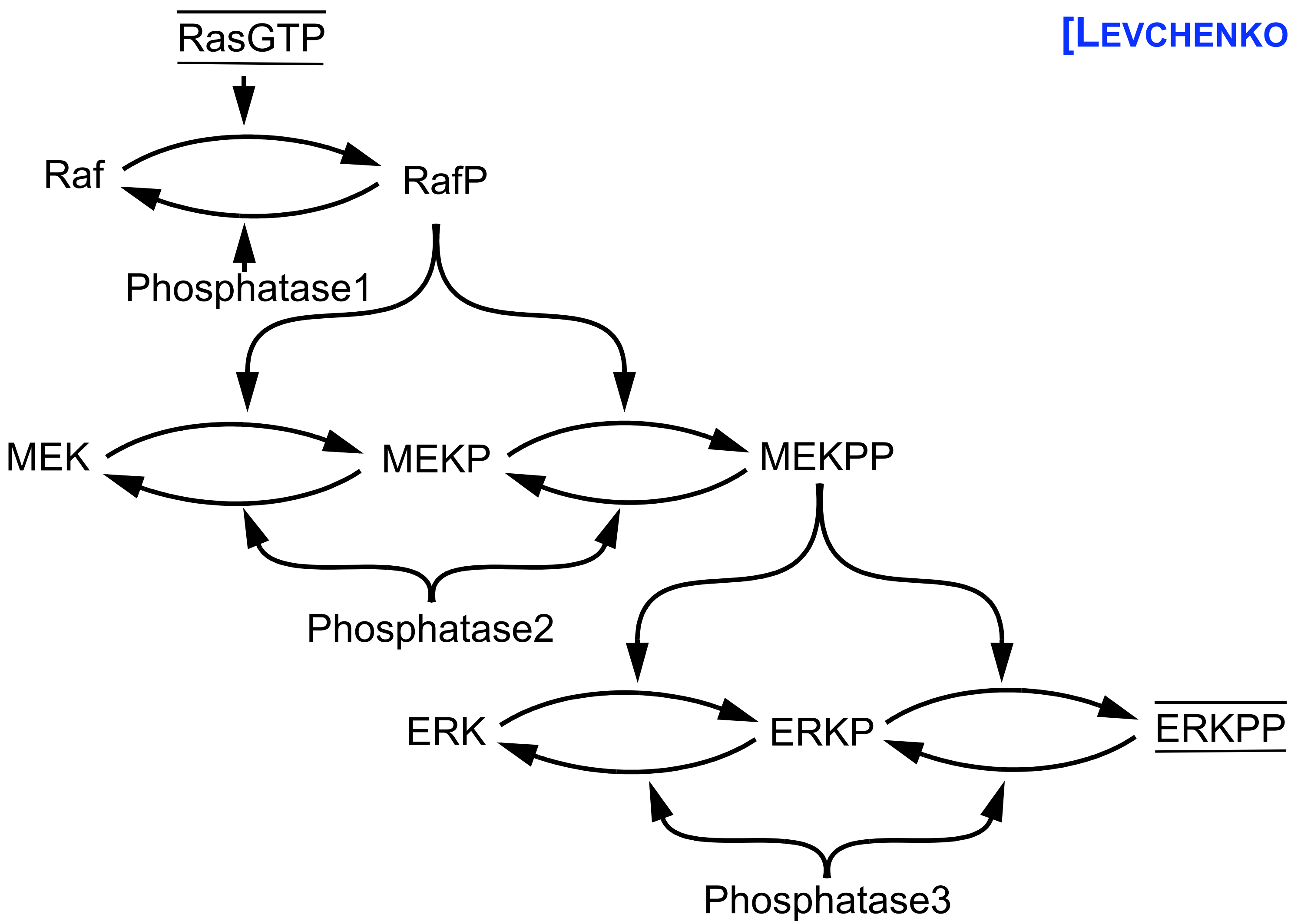


[UNPUBLISHED]

