

A PETRI NET FRAMEWORK FOR SPATIAL AND MULTISCALE MODELLING OF PLANT BIOCHEMISTRY

- A PETRI NET PERSPECTIVE ON SYSTEMS AND SYNTHETIC BIOLOGY -

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BRANDENBURG TECHNICAL UNIVERSITY COTTBUS-SENFTENBERG
COMPUTER SCIENCE INSTITUTE

❑ FRAMEWORK

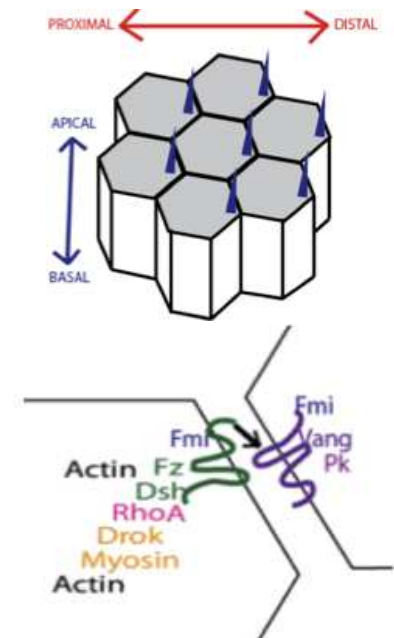
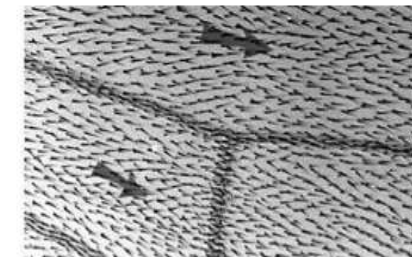
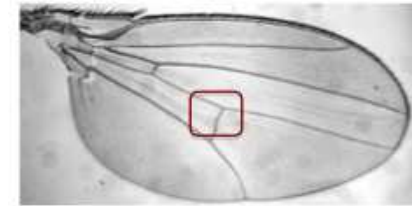
- > *unifying four paradigms:*
QPN - SPN - CPN - HPN
- > *our toolbox:*
Snoopy - Marcie - Charlie - Patty - Strike

❑ COLOURED PETRI NETS -> PDE

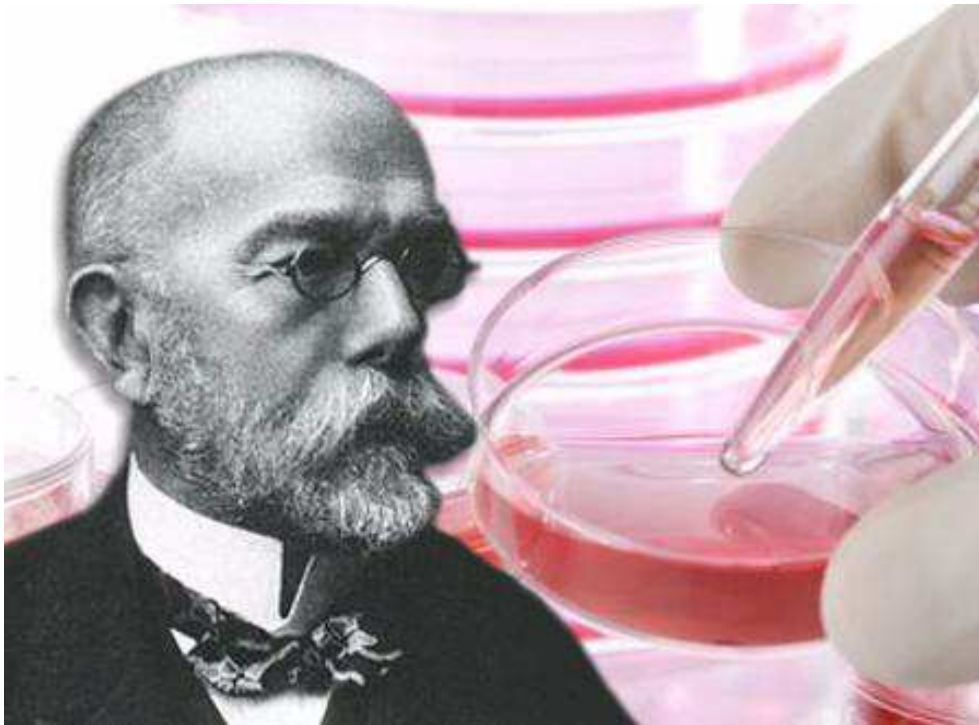
- > *Turing patterns (CPN)*
- > *phase variation (SPN)*
- > *planar cell polarity (hierarchical space)*

❑ MODELLING BIO PETRI NETS

- > *composition from standard components*
- > *bottom-up (reverse engineering)*
- > *modular modelling*
- > *genome-controlled model generation*



THE PETRI NET FRAMEWORK



Julius Richard Petri, 1852 - 1921

Carl Adam Petri, 1926 - 2010

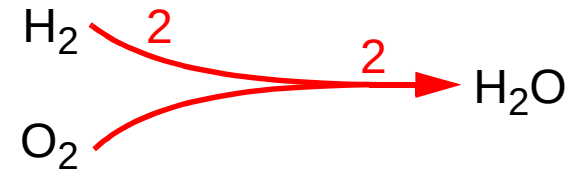
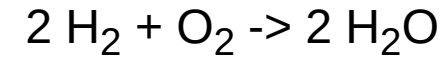


NOVEMBER 2006



...

**ARE NETWORKS
OF BIOCHEMICAL
REACTIONS**

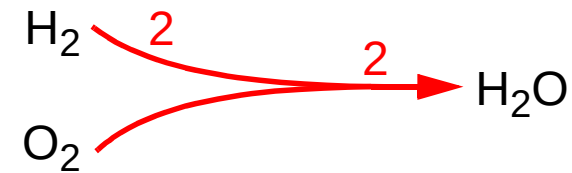
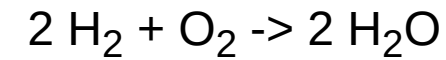


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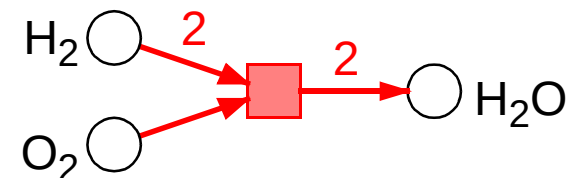
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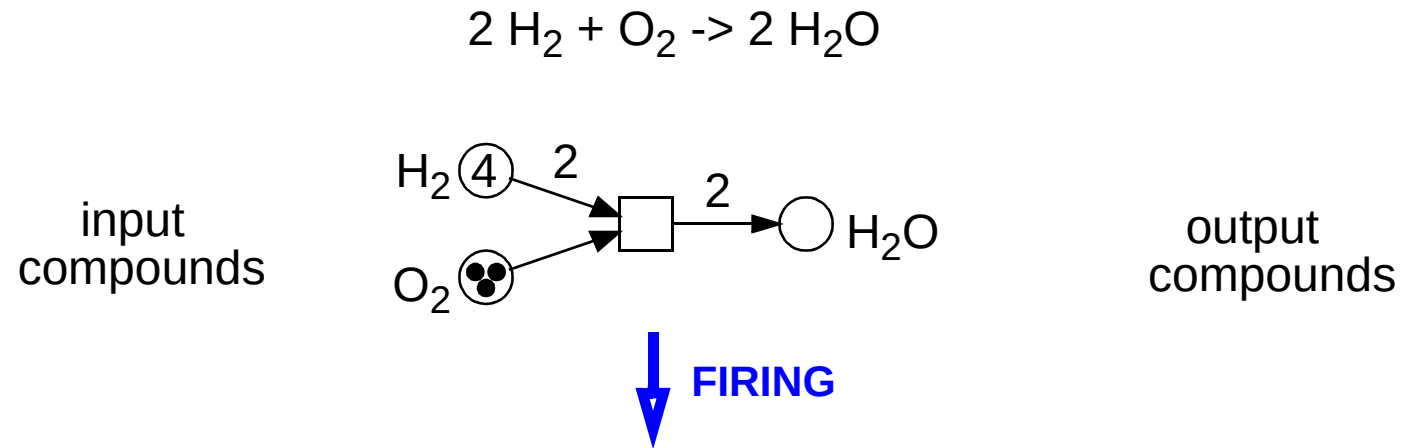
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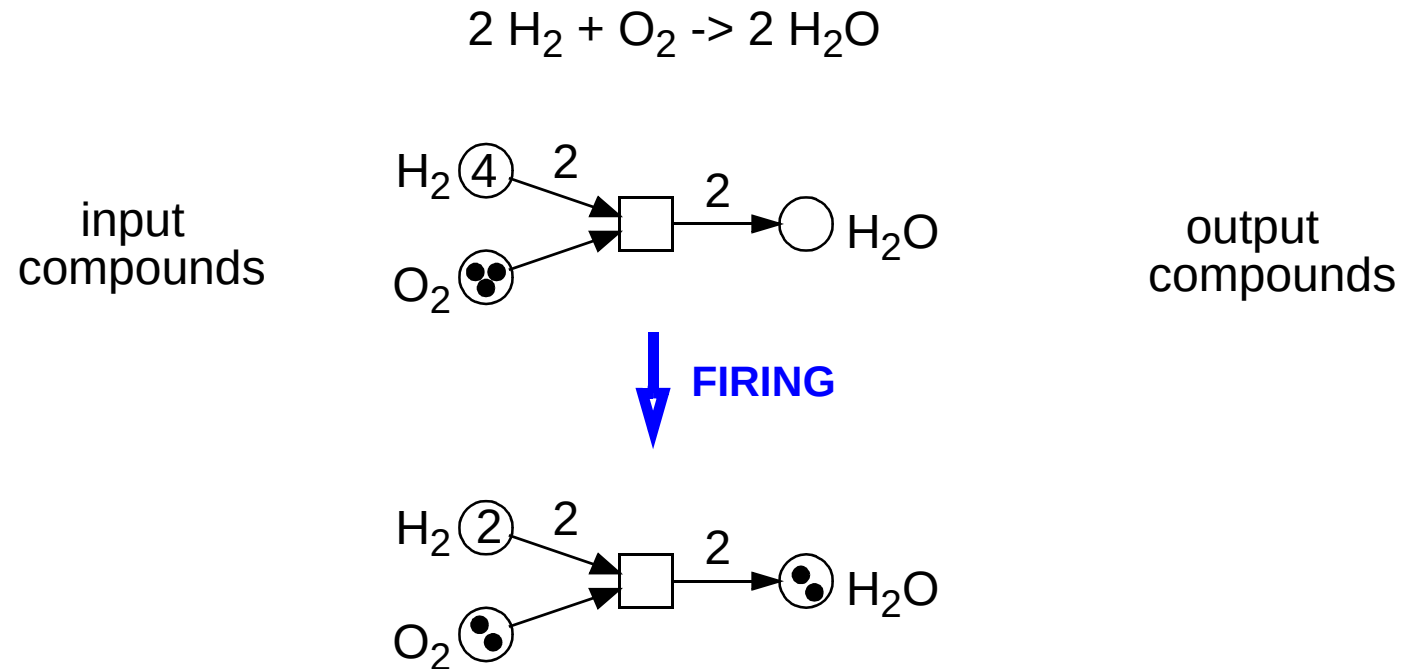
**NATURALLY
EXPRESSIBLE AS
PETRI NETS**

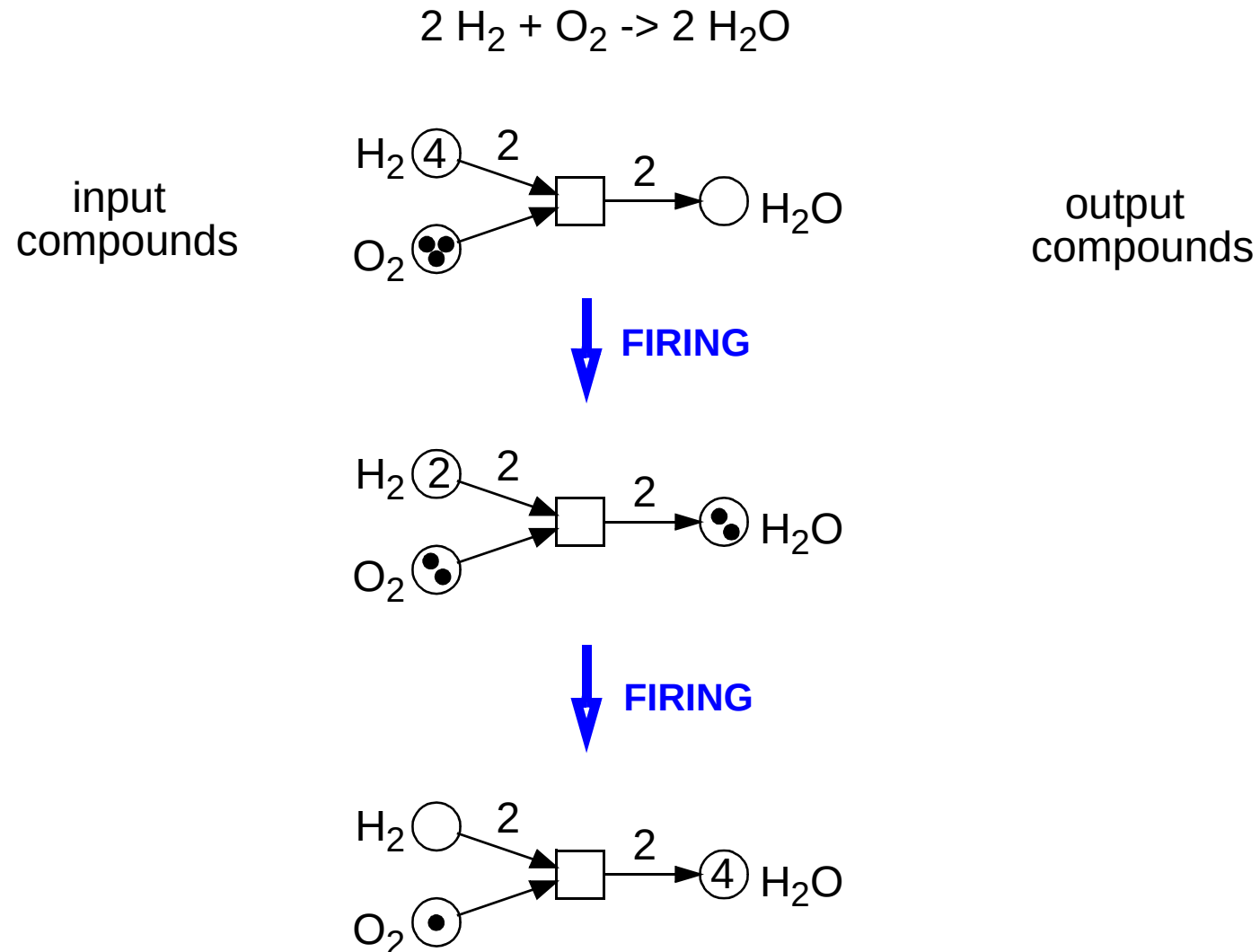


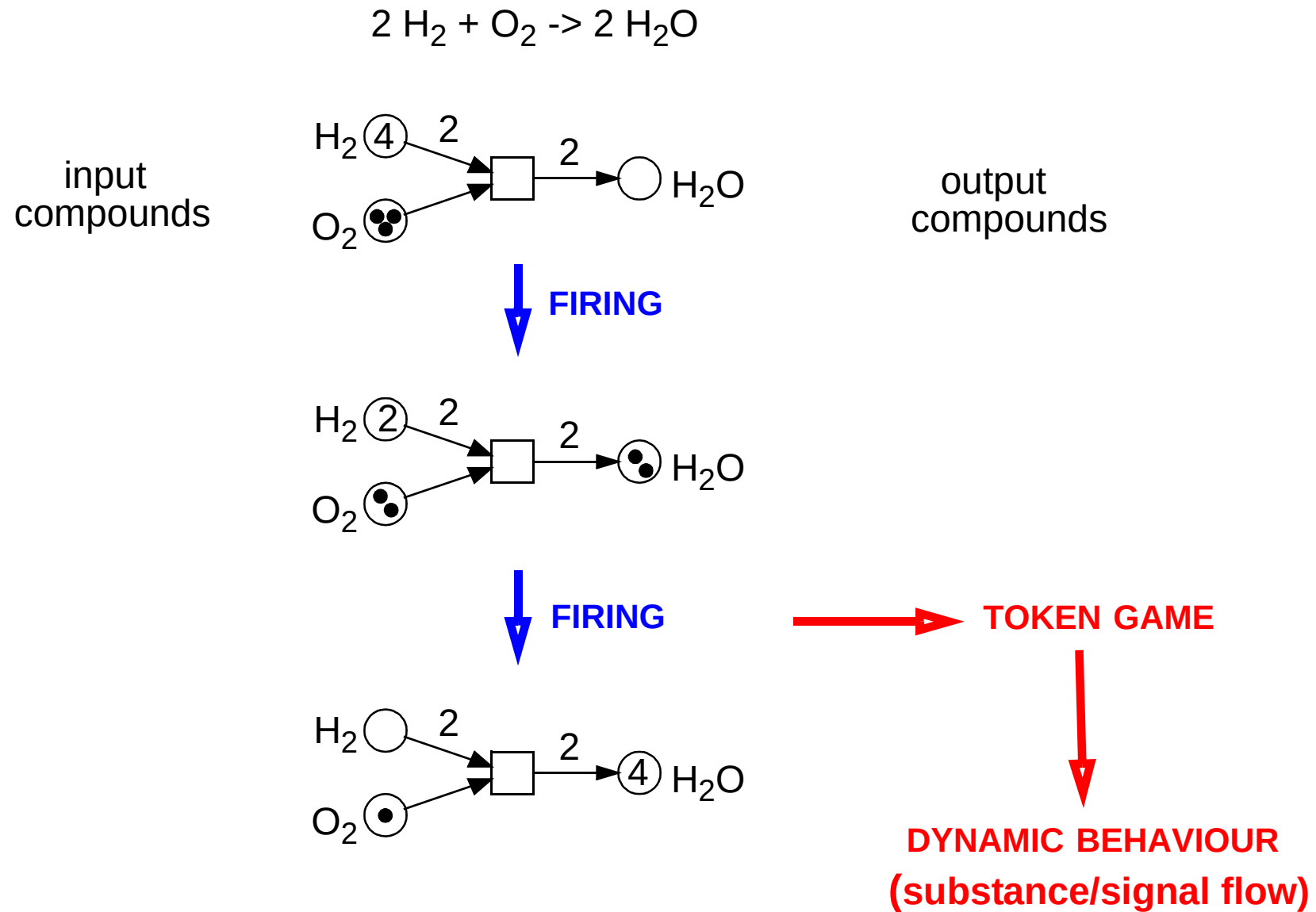
↑
hyper-arcs
↓







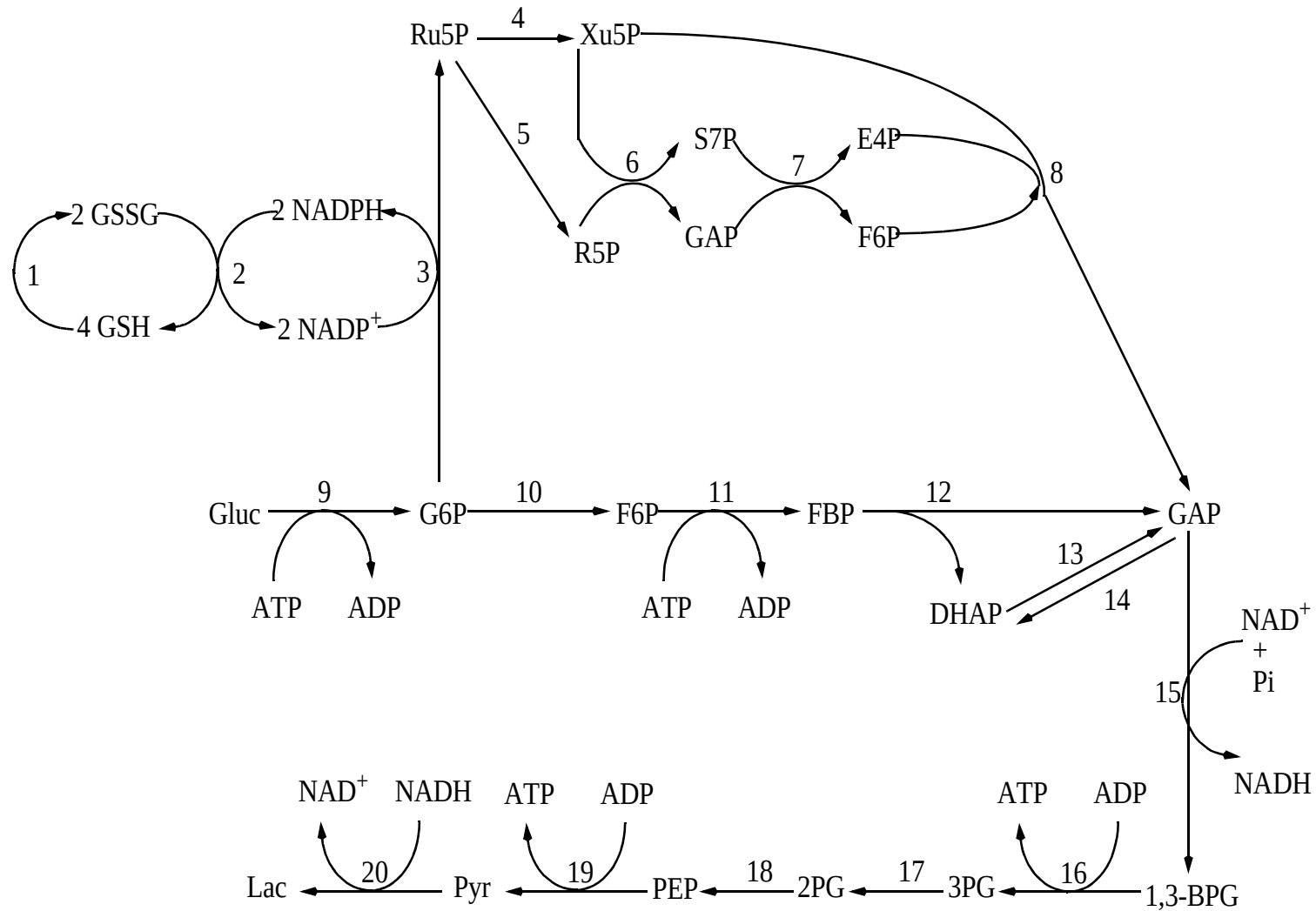




Ex: Glycolysis and Pentose Phosphate Pathway

PN & BioModel Engineering

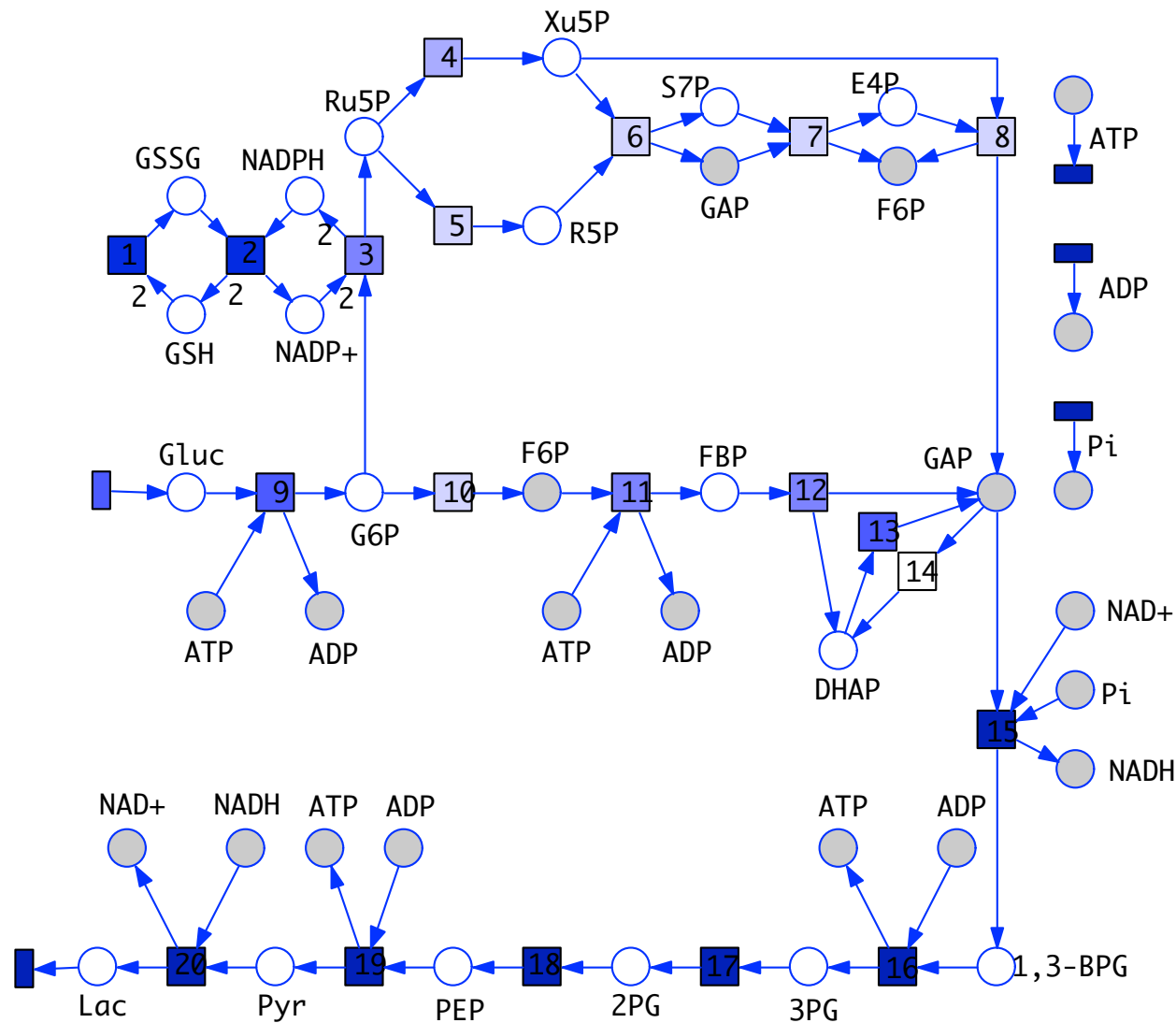
[Reddy 1993]



Ex: Glycolysis and Pentose Phosphate Pathway

PN & BioModel Engineering

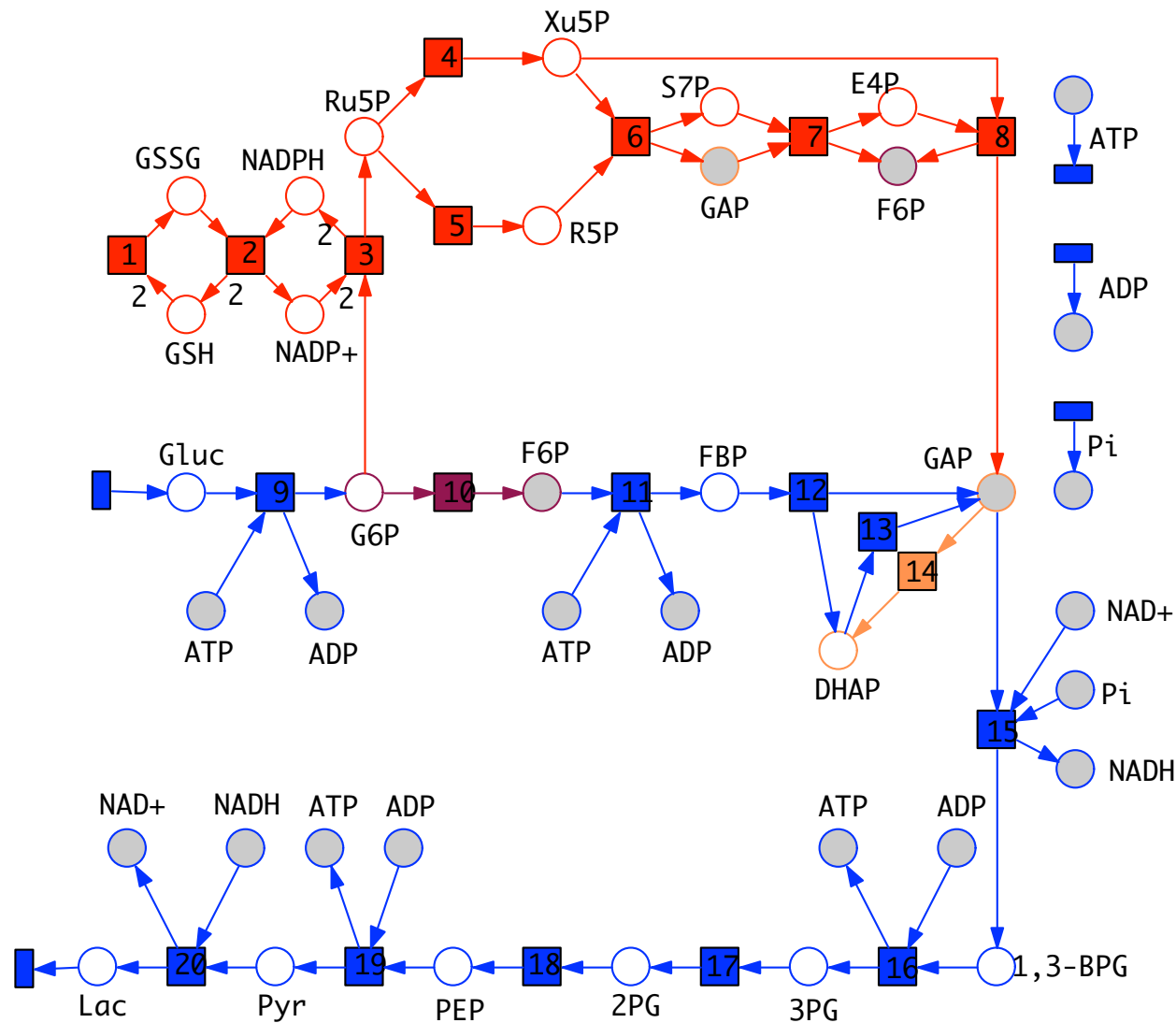
[Reddy 1993]
[Heiner 1998]



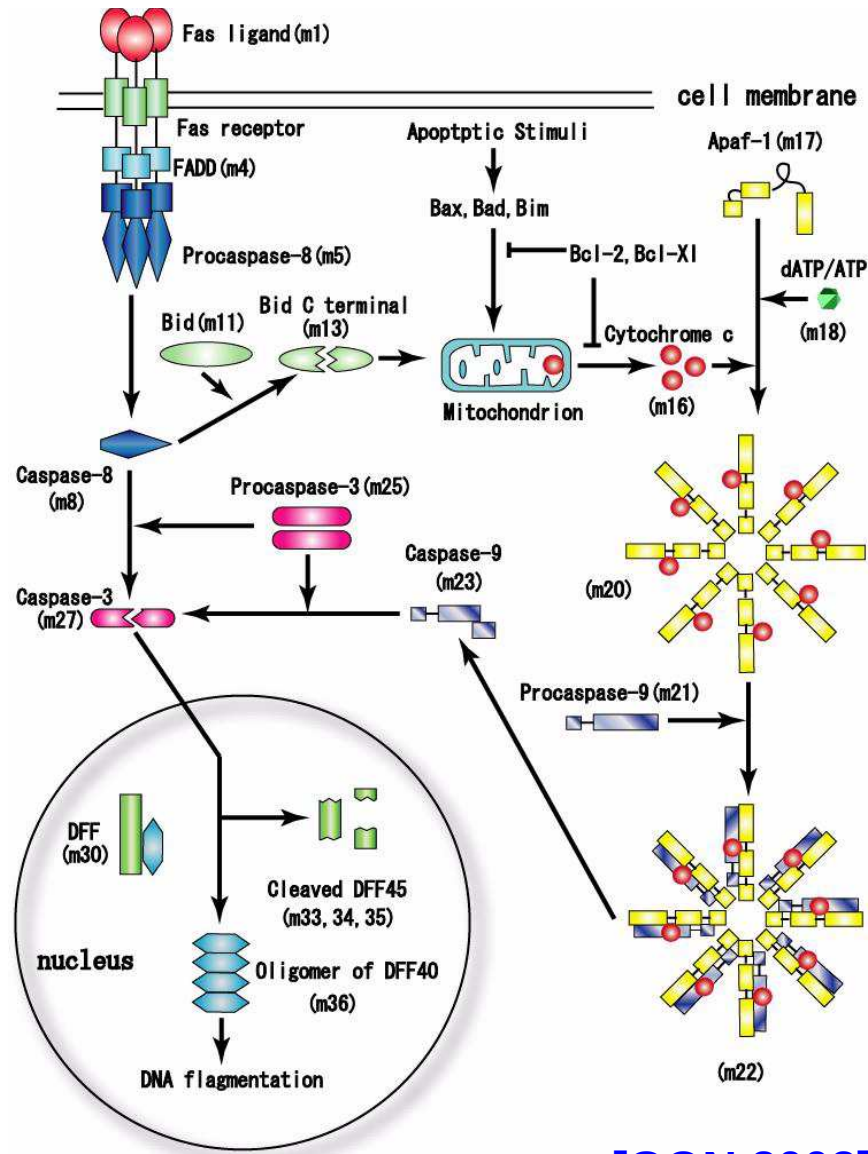
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PN & BioModel Engineering

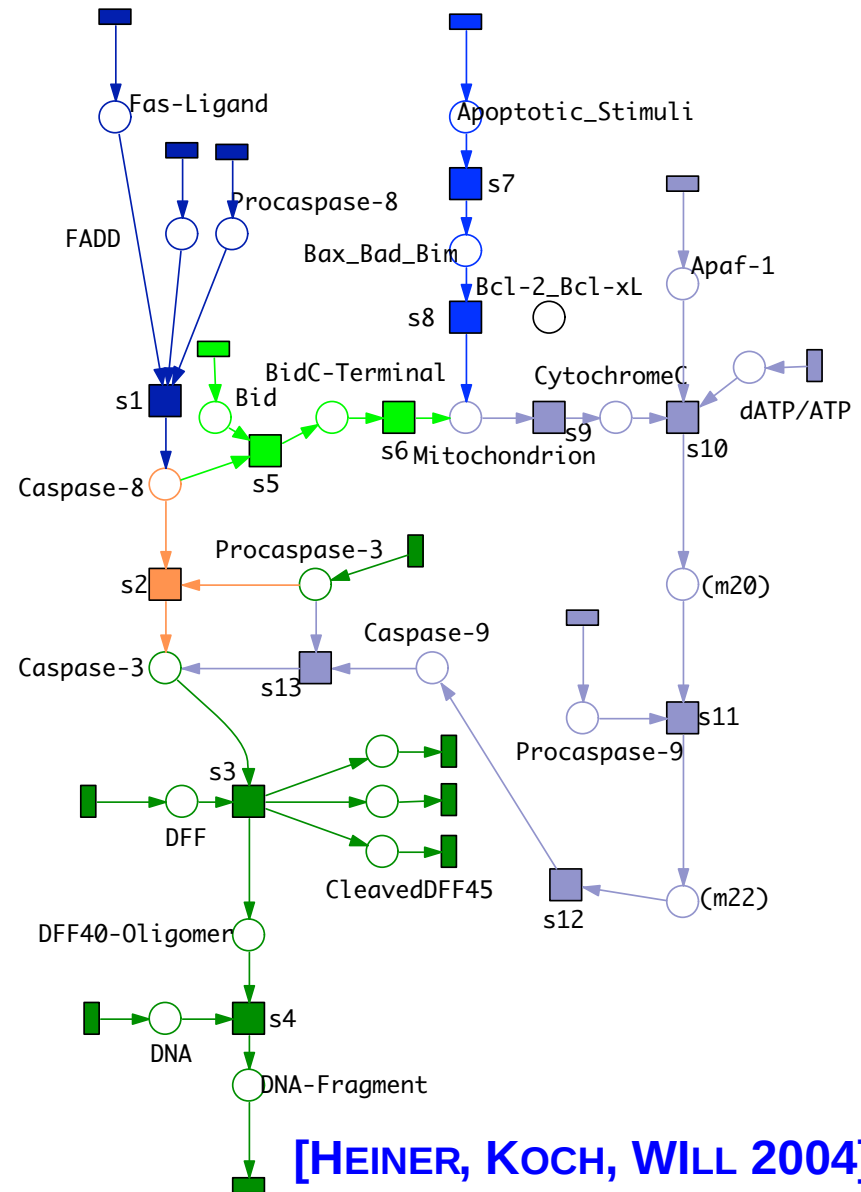
[Reddy 1993]
[Heiner 1998]
[Heiner 2009]



Ex: APOPTOSIS IN MAMMALIAN CELLS



[GON 2003]



[HEINER, KOCH, WILL 2004]

□ places -> model variables

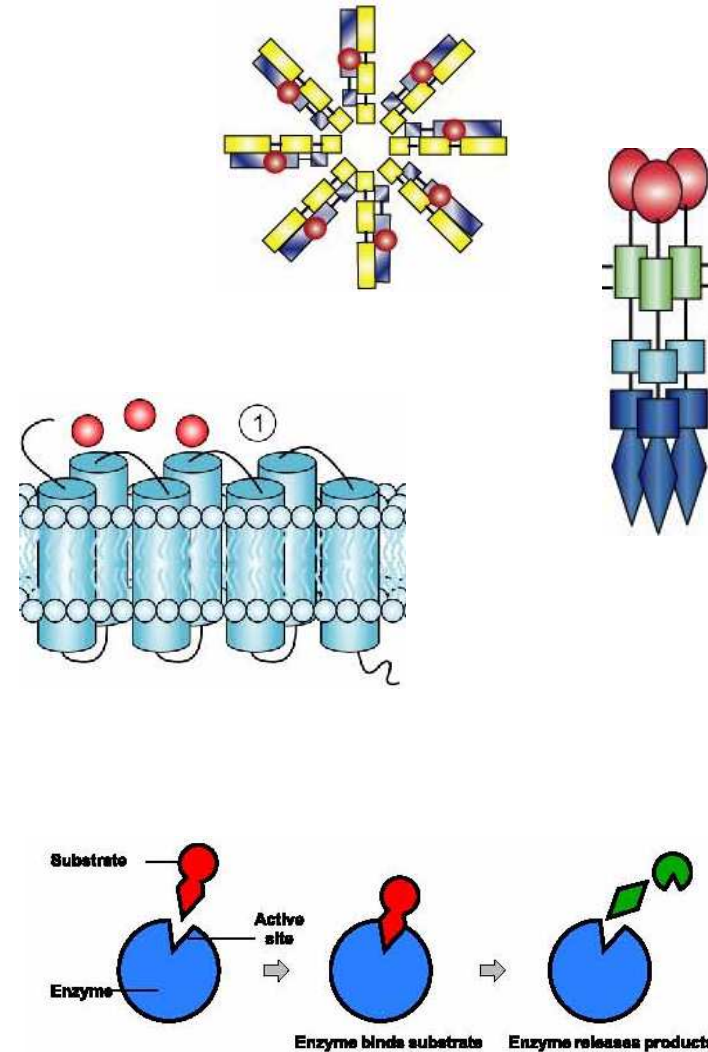
- > (bio-) chemical compounds
- > proteins
- > protein conformations
- > complexes
- > genes, . . . , etc.

. . . in different locations

□ transitions -> atomic events

- > (stoichiometric) chemical reaction
- > complexation / decomplexation
- > phosphorylation / dephosphorylation
- > conformational change
- > transport step, . . . , etc.

. . . in different locations



- ❑ **graphics may support communication between professionals not familiar with modelling techniques**

Genomic, Biochemical, and Modeling Analyses of Asparagine Synthetases from Wheat

Hongwei Xu^{1,2}, Tanya Y. Curtis², Stephen J. Powers³, Sarah Raffan², Runhong Gao^{1,2}, Jianhua Huang^{1}, Monika Heiner⁴, David R. Gilbert⁵ and Nigel G. Halford^{2*}*

¹ Biotechnology Research Institute, Shanghai Academy of Agricultural Sciences, Shanghai, China, ² Department of Plant Sciences, Rothamsted Research, Harpenden, United Kingdom, ³ Department of Computational and Analytical Sciences, Rothamsted Research, Harpenden, United Kingdom, ⁴ Department of Computer Science, Brandenburg University of Technology Cottbus-Senftenberg, Cottbus, Germany, ⁵ Department of Computer Science, College of Engineering, Design and Physical Sciences, Brunel University London, Uxbridge, United Kingdom

Asparagine synthetase activity in cereals has become an important issue with discovery that free asparagine concentration determines the potential for

Front. Plant Sci., 15 January 2018
<https://doi.org/10.3389/fpls.2017.02237>



r1: Gln +ASNe -> ASNe_Gln

r2: ASNe_Gln -> Glu + ASeNH3

r3: ASNe_NH3 + Asp + ATP + Mg2 -> bAsp_AMP_ASNe_NH3 +ATP +Mg2

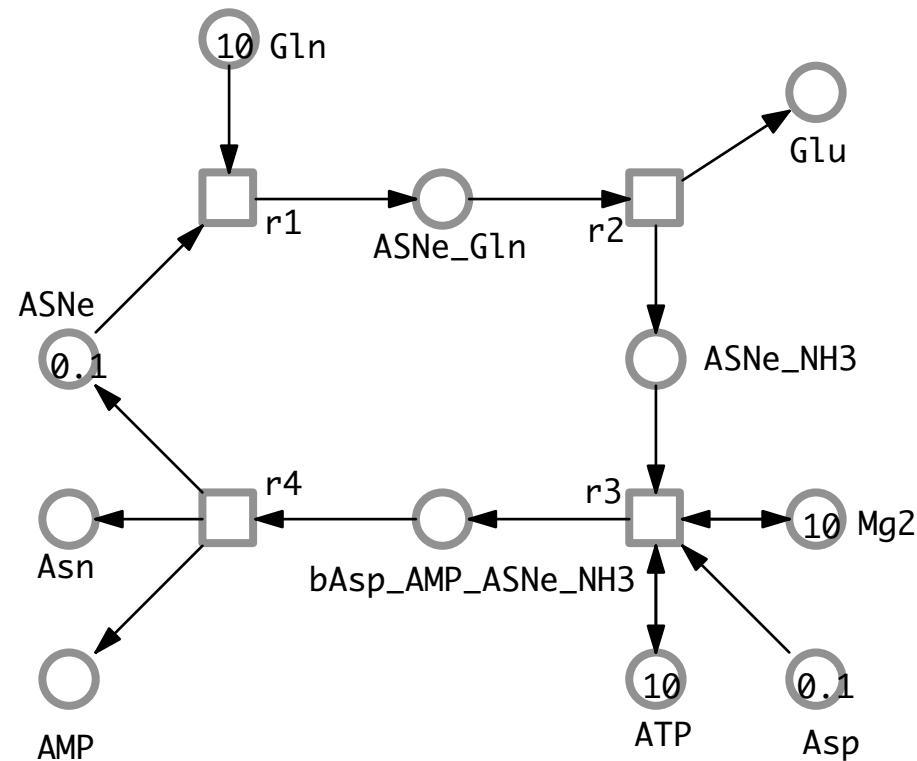
r4: bAsp_AMP_ASNe_NH3 -> ASNe + Asn + AP

r1: Gln + ASNe $\rightarrow$ ASNe_Gln

r2: ASNe_Gln $\rightarrow$ Glu + ASNe_NH3

r3: ASNe_NH3 + Asp + ATP + Mg2 $\rightarrow$ bAsp_AMP_ASNe_NH3 + ATP + Mg2

r4: bAsp_AMP_ASNe_NH3 $\rightarrow$ ASNe + Asn + AP



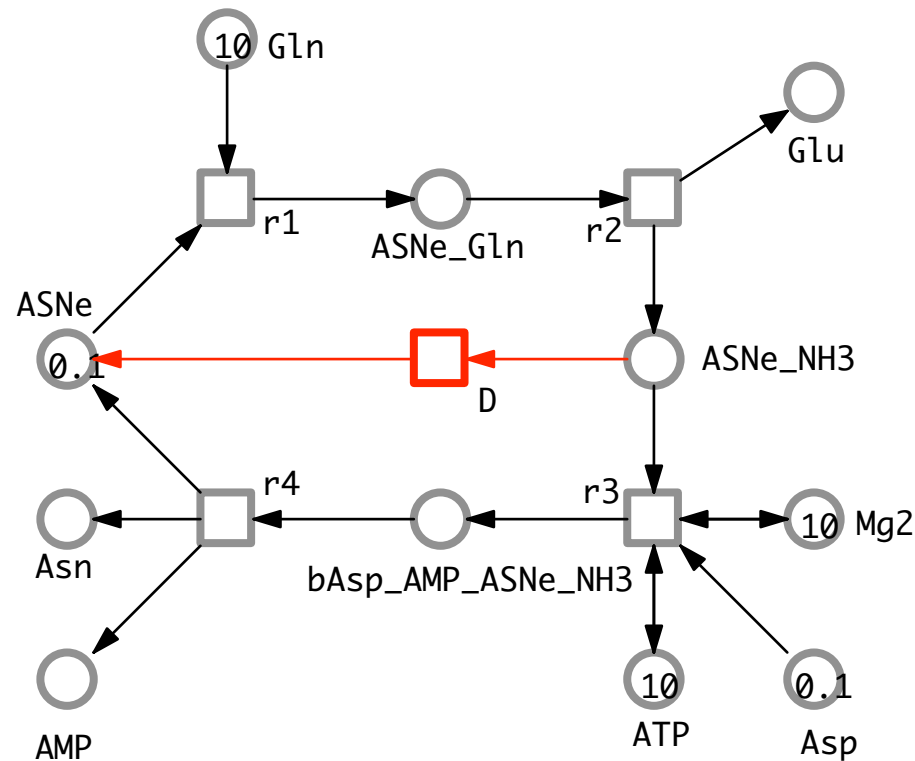
Ex: WHEAT - FINITE SUPPLY

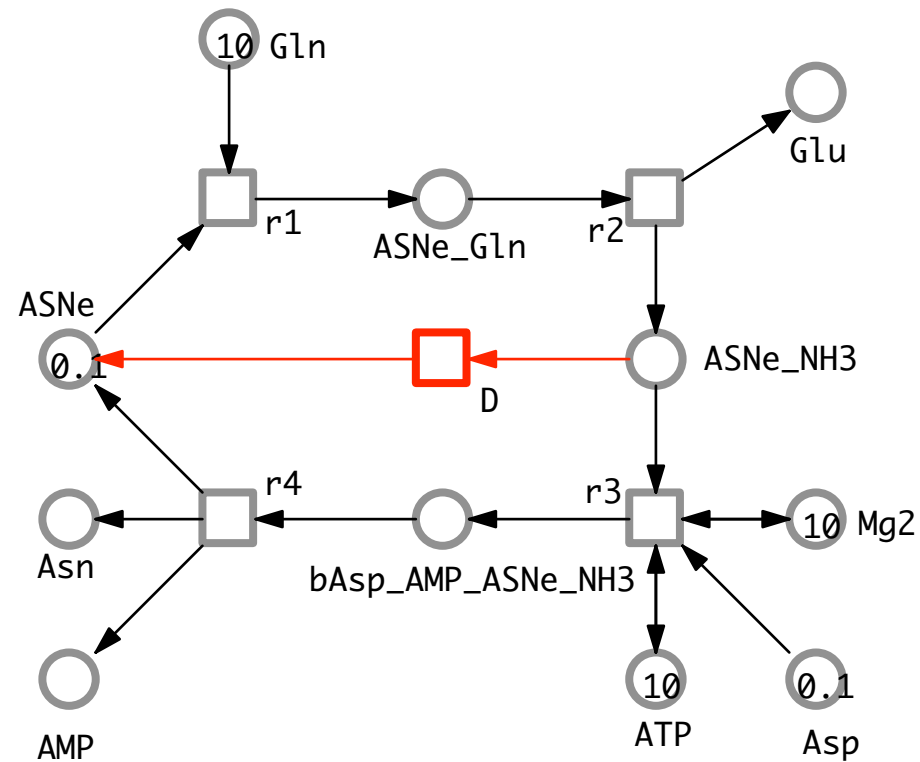
r1: Gln + ASNe $\rightarrow$ ASNe_Gln

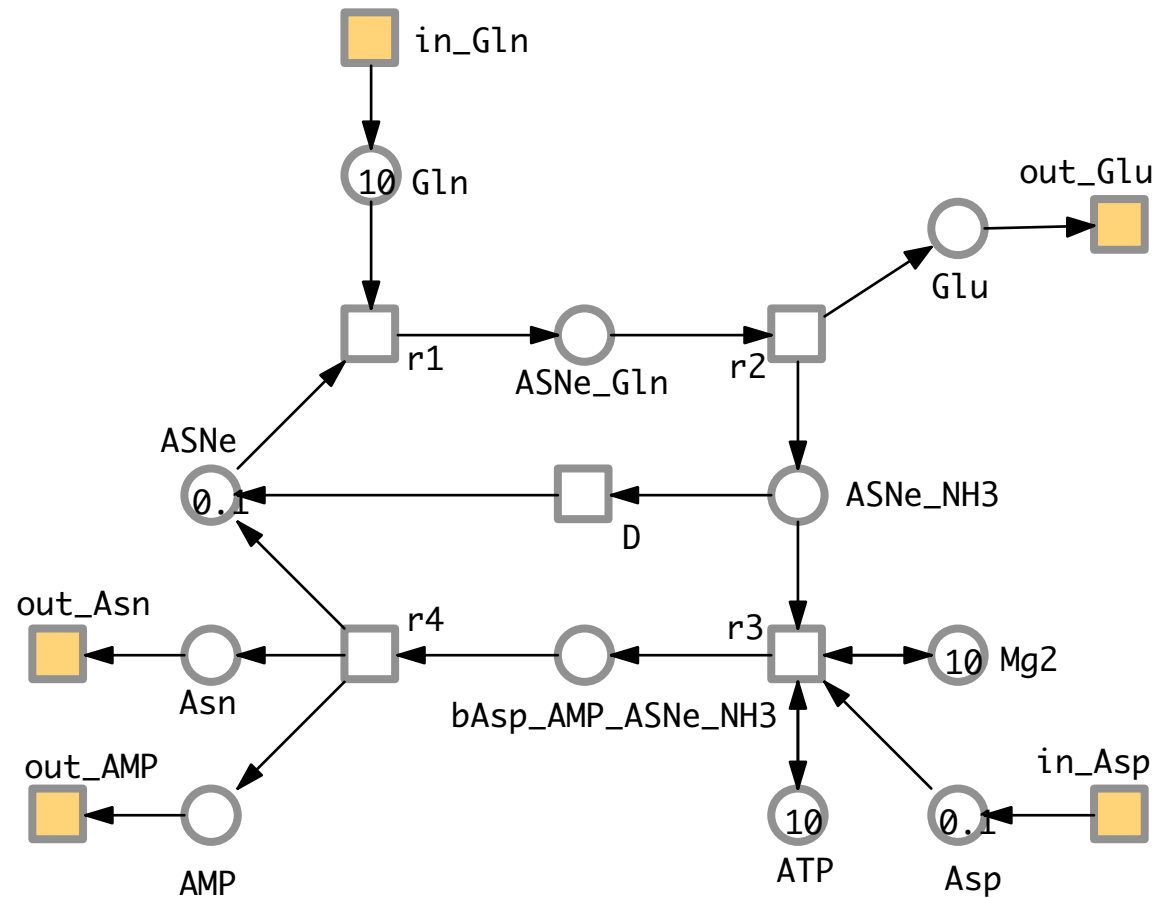
r2: ASNe_Gln $\rightarrow$ Glu + ASNe_NH3

