# Efficient Unfolding of Coloured Petri Nets using Interval Decision Diagrams

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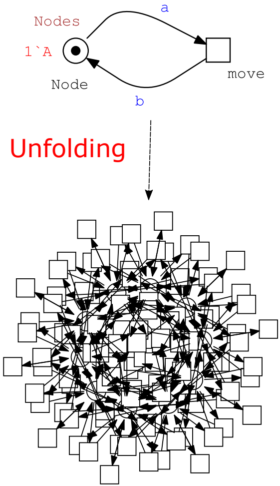
Brandenburg Technical University
Petri Nets 2020 - Paris

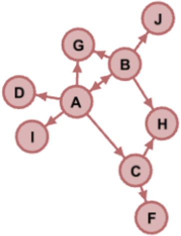
25 June 2020

# Outline





# Powerful modelling - Coloured Petri nets



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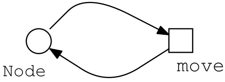


## Color definitions:

\\ Simple CS
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\\ Product CS
Matrix = Prod(Nodes,Nodes);
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Connections = Matrix[(a=A &
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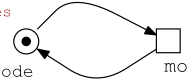


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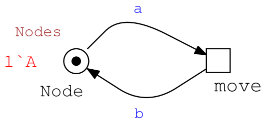


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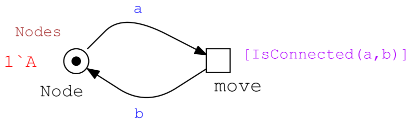


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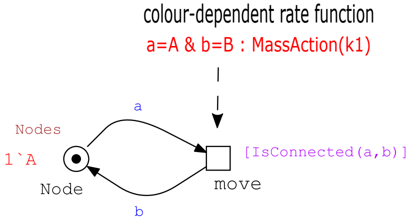


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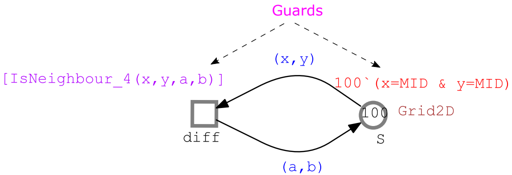
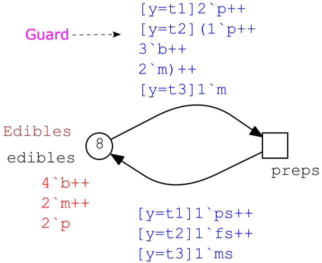



coloured continuous Petri net




coloured stochastic Petri net

# State of the Art

- Coloured Petri nets are in use for a wide range of applications, covering natural/engineering/life sciences.

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- Unfolding tends to be time consuming.

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- Example of a scaleable model to adjust grid size.

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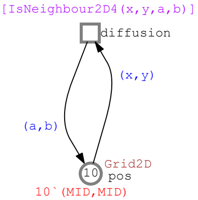
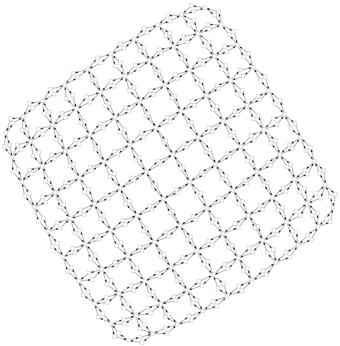
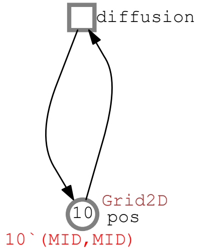
- 2D Diffusion in space.



# The Problem

- The core problem of efficient unfolding is to determine the transition instances, i.e., all bindings of the involved variables.



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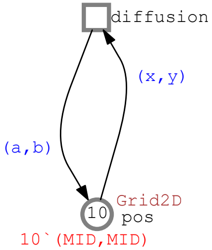

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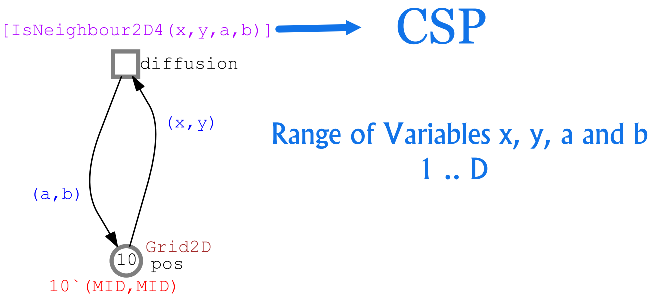
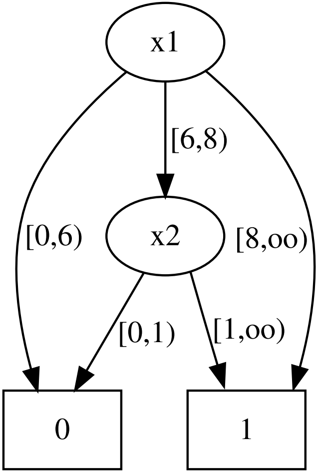


# Solution

Interval decision diagrams → CSP.

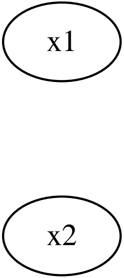
# What are IDD?

- Directed acyclic graphs (DAGs) to encode interval logic functions in a symbolic data structure.



# What are IDD?

- There are two types of nodes:
  non-terminal nodes (ellipses) and terminal nodes (boxes).



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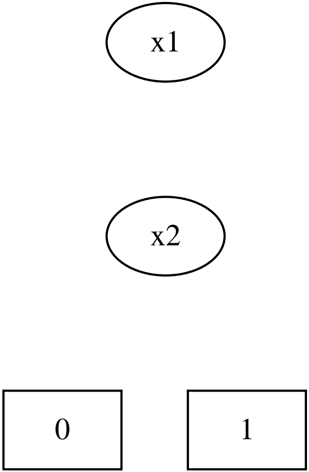
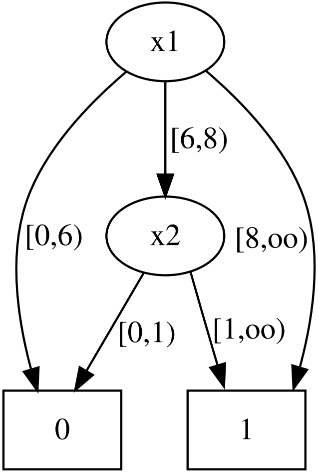


# What are IDD?

- Non-terminal nodes may have an arbitrary number of outgoing arcs labelled with intervals of natural numbers in the form [a,b).



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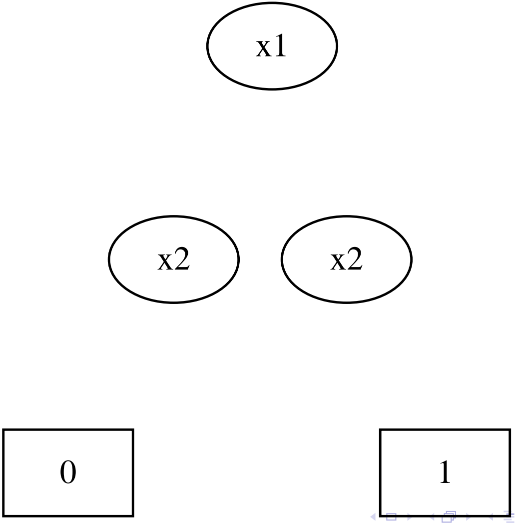
- Typically, one path encodes more than one solution.
- Thus, we can easily pick all CSP