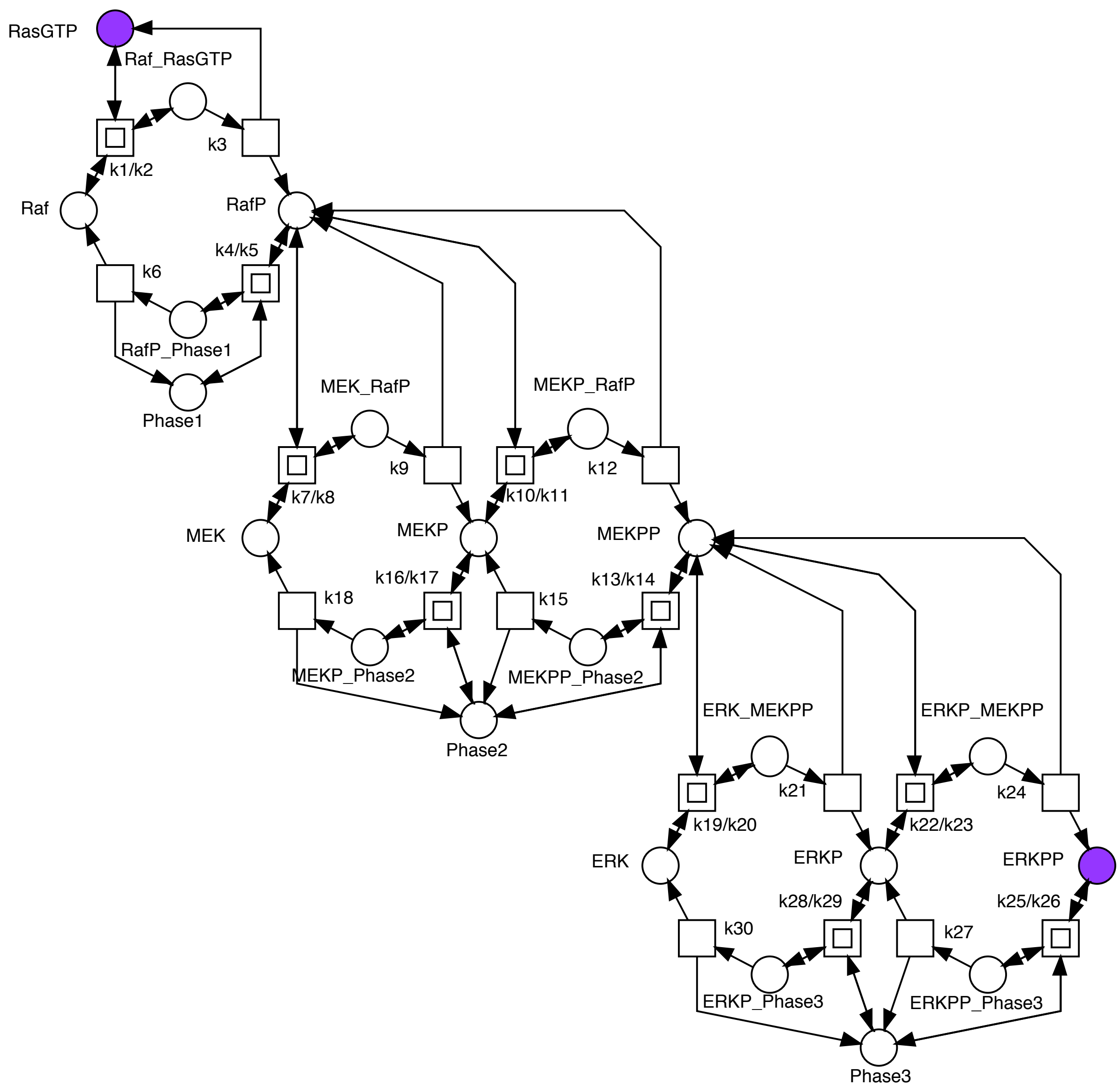
monika.heiner(at)informatik.tu-cottbus.de

EXAMPLE - MAPK SIGNALLING CASCADE

[LEVCHENKO 2000]



Example - MAPK Signalling Cascade



[Gilbert, Heiner, Lehrack 2007]