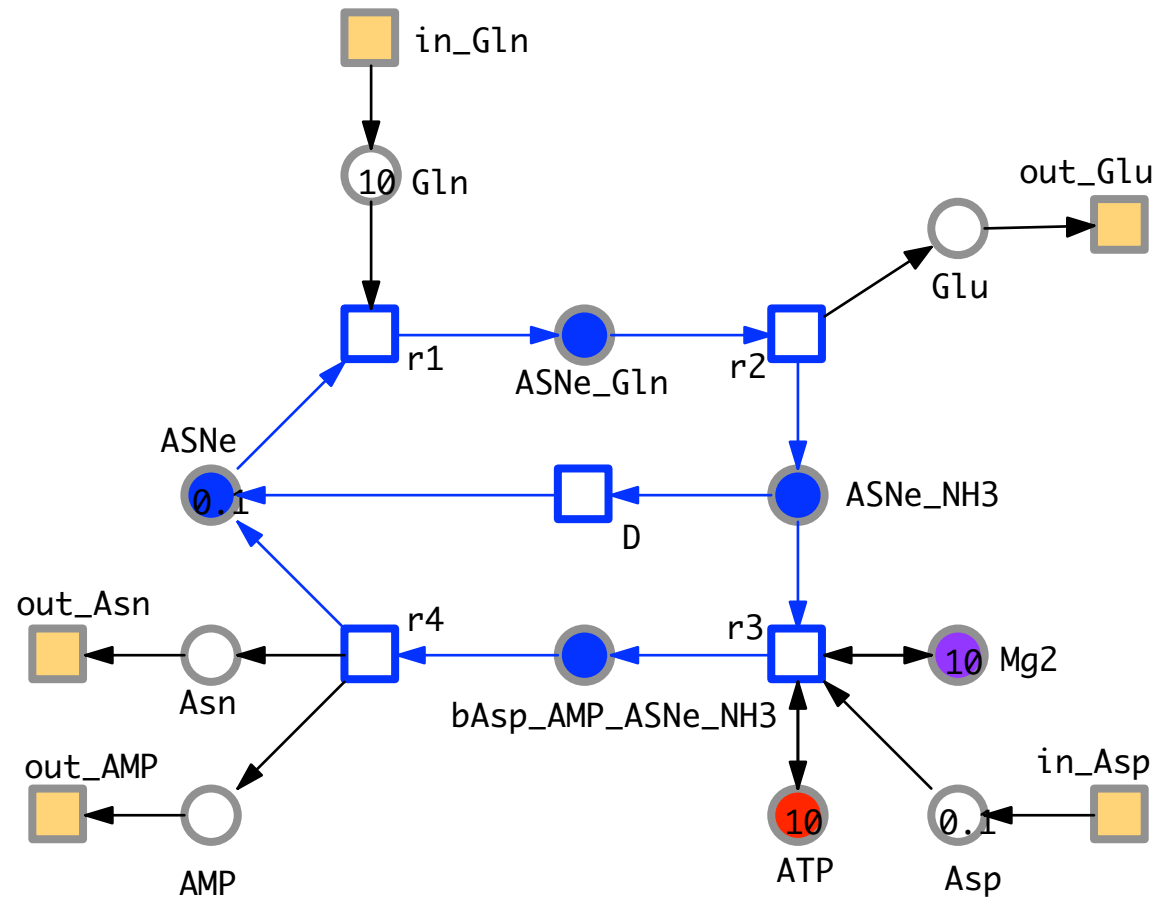
r3: ASNe_NH3 + Asp + ATP + Mg2 $\rightarrow$ bAsp_AMP_ASNe_NH3 + ATP + Mg2

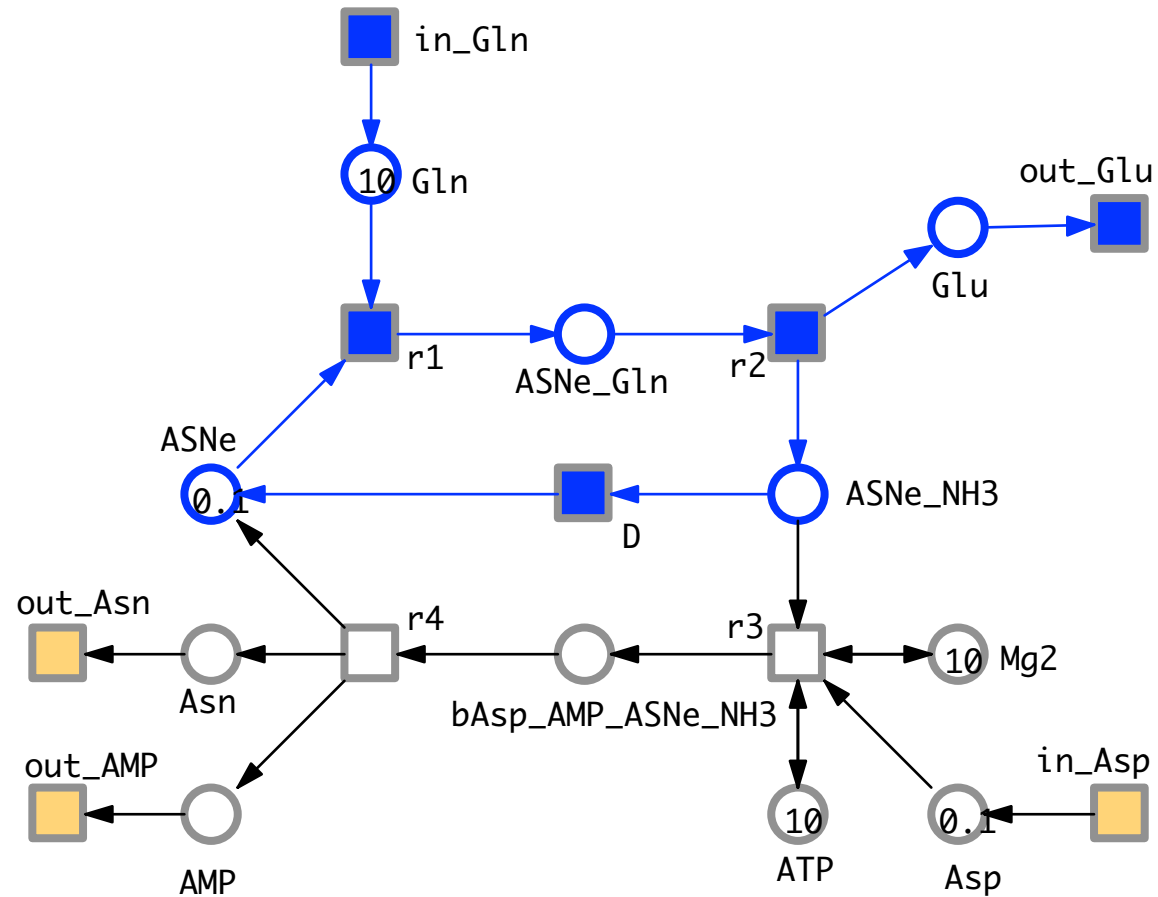
r4: bAsp_AMP_ASNe_NH3 $\rightarrow$ ASNe + Asn + AP

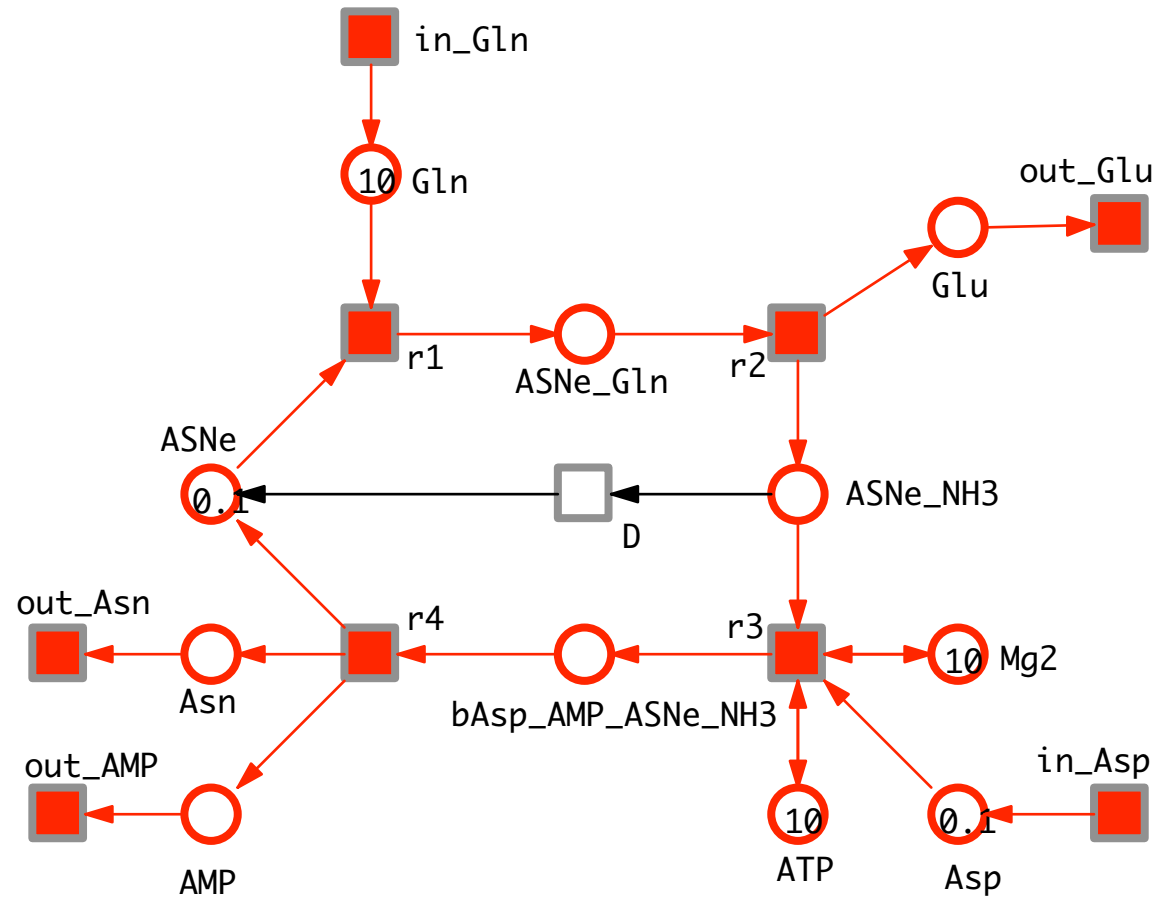




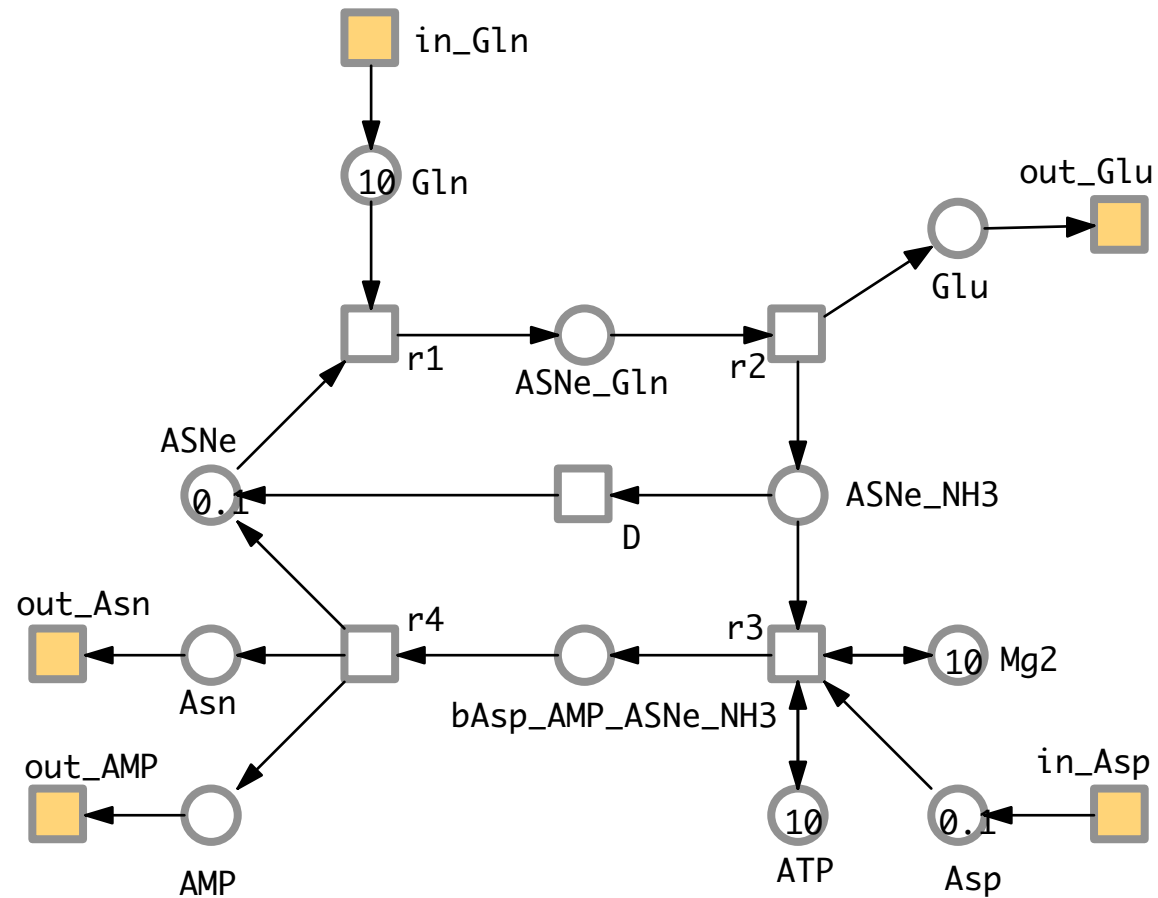




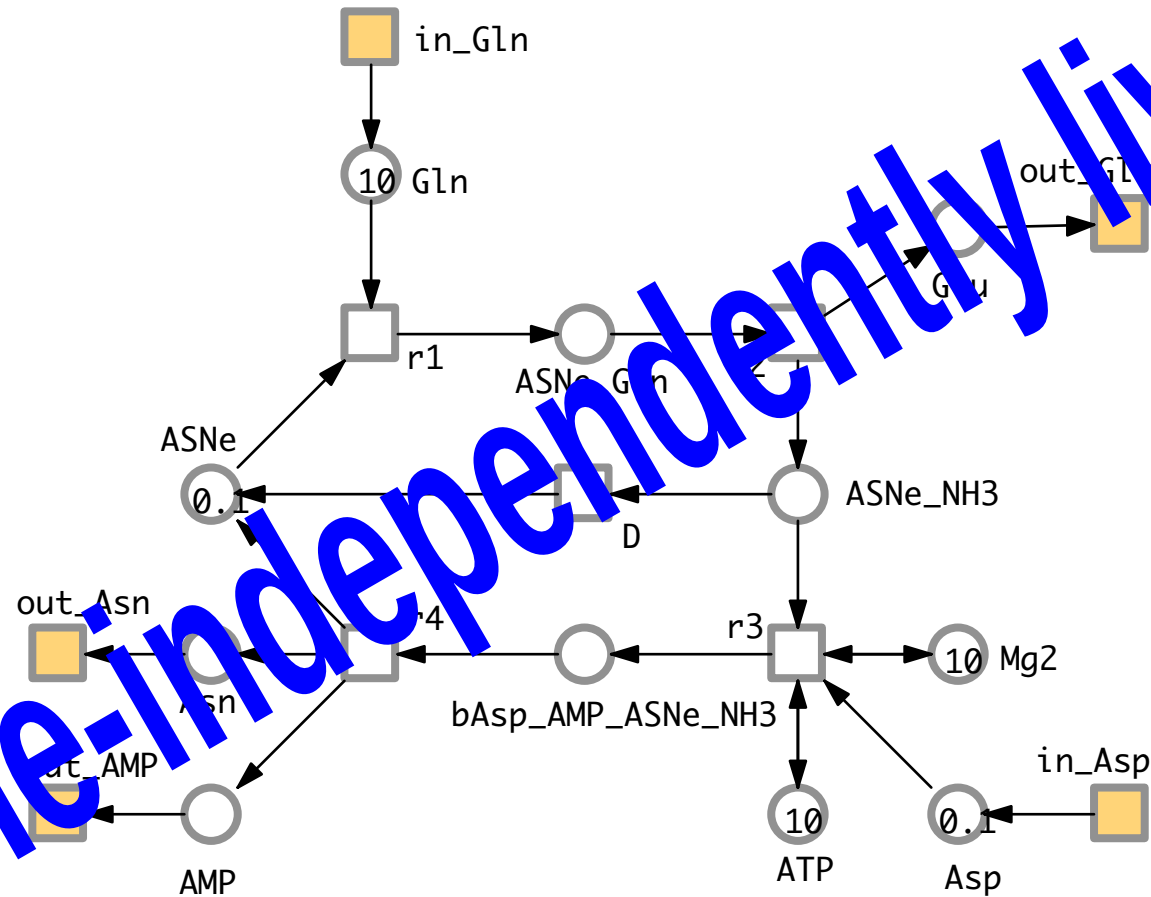




STP & ES -> LIVE



STP & ES -> LIVE



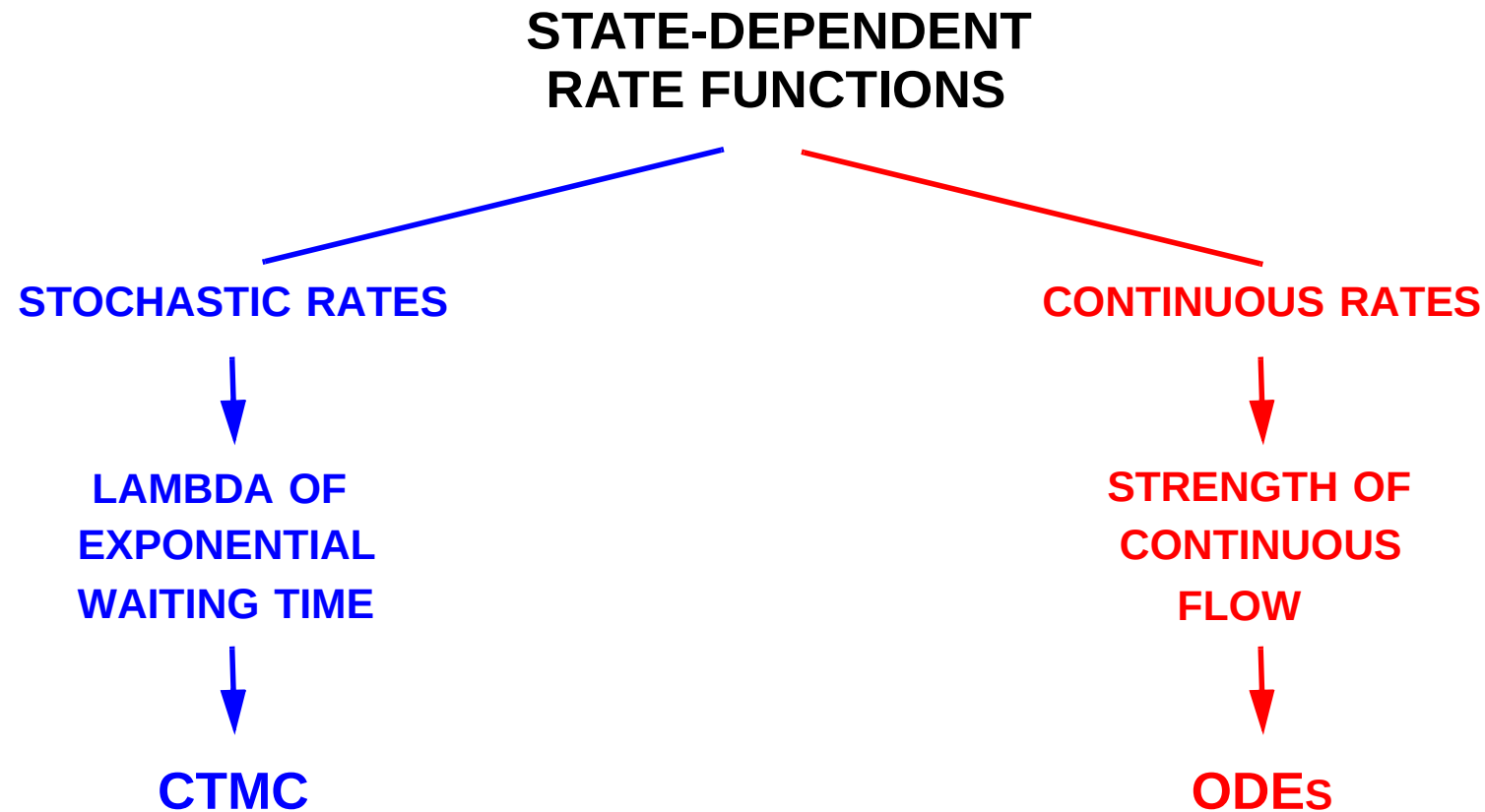
... AND THEN THERE WAS TIME

PN & BioModel Engineering



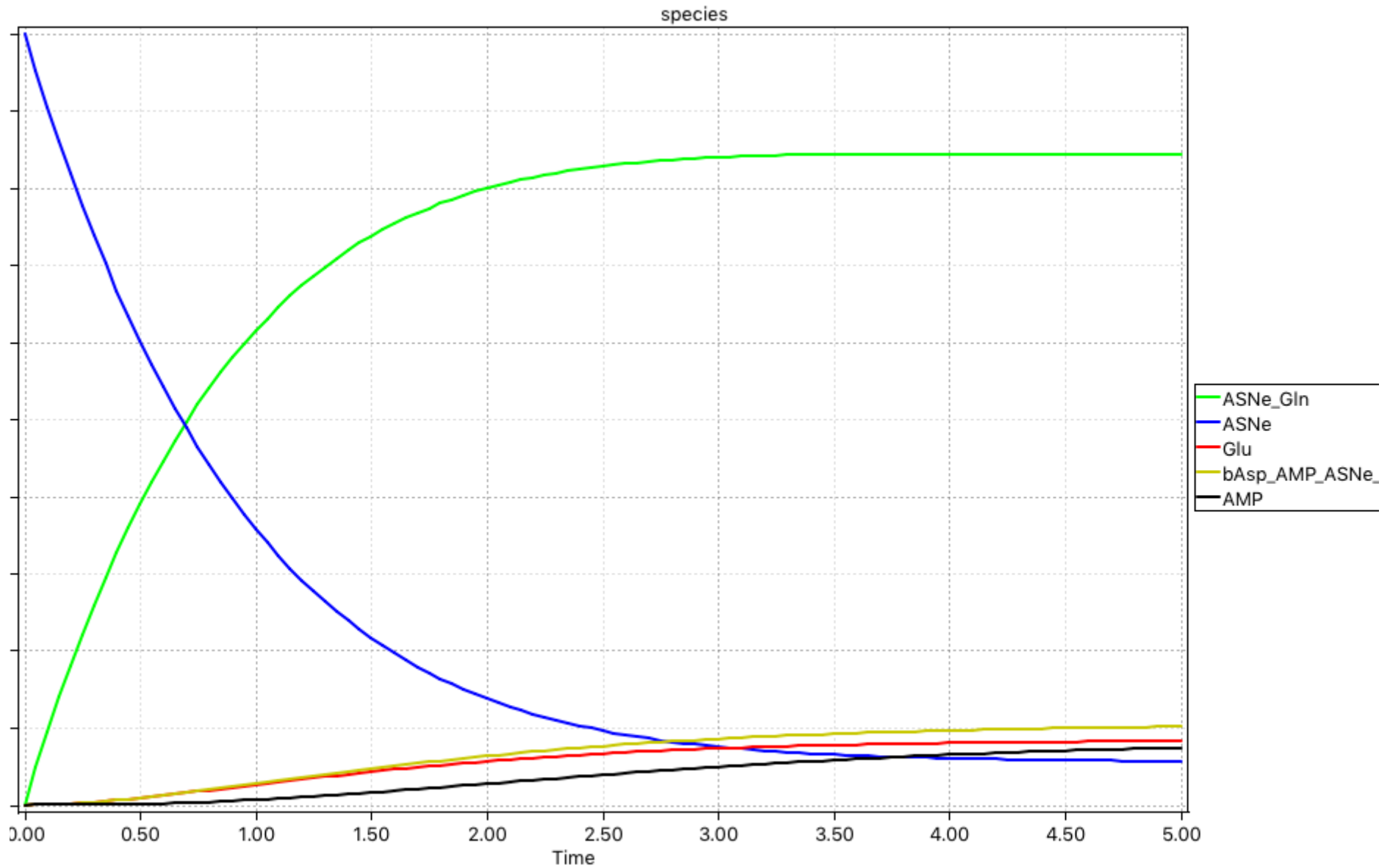
Canary Wharf, 20/08/2011

STATE-DEPENDENT RATE FUNCTIONS

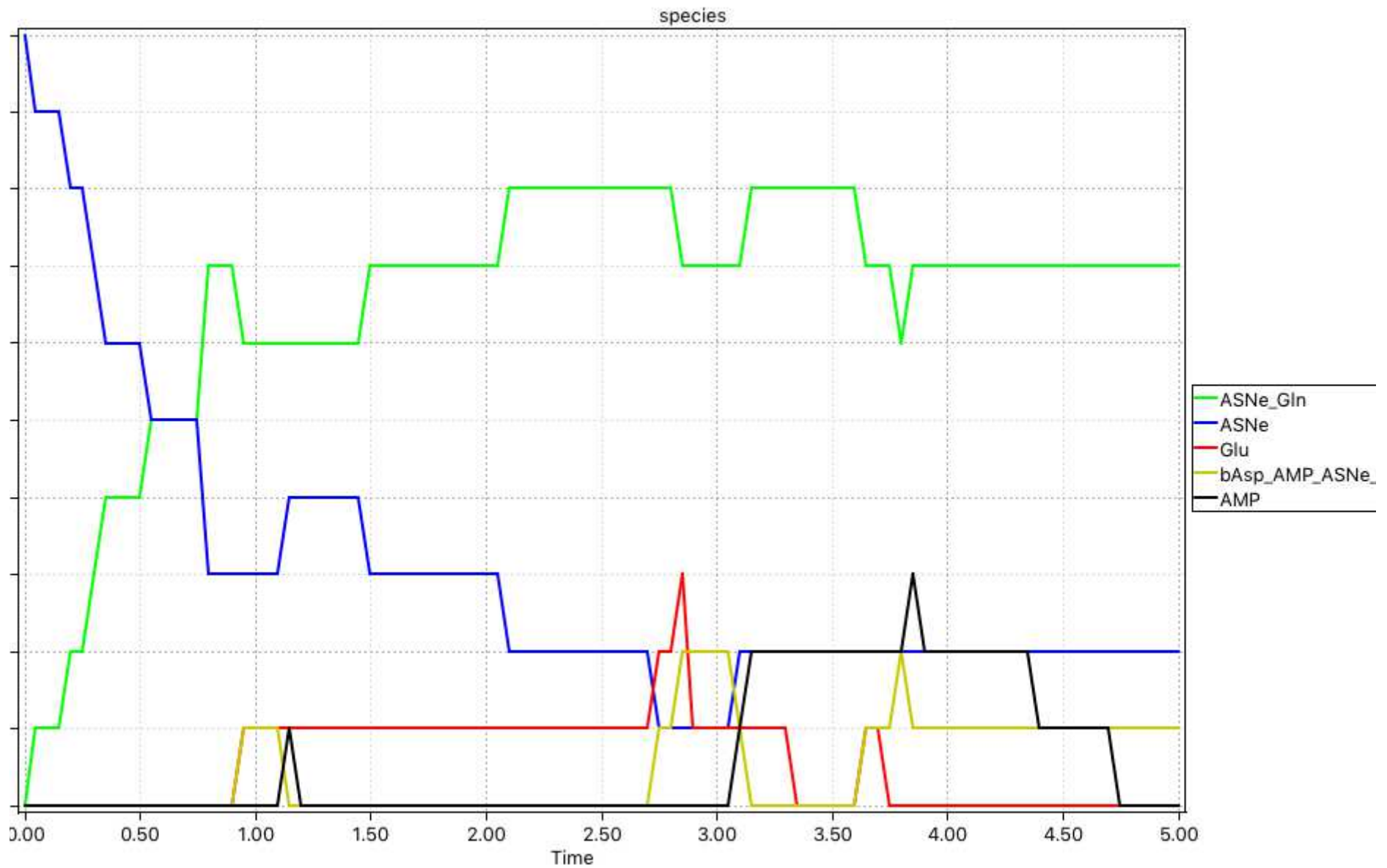


-> supported by, e.g., COPASI, Dizzy, ..., Snoopy

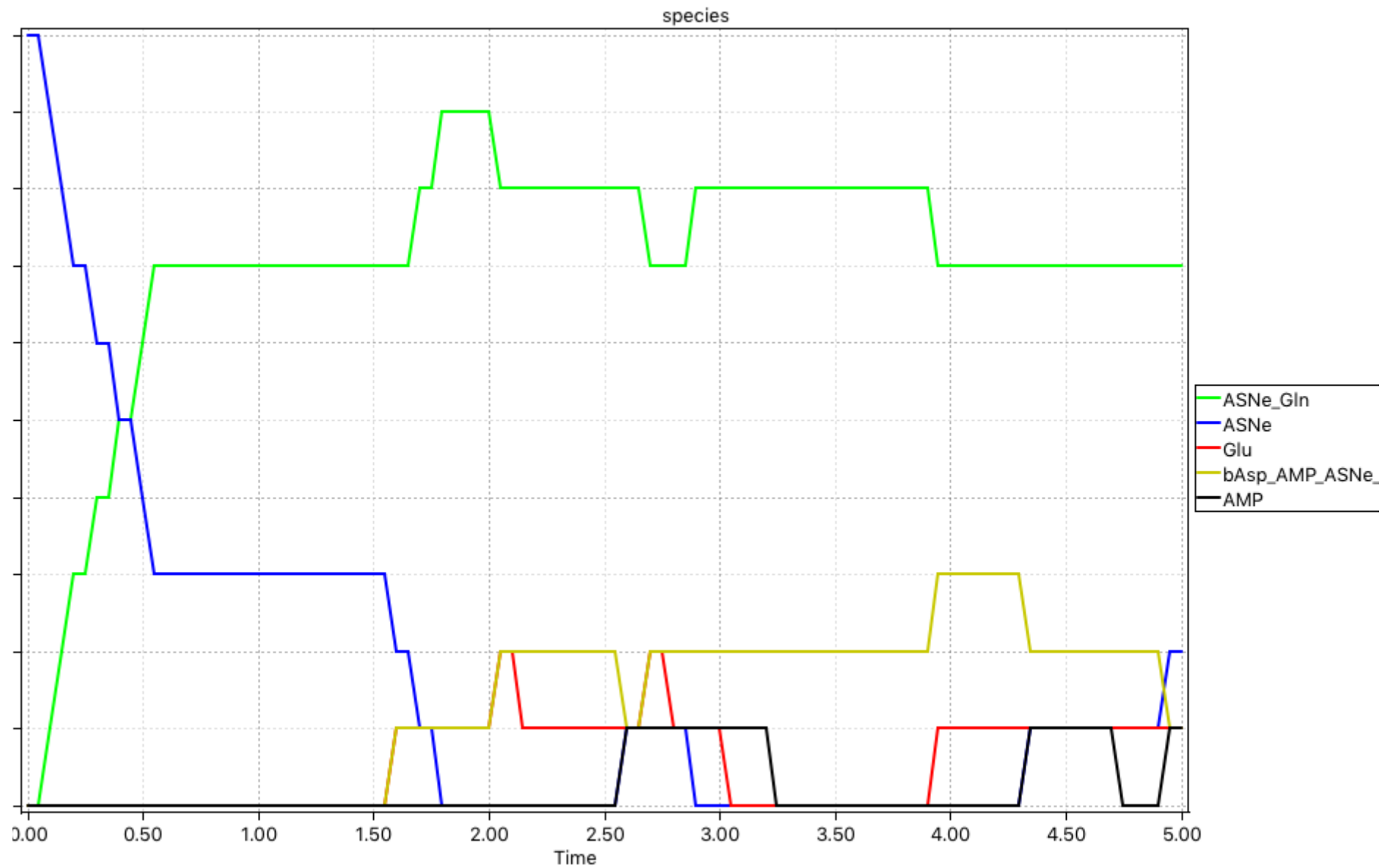
Ex: WHEAT - DETERMINISTIC RUN (ODEs)