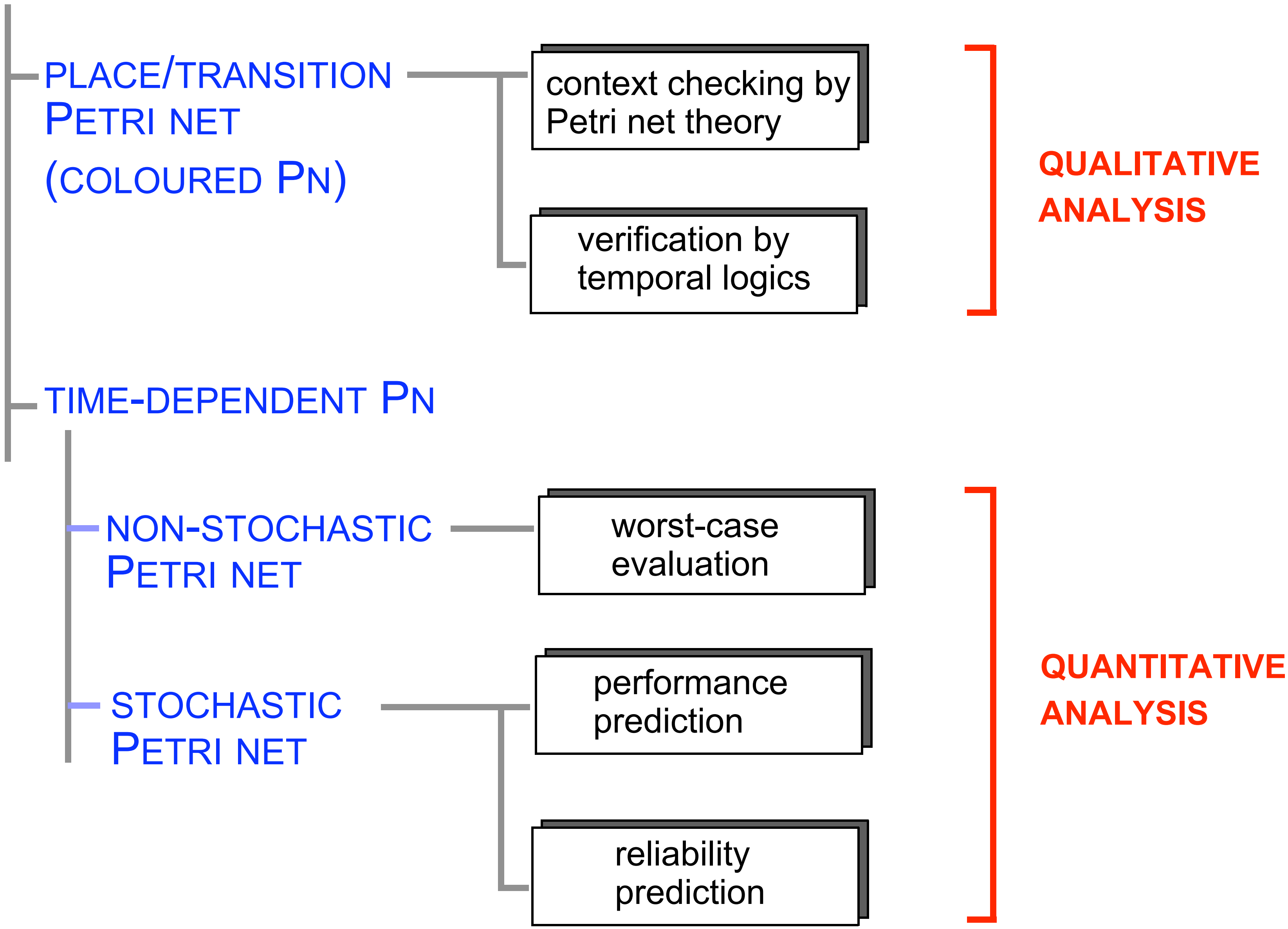
[Heiner, Gilbert, Donaldson 2008]

Petri nets, summary

WHY PETRI NETS?

- **a suitable intermediate representation for**
  - -> *different (specification/programming) languages,*
  - -> *different phases of software development cycle,*
  - -> *different validation methods;*
  - -> *technical & natural systems*
- **modelling power**
  - -> *partial order (true concurrency) semantics*
  - -> *applicable on any abstraction level*
  - -> *specification of limited resources possible*
- **analyzing power**
  - -> *combination of static and dynamic analysis techniques*
  - -> *rich choice of methods, algorithms, tools*
- **BUT: modelling power <-> analyzing power**

PETRI NETS, MODEL CLASSES



Petri nets, typical properties

TYPICAL PETRI NET QUESTIONS

- **How many tokens can reside at most in a given place ?**
  - -> *(0, 1, k, oo)* -> ***BOUNDEDNESS***
- **How often can a transition fire ?**
  - -> *(0-times, n-times, oo-times)* -> ***LIVENESS***
- **How often can a system state be reached ?**
  - -> *never* -> ***UNREACHABLE* -> *SAFETY PROPERTIES***
  - -> *n-times* -> ***REPRODUCIBLE***
  - -> *always reachable again* -> ***REVERSIBLE (HOME STATE)***
  - -> *reversible initial state* -> ***REVERSIBILITY***

GENERAL BEHAVIOURAL PROPERTIES

- **Are there behaviourally invariant subnet structures ?**
  - -> *token conservation* -> *P* - ***INVARIANTS***
  - -> *token distribution reproduction* -> *T* - ***INVARIANTS***

SPECIAL BEHAVIOURAL PROPERTIES

- **. . . and many more -> temporal logics (CTL, LTL)**

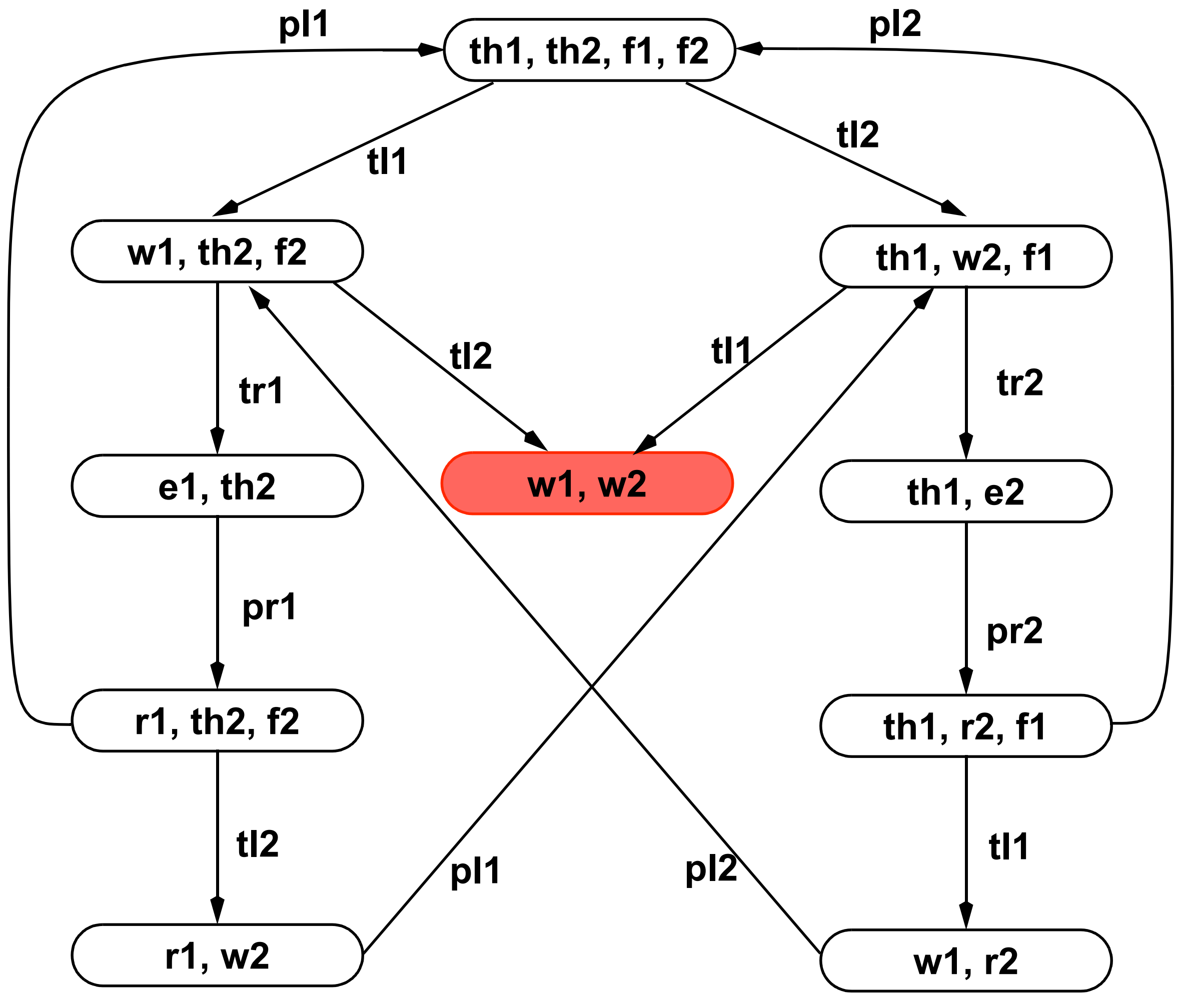
Petri nets, typical analysis techniques

MODEL ANIMATION (?)

Dynamic analyses

- **reachability / occurrence graph,**
  - -> *(labelled) state transition system (-> graph)*
  - -> *Kripke structure, CTMC, . . .*
- **nodes**
  - -> *system states / markings*
- **arcs**
  - -> *the (single) firing transition*
  - -> *single step firing*
- **interleaving semantics**
  - -> *(sequential) finite automaton*
  - -> *concurrency == enumerating all interleaving sequences*
- **reachability graph construction - simple algorithm**