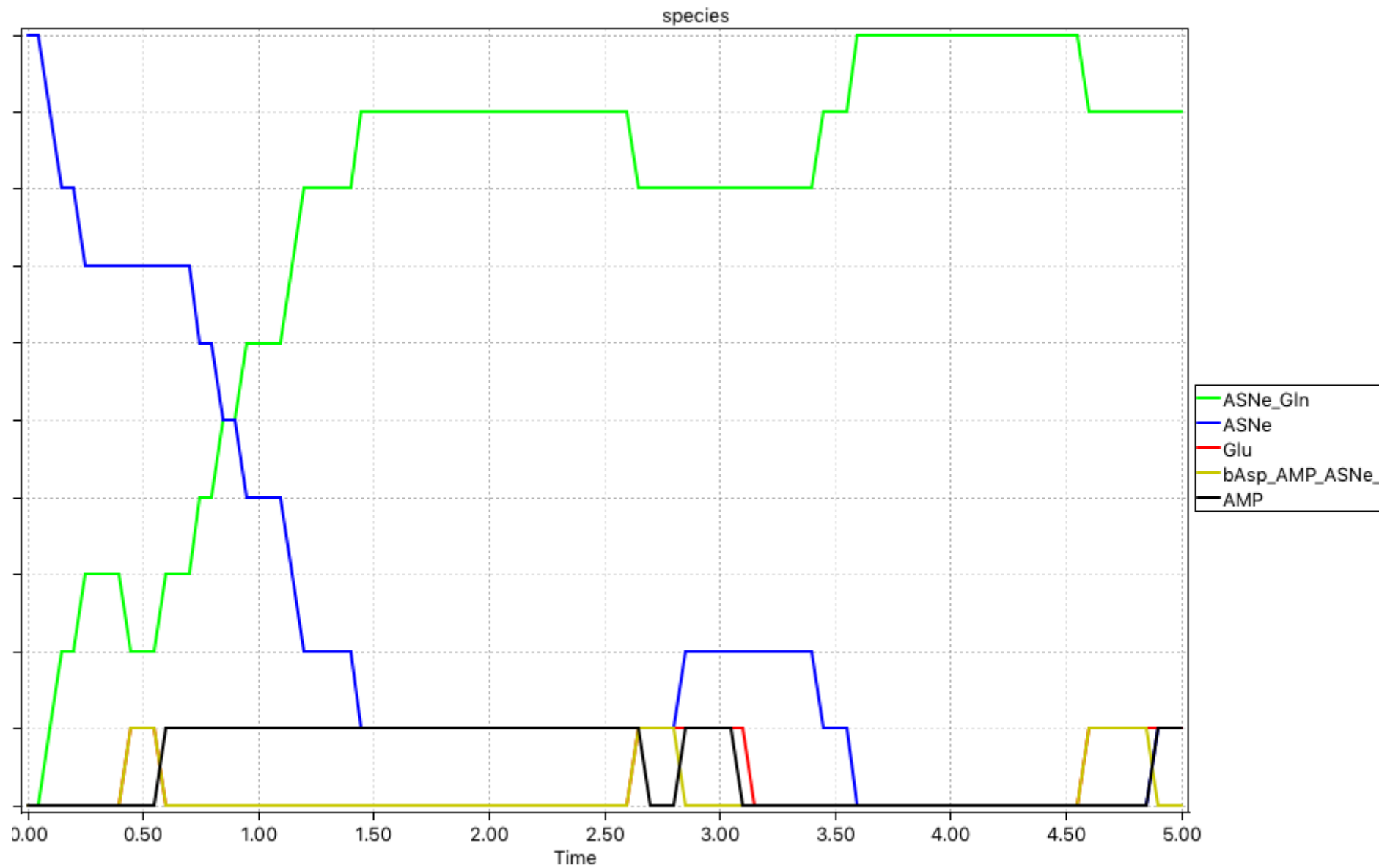
Ex: WHEAT - STOCHASTIC RUNS - 10 A



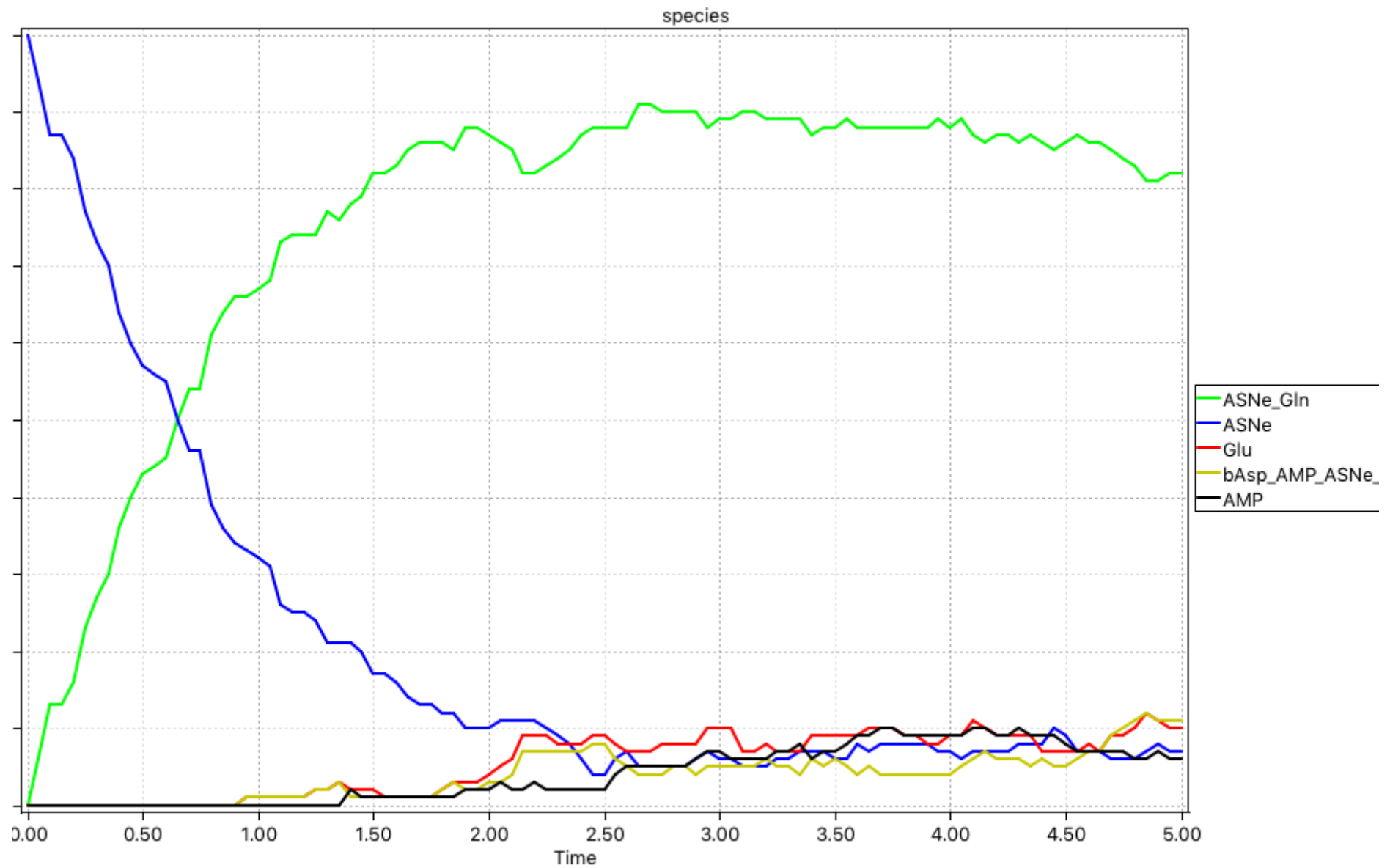
Ex: WHEAT - STOCHASTIC RUNS - 10 B



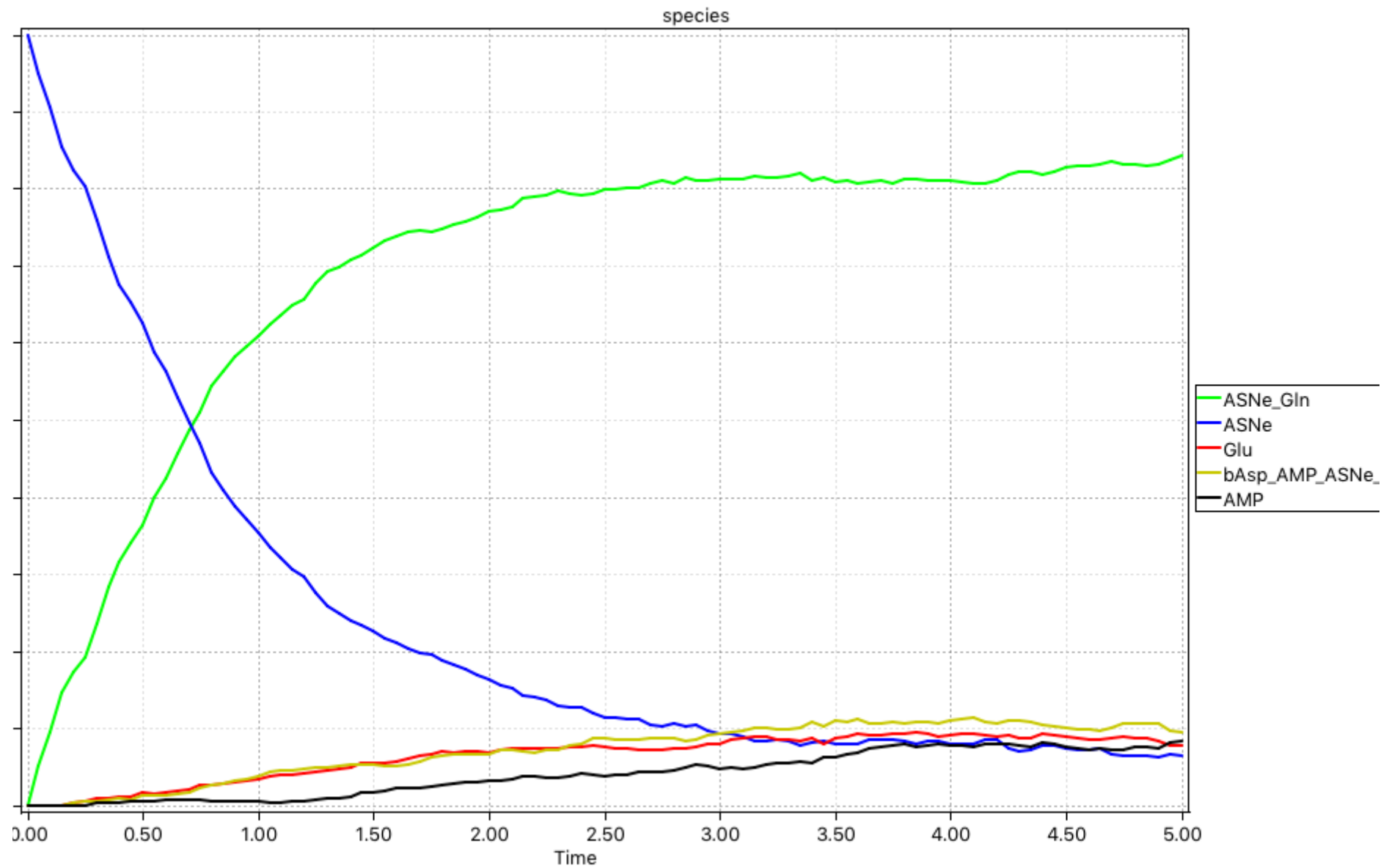
Ex: WHEAT - STOCHASTIC RUNS - 10 c



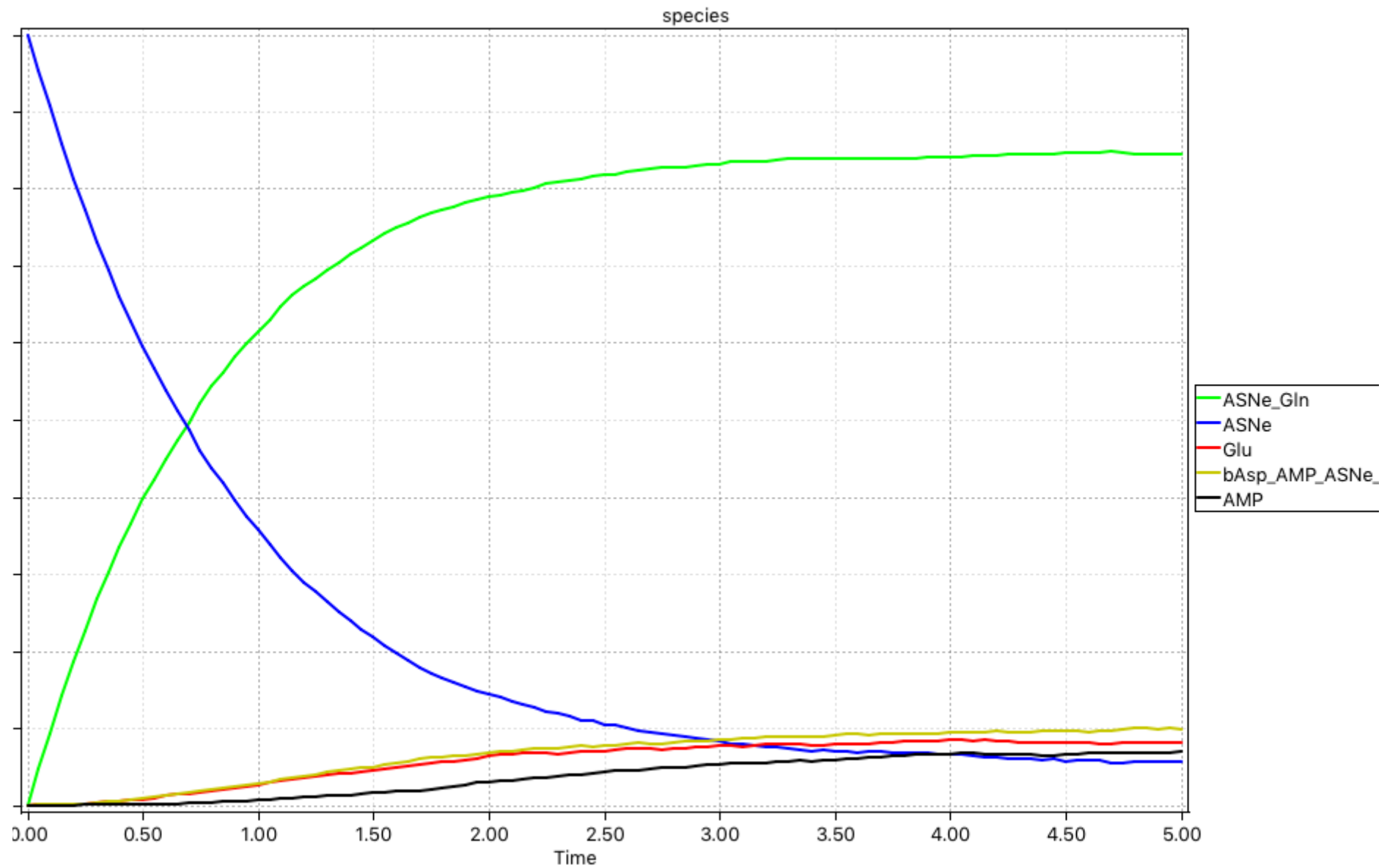
Ex: WHEAT - STOCHASTIC RUNS - 100



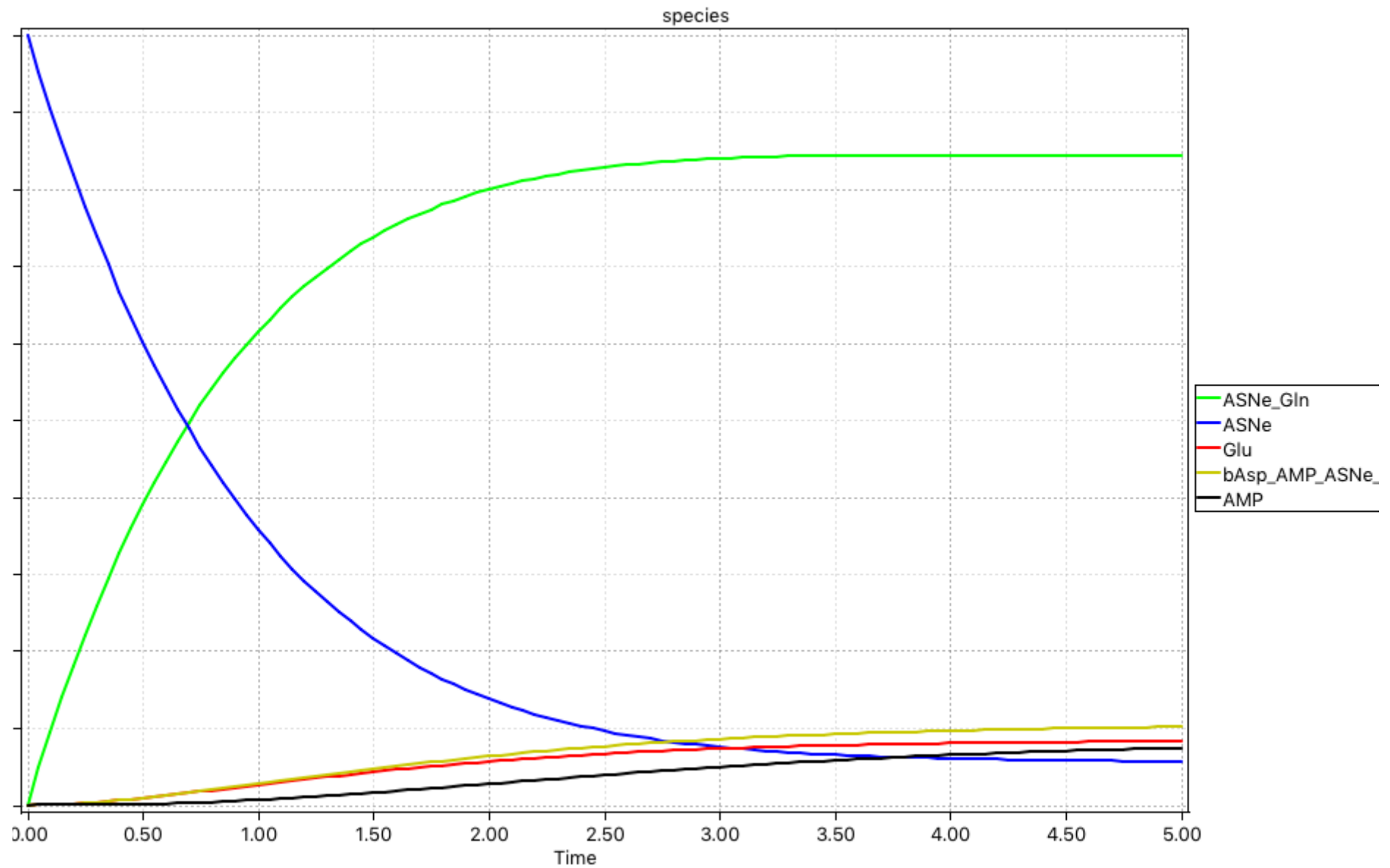
Ex: WHEAT - STOCHASTIC RUNS - 1,000



Ex: WHEAT - STOCHASTIC RUNS - 10,000



Ex: WHEAT - DETERMINISTIC RUN (ODEs)

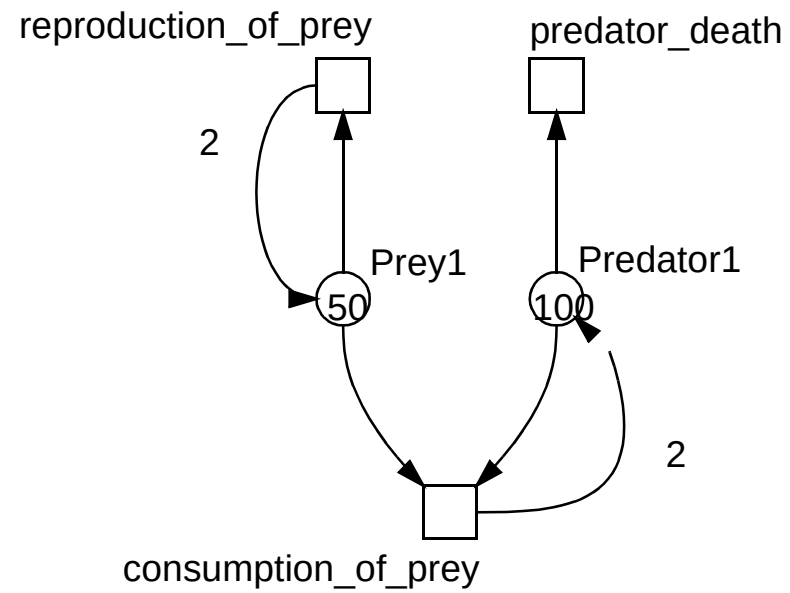


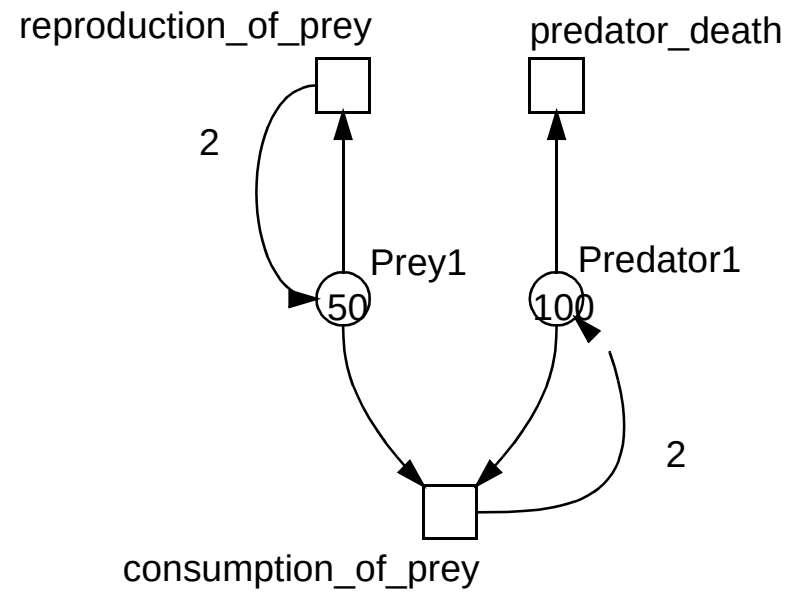
... AND THEN THERE WAS COLOUR

PN & BioModel Engineering

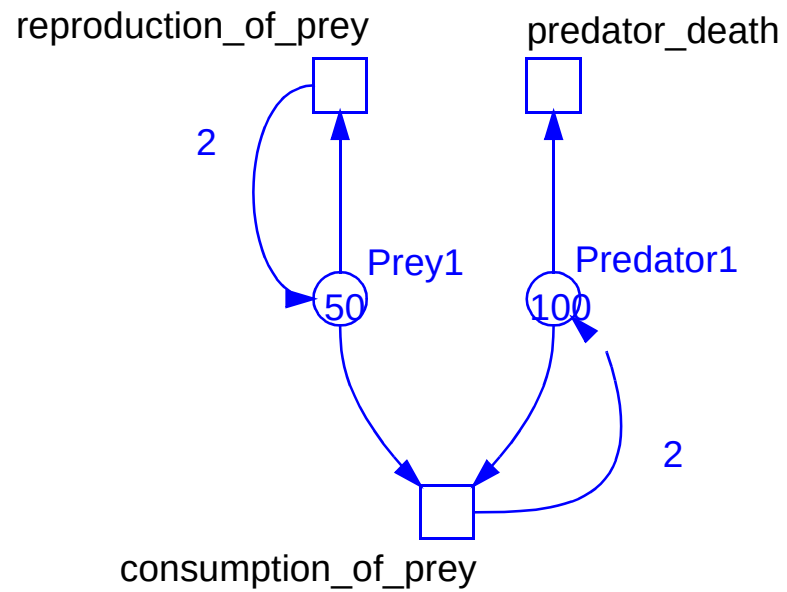


Kew Gardens, 24/04/2011

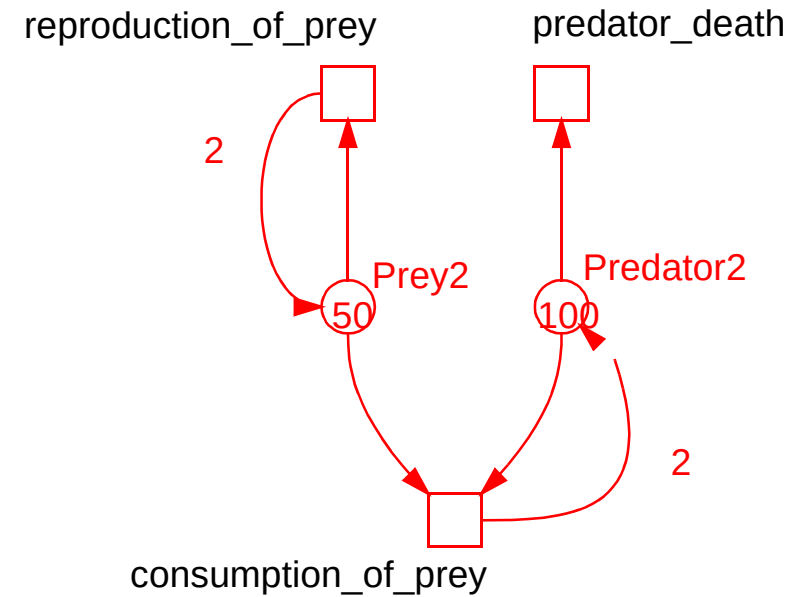




sub-system1



sub-system2



❑ definitions

colourset CS = 1-2;

var x : CS;

❑ better:

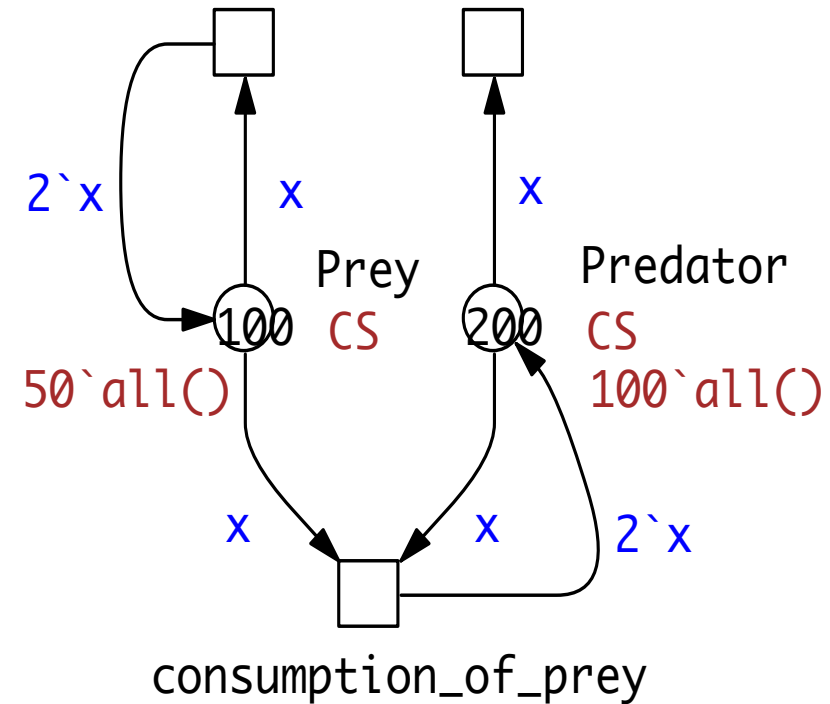
const SIZE = 2;

colourset CS = 1-SIZE;

var x : CS;



reproduction_of_prey predator_death



❑ definitions

colourset CS = 1-2;

var x : CS;

❑ better:

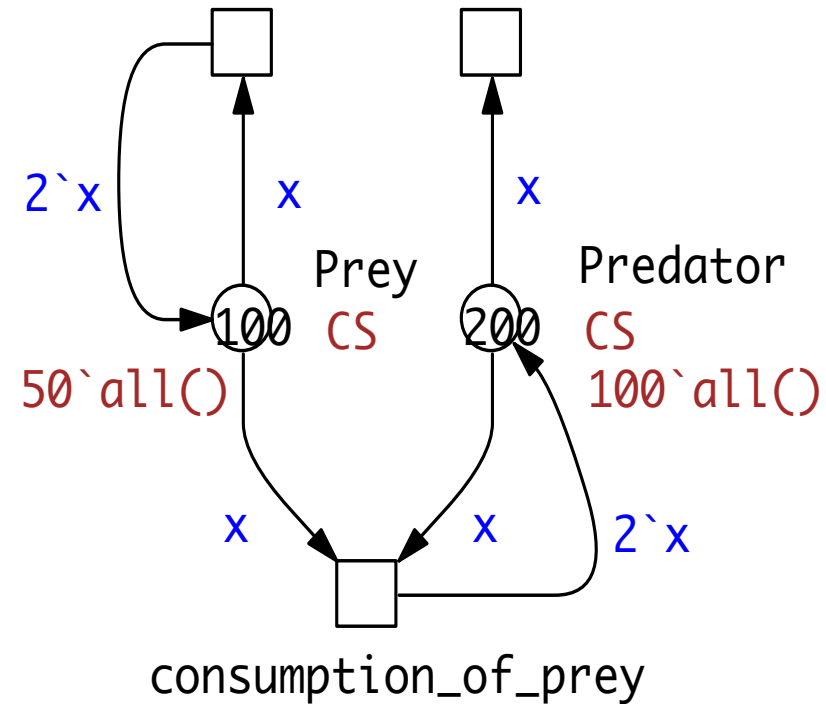
const SIZE = 2;

colourset CS = 1-SIZE;

var x : CS;



reproduction_of_prey predator_death



❑ changing SIZE adapts the model to various scenarios

COLOURING SPACE

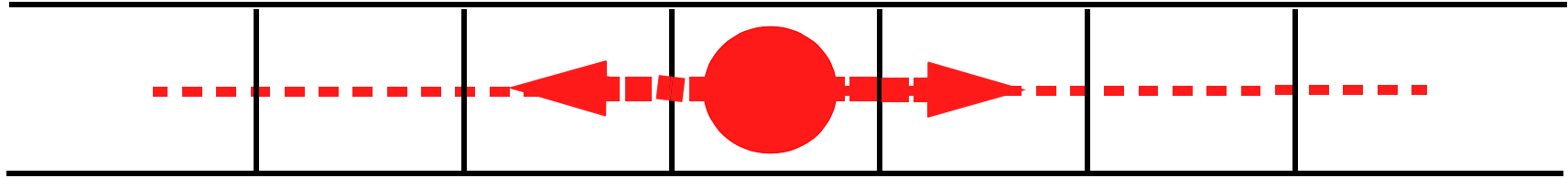
EXAMPLE 1:

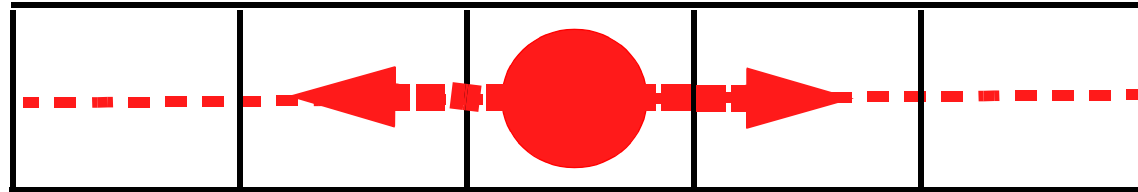
DIFFUSION IN SPACE

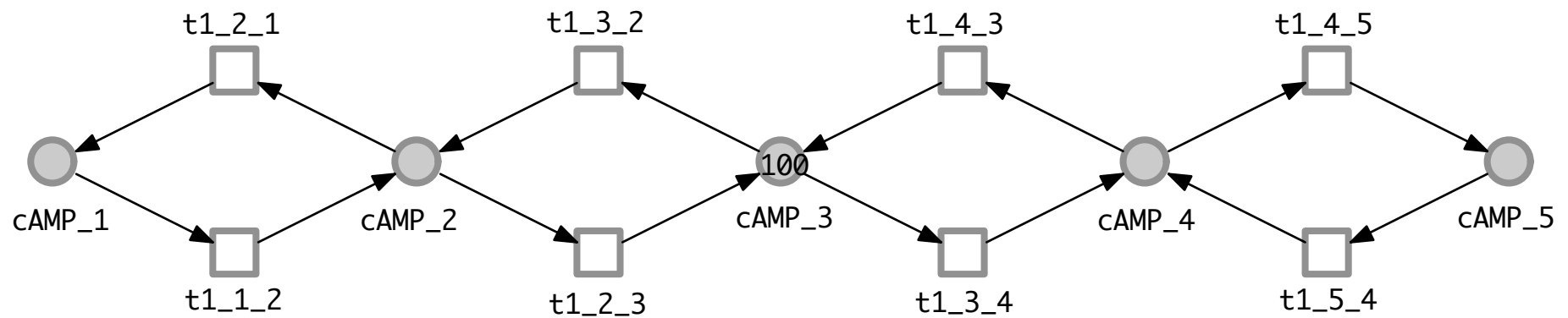
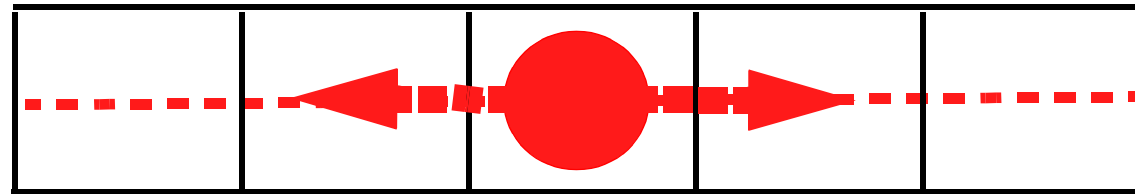
Heiner, Gilbert, Liu, Saunders:
Colouring Space - A Coloured Framework for Spatial Modelling in Systems Biology;
Proc. PETRI NETS 2013, LNCS 7927, 2013.



Richmond, 13/09/2011







❑ definitions

```
const D1 = 5;           // grid size  
const MIDDLE = D1/2;  
colorset Grid1D = 1-D1; // grid positions  
var x,y : Grid1D;
```

❑ definitions

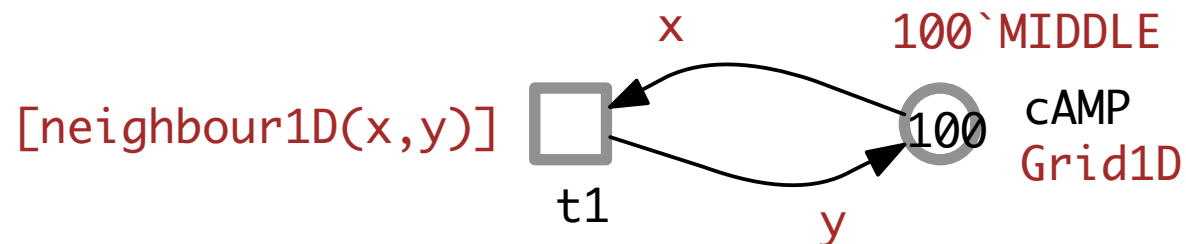
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var x,y : Grid1D;

function neighbour1D (Grid1D x,a) bool:
  // a is neighbour of x
  ( a=x-1 | a=x+1 ) & (1<=a) & (a<=D1);
```

❑ definitions

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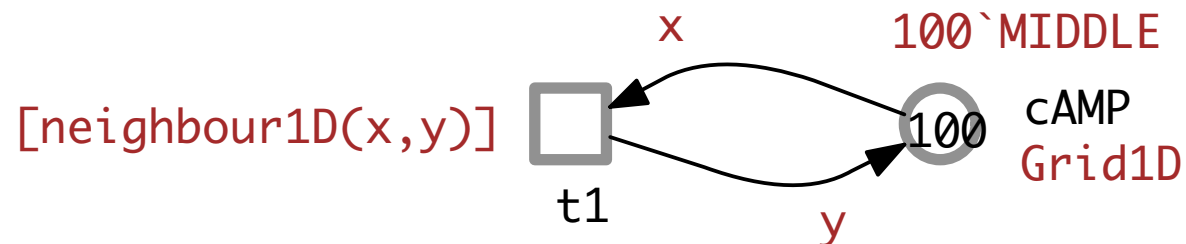
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```



❑ movement = changing colour

$$\frac{dc_1}{dt} = k \cdot c_2 - k \cdot c_1$$

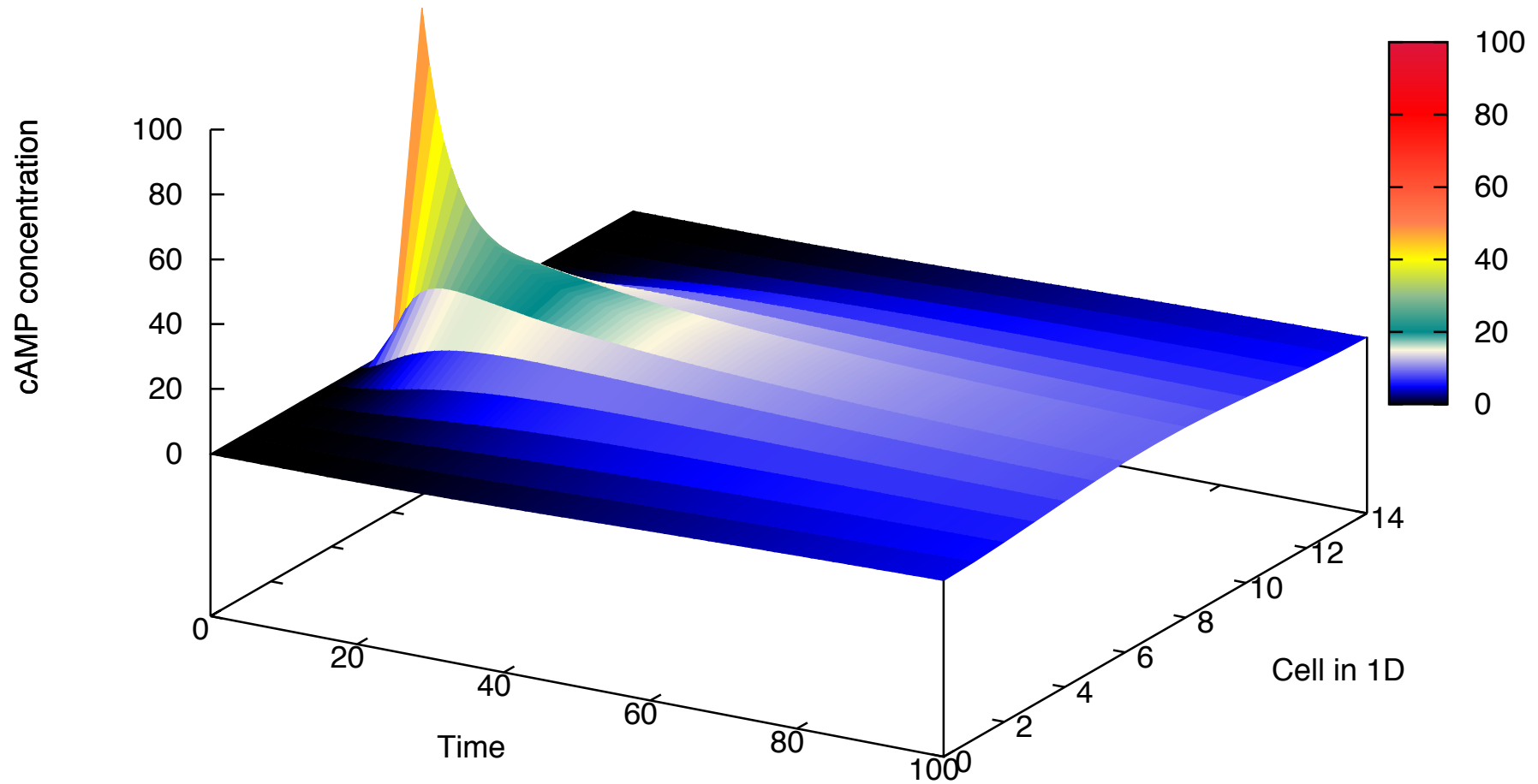
$$\frac{dc_2}{dt} = k \cdot c_1 + k \cdot c_3 - 2 \cdot k \cdot c_2$$

$$\frac{dc_3}{dt} = k \cdot c_2 + k \cdot c_4 - 2 \cdot k \cdot c_3$$

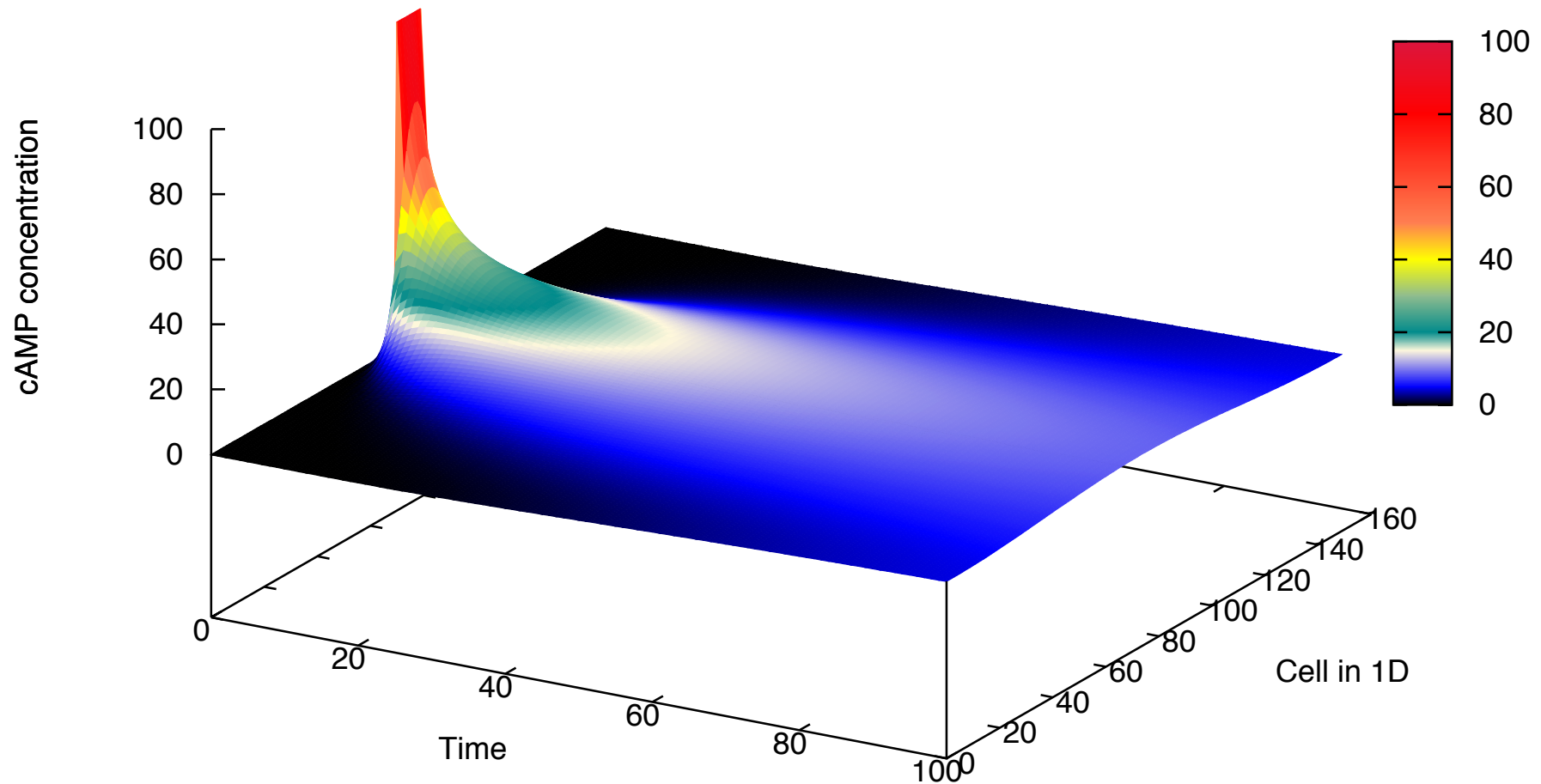
$$\frac{dc_4}{dt} = k \cdot c_3 + k \cdot c_5 - 2 \cdot k \cdot c_4$$

$$\frac{dc_5}{dt} = k \cdot c_4 - k \cdot c_5$$

with $c_i := cAMP_i$

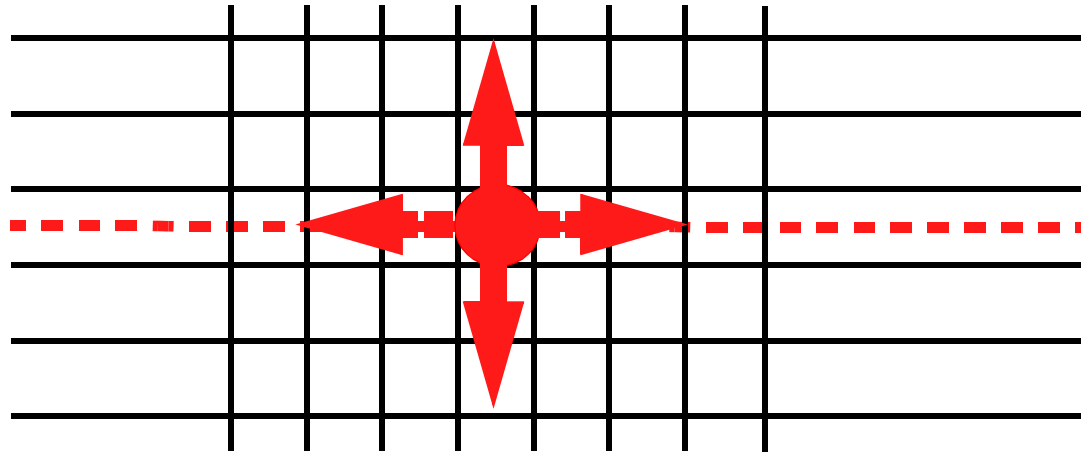


15 GRID POSITIONS

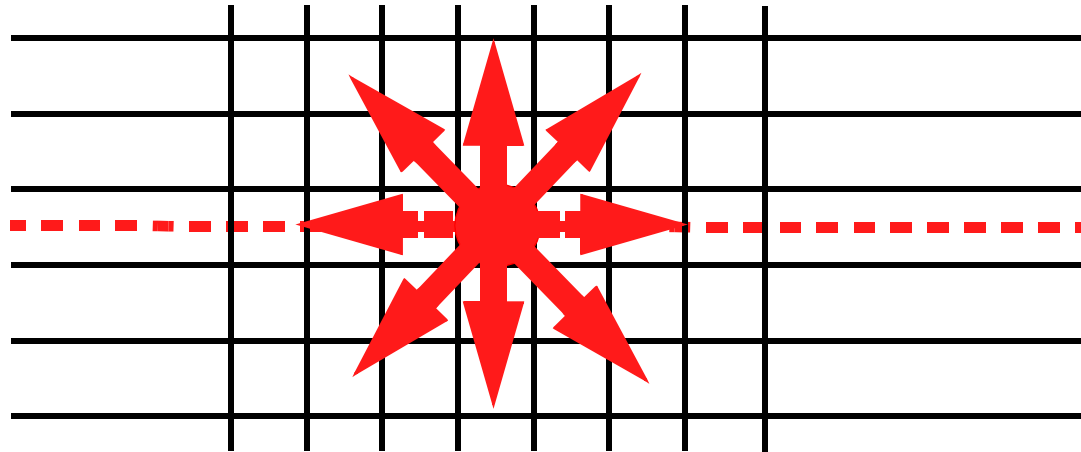


150 GRID POSITIONS, SCALING OF INITIAL MARKING AND RATES

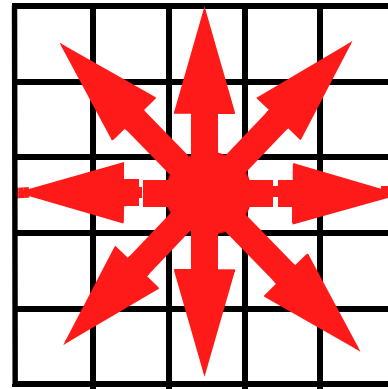
❏ SCHEME



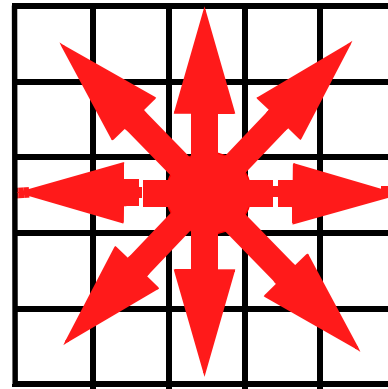
❑ SCHEME



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❑ SCHEME



❑ definitions

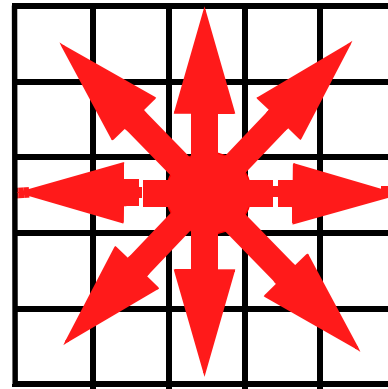
const D1 = 5;

// grid size first dimension

const D2 = D1;

// grid size second dimension

❑ SCHEME



❑ definitions

const *D1* = 5;

// grid size first dimension

const *D2* = *D1*;

// grid size second dimension

const *MIDDLE* = *D1*/2;

colorset *CD1* = 1-*D1*;

// row index

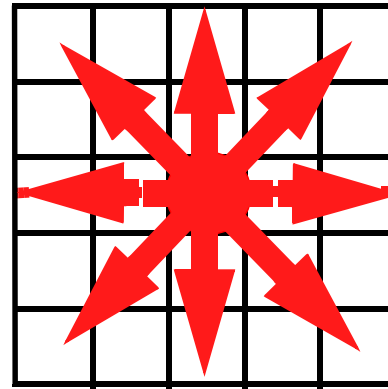
colorset *CD2* = 1-*D2*;

// column index

colorset *Grid2D* = *CD1* x *CD2*;

// 2D grid

❑ SCHEME



❑ definitions

const D1 = 5;

// grid size first dimension

const D2 = D1;

// grid size second dimension

const MIDDLE = D1/2;

colorset CD1 = 1-D1;

// row index

colorset CD2 = 1-D2;

// column index

colorset Grid2D = CD1 x CD2;

// 2D grid

var x, a : CD1;

var y, b : CD2;

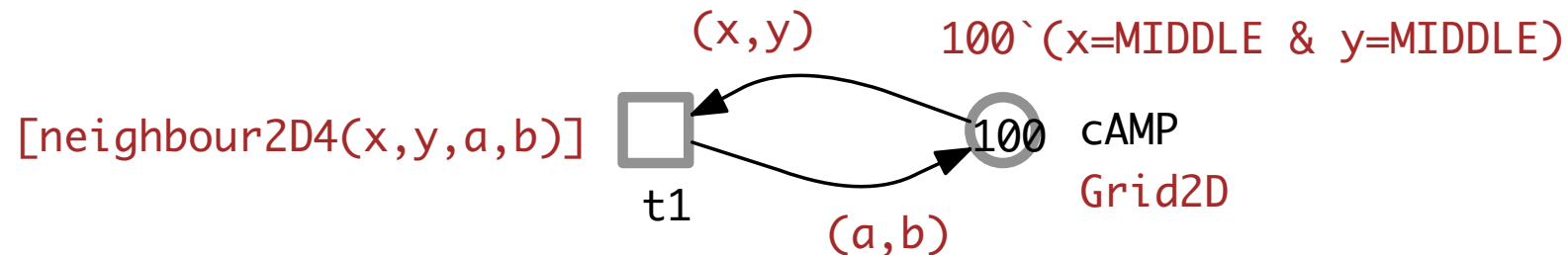
❑ four neighbours

function neighbour2D4 (CD1 x, CD2 y, CD1 a, CD2 b) **bool**:

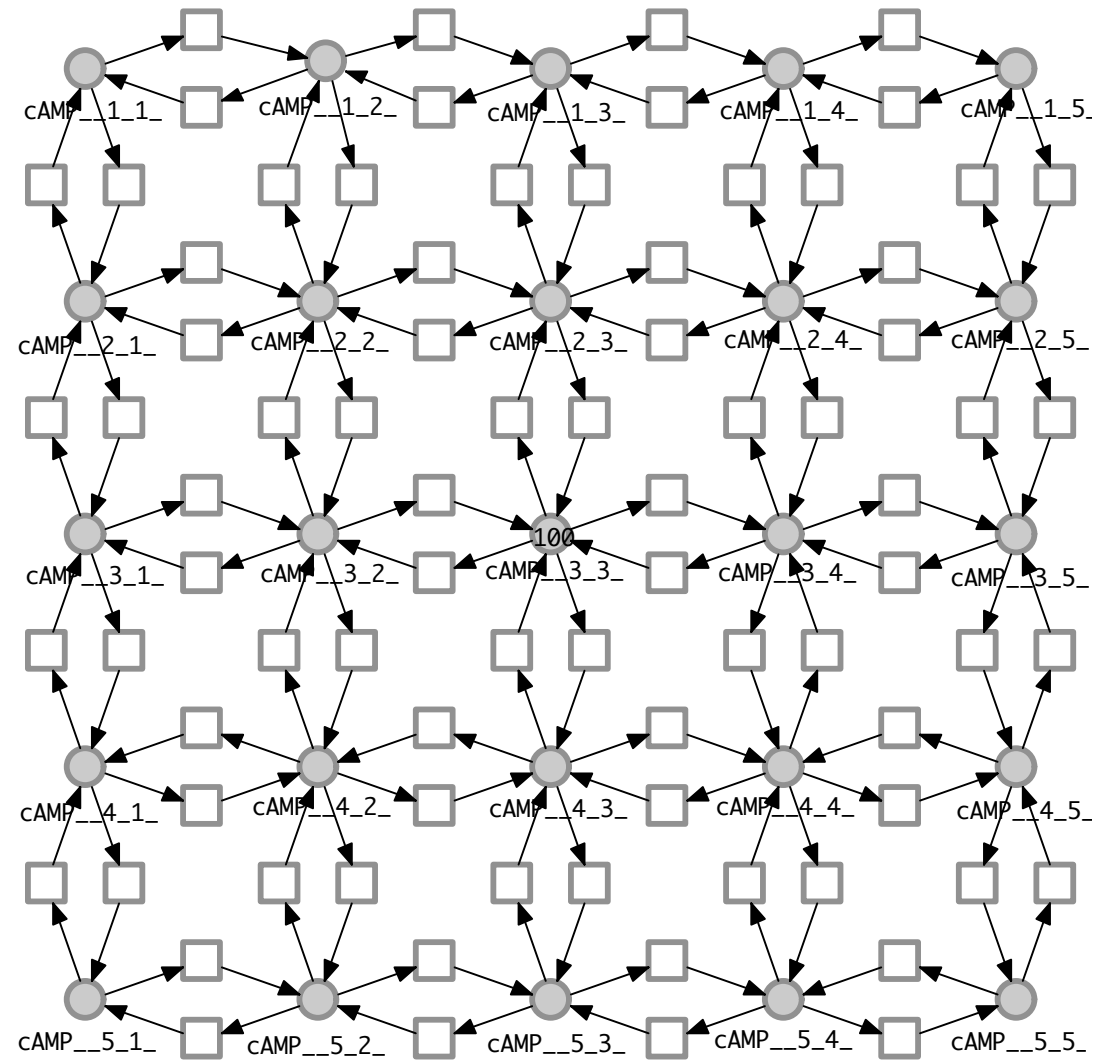
// (a,b) is one of the up to four neighbours of (x,y)

$(a=x \ \& \ b=y-1) \mid (a=x \ \& \ b=y+1)$

$\mid (b=y \ \& \ a=x-1) \mid (b=y \ \& \ a=x+1);$



Ex1: DIFFUSION - 2D4 NEIGHBOURHOOD



❑ eight neighbours

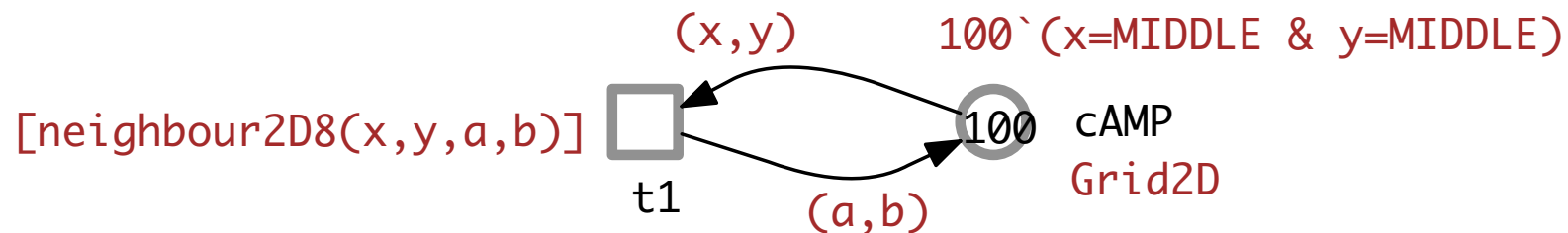
function neighbour2D8 (CD1 x, CD2 y, CD1 a, CD2 b) **bool**:

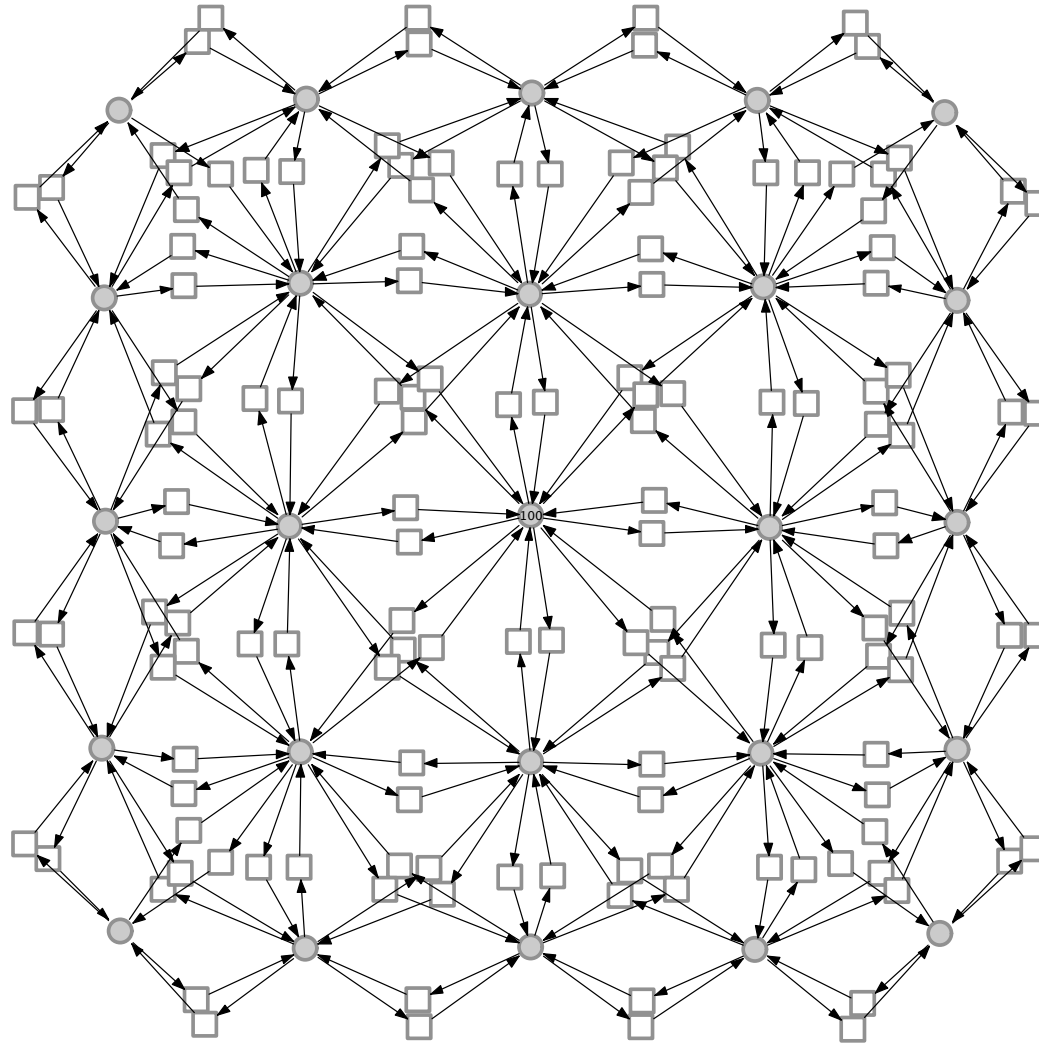
// (a,b) is one of the up to eight neighbours of (x,y)

$(a=x-1 \mid a=x \mid a=x+1) \ \& \ (b=y-1 \mid b=y \mid b=y+1)$

$\& \ !(a=x \ \& \ b=y)$

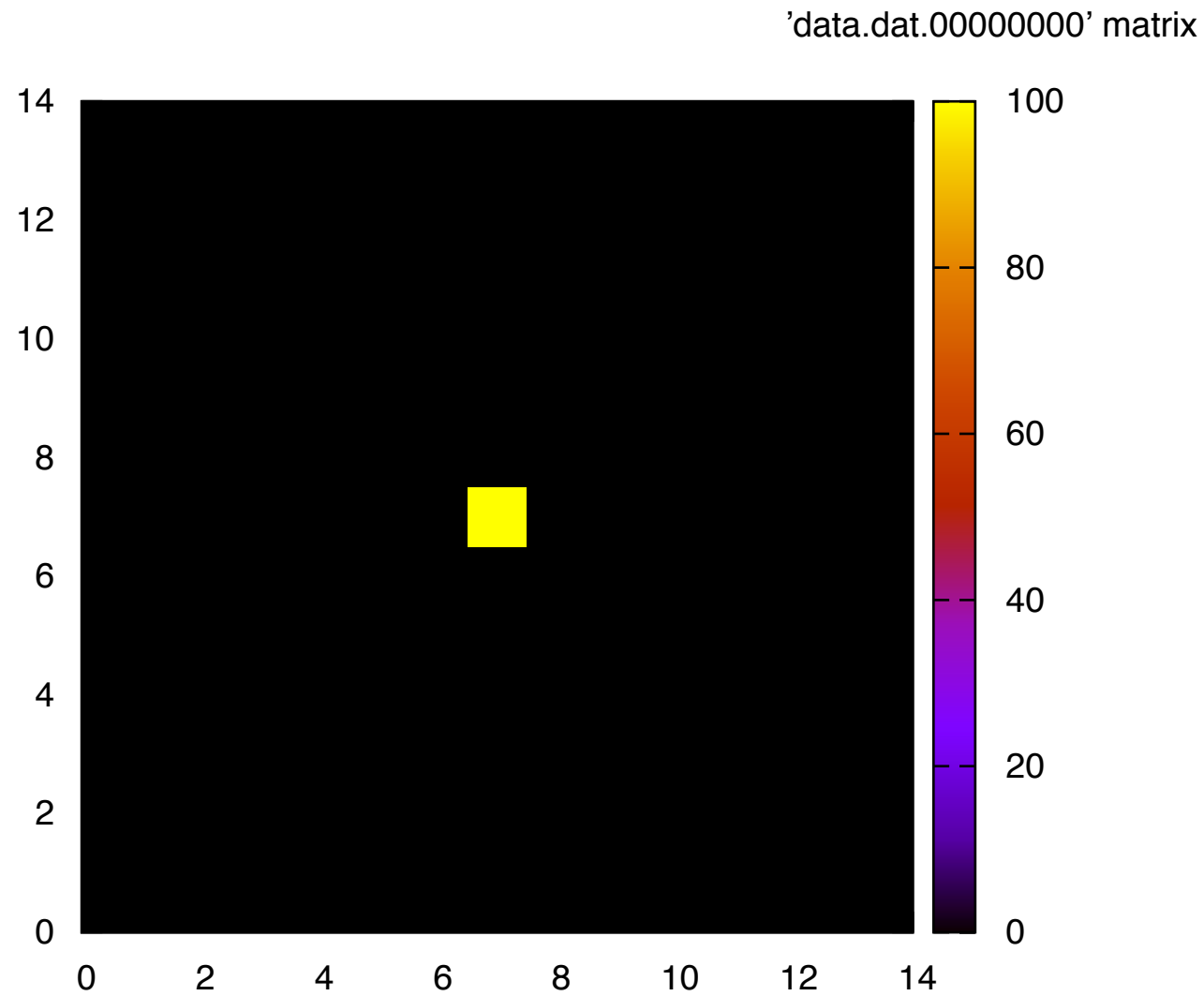