Reachability Graph, Dining Philosophers (2 phils),



REACHABILITY GRAPH EVALUATION

- **boundedness**

  -> *finite graph*

- **reversibility**

  -> *one Strongly Connected Component (SCC)*

- **liveness**

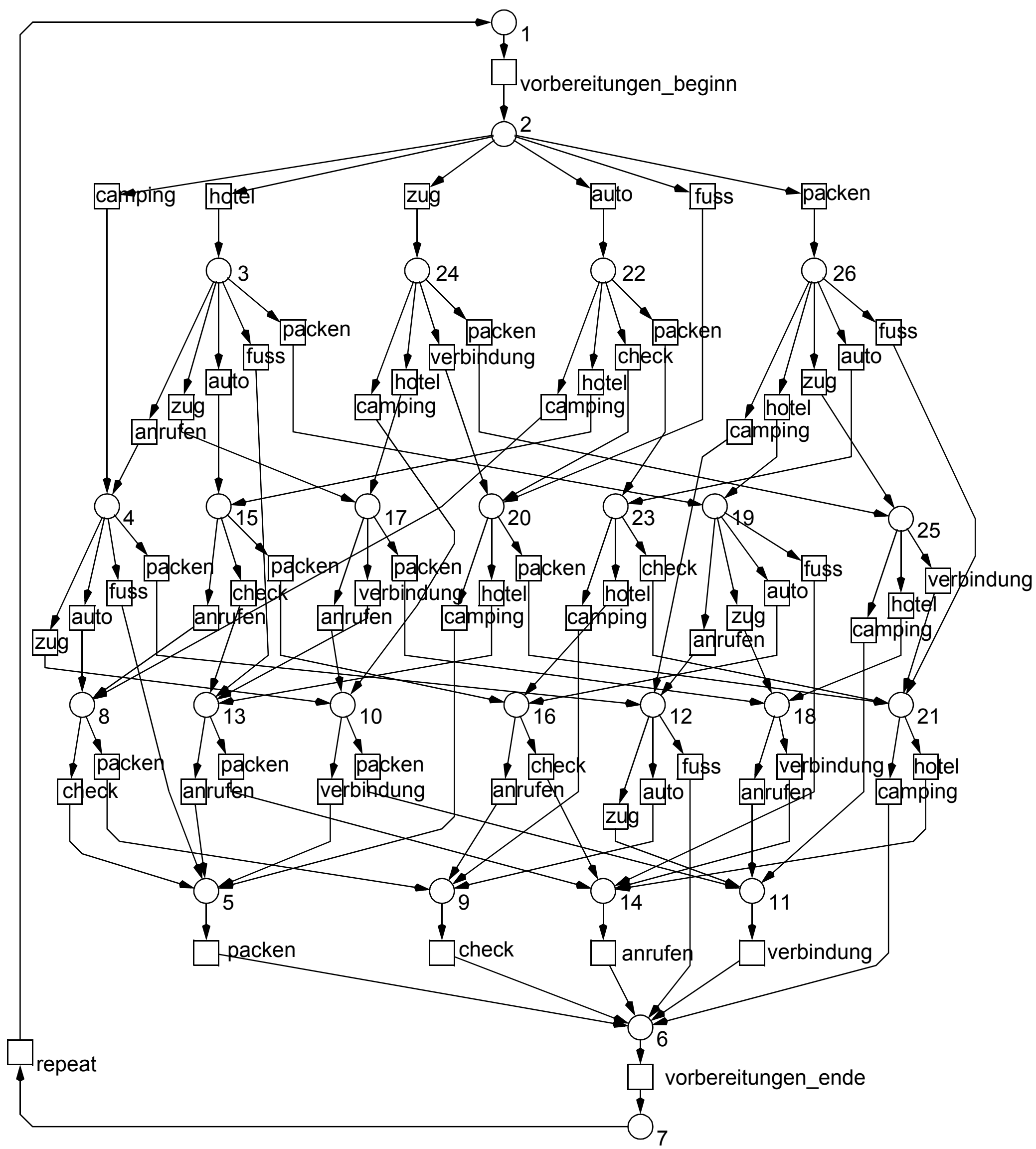
  -> *every transition contained in all terminal SCC*