$\& \ (1 \leq a \ \& \ a \leq D1) \ \& \ (1 \leq b \ \& \ b \leq D2);$

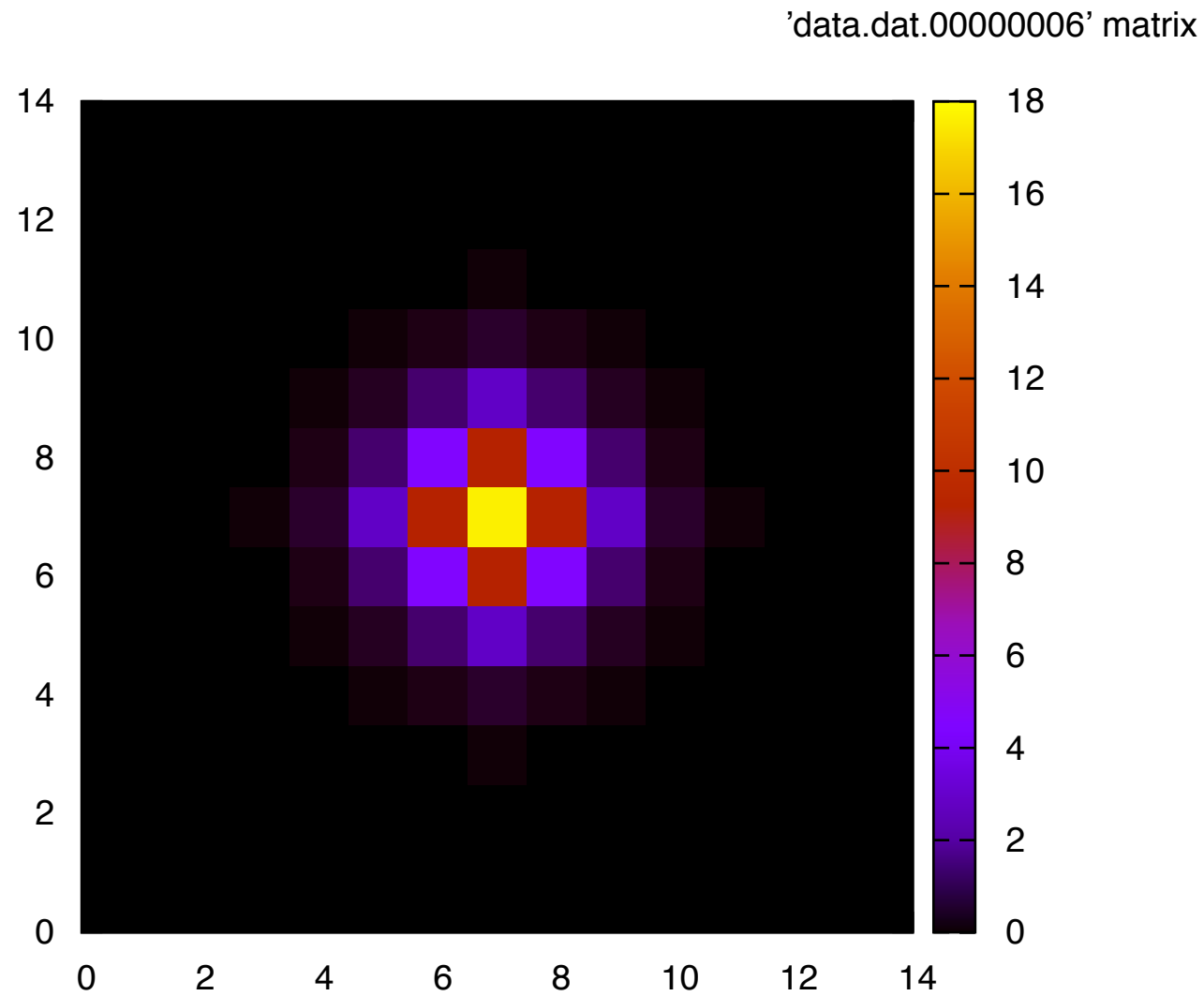




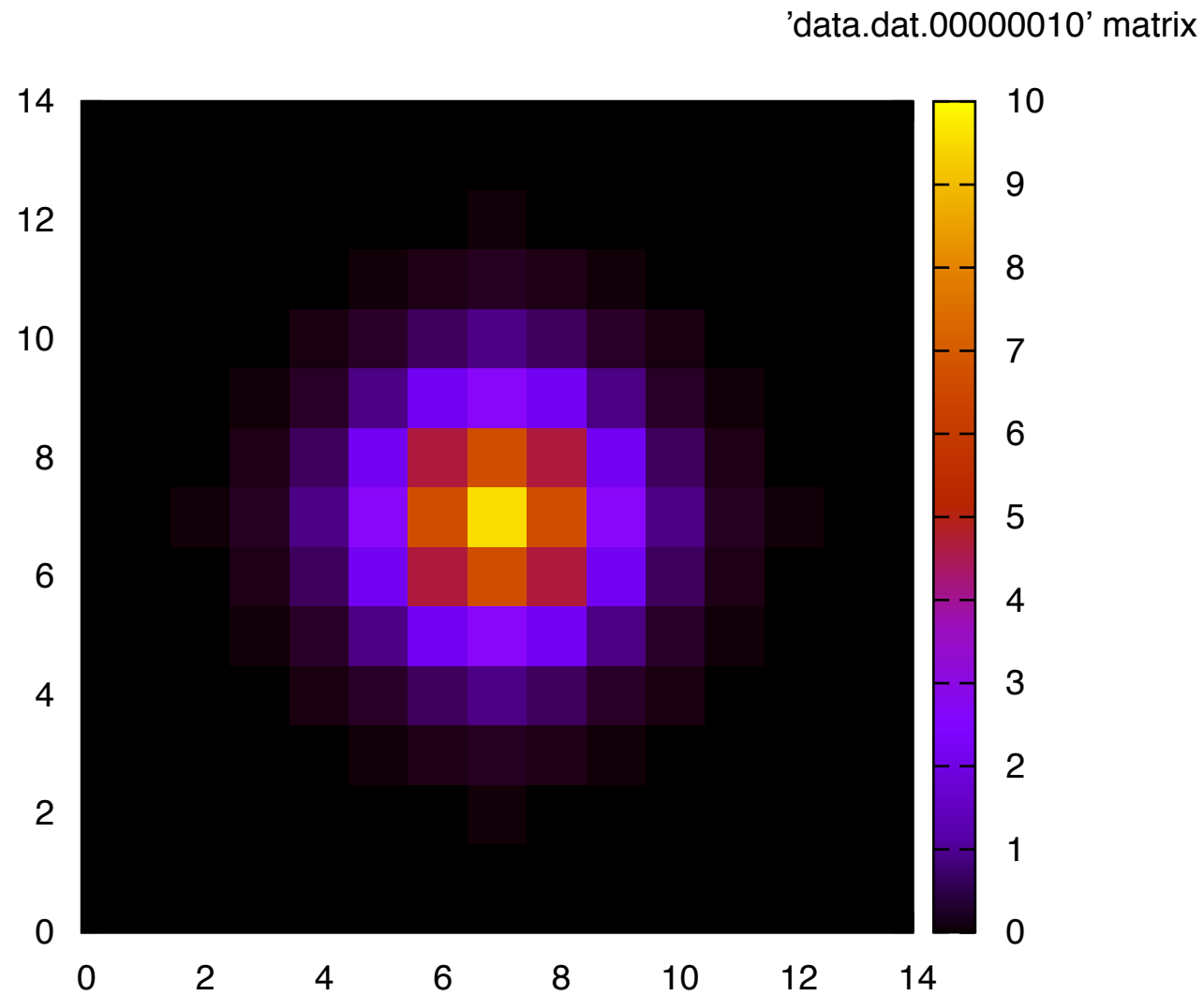
Ex1: DIFFUSION - 2D4 NEIGHBOURHOOD, 15X15

PN & BioModel Engineering

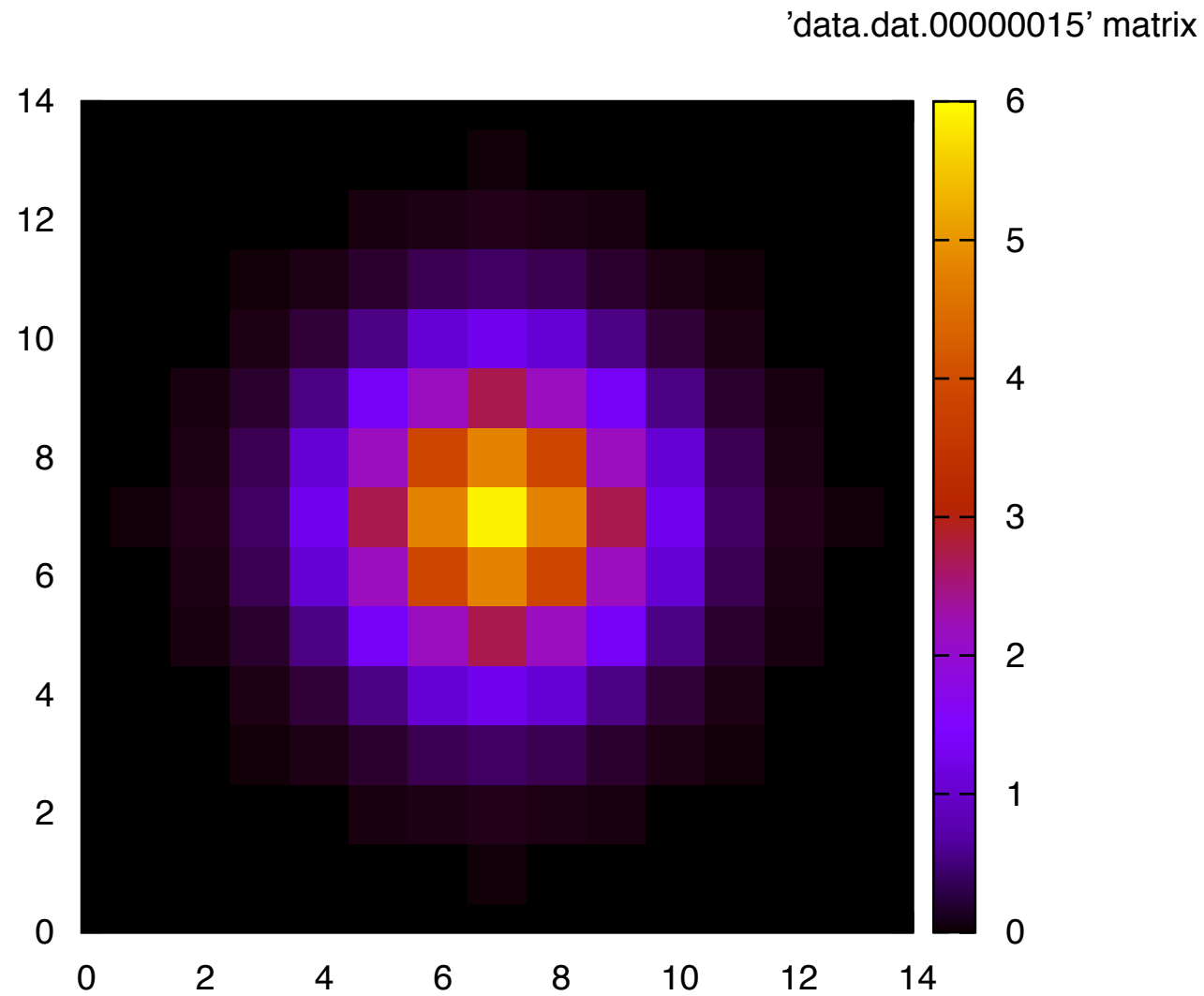




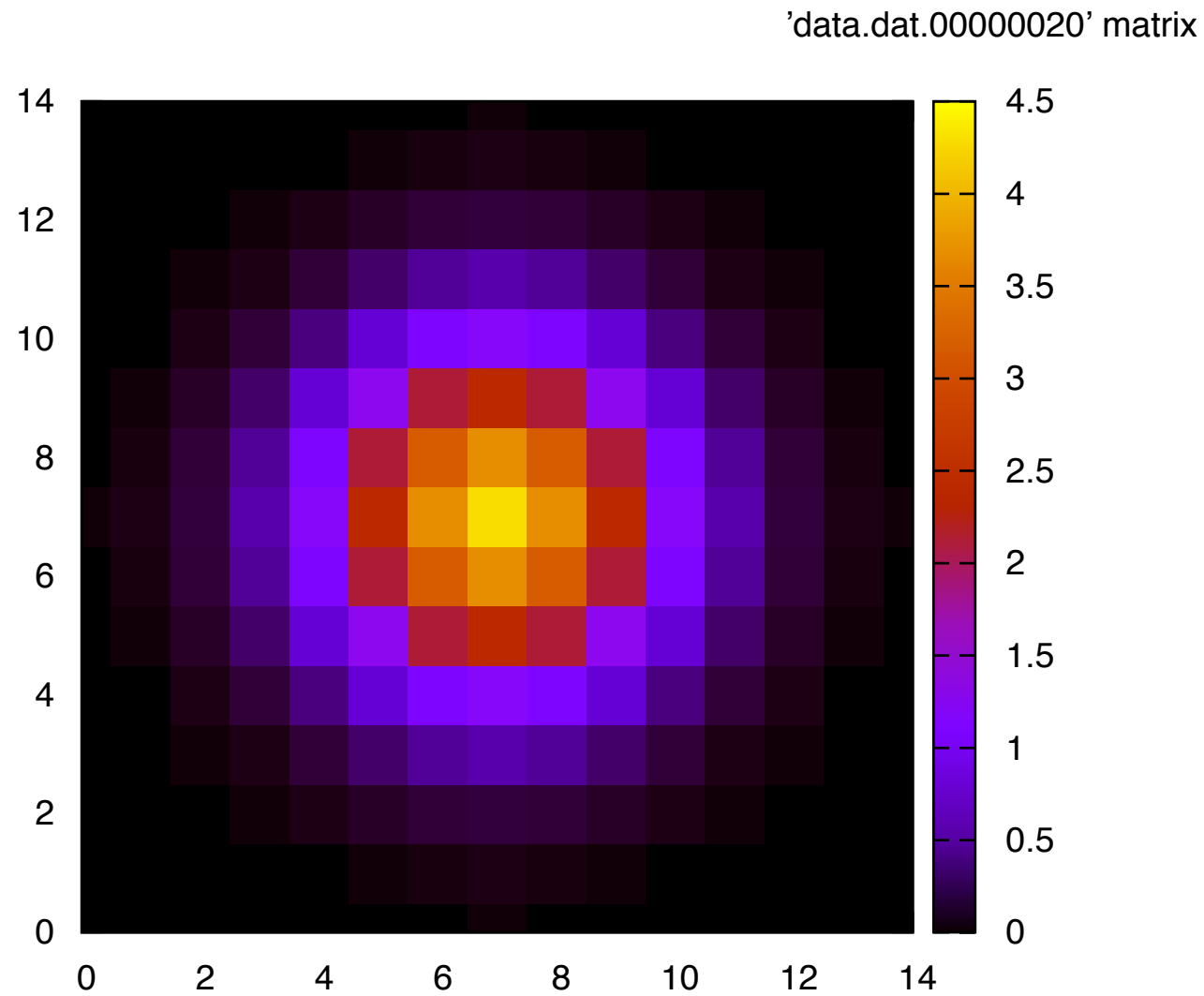
Ex1: DIFFUSION - 2D4 NEIGHBOURHOOD, 15X15



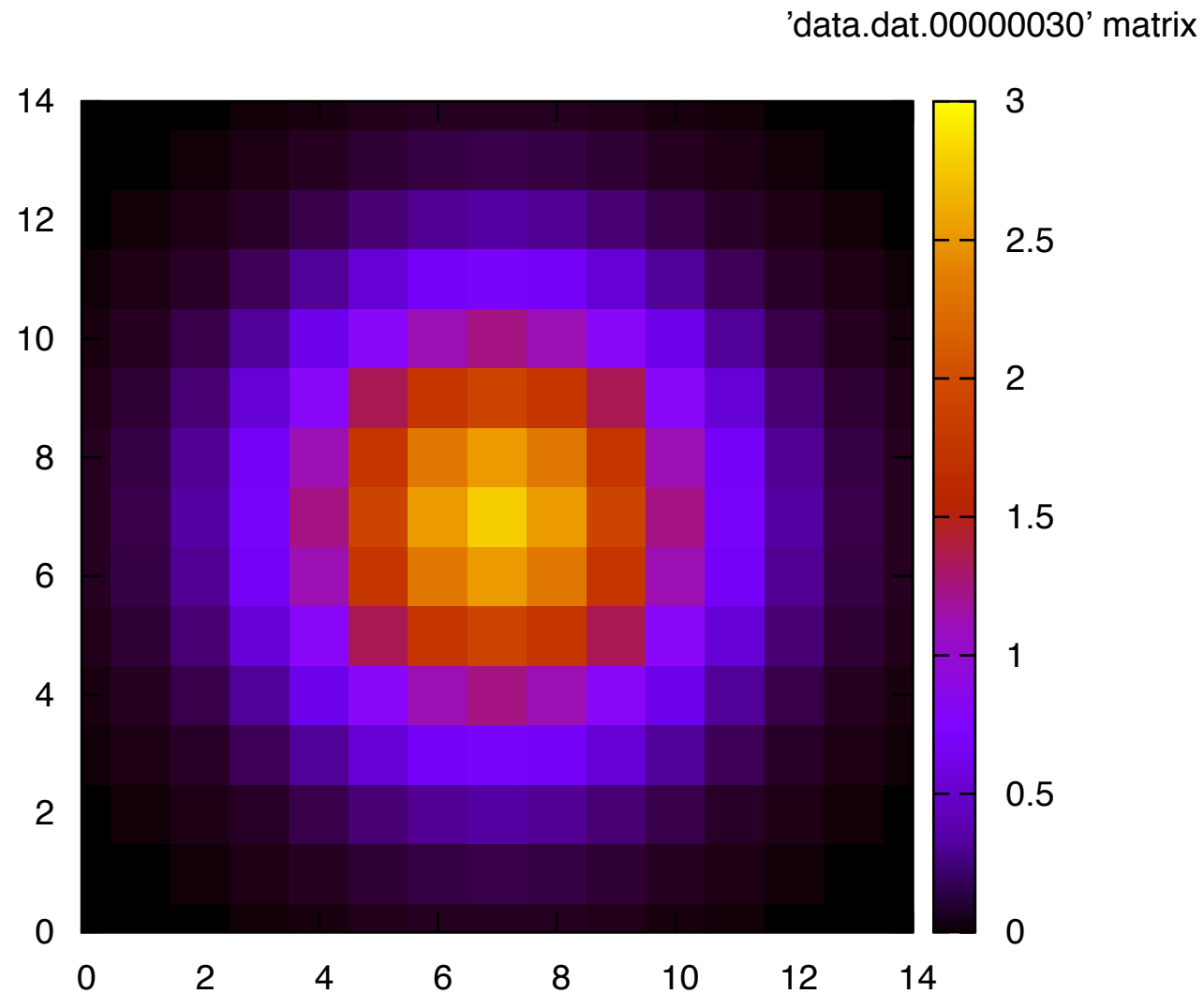
Ex1: DIFFUSION - 2D4 NEIGHBOURHOOD, 15X15



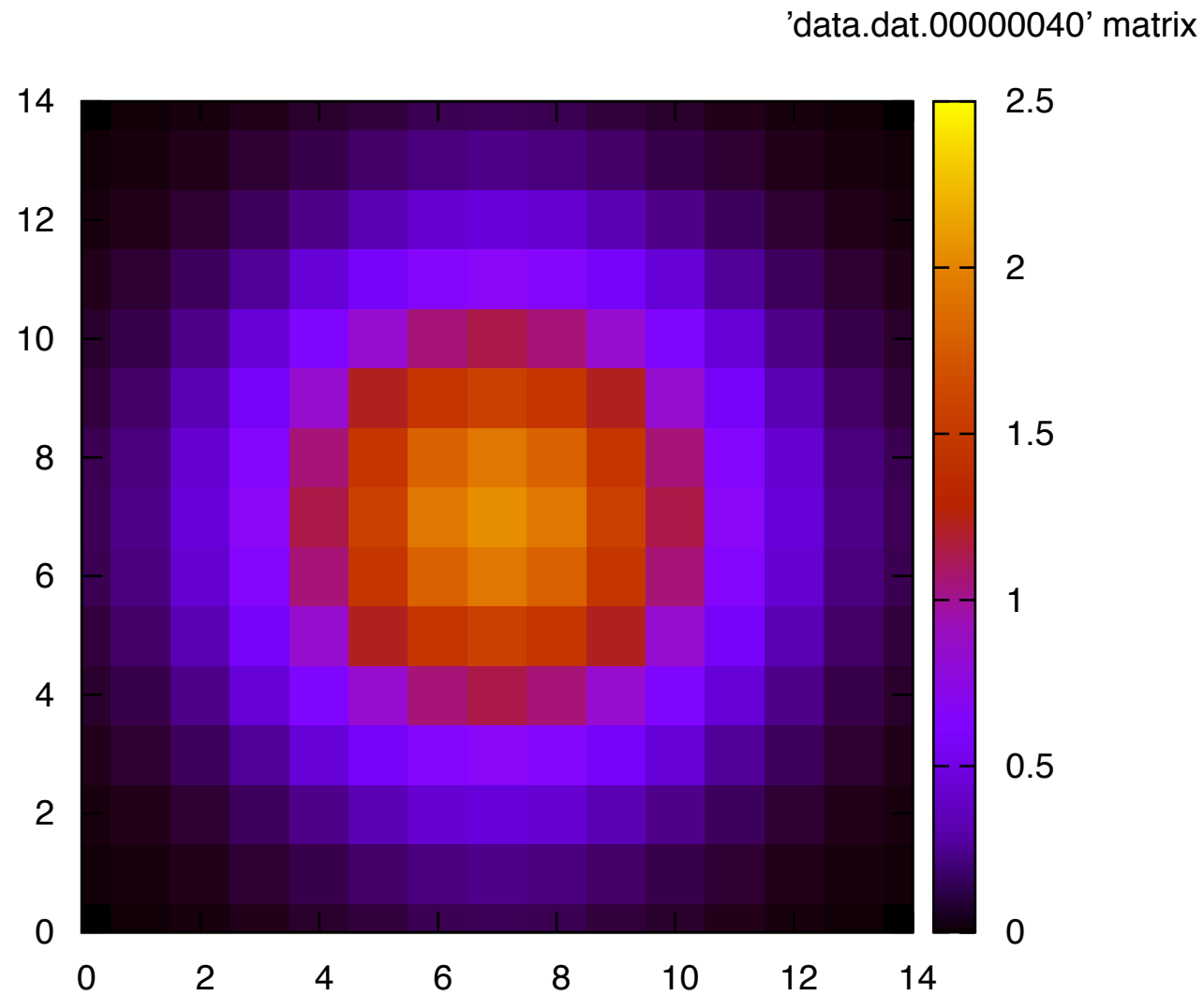
Ex1: DIFFUSION - 2D4 NEIGHBOURHOOD, 15X15



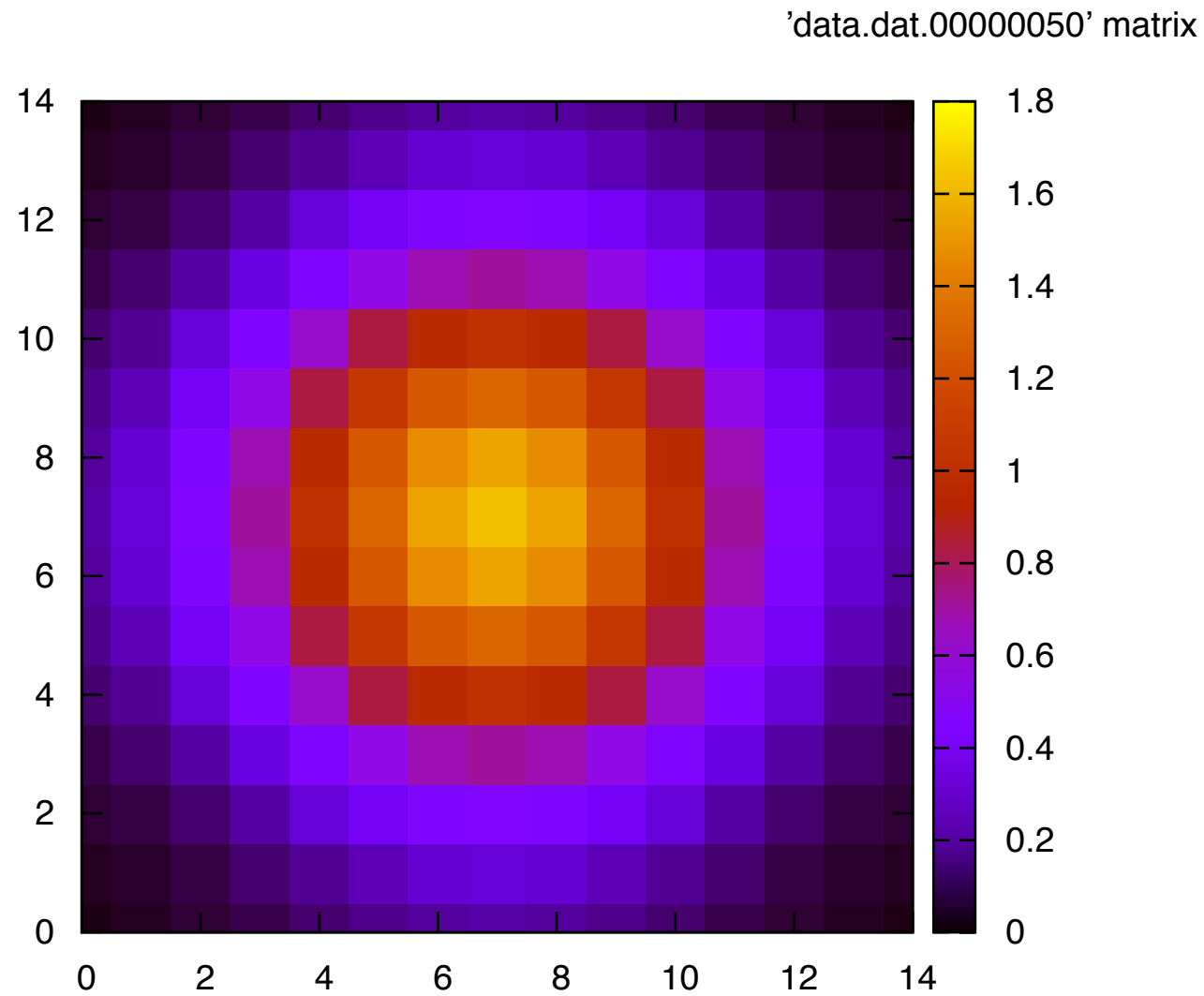
Ex1: DIFFUSION - 2D4 NEIGHBOURHOOD, 15X15

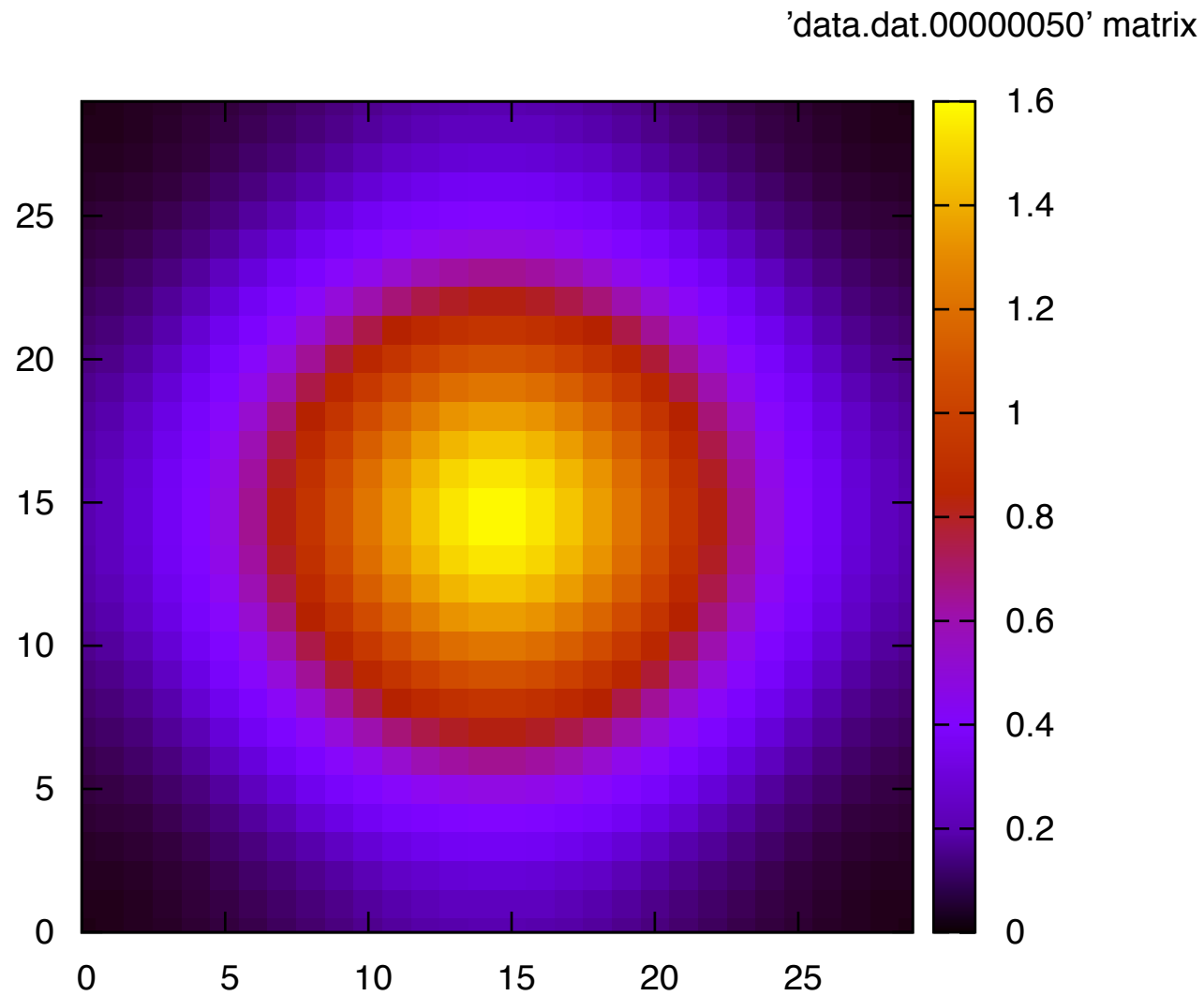


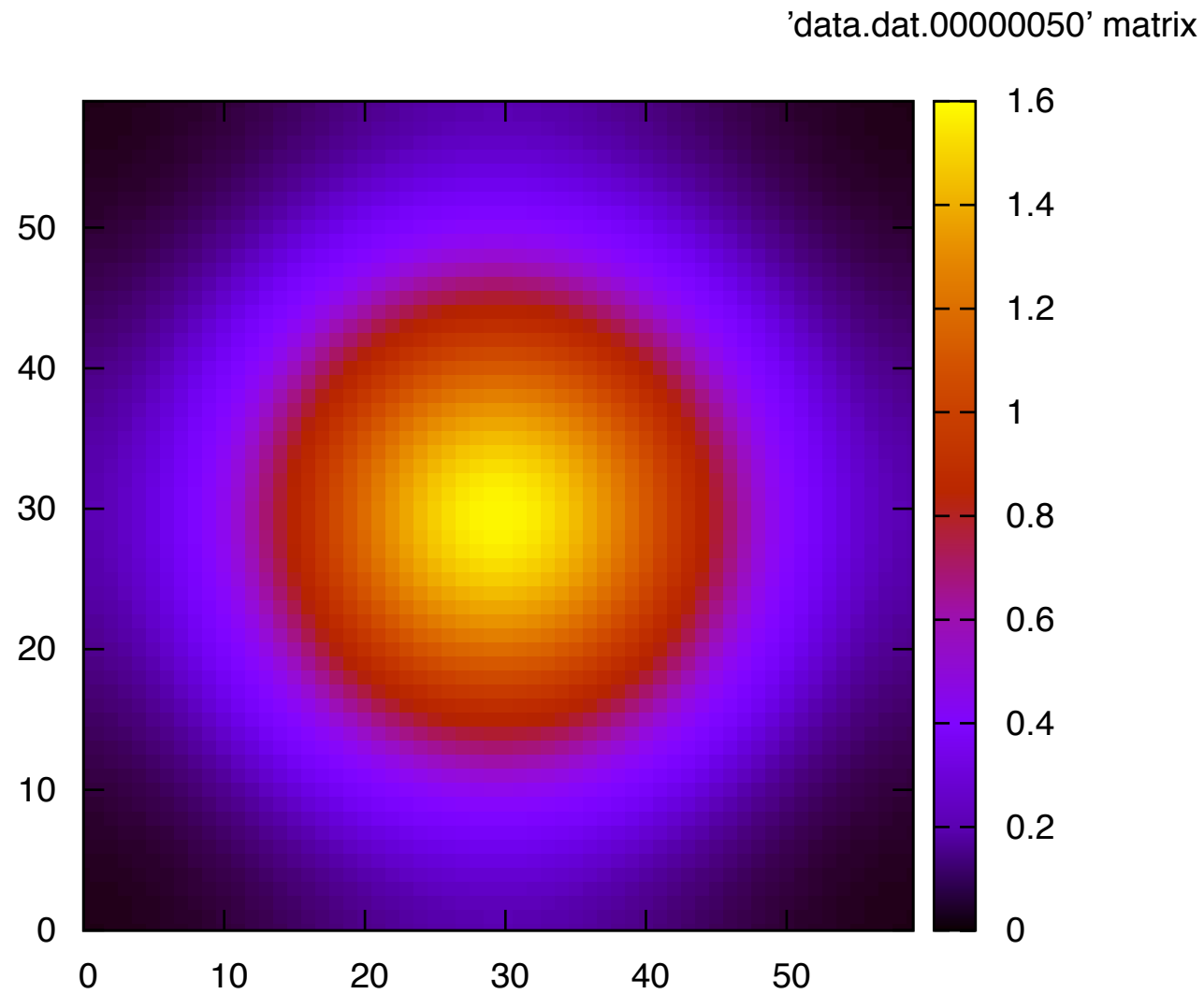
Ex1: DIFFUSION - 2D4 NEIGHBOURHOOD, 15X15

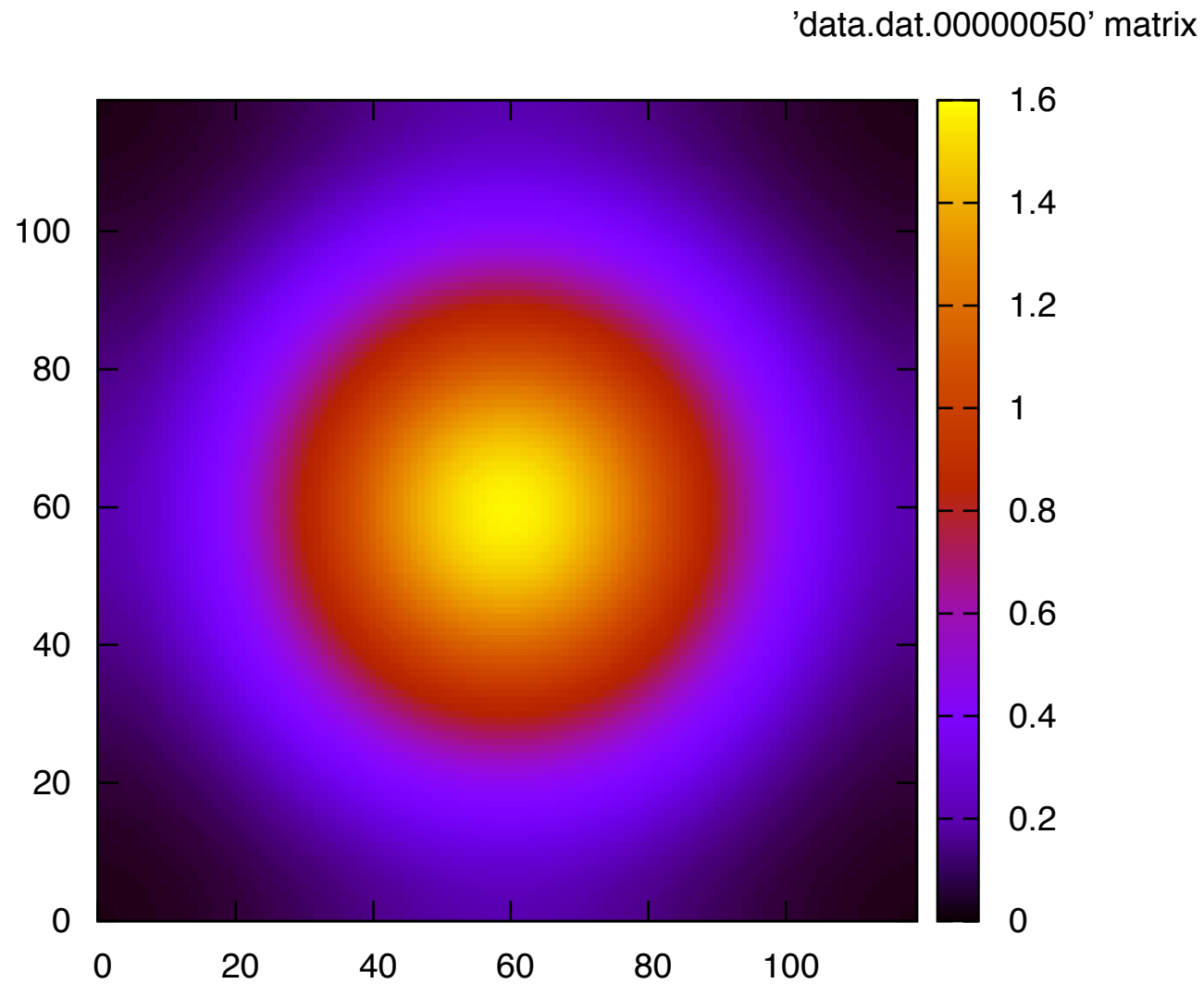


Ex1: DIFFUSION - 2D4 NEIGHBOURHOOD, 15X15









EXAMPLE 2:

TURING PATTERNS

Liu, Blätke, Heiner, Yang:
Modelling and simulating reaction–diffusion systems using coloured Petri nets;
Computers in Biology and Medicine, 2014.

“How the Leopard Got Its Spots”

❑ **morphogenesis**

- > *developmental pattern formation in bio systems*
- > *the process that controls the organized spatial distribution of cells*
- > *tiger stripes, leopard spots, the precisely spaced rows of alligator teeth, etc.*

❑ **Turing's theory of biological pattern formation, 1952**

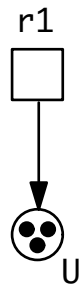
- > *patterns form as result of*
the interactions between two chemicals
that spread throughout a system at different rates

❑ **reactions**

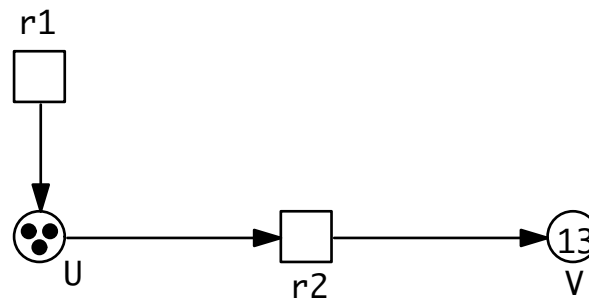
- > *version of Brusselator model, <http://en.wikipedia.org/wiki/Brusselator>*

❑ **highly simplified and idealised take on biological patterning**

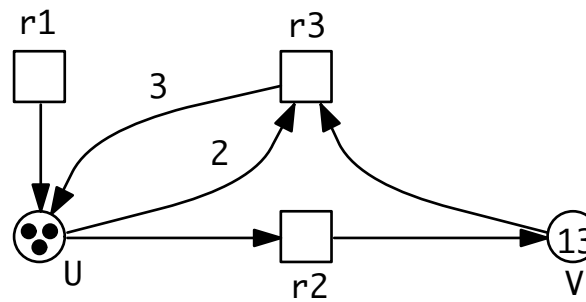
r1: -> U



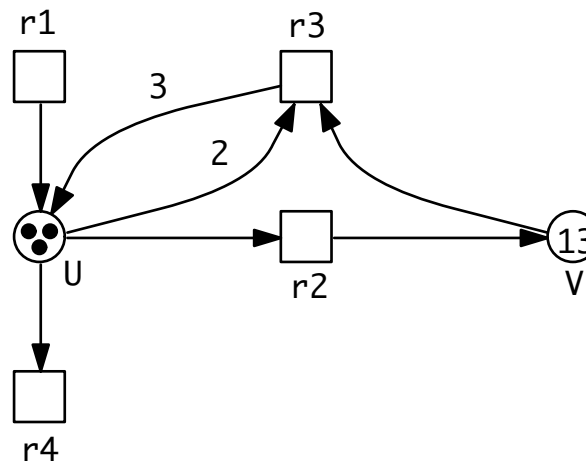
r1: $\rightarrow U$
r2: $U \rightarrow V$



r1: $\rightarrow U$
r2: $U \rightarrow V$
r3: $2U + V \rightarrow 3U$

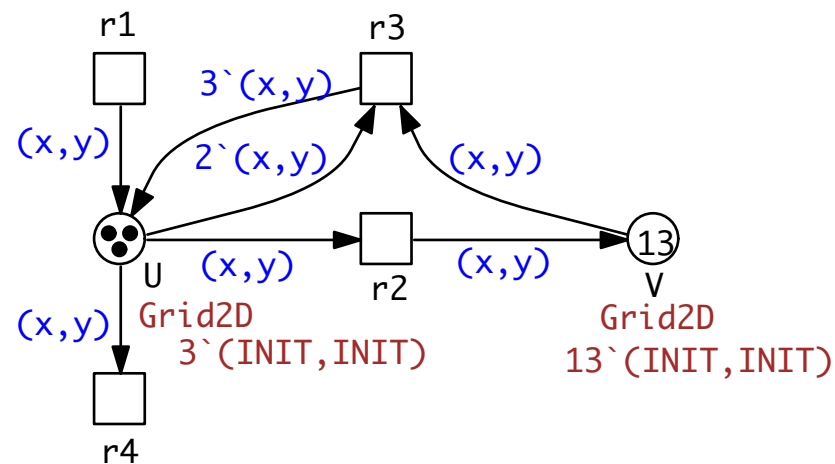


r1: $\rightarrow U$
r2: $U \rightarrow V$
r3: $2U + V \rightarrow 3U$
r4: $U \rightarrow$



$r1: \quad \rightarrow U$
 $r2: \quad U \rightarrow V$
 $r3: \quad 2U + V \rightarrow 3U$
 $r4: \quad U \rightarrow$

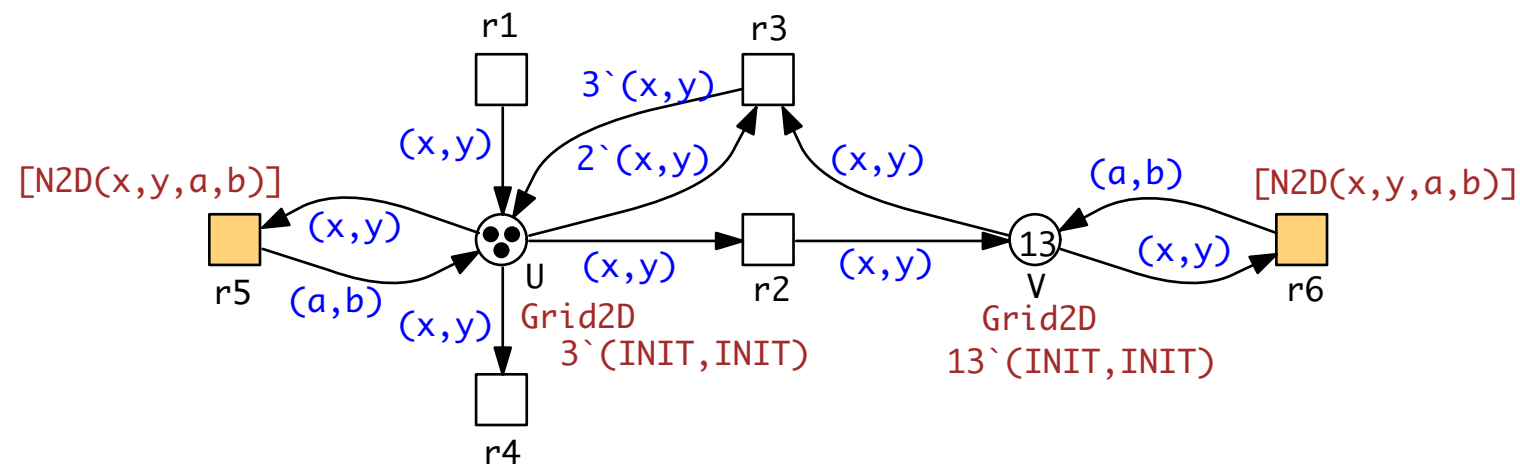
adding SPACE



$r1: \quad \quad \rightarrow U$
 $r2: \quad \quad U \rightarrow V$
 $r3: \quad 2U + V \rightarrow 3U$
 $r4: \quad \quad U \rightarrow$

diffusion:

$r5: \quad U_{xy} \xrightarrow{1/h^2} U_{ab}$
 $r6: \quad V_{xy} \xrightarrow{D/h^2} V_{ab}$



$r1 - r4$ follow mass action kinetics with rate constants:

$r1: A, r2: B, r3: 1, r4: 1;$

❑ parameters

- > *Pena, Perez-Garcia: Stability of Turing patterns in the Brusselator model, Physical Review 2001*
- > $A = 4.5$, $B = 0.04 \dots 0.98$, $D = 128$, $h = 0.8$

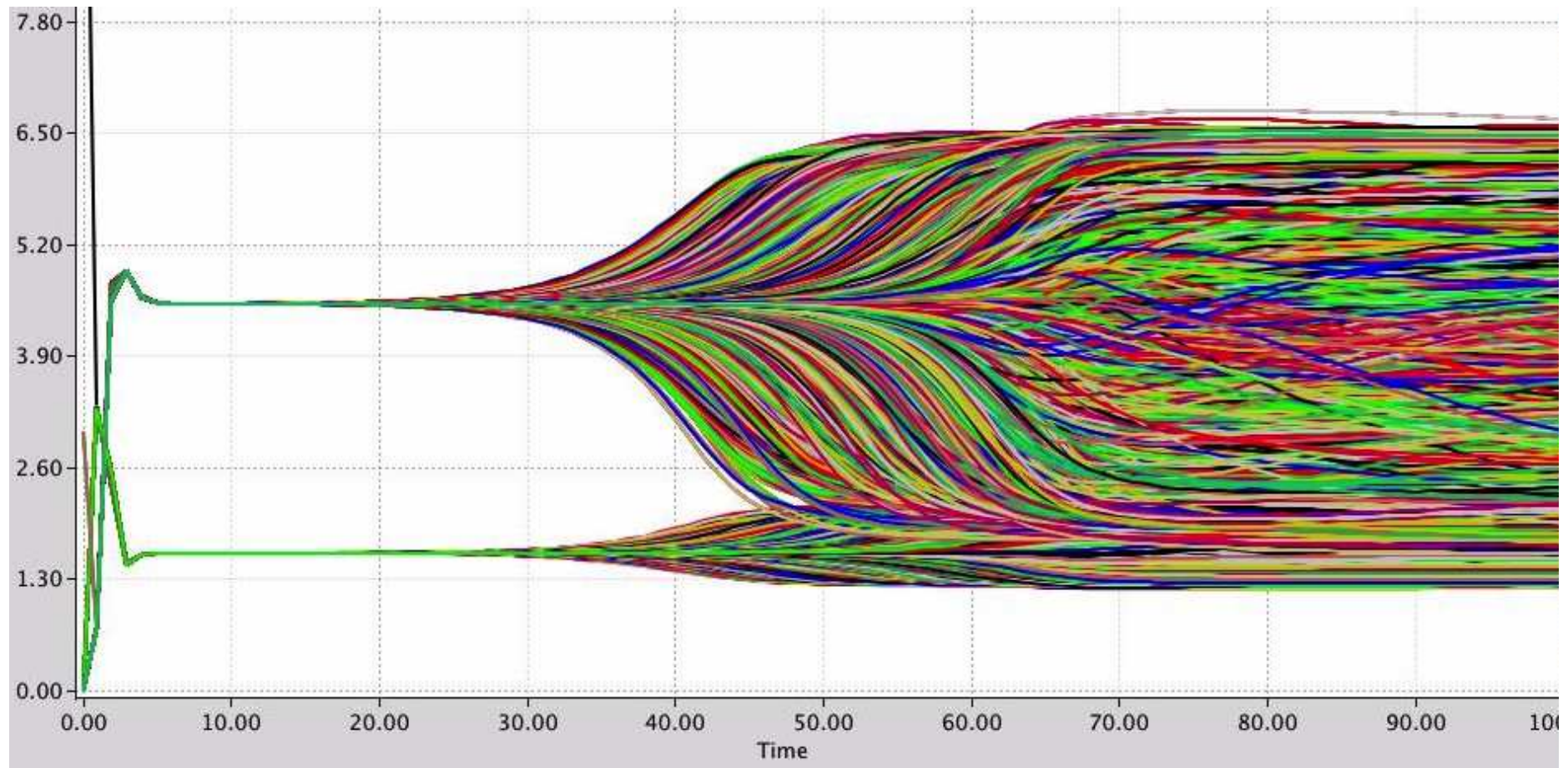
❑ unfolding

- > *runtime (constraint solver, 4 threads): 128 sec*
- > *places: 32,768, transitions: 324,616*

❑ continues simulation

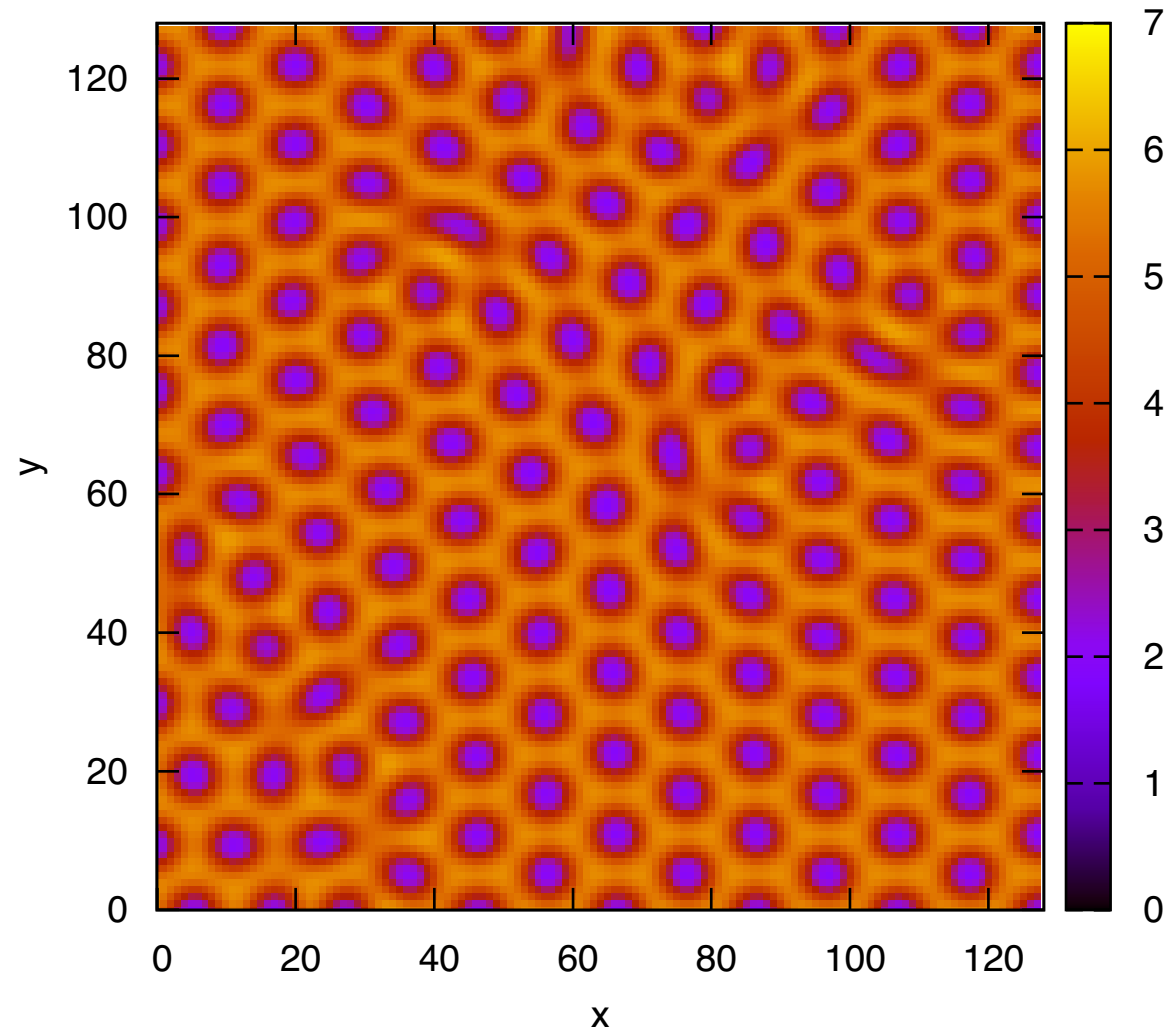
- > *BDF (Backward Differentiation Formulae, higher-order stiffly stable solver)*
- > *simulation time: 5,000 -> runtime: about 30h*

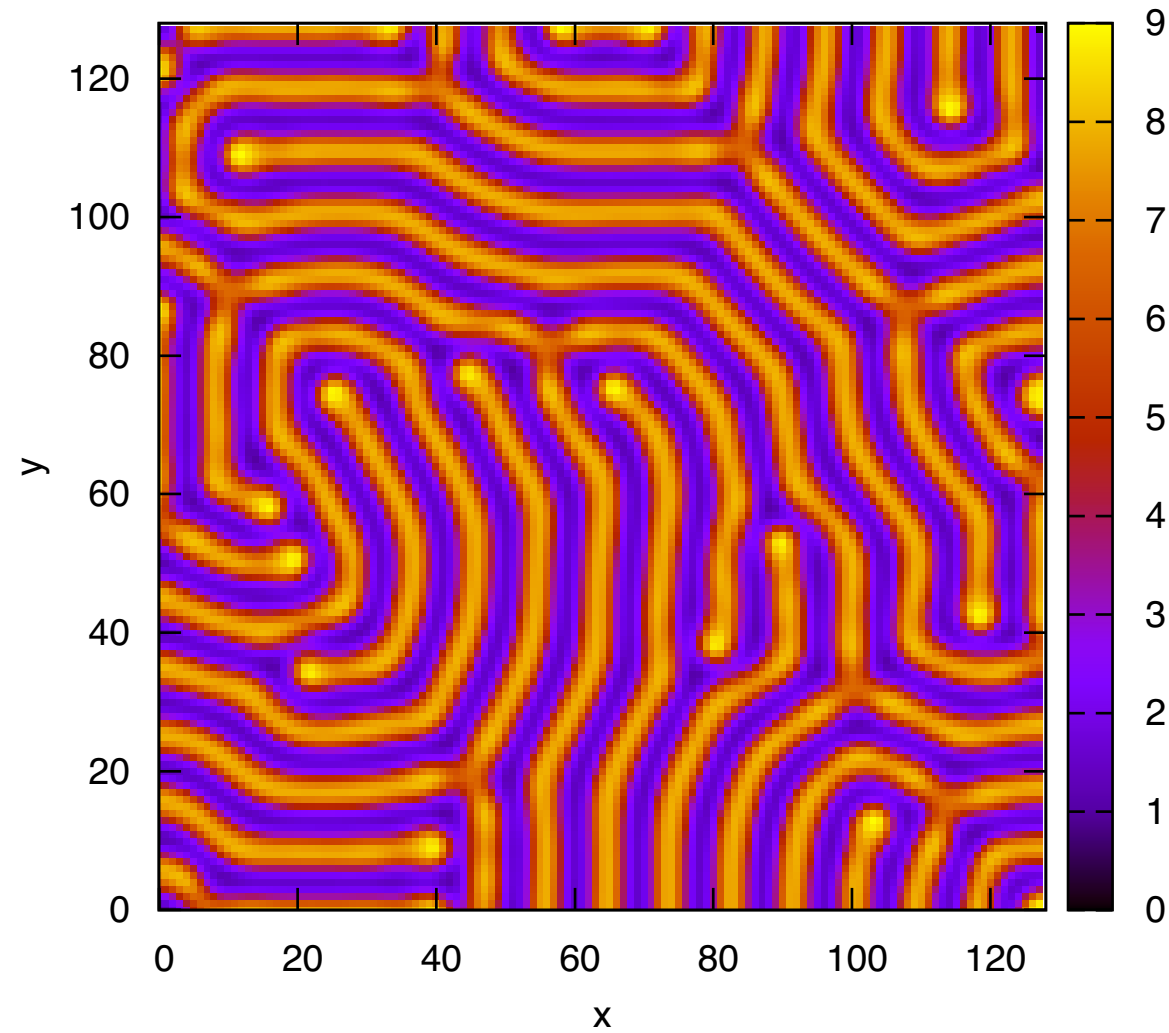
-> EVOLUTION OF U, V IN ALL GRID POSITIONS OVER TIME

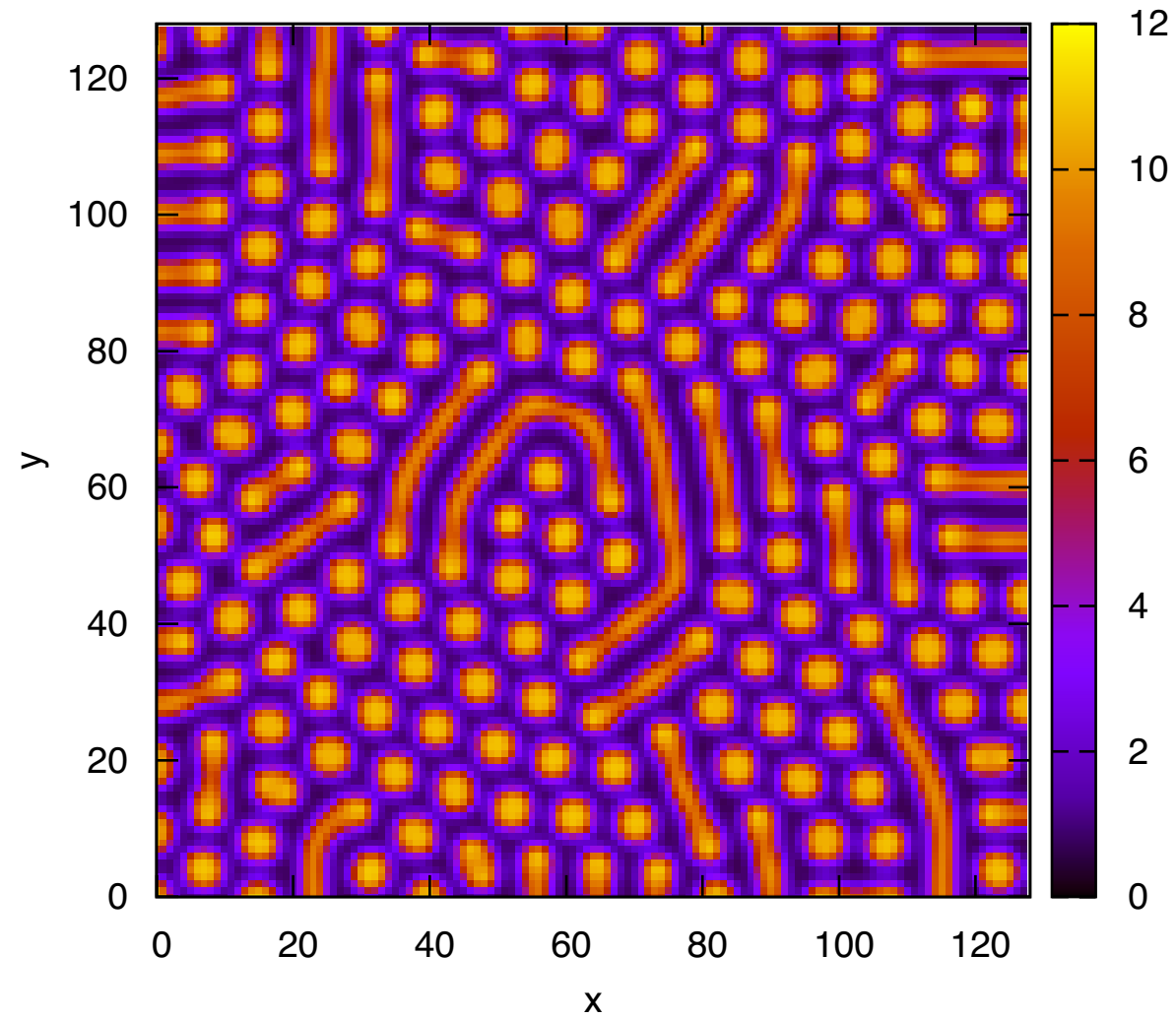


Ex2: TURING PATTERNS

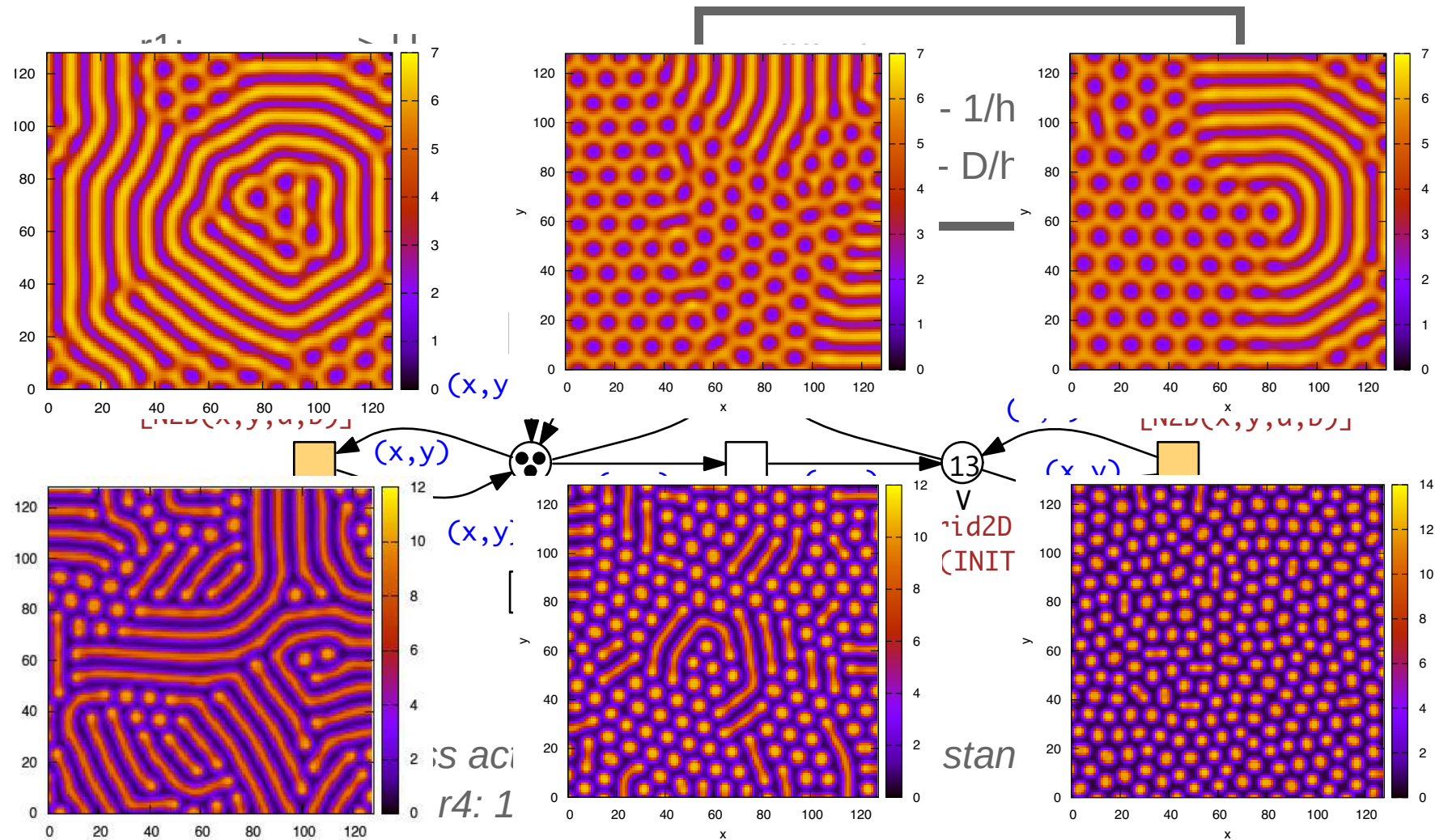
-> U IN ALL GRID POSITIONS AT TIME 5,000







Ex2: TURING PATTERNS



[LIU ET AL 2014]

EXAMPLE 3:

PHASE VARIATION IN MULTISTRAIN CELL COLONIES

Pârvu, Gilbert, Heiner, Liu, Saunders, Shaw:
Spatial-temporal modelling and analysis of bacterial colonies with phase variable genes;
ACM TOMACS, 2015.

❑ phase variation

-> *method for dealing with rapidly varying environments without requiring random mutations*

❑ contingency genes

-> *populations include variants adapted to “foreseeable” frequently encountered environmental or selective conditions*

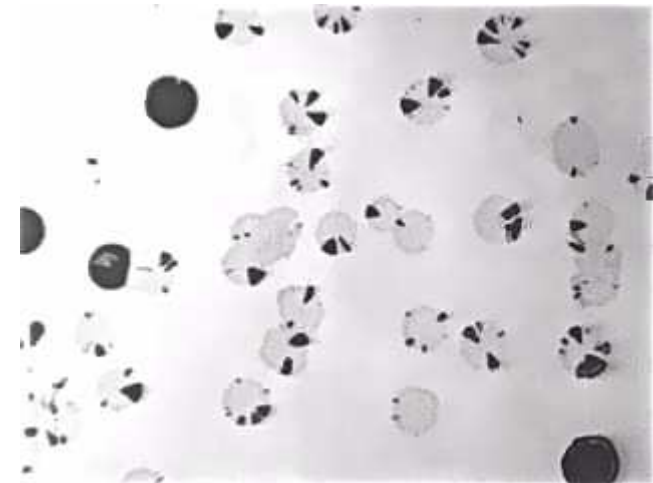
❑ stochastic gene switching process

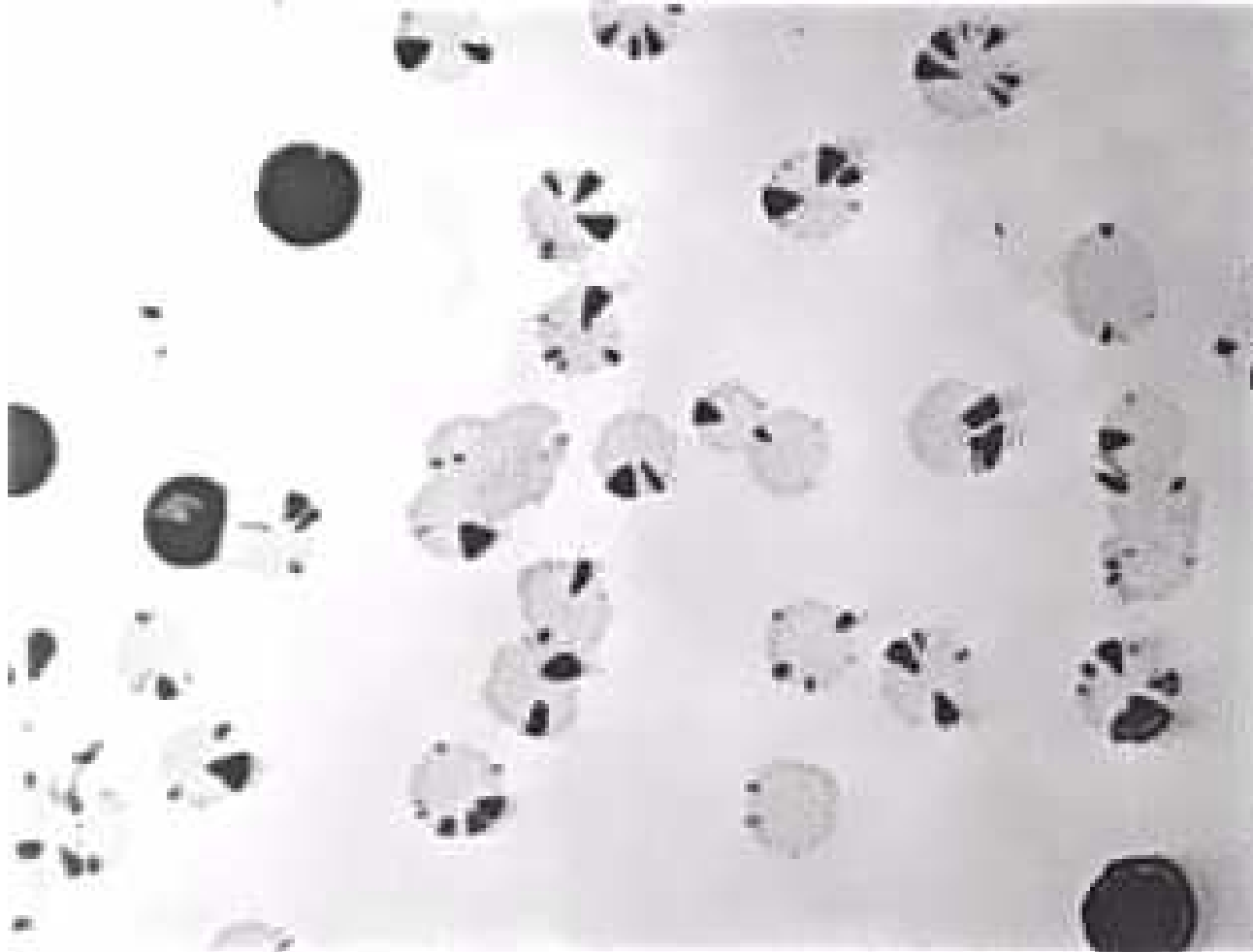
-> *controlled by reversible gene mutations, inversions, or epigenetic modification*

-> e.g. *switch between two phenotypes A, B*

❑ colonial sectoring

-> *observable effect in cultures grown in vitro*





(courtesy of N Saunders)

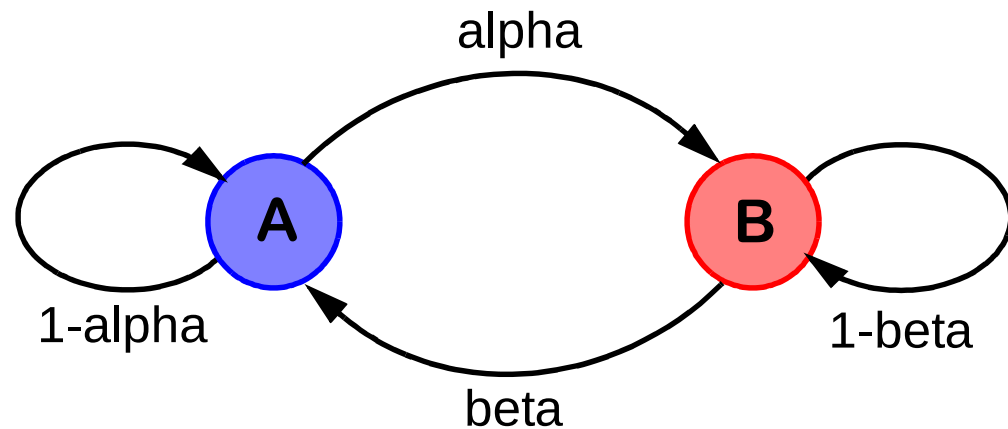
❑ two phenotypes: A and B

❑ cell divide

- > cell division may involve mutation of the offspring
- > parent cell keeps its phenotype

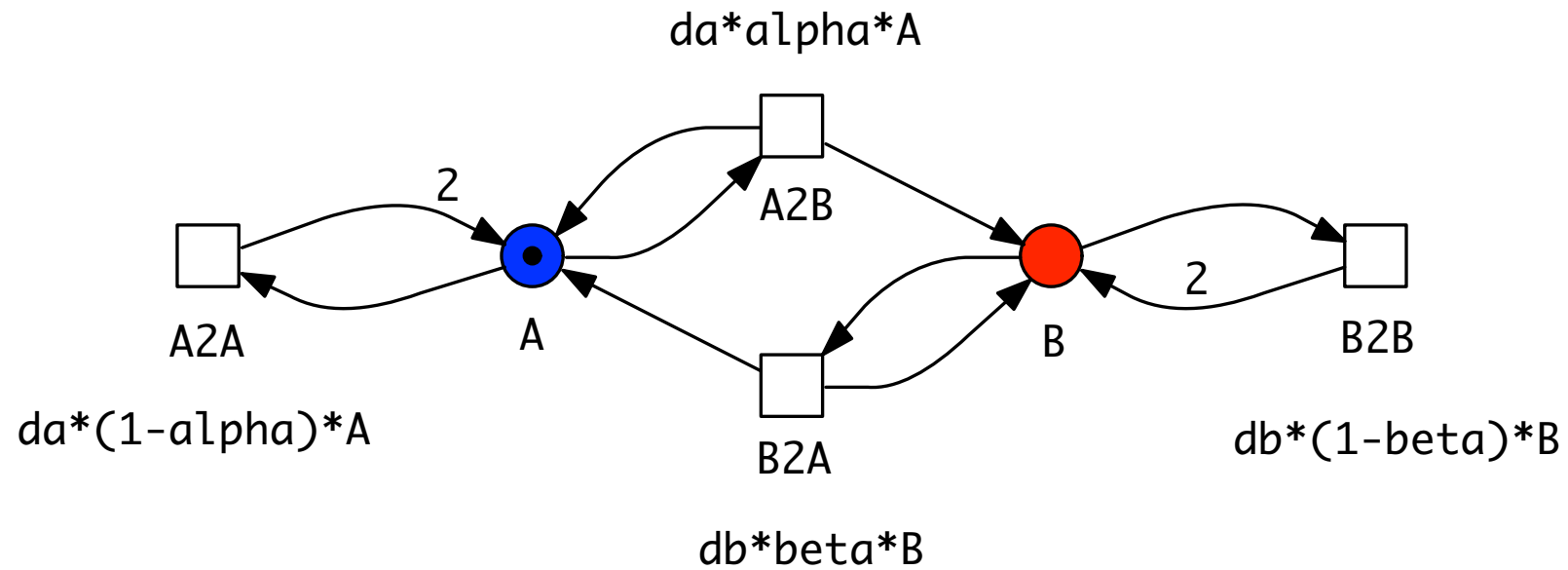
❑ model parameters

- > $\alpha = \beta$ - mutation rates
- > d_a, d_b - fitness of A, B
- > d_a/d_b - relative fitness

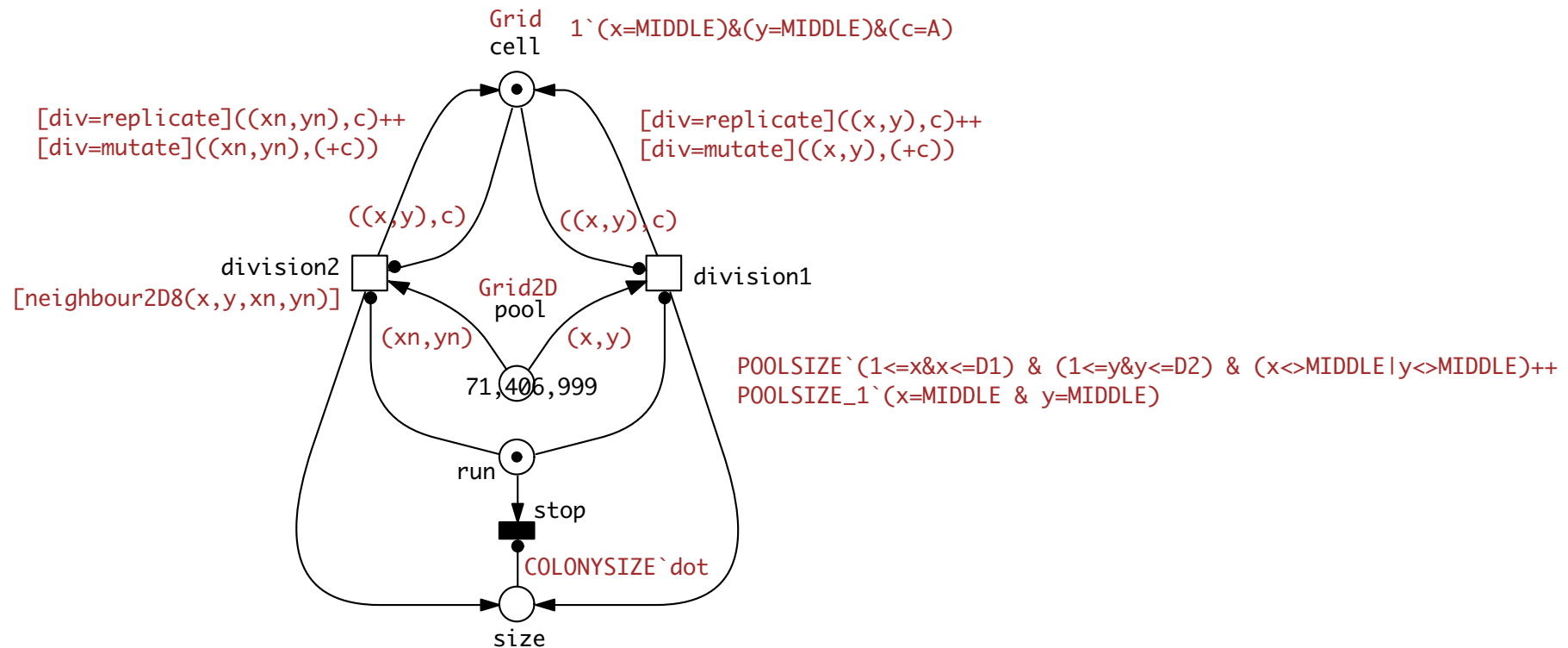


❑ output

- > total number of cells
- > proportion of A = $A / (A + B)$
- > proportion of B = $B / (A + B)$



colorset Grid = product with Grid2D x Phenotype;



❑ model assumptions

- > *"If phase variation occurs, the progeny consists of one A and one B"*
(Saunders 2003)
- > *It is always the mutant who goes to a neighbouring position, if any.*
- > *constant biofilm thickness (so far)*

❑ colony size - 24 h

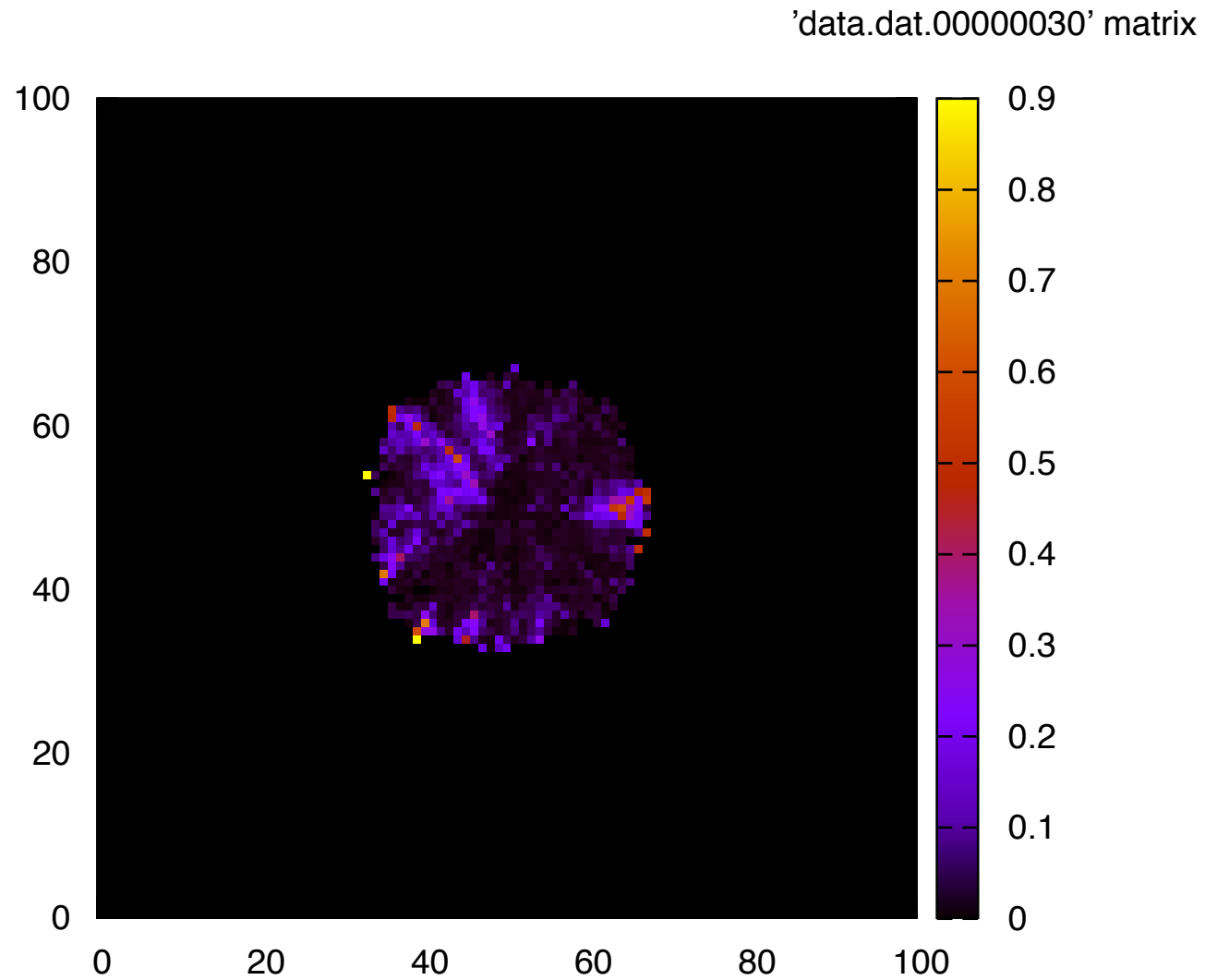
- > 25 generations: 33.5 E+06
- > 26 generations: 67 E+06
- > COLONYSIZE = 70,000,000

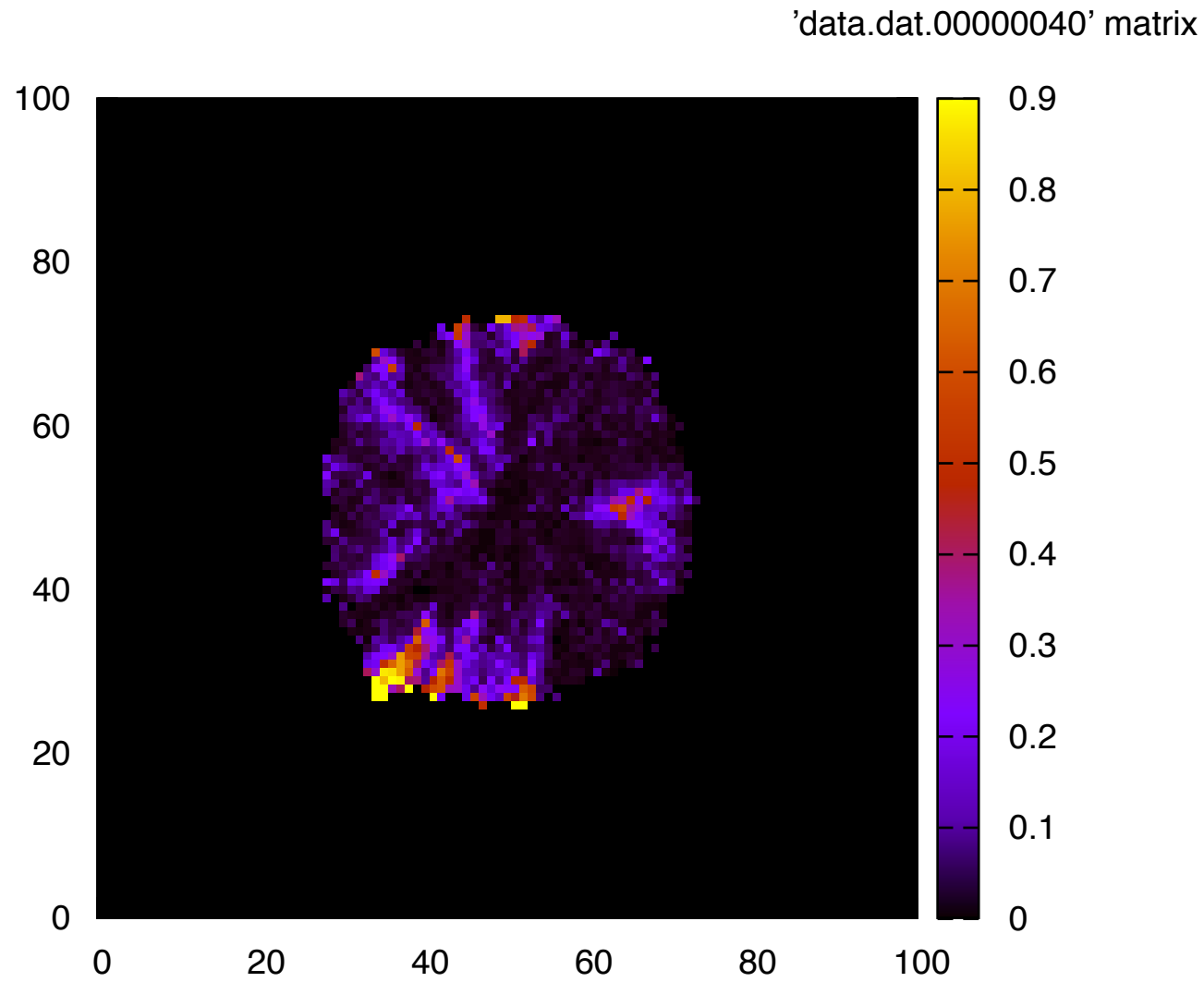
❑ grid size

- > 61 x 61 grid: 11,163 P / 131,044 T; unfolding: 152 sec;
- > 101 x 101 grid: 30,603 P / 362,404 T; unfolding: 9 min;
- > runtime 1 stoch. simulation: 35-40 minutes

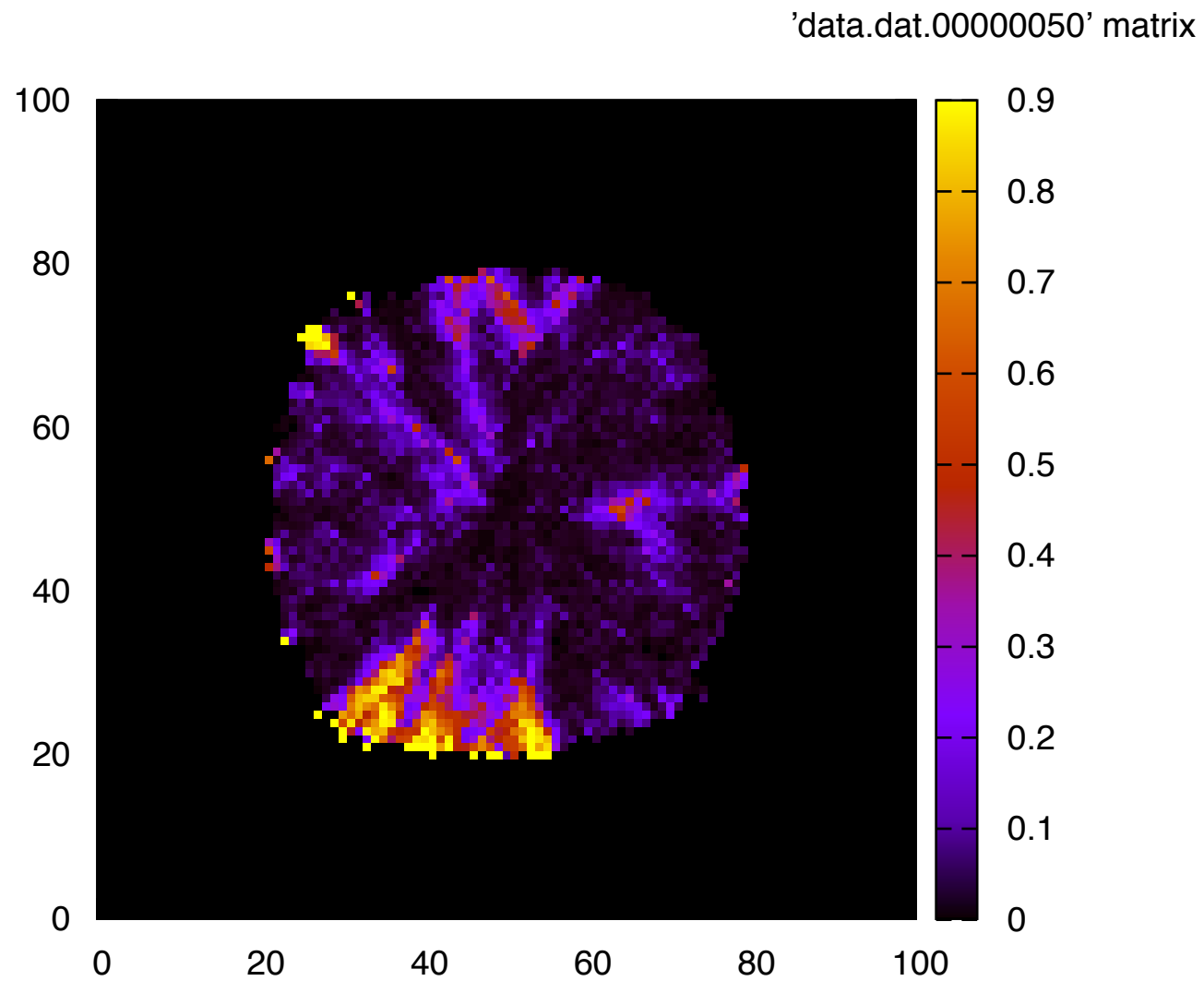
. . . SOME EXPERIMENTS

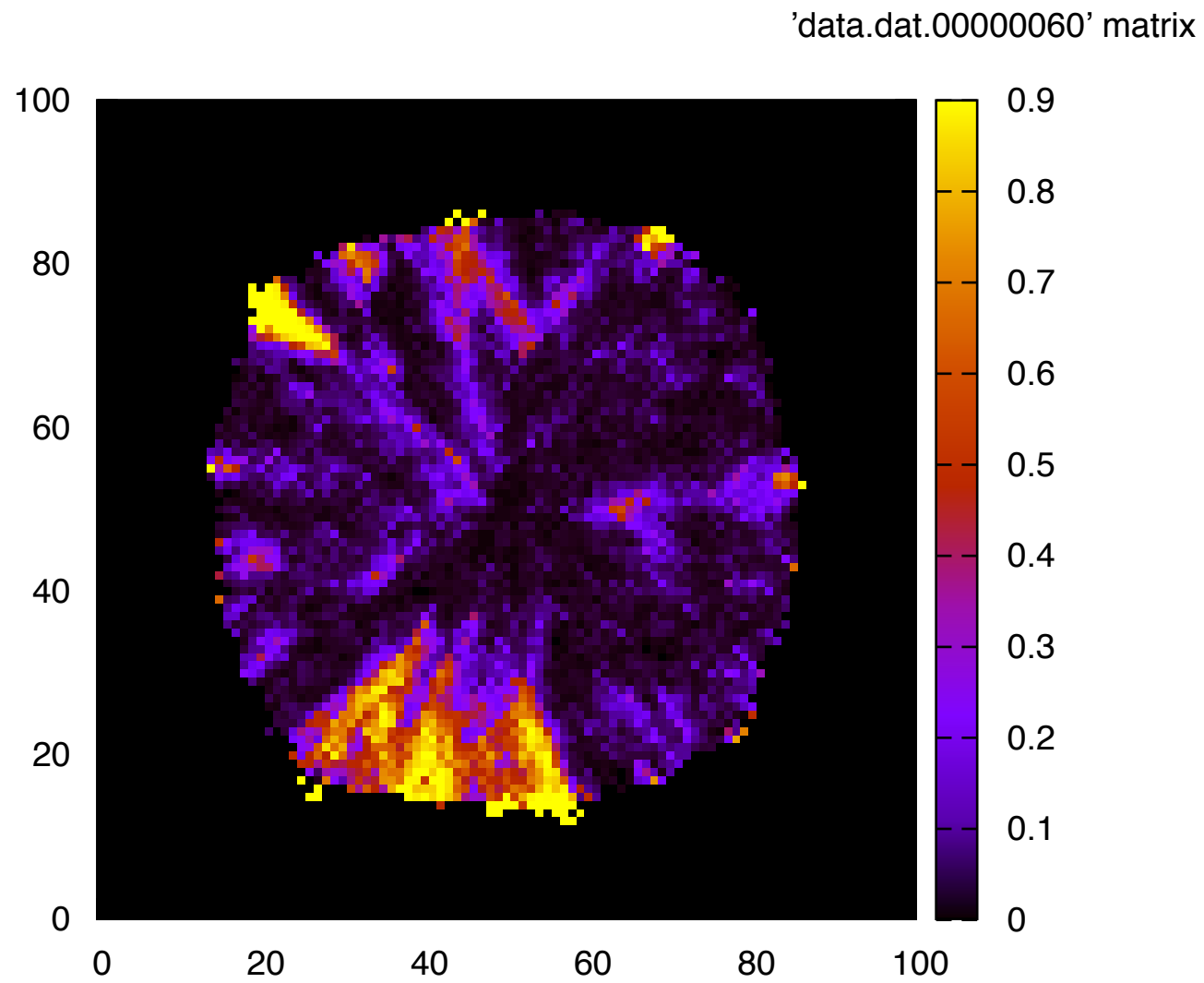
Ex3: 2D - TRACE 1 (HIGH, F=1)

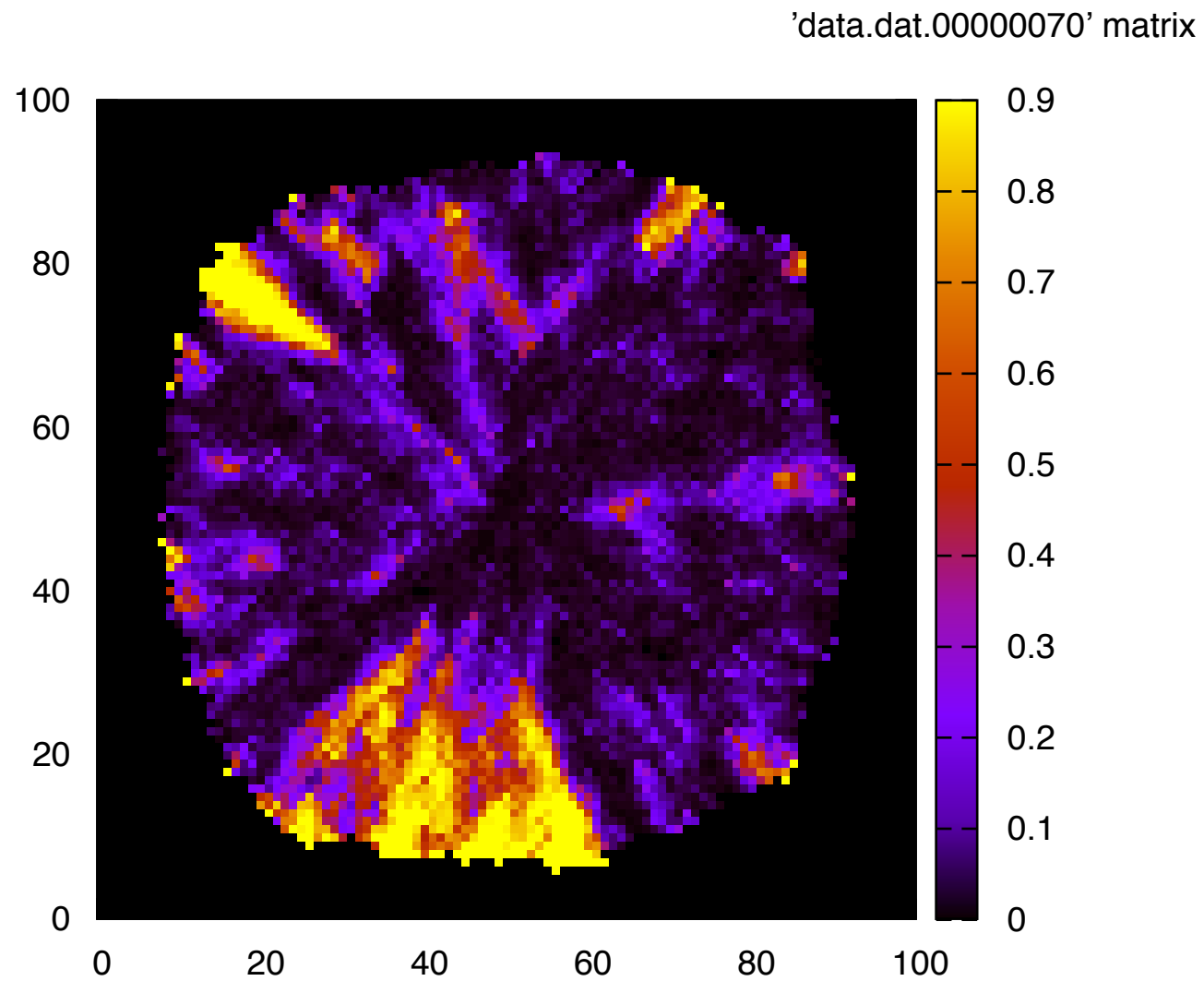


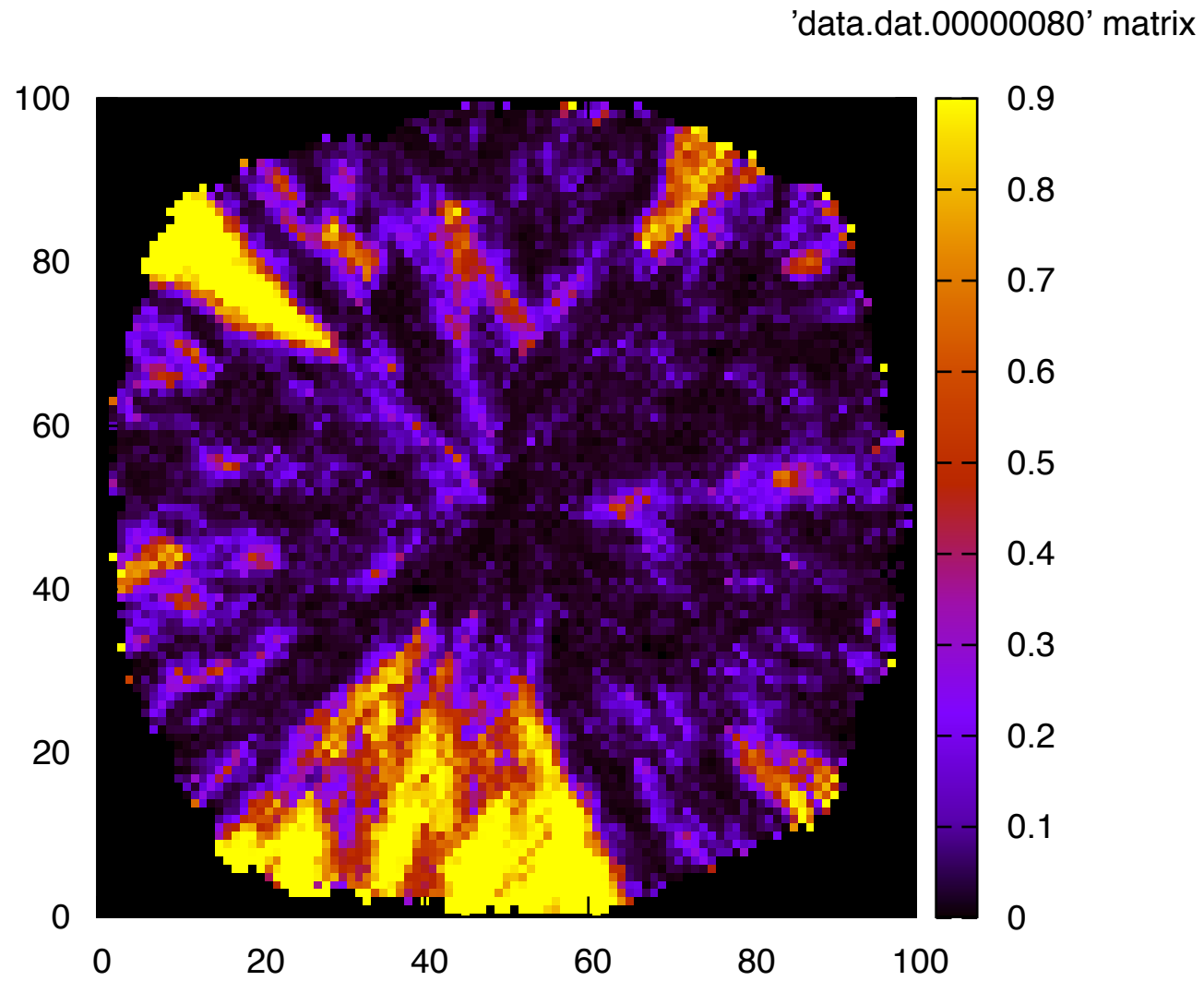


Ex3: 2D - TRACE 1 (HIGH, F=1)

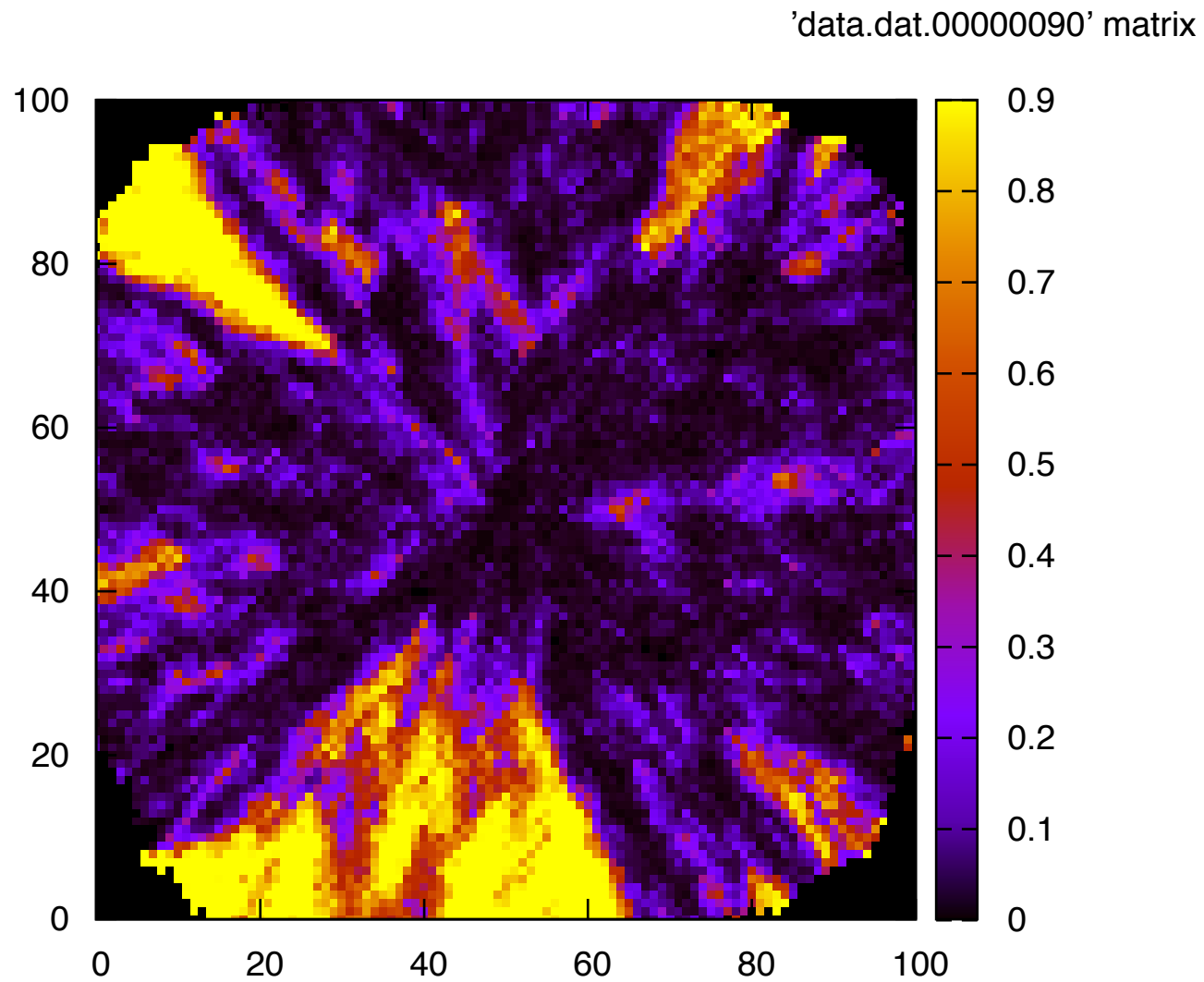




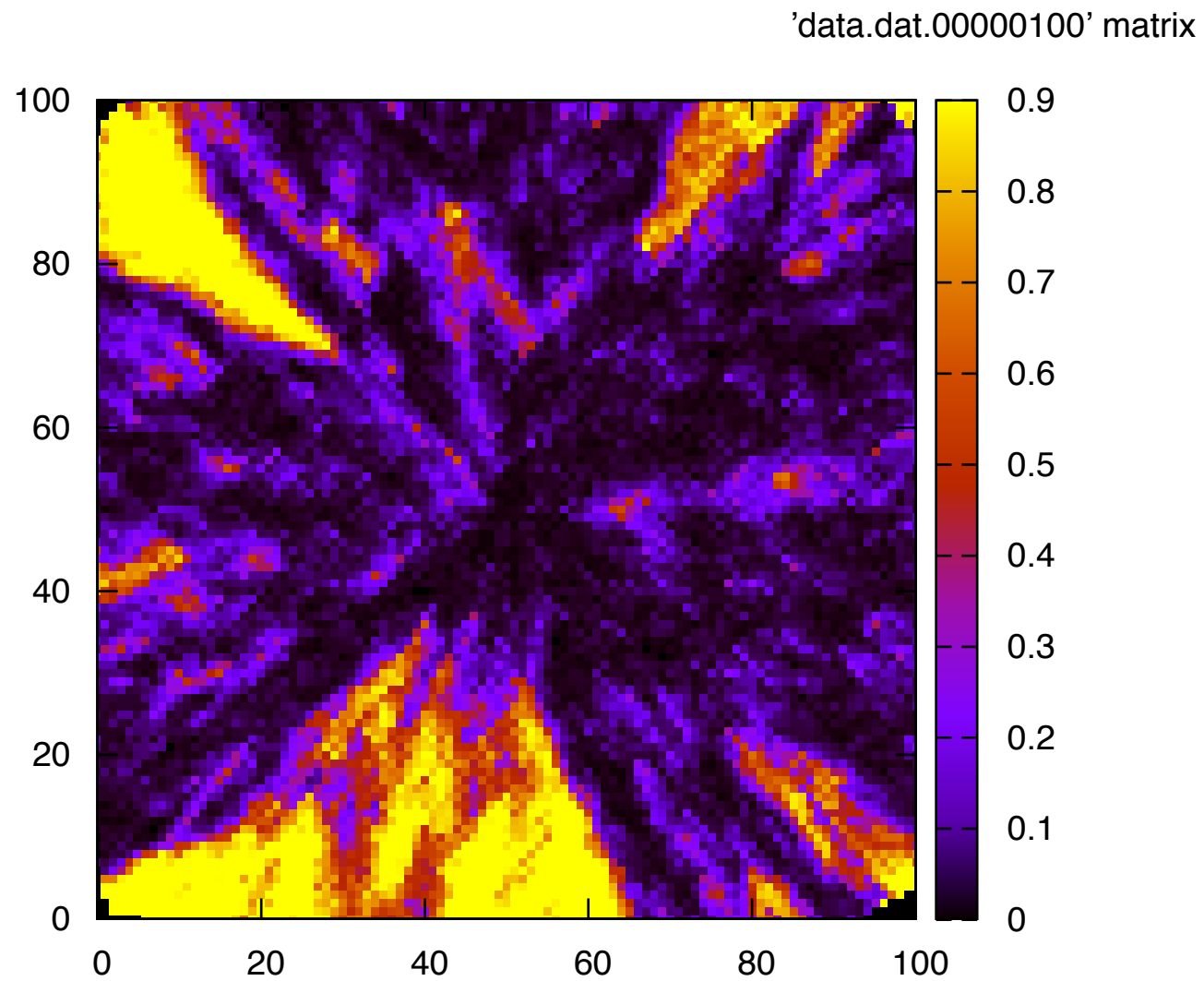


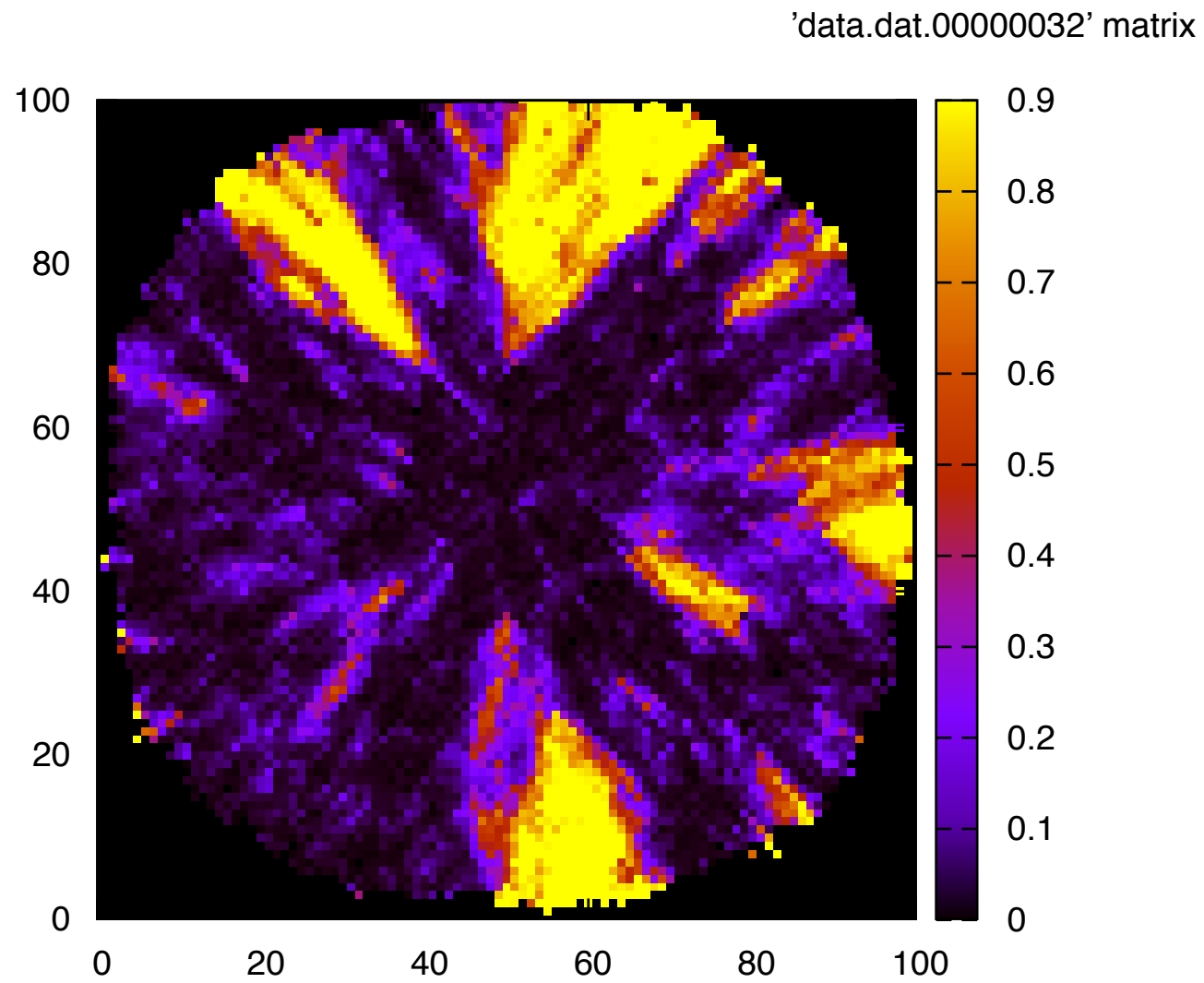


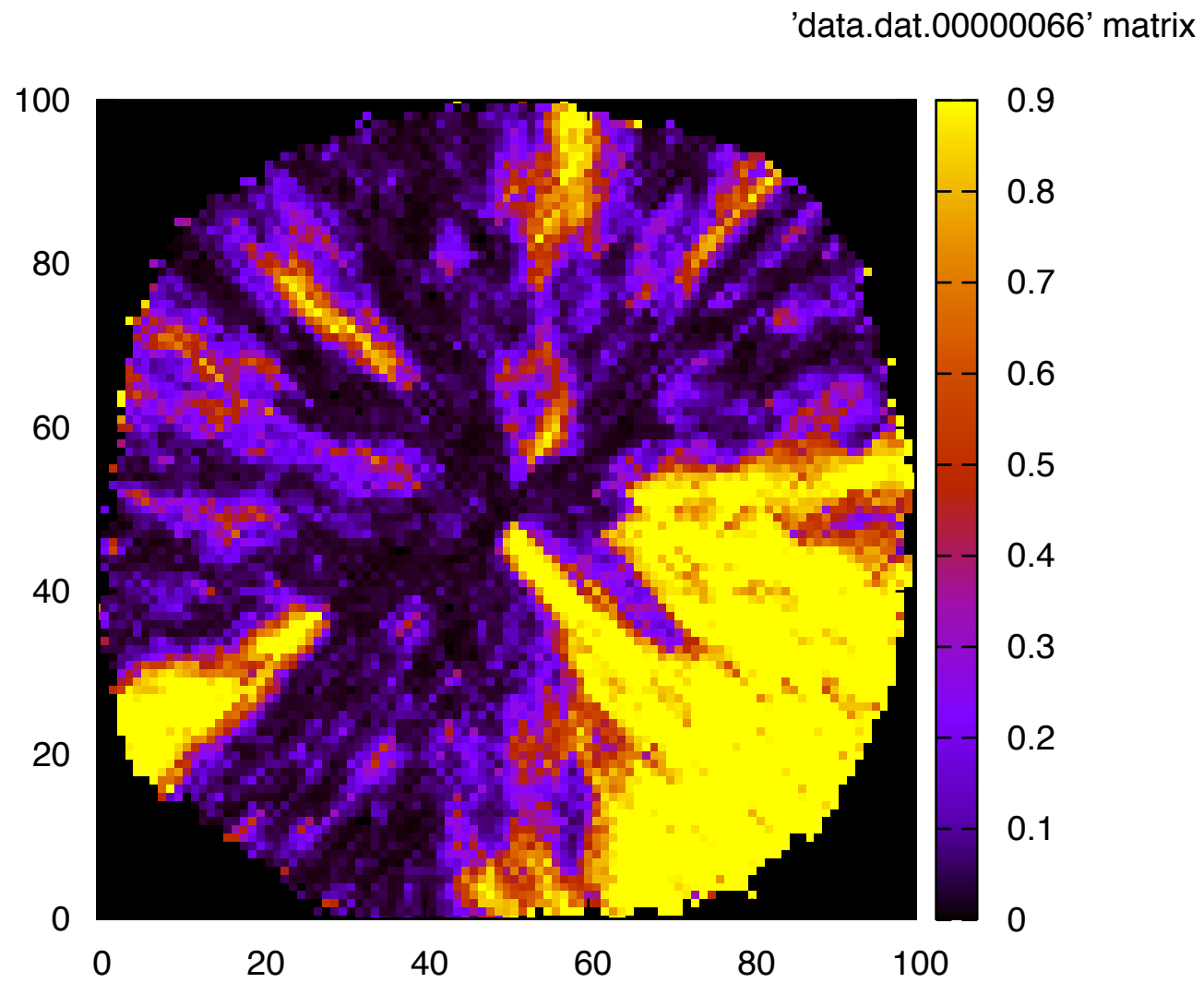
Ex3: 2D - TRACE 1 (HIGH, F=1)



Ex3: 2D - TRACE 1 (HIGH, F=1)

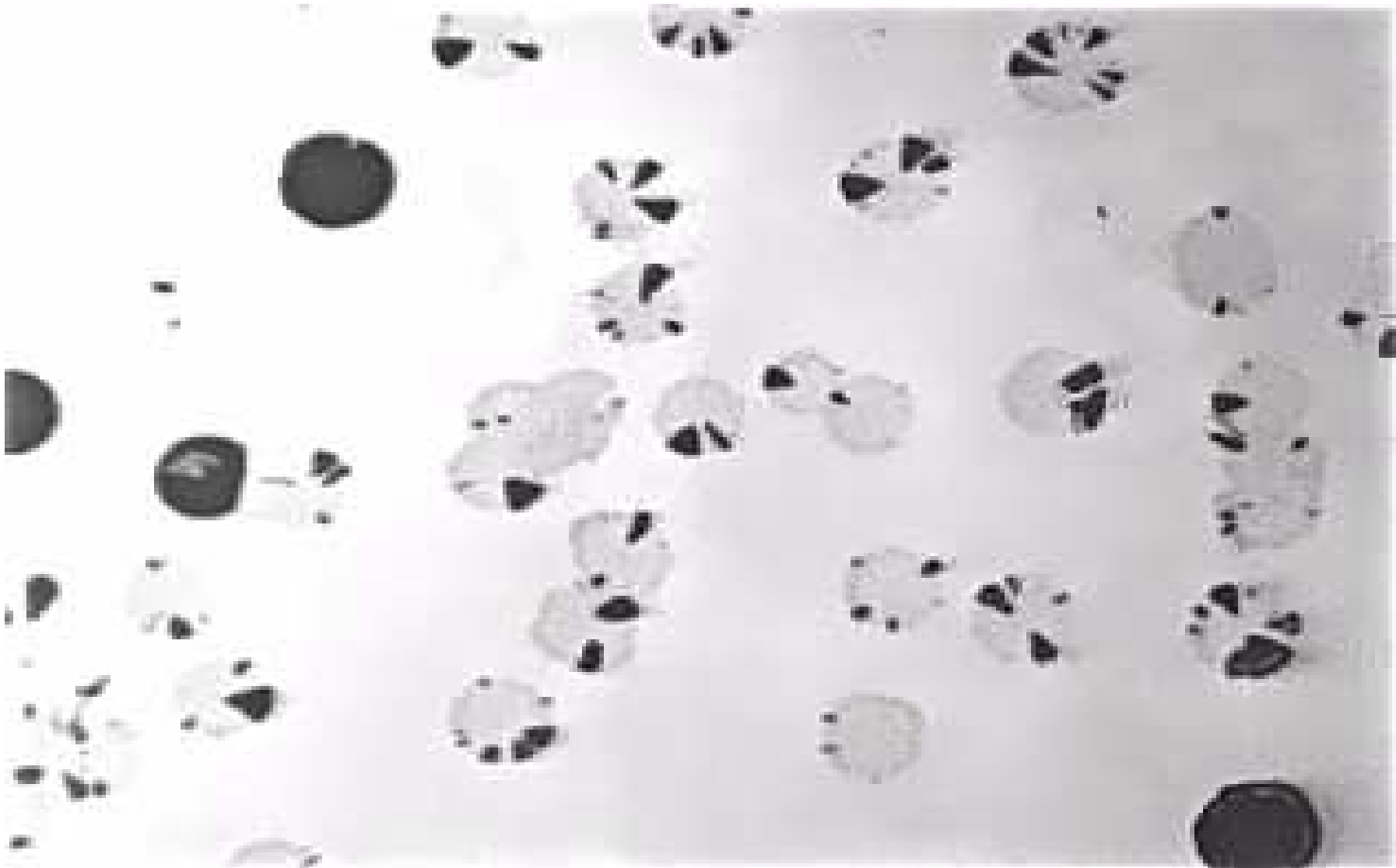




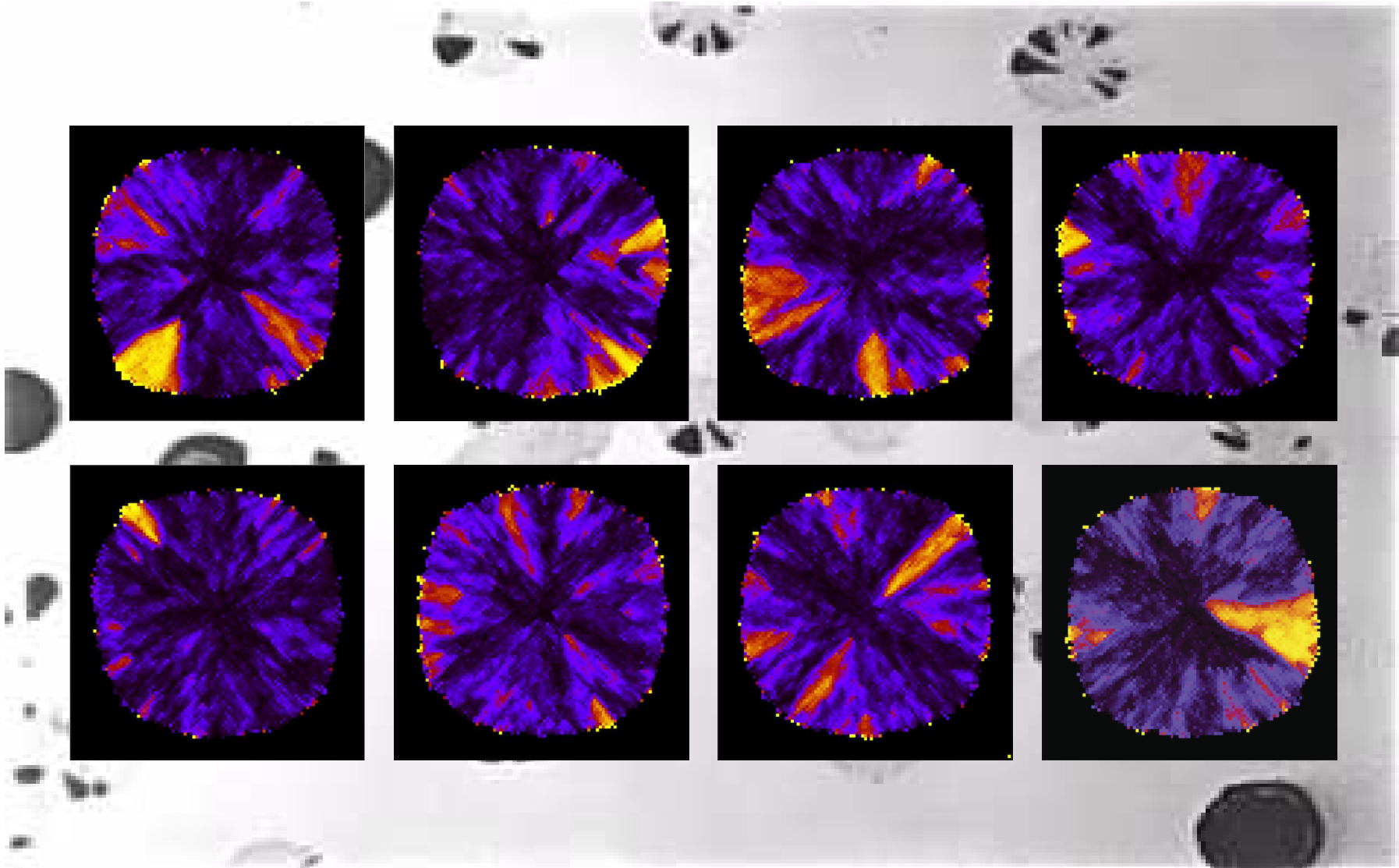


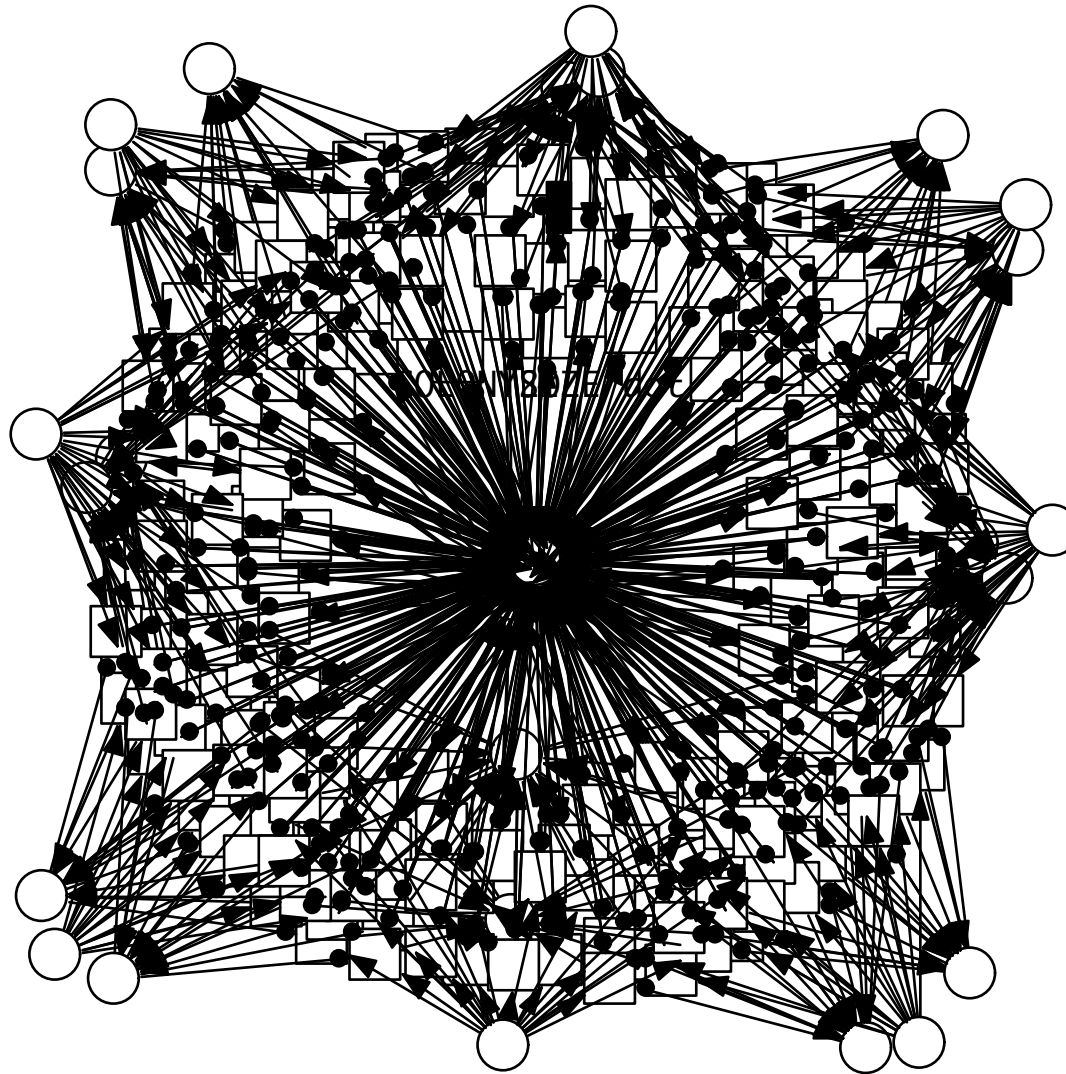
Ex3: SOME FINAL STATES (HIGH, $F=1$)

PN & BioModel Engineering



Ex3: SOME FINAL STATES (HIGH, $F=1$)



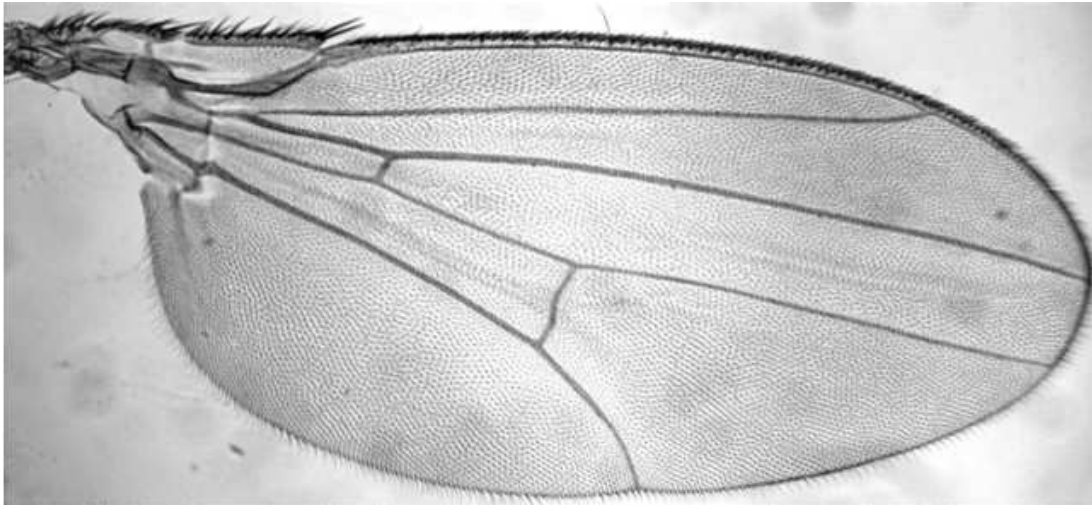


EXAMPLE 4:

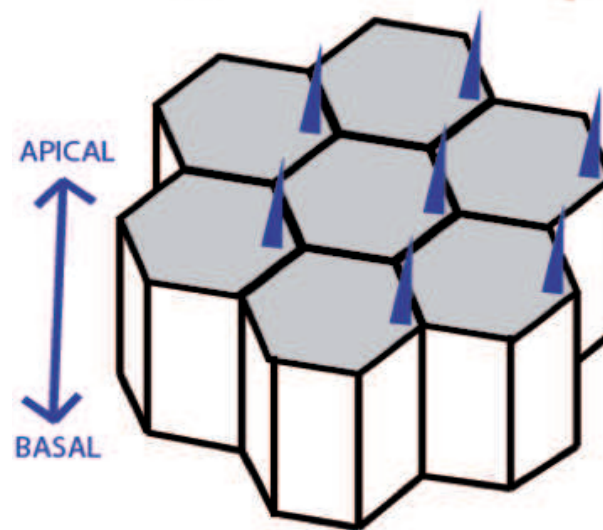
PLANAR CELL POLARITY IN FLY WING

Gao, Gilbert, Heiner, Liu, Maccagnola, Tree:
Multiscale Modelling and Analysis of Planar Cell Polarity in the Drosophila Wing;
IEEE/ACM Transactions on Computational Biology and Bioinformatics, 2013.

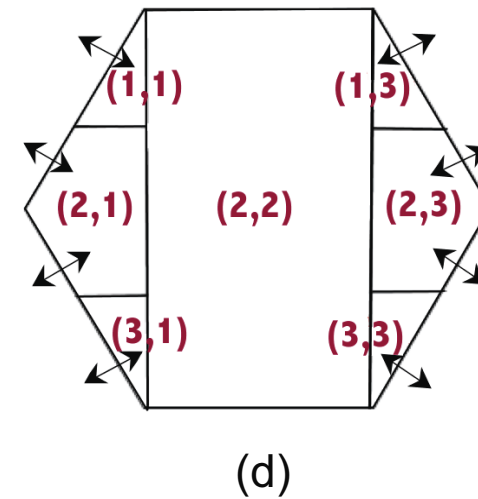
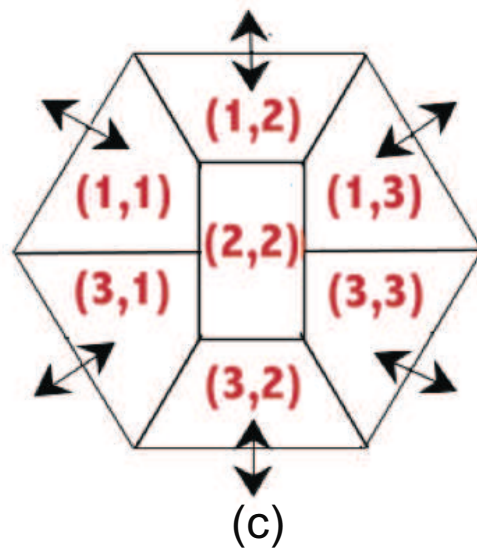
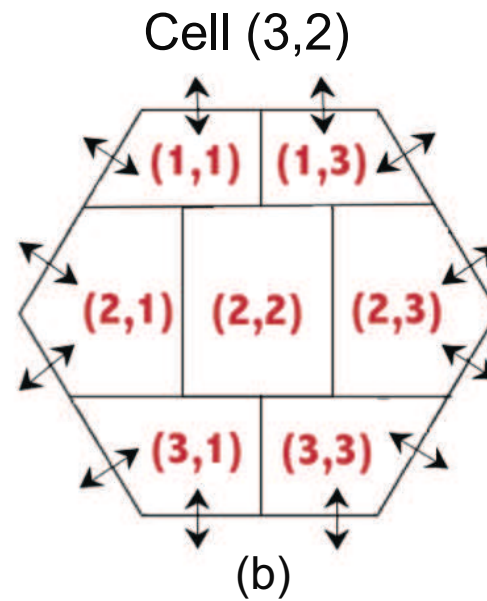
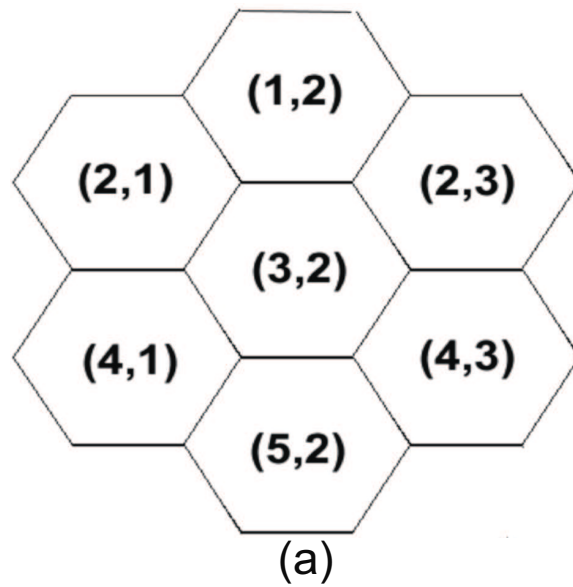
Ex4: PLANAR CELL POLARITY



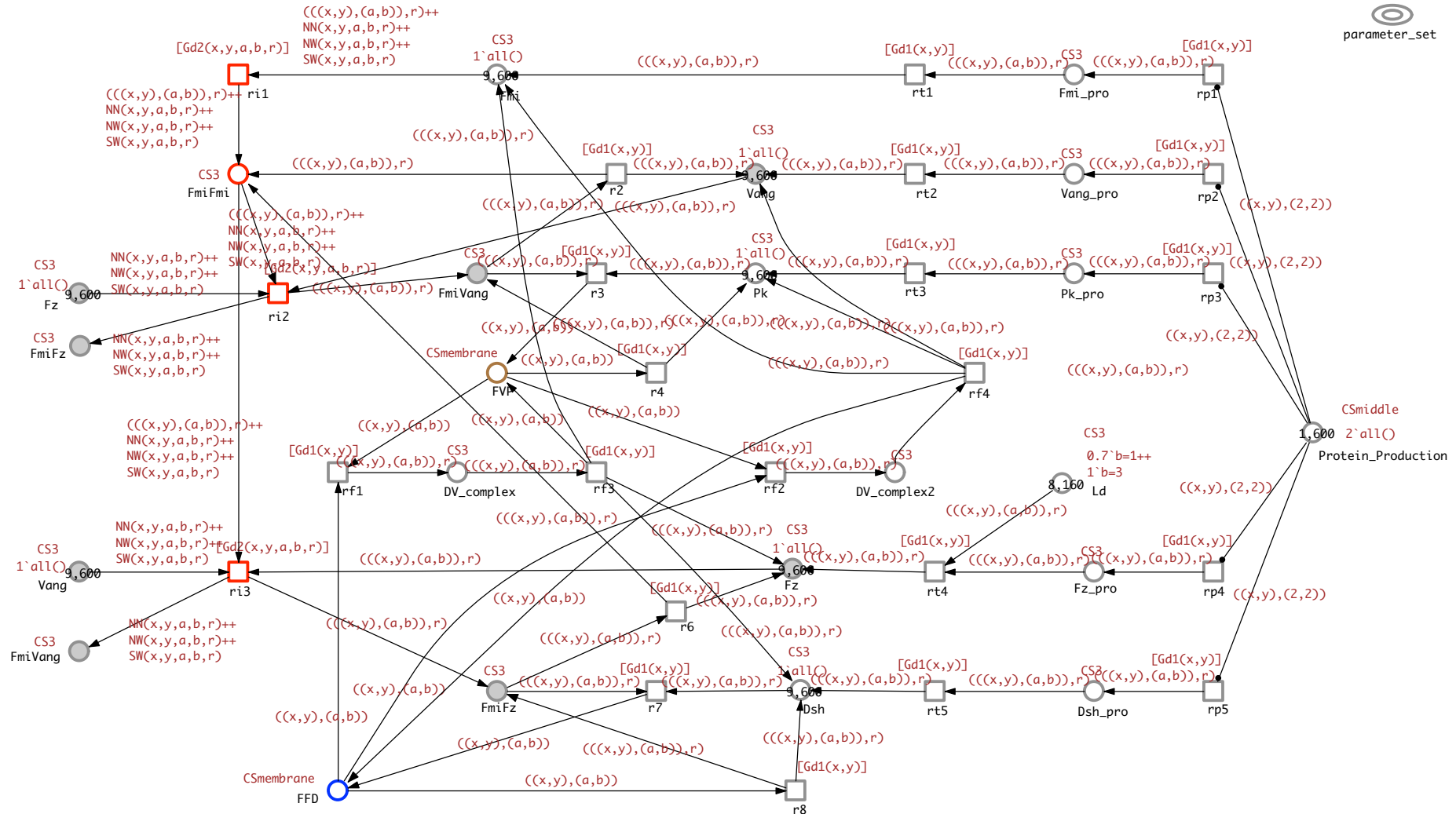
PROXIMAL $\longleftrightarrow$ DISTAL



[BioPPN 2011]
[CMSB 2011]



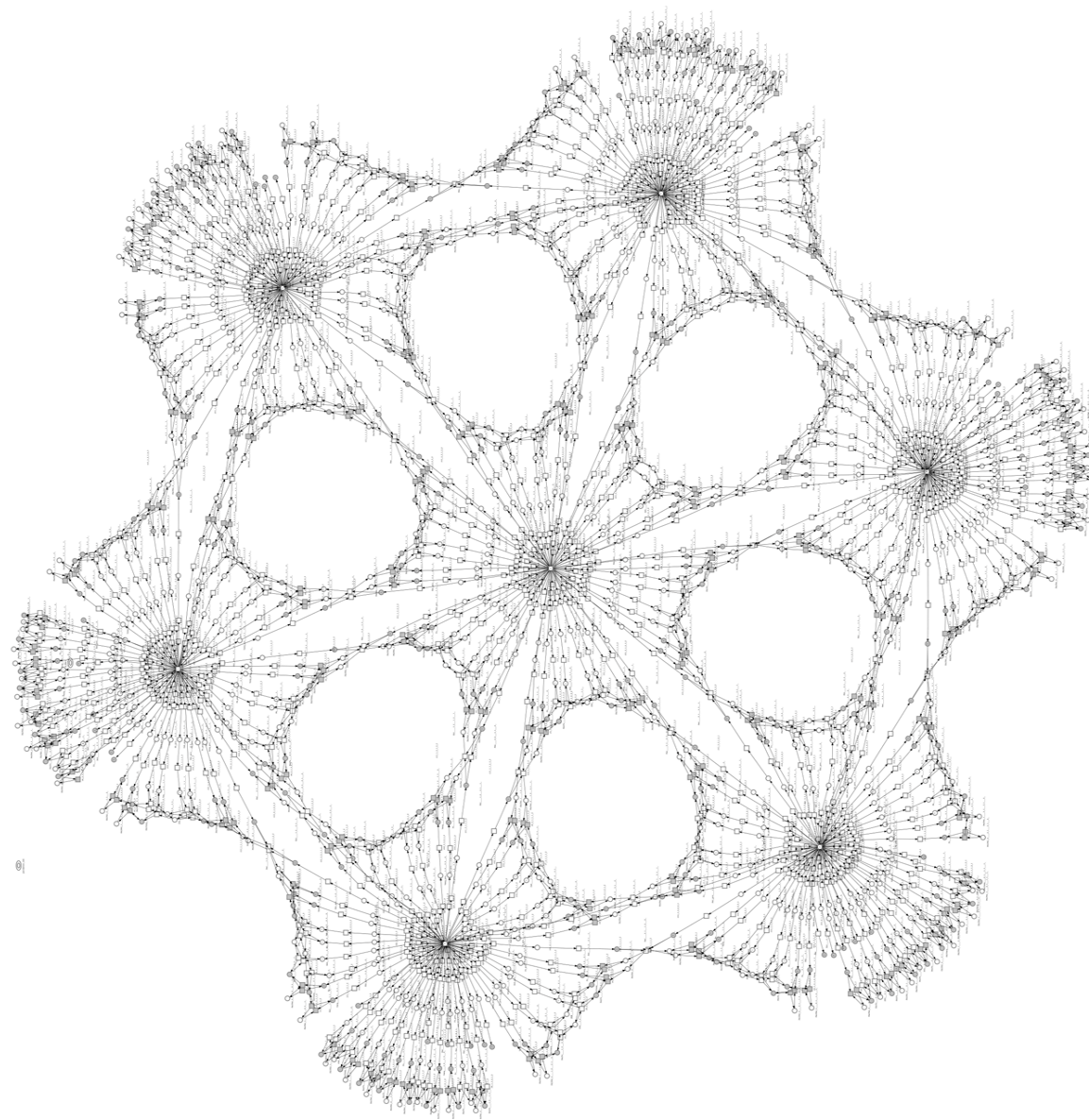
Ex4: PLANAR CELL POLARITY



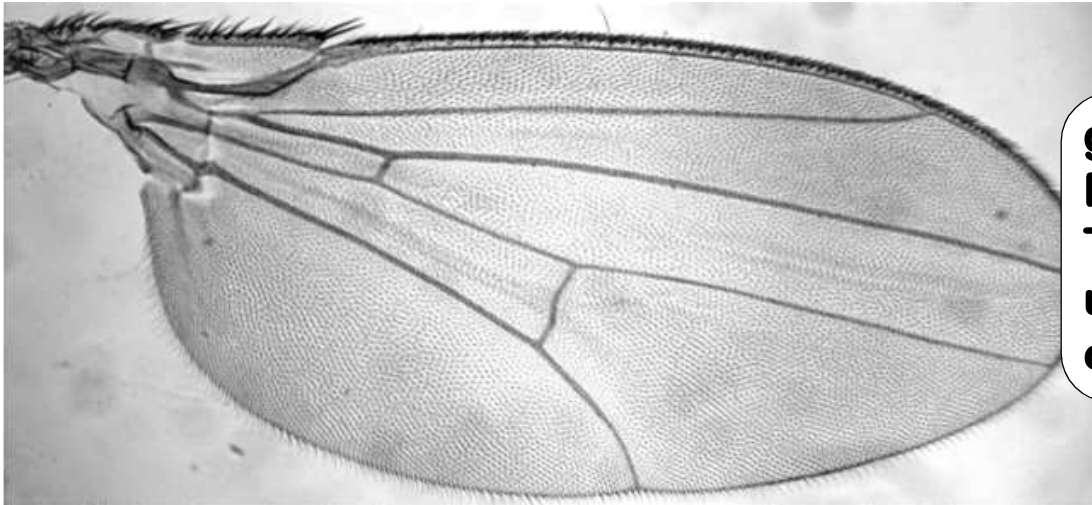
[QIAN GAO, PHD THESIS 2013]

Ex4: PLANAR CELL POLARITY, PLAIN MODEL (7 CELLS)

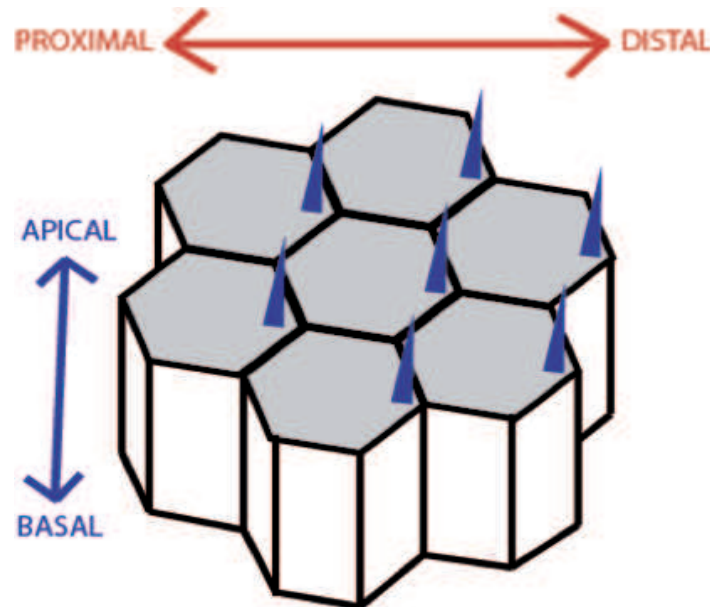
PN & BioModel Engineering



Ex4 - PLANAR CELL POLARITY



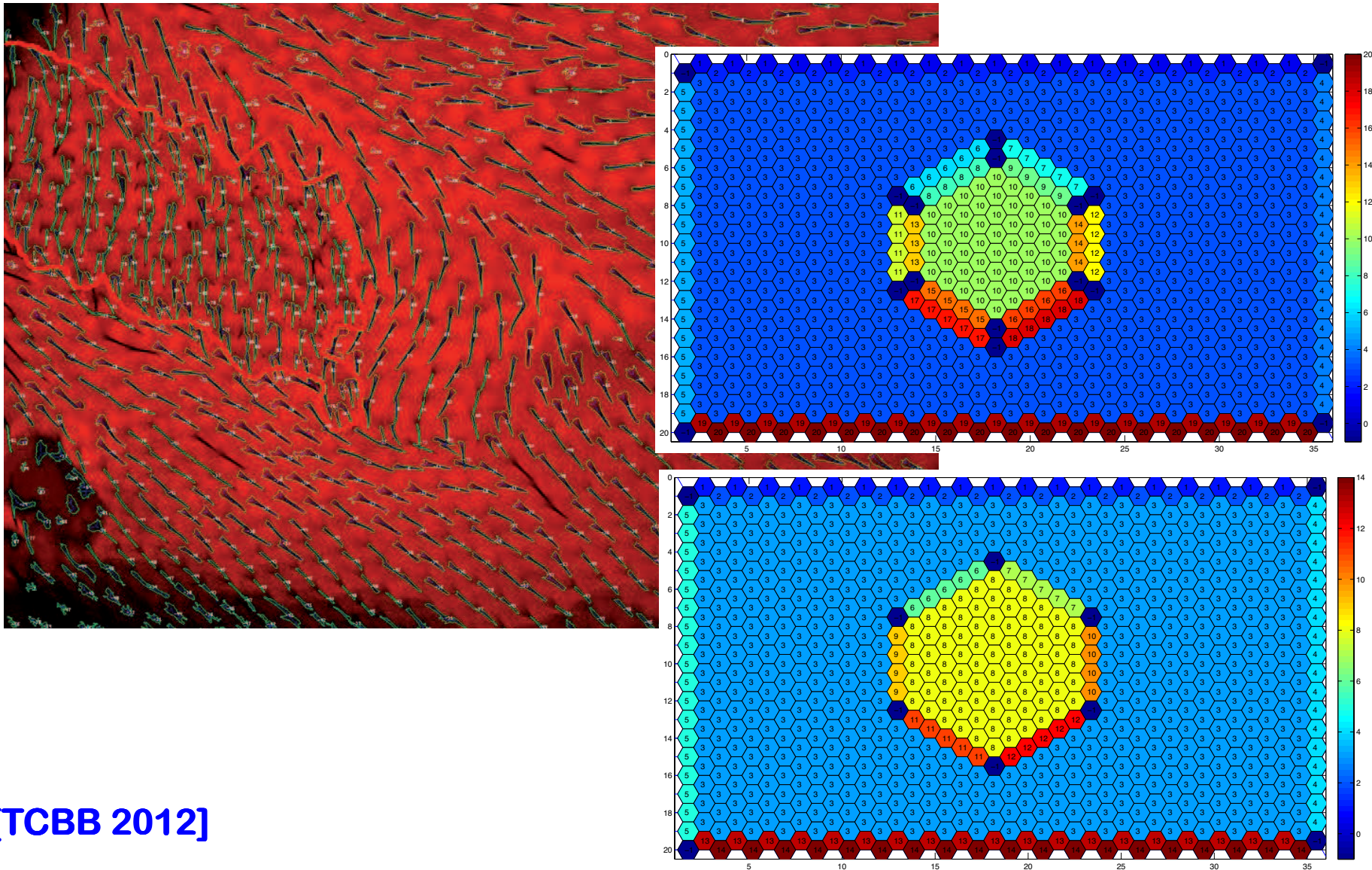
grid size:	40 x 40
PLACES:	164,000
TRANSITIONS:	229,686
unfolding:	4 min
cont. simulation:	2 h



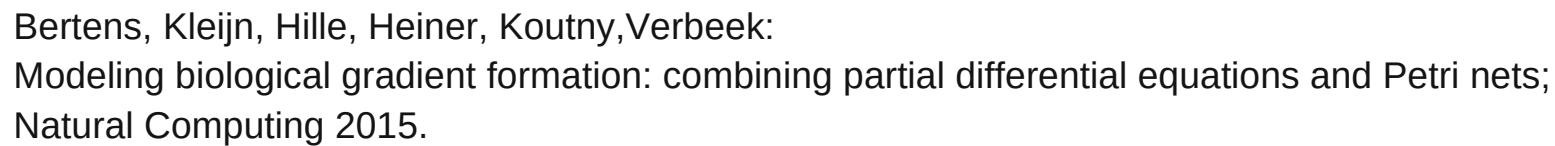
[BioPPN 2011]
[CMSB 2011]



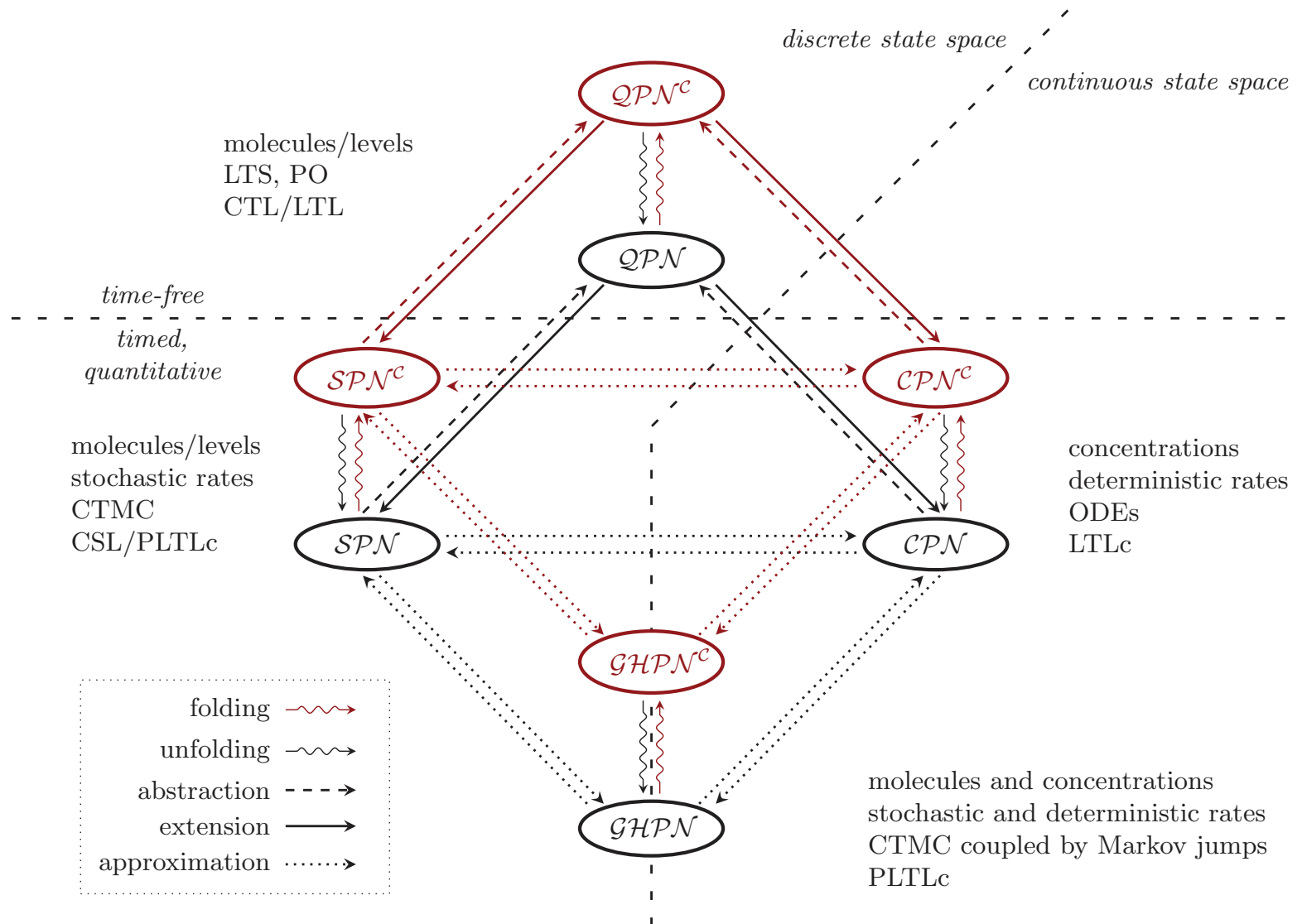
Ex4 - PLANAR CELL POLARITY



[TCBB 2012]

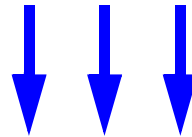


FRAMEWORK SUMMARY

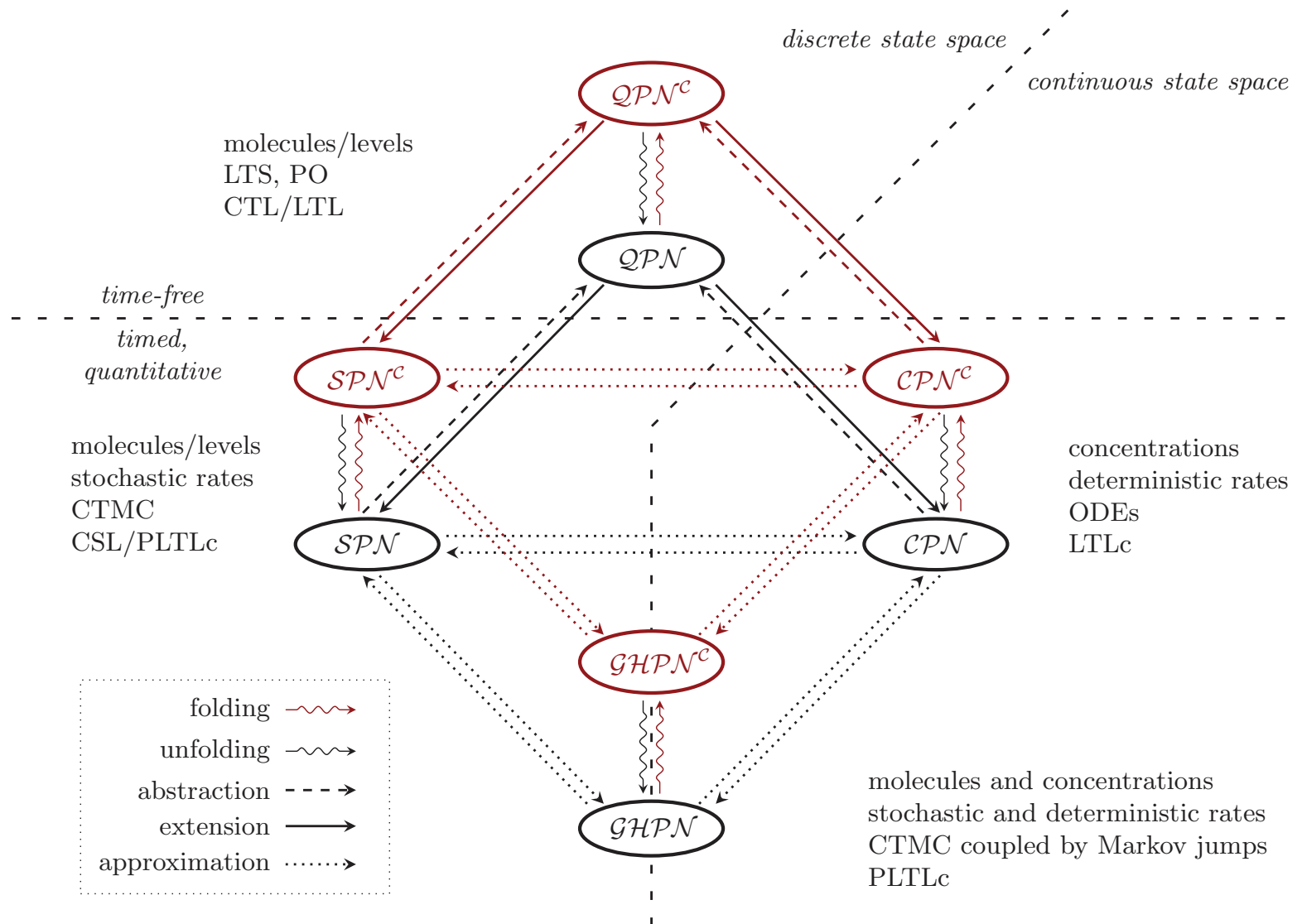


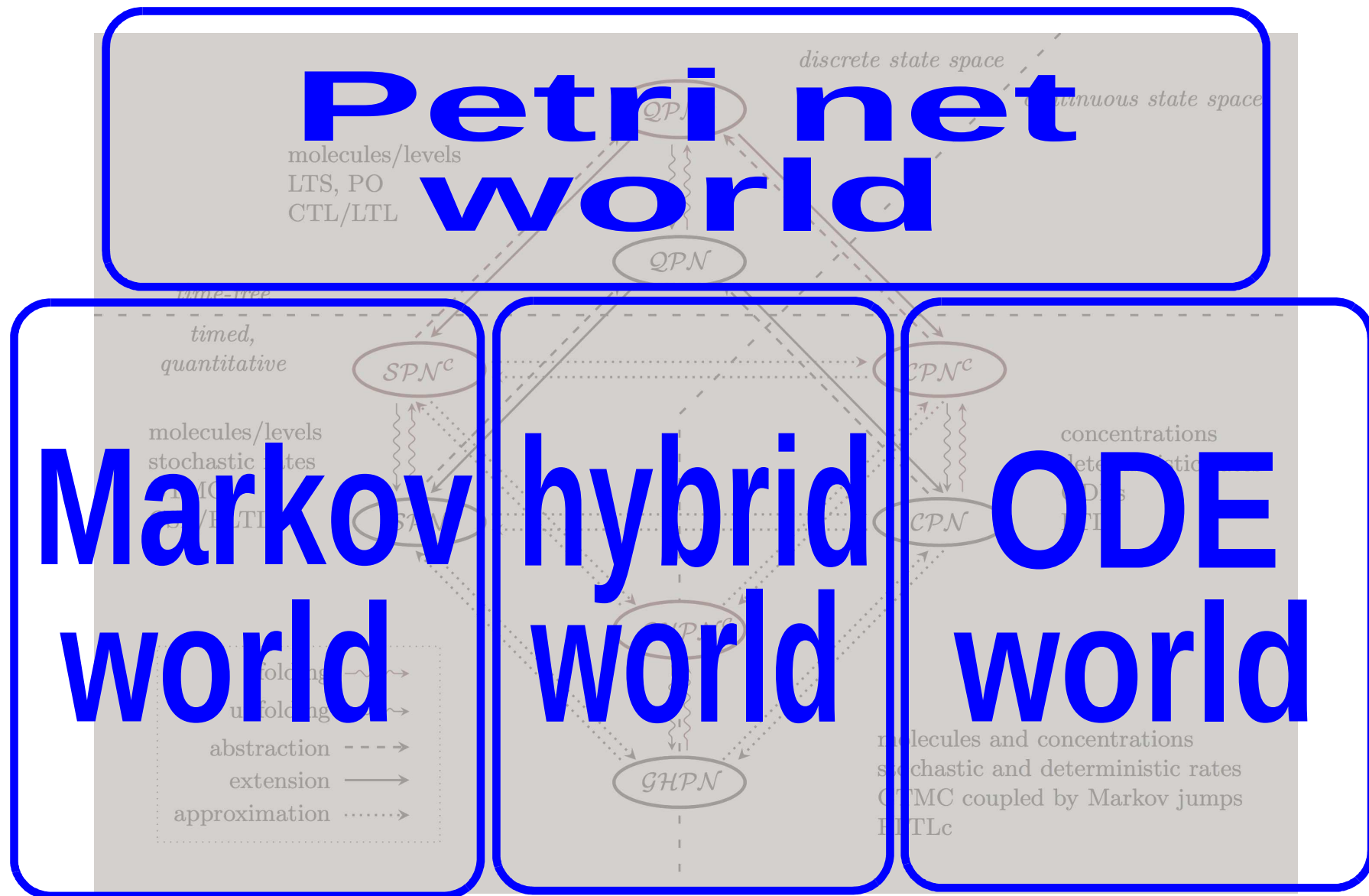
4x2

MODELS SHARING STRUCTURE



**QUANTITATIVE MODEL = QUALITATIVE MODEL
+
RATE FUNCTIONS
(KINETICS)**

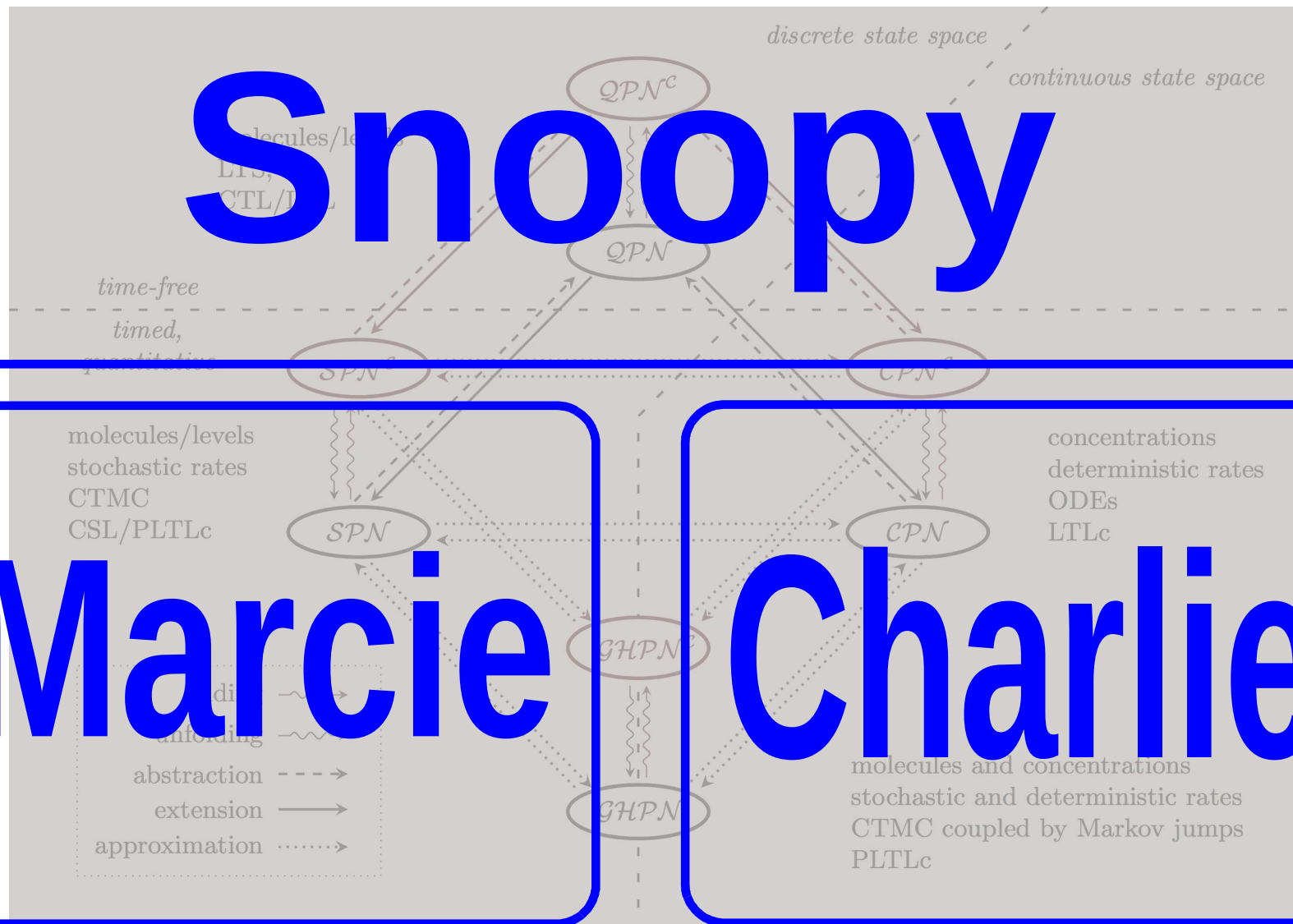




Snoopy

Marcie

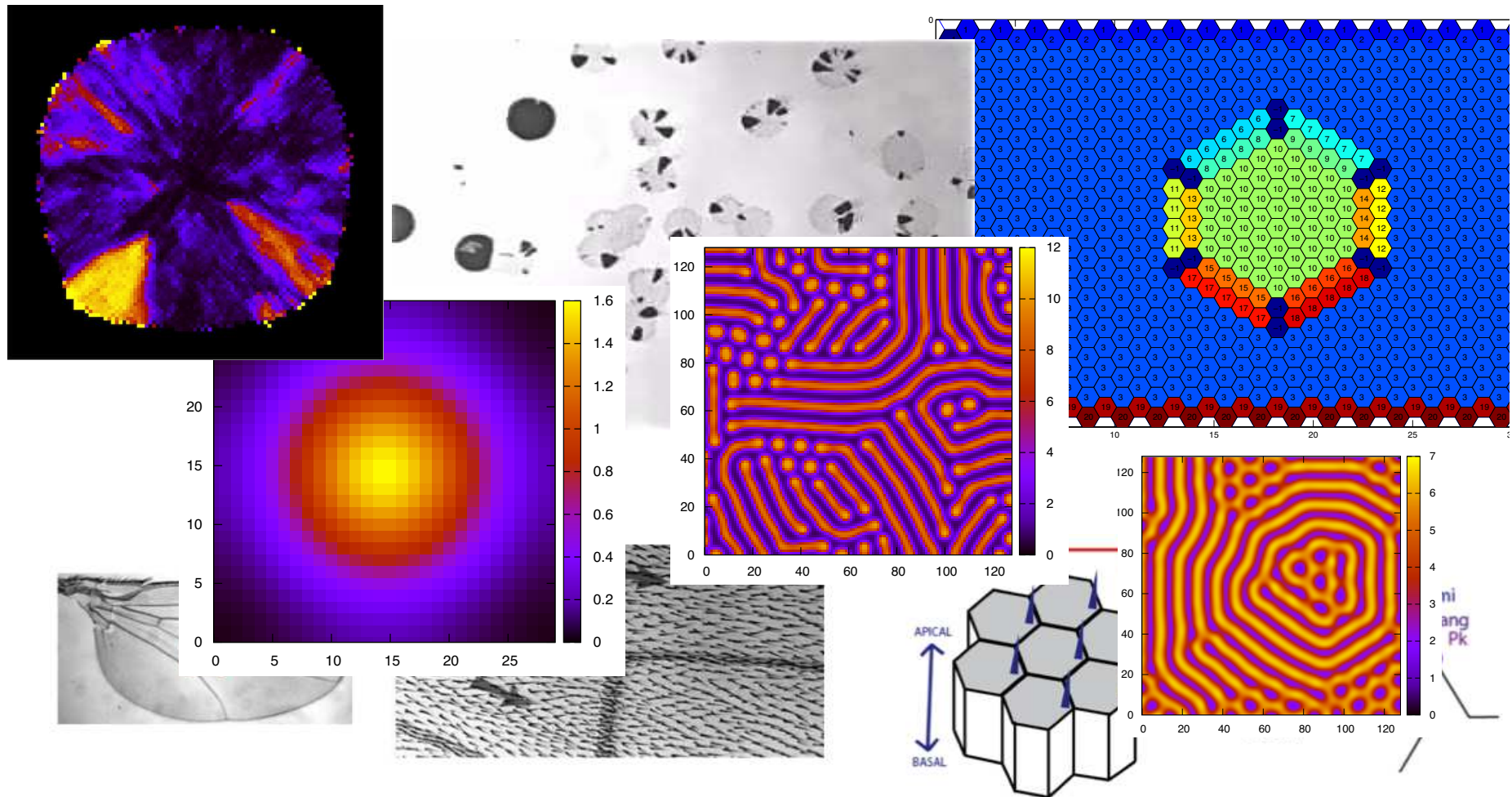