- **dead states (deadlock)**

  -> *terminal nodes*

-> reachability graphs tend to be huge <-

REACHABILITY GRAPH, TRAVEL PLANING



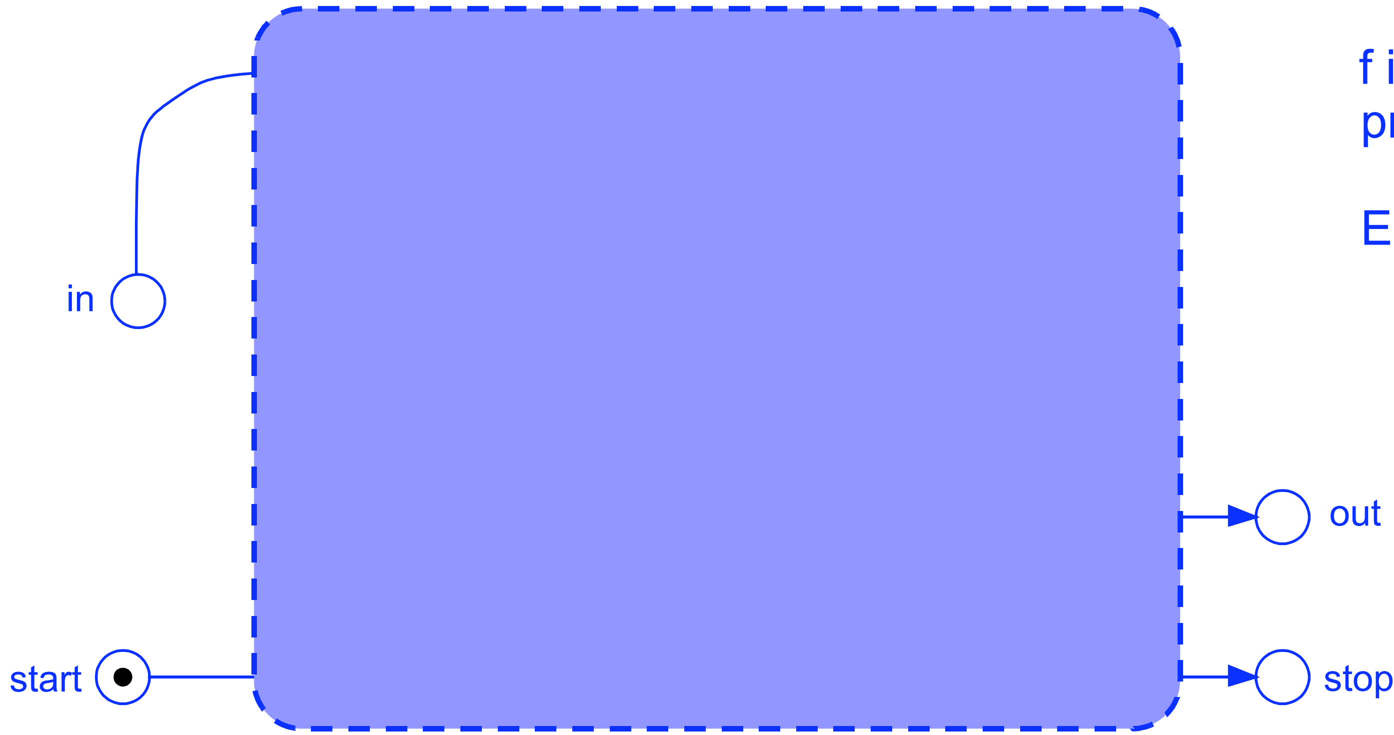
monika.heiner(at)informatik.tu-cottbus.de

- **infinite for unbounded nets**

- **worst-case for finite state spaces** **[Priese, Wimmel 2003]**

  ... cannot be bounded by a primitive recursive function ...

- **proof** -> Petri net computer for a function **f:** $\mathbf{N_0}^{m} \to \mathbf{N_0}$



f is weakly pn-computable:

EF(out = f(in) &
stop = 1)