Charlie





- ❑ **David Gilbert**
Brunel University London, UK
- ❑ **Wolfgang Marwan,**
Otto-von-Guericke University Magdeburg,
Germany
- ❑ **Fei Liu**
South China University of Technology
- ❑ **Mostafa Herajy**
Port Said Universit, Egypt
- ❑ **Mary Ann Blätke**
IPK Gatersleben, Germany





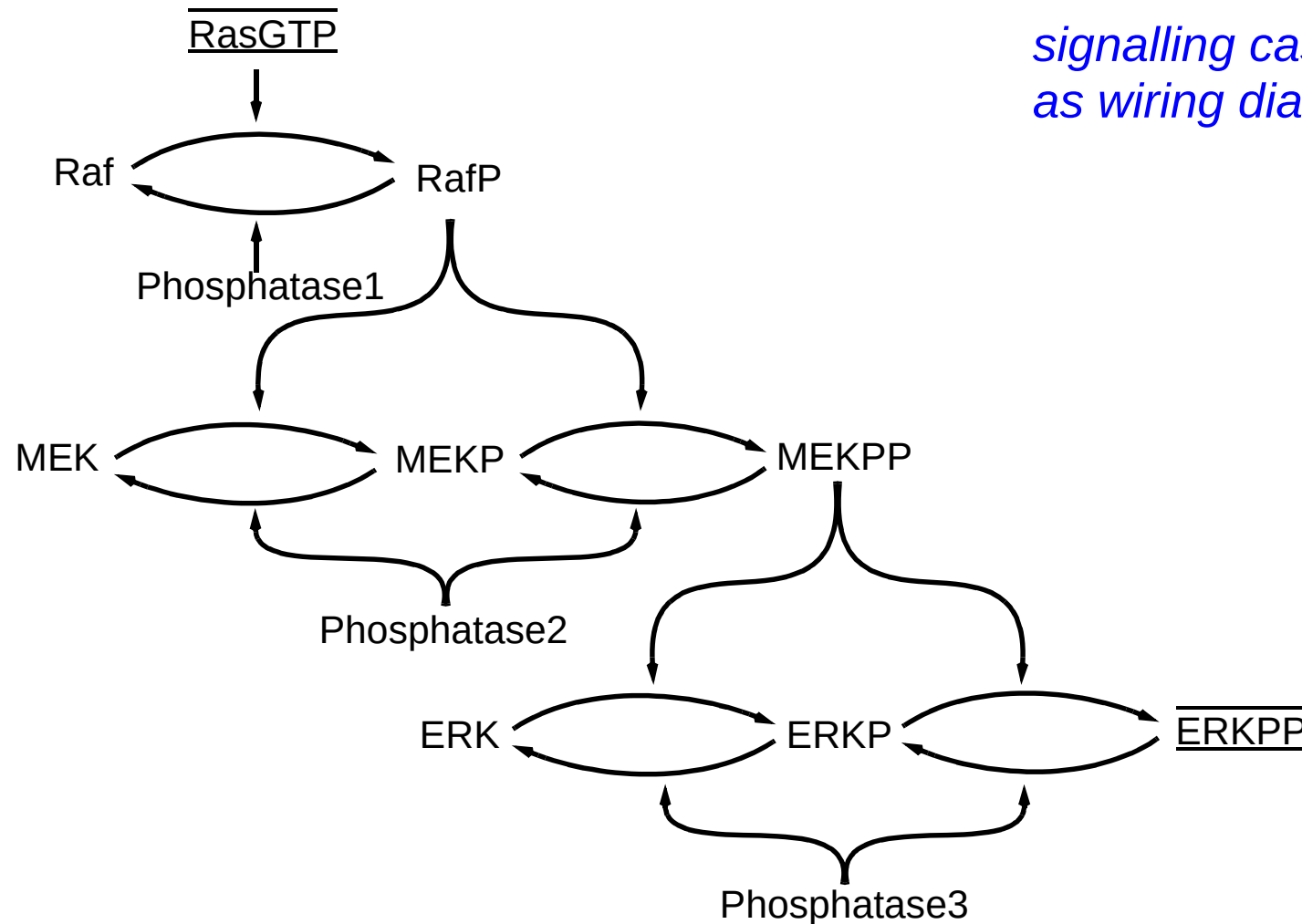
[HTTP://WWW-DSSZ.INFORMATIK.TU-COTTBUS.DE](http://www-dssz.informatik.tu-cottbus.de)

[HTTP://MULTISCALEPN.BRUNEL.AC.UK](http://multiscalepn.brunel.ac.uk)



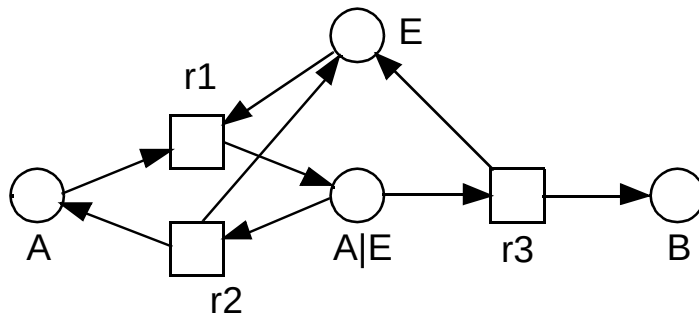
MODELLING Bio (PETRI) NETS

APPROACH 1



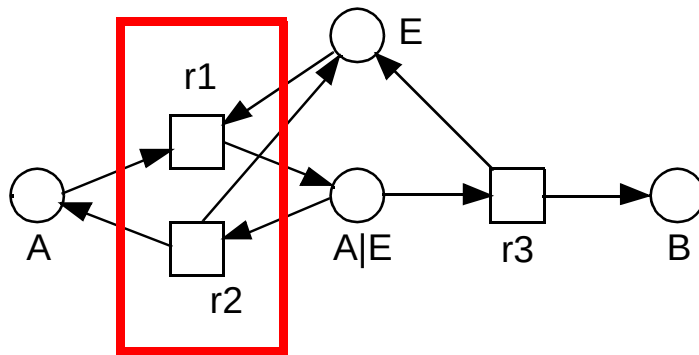


*enzymatic reaction,
mass-action kinetics*



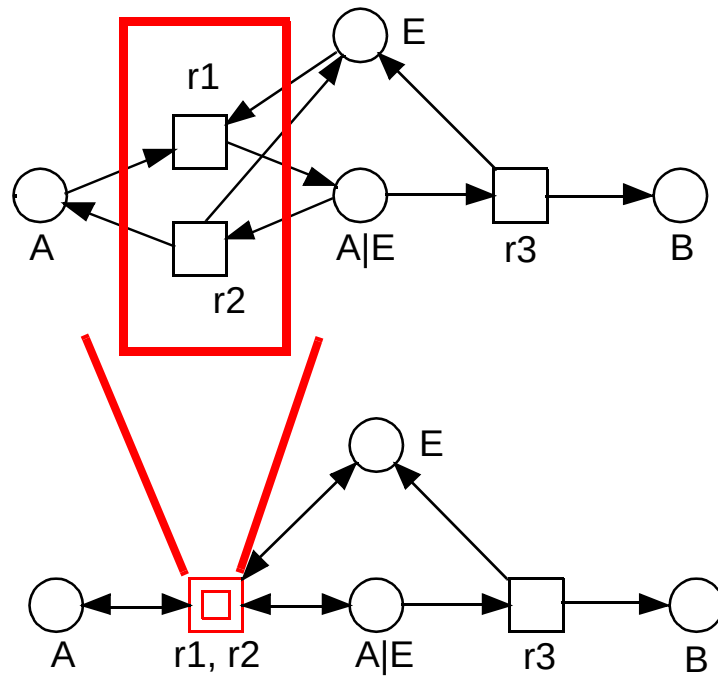


*enzymatic reaction,
mass-action kinetics*



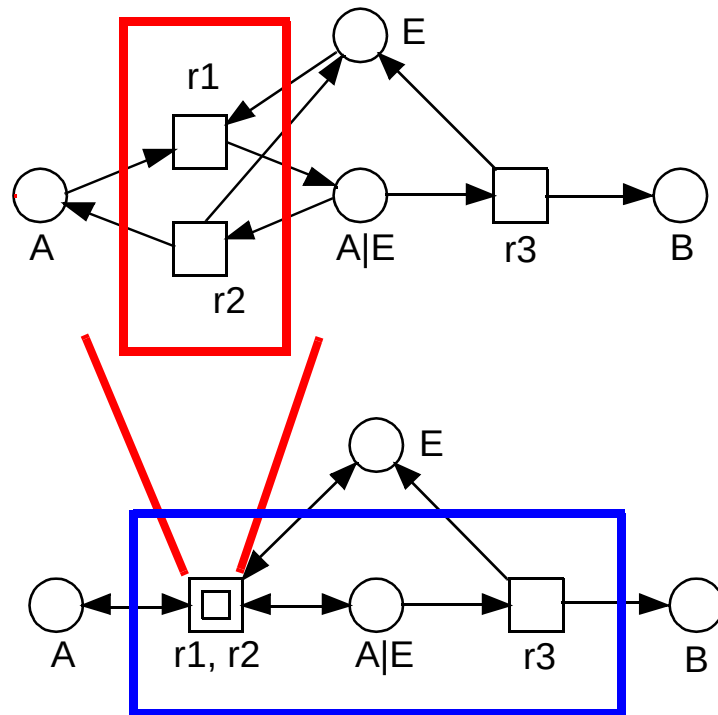


*enzymatic reaction,
mass-action kinetics*



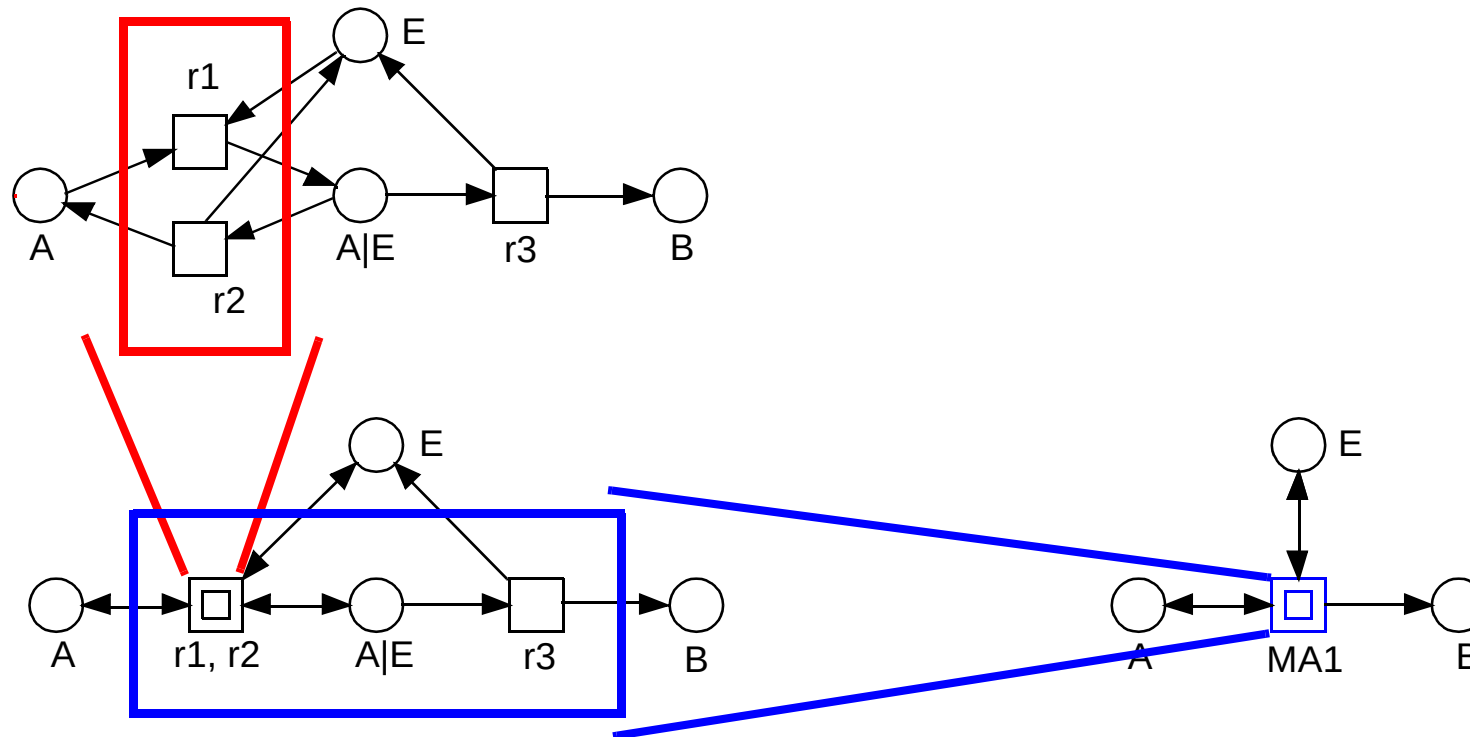


*enzymatic reaction,
mass-action kinetics*



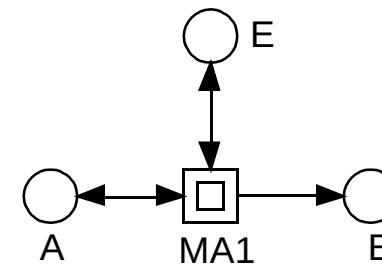
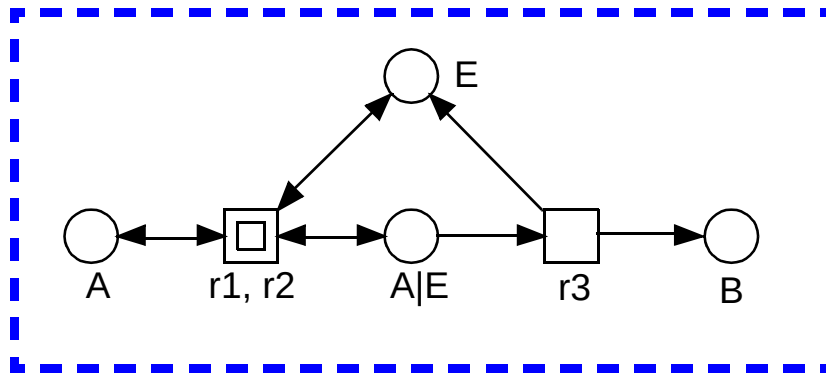
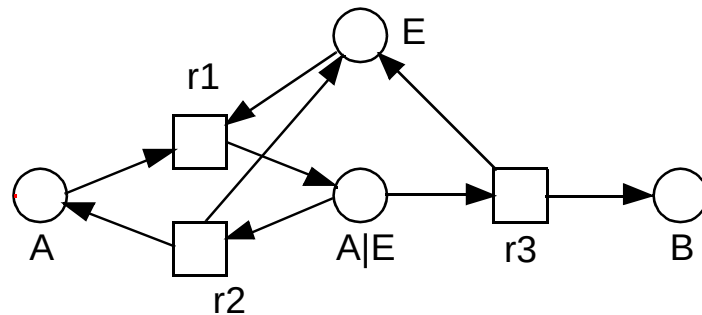


*enzymatic reaction,
mass-action kinetics*

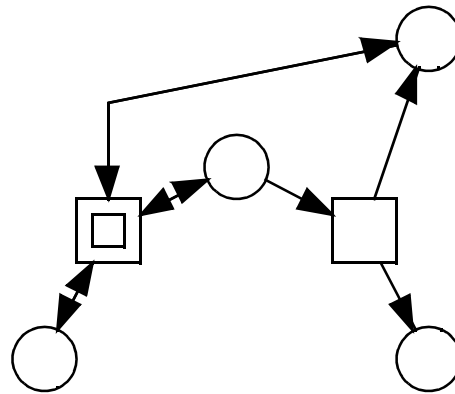


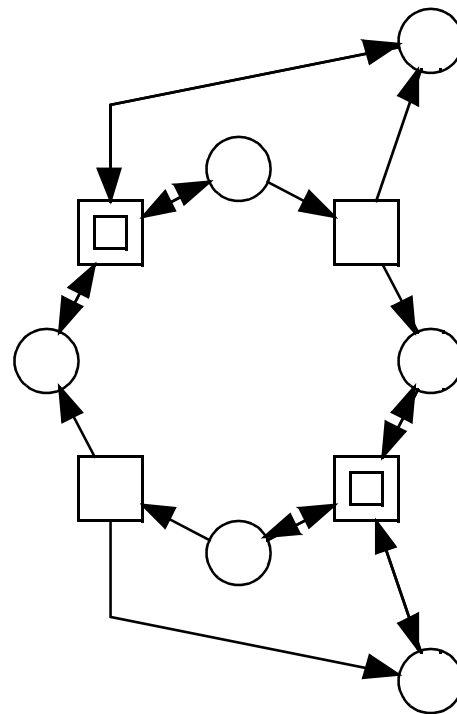


*enzymatic reaction,
mass-action kinetics*



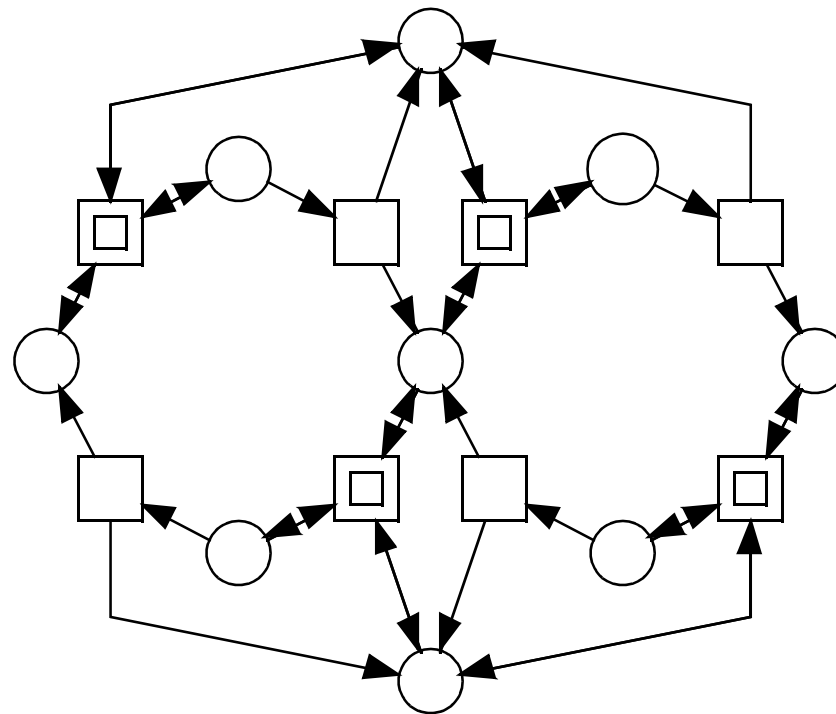
*SINGLE
MASS-ACTION STEP*

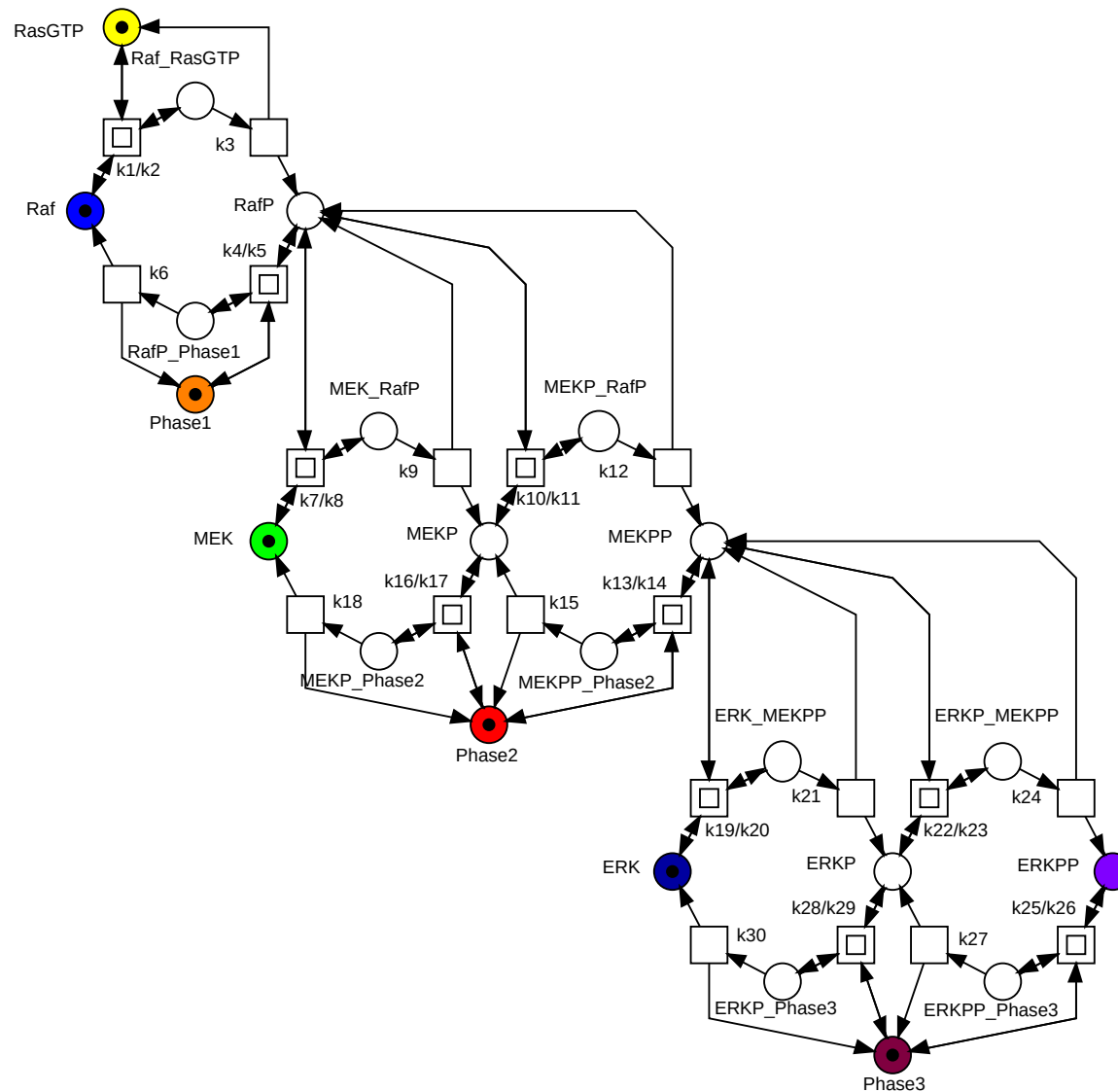




SINGLE
PHOSPHORYLATION / DEPHOSPHORYLATION

DOUBLE PHOSPHORYLATION / DEPHOSPHORYLATION



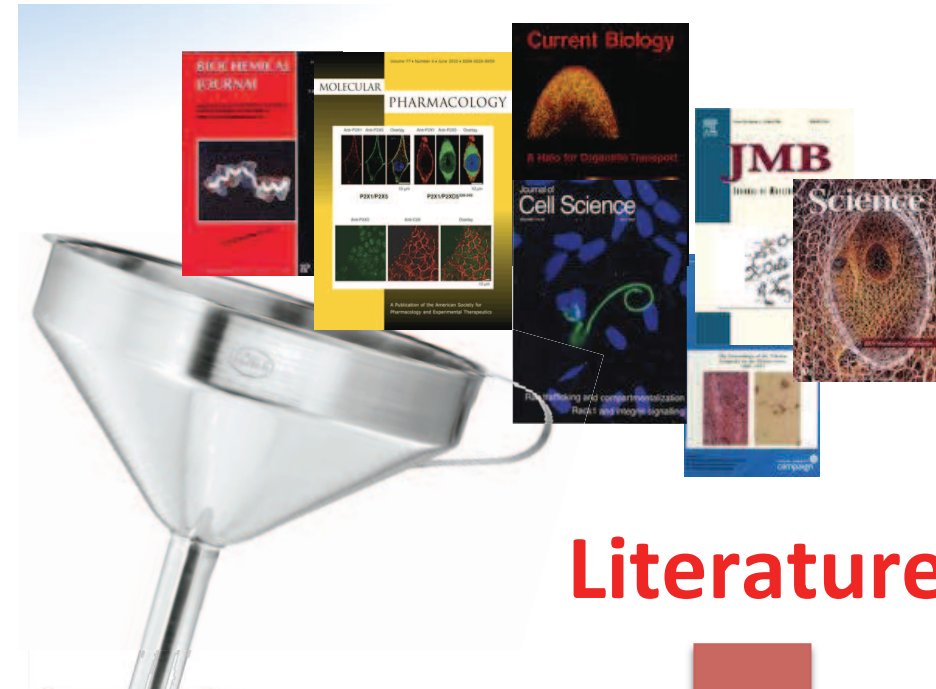
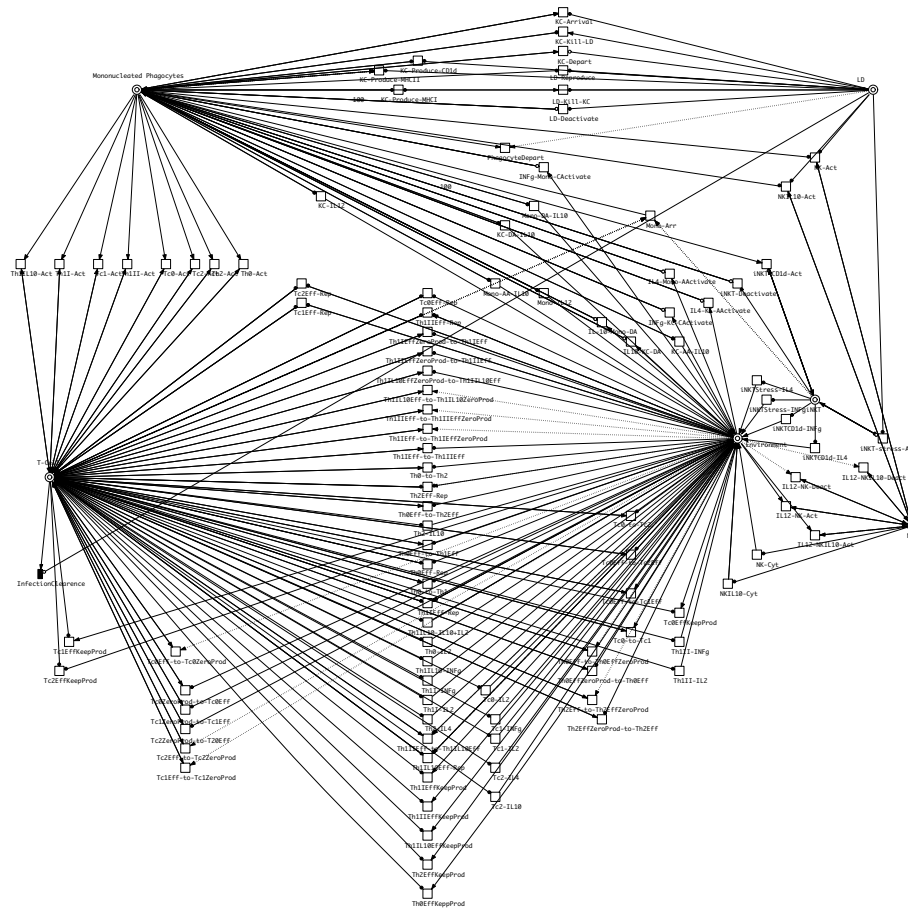


[GILBERT,
HEINER,
LEHRACK 2007]

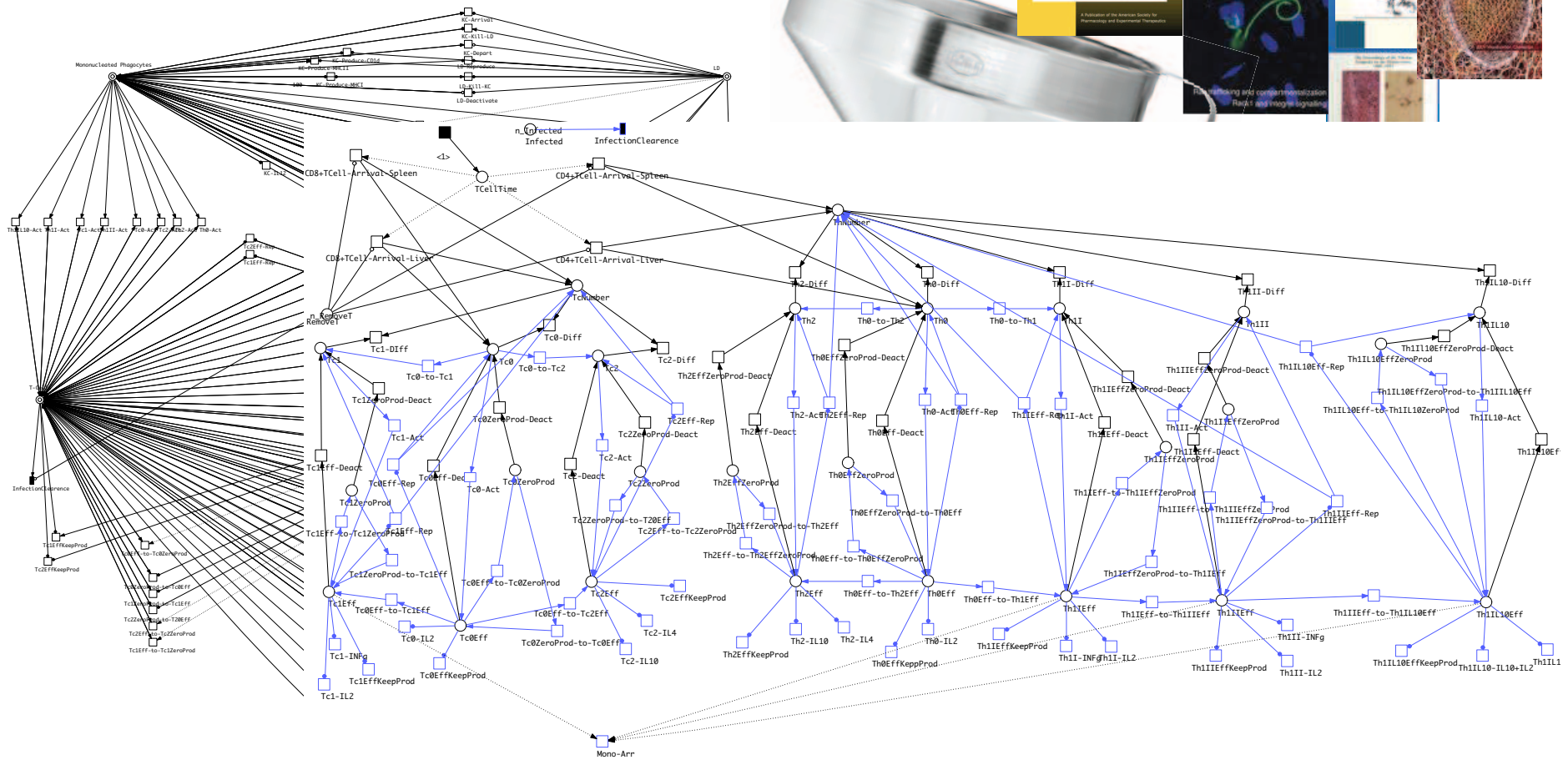
[HEINER,
GILBERT,
DONALDSON 2008]

APPROACH 2

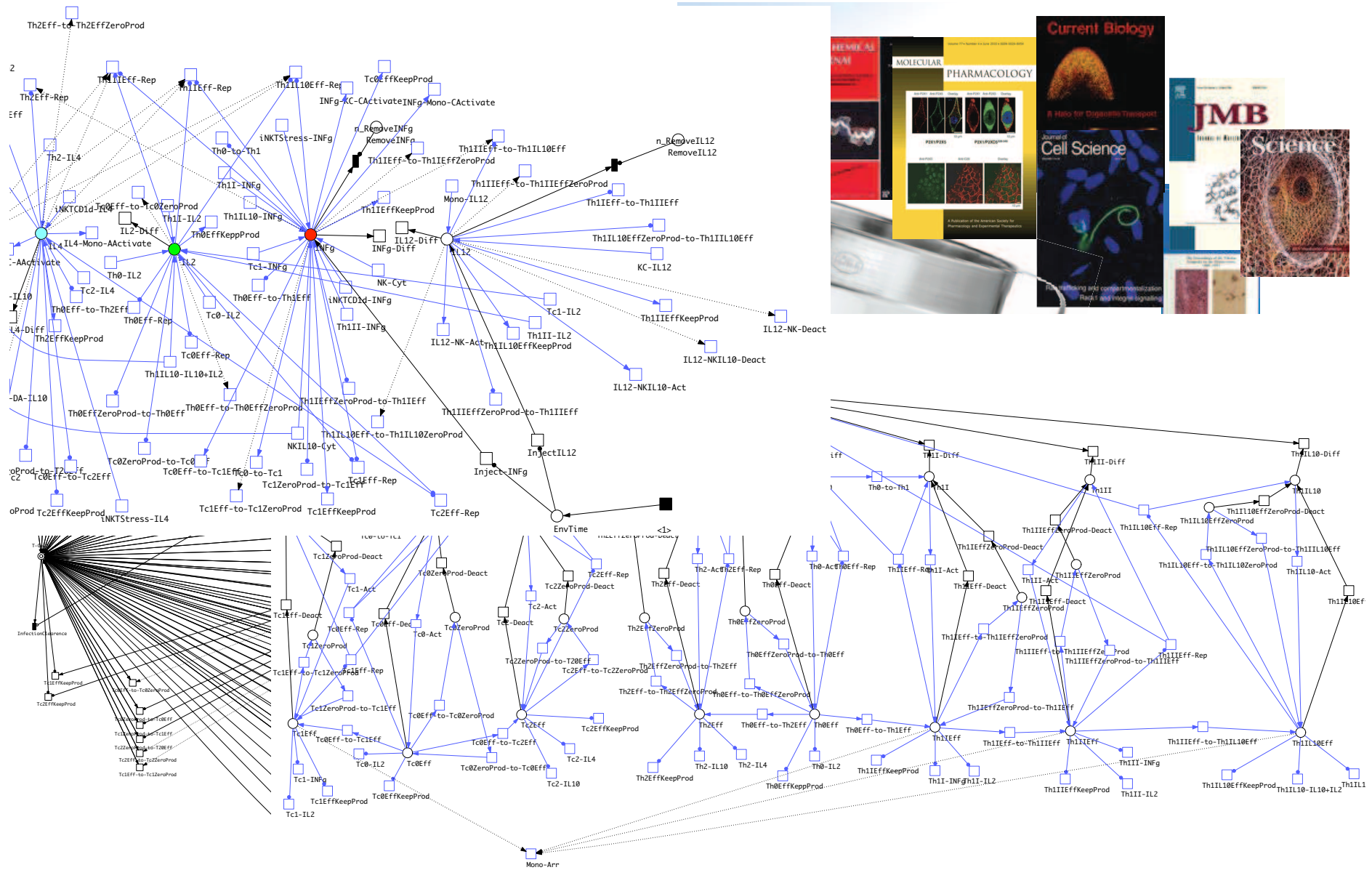




Literature

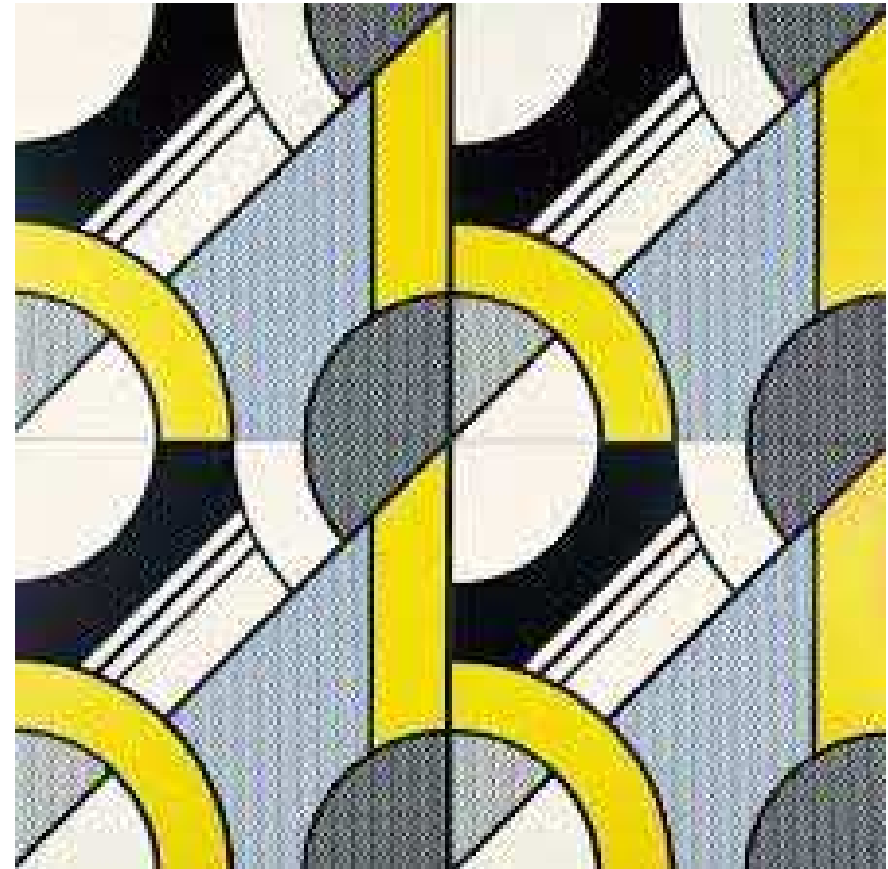


FROM LITERATURE TO MODELS



APPROACH 3

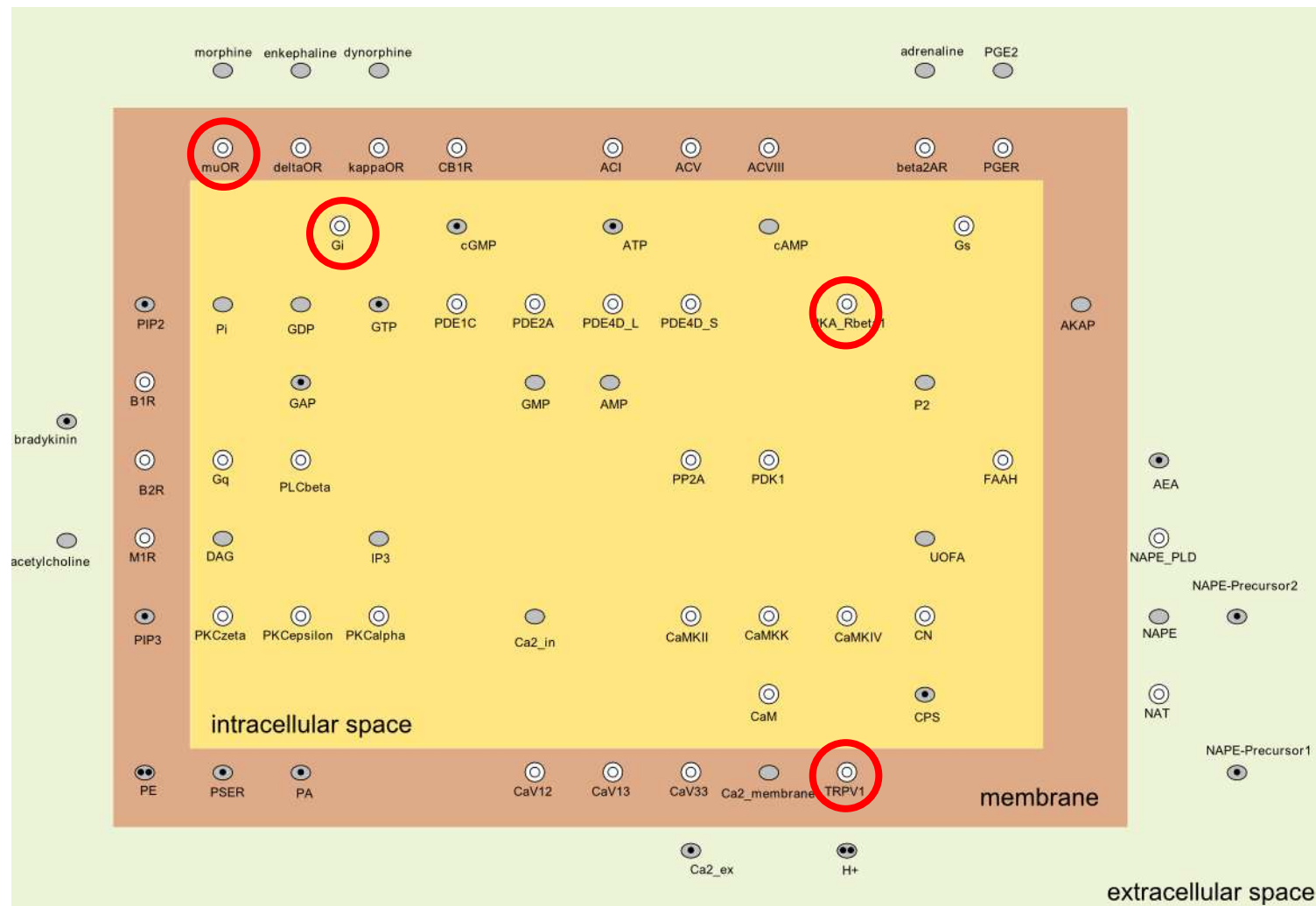
MODULAR MODELLING



**JOINT WORK WITH
MARY ANN BLÄTKE, WOLFGANG MARWAN**

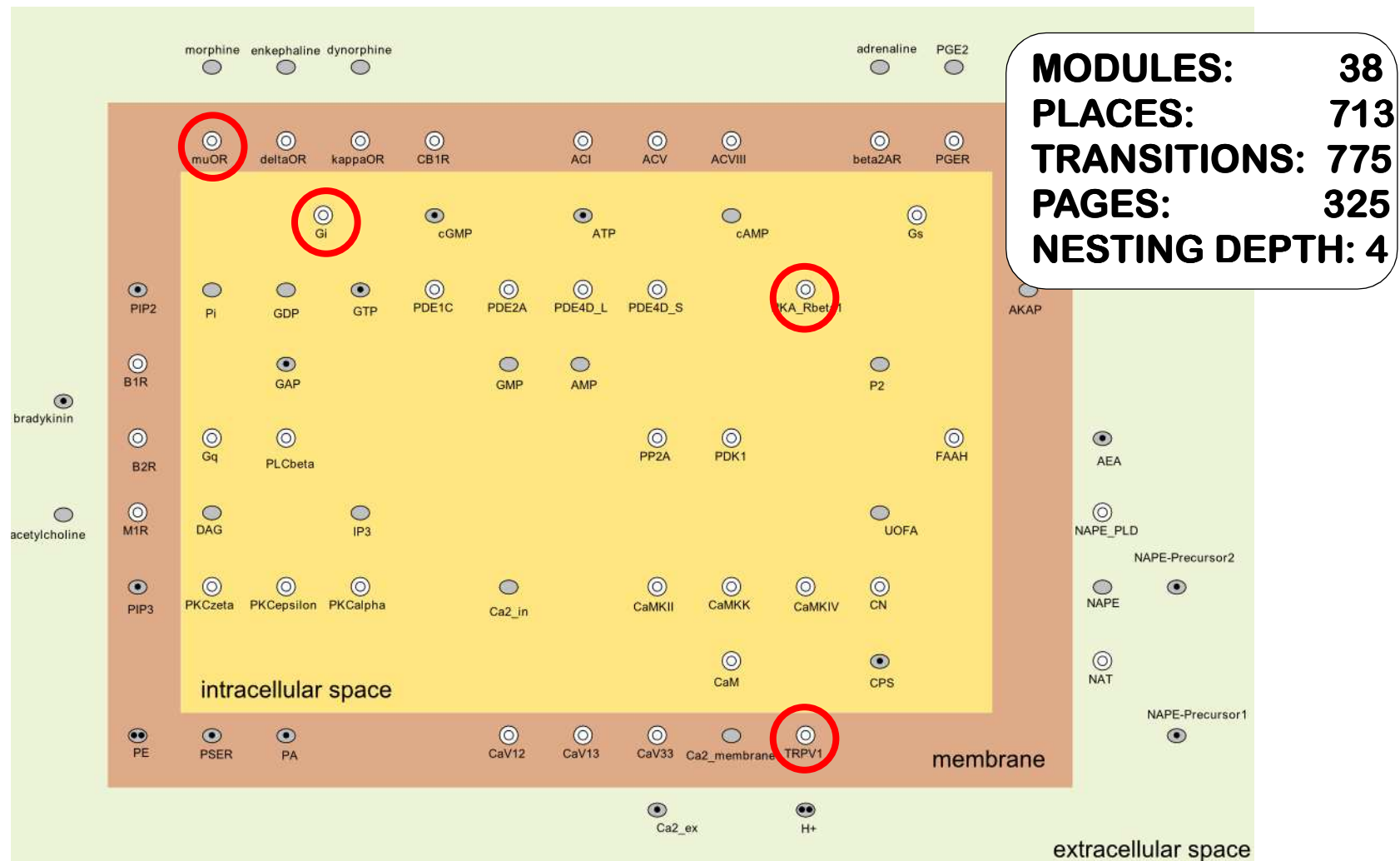
[BLÄTKE, MEYER, MARWAN 2011]

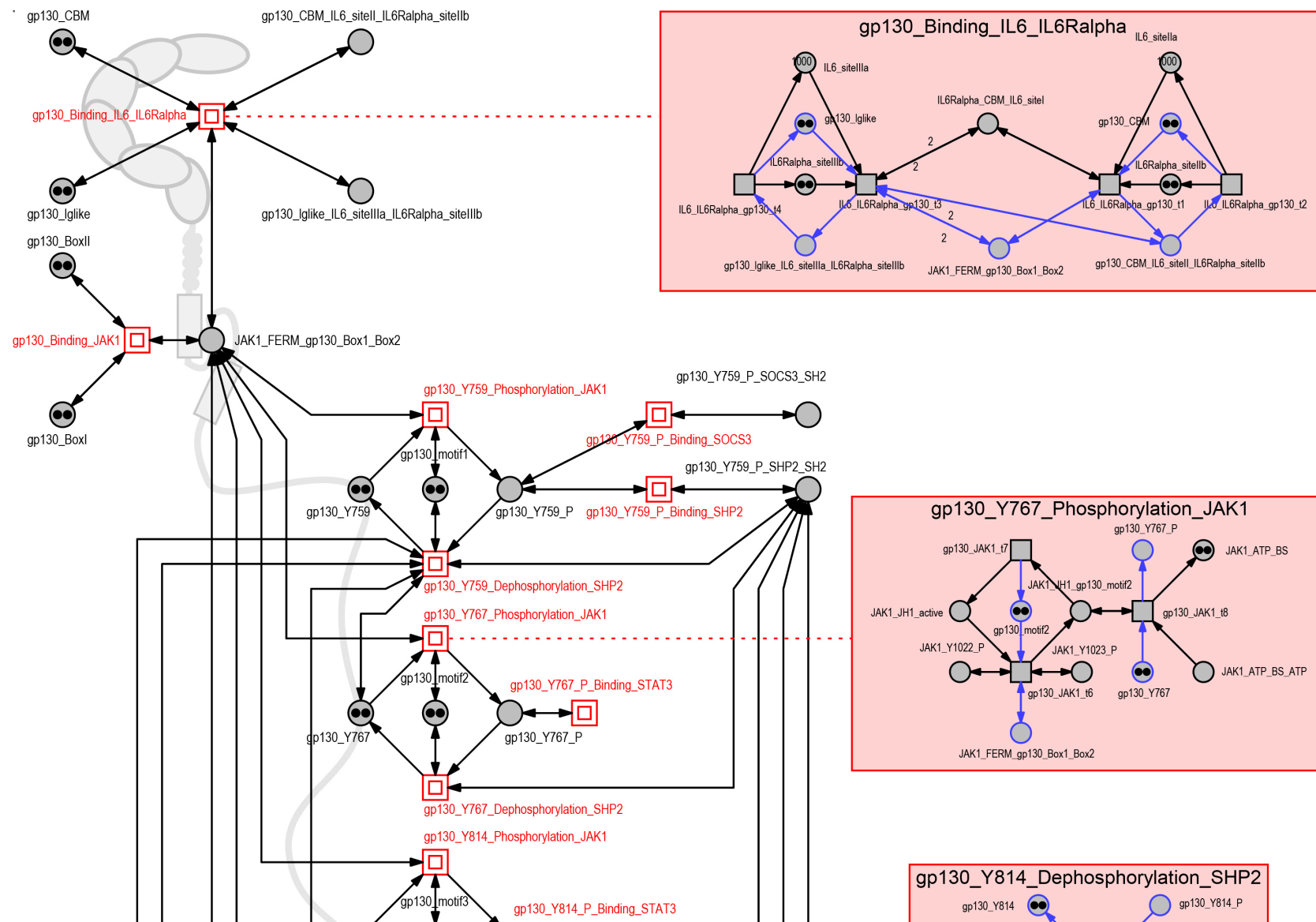
-> A PROTEIN-ORIENTED MODULAR MODELLING CONCEPT



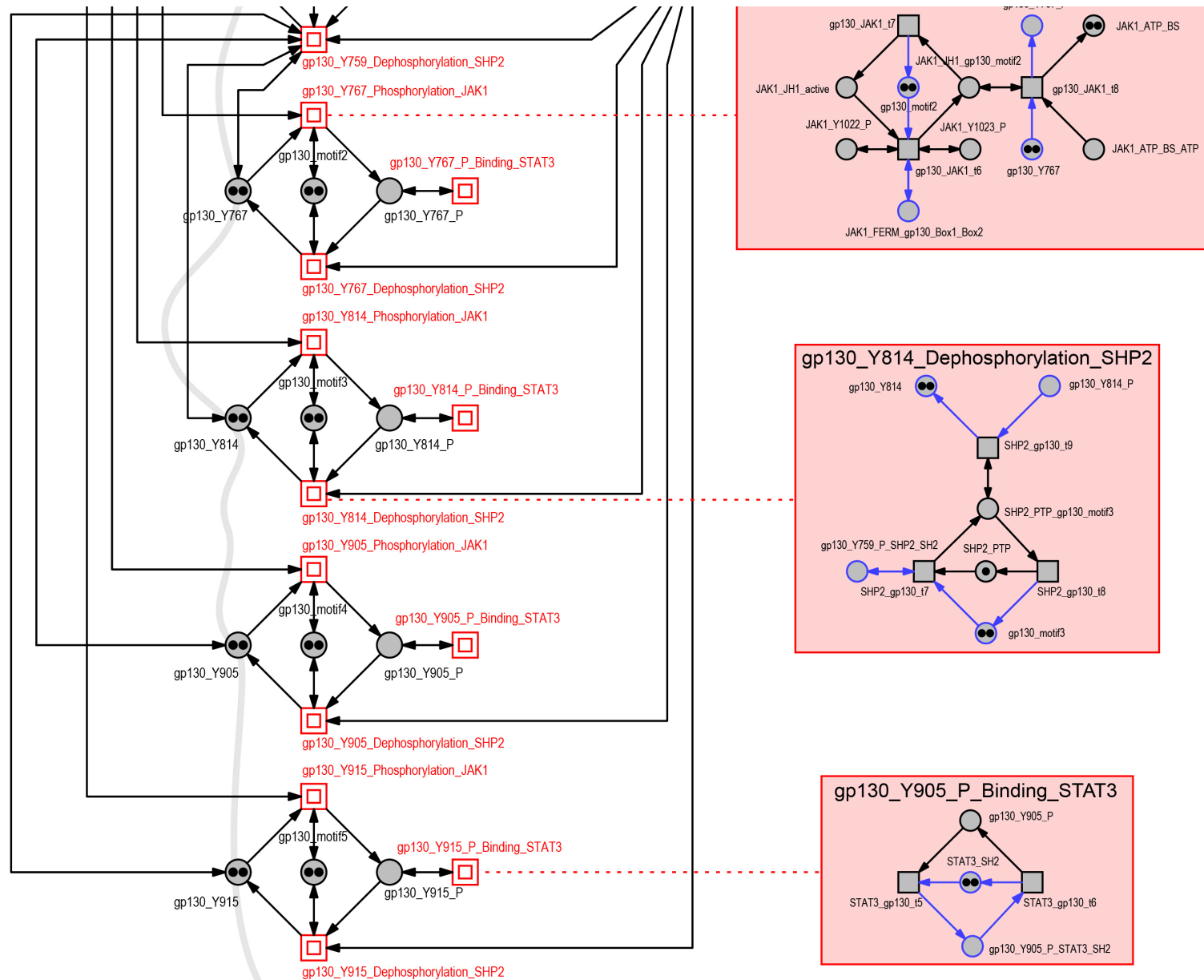
[BLÄTKE, MEYER, MARWAN 2011]

-> A PROTEIN-ORIENTED MODULAR MODELLING CONCEPT



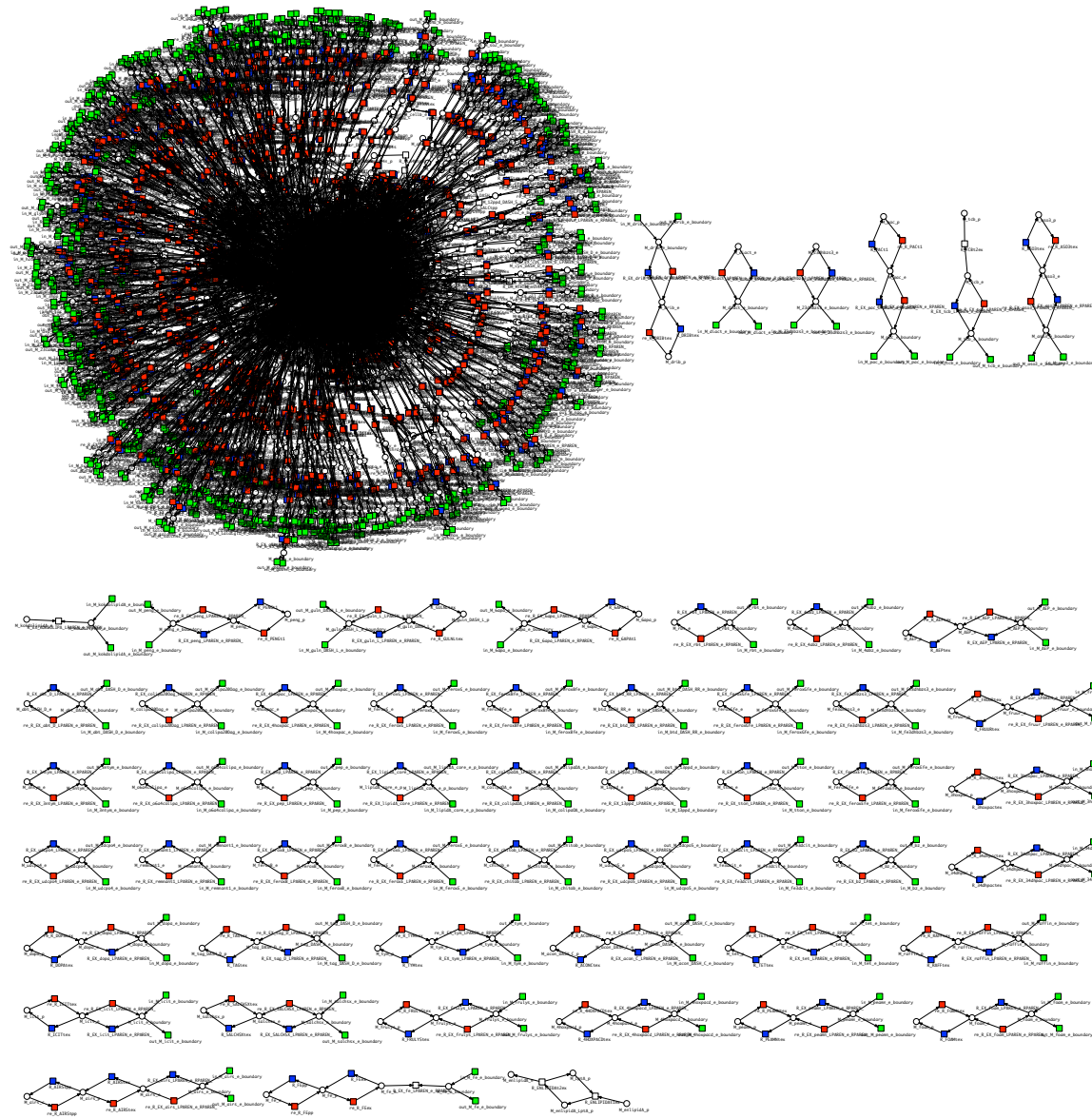


MODULE GP130 TRANSMEMBRANE RECEPTOR



APPROACH 4

[Monk 2013]



[illegible]

Nodeclass	Count
Place	2330
Transition	4168
Immediate Transition	0
Deterministic Transition	0
Scheduled Transition	0
LookupTable	0

Edgeclass	Count
Arc	13558
Read Arc	0
Inhibitor Arc	0
Reset Arc	0
Equal Arc	0
Modifier Arc	0

OK