- **infinite for unbounded nets**
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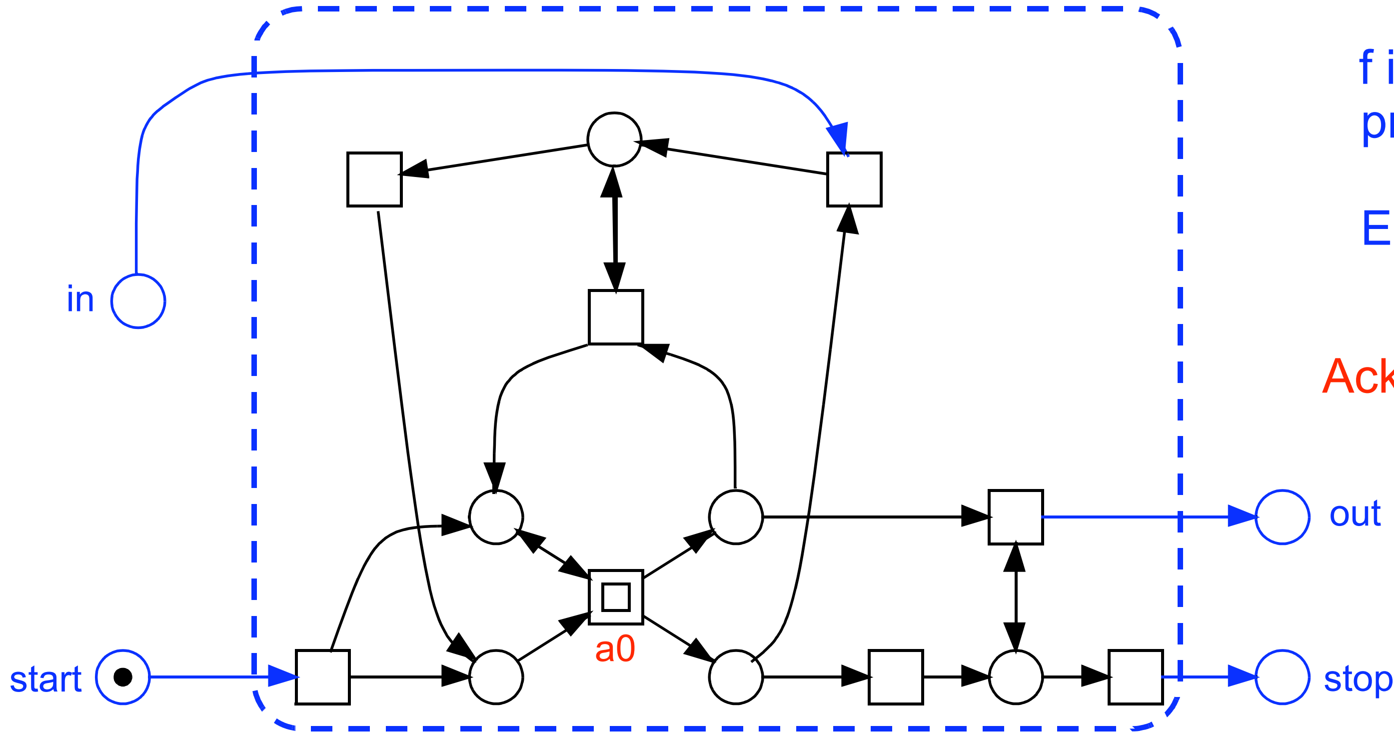

f is weakly pn-computable:

EF(out = f(in) &
stop = 1)

Ackermann function a1

REACHABILITY GRAPH, STATE SPACE COMPLEXITY

- **infinite for unbounded nets**

- **worst-case for finite state spaces** **[Priese, Wimmel 2003]**

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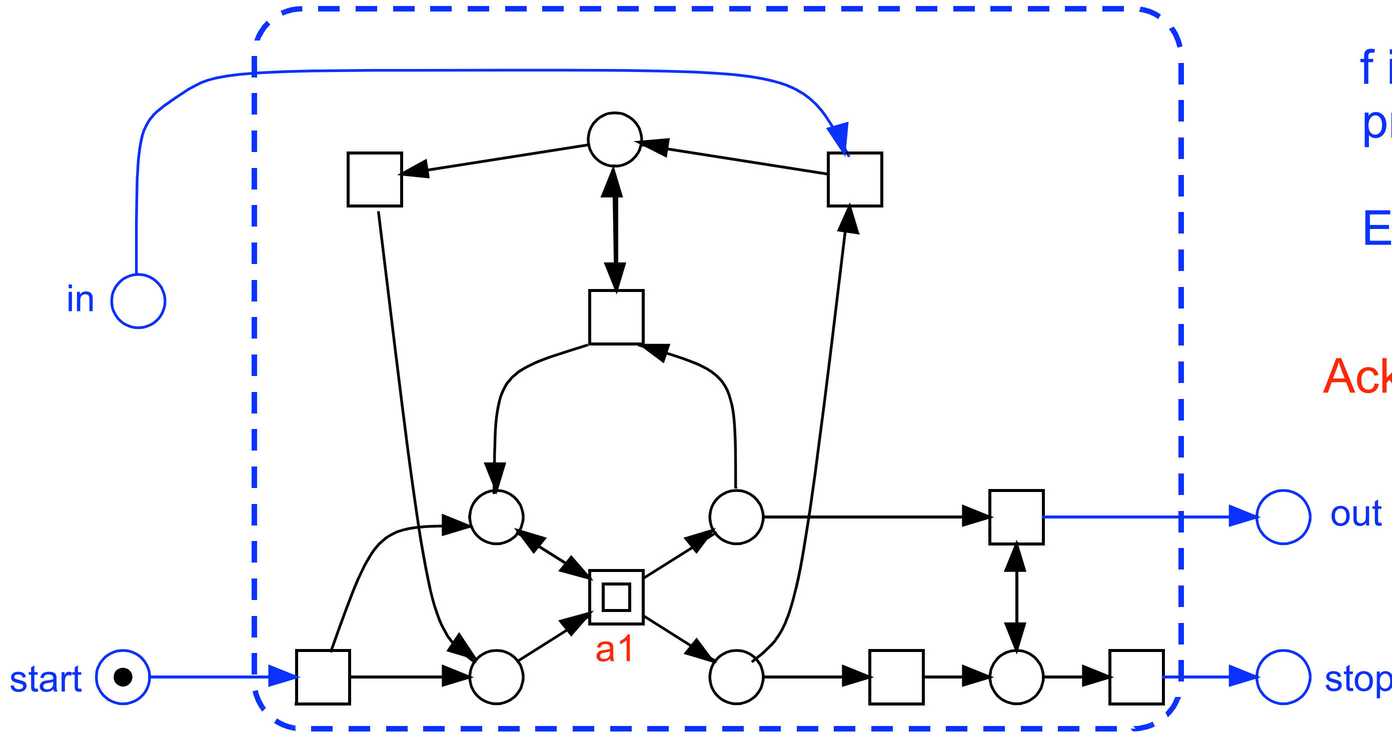
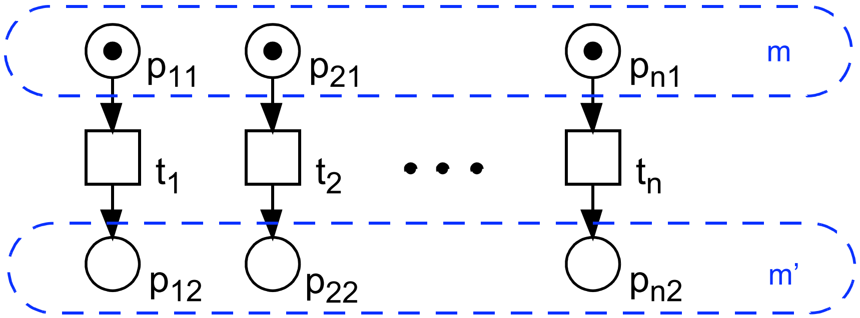


f is weakly pn-computable:

EF(out = f(in) &
stop = 1)

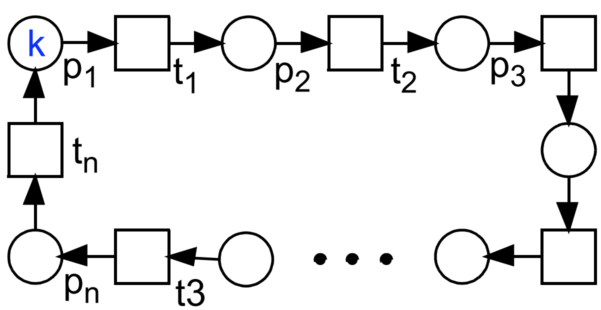
Ackermann function a2



n! interleaving sequences

m -> m'

2^n - 2 intermediate states



$$\frac{(n + k - 1)!}{(n - 1)!\, k!} \text{ states}$$

(combination with repetition)

ANALYSIS TECHNIQUES

- **static analyses** **-> no state space construction**
  - -> *structural properties (graph theory)*
  - -> *P / T - invariants (linear algebra)*
  - -> *state equation*
- **dynamic analyses** **-> total / partial state space construction**
  - -> *analysis of general behavioural system properties,*
    i.e. boundedness, liveness, reversibility
  - -> *model checking of special behavioural system properties,*
    e.g. reachability of a given (sub-) system state (with constraints),
    reproducability of a given (sub-) system state (with constraints)

  => expressed in temporal logics (CTL / LTL),
  as very flexible & powerful query language

ANALYSIS TOOLS

- **Petri net theory**
  - -> *INA (HU Berlin)*
  - -> *TINA (LAAS/CNRS)*
  - -> *Charlie*

- **model checking**

		CTL	**LTL**
-> *reachability graph*	->	*INA, Charlie*	*Charlie*
		PROD, MARIA	*PROD, MARIA*
-> *lazy state spaces*			
- stubborn set reduction	->	*LoLA*	*PROD (LTL\X)*
- symmetry reduction	->	*LoLA*	
-> *compressed state spaces (BDD, NDD, ... , IDD)*	->	*bdd-CTL, SMART* *idd-CTL*	*bdd-LTL* *idd-LTL*
-> *Kronecker algebra*	->	*[Kemper]*	
-> *prefix*	->	*PEP (CTL_0)*	*QQ (LTL\X)*
-> *process automata*	->	*[pd]*	

-> TOOL BOX <-

to be continued:

Temporal Logics,

CTL - a crash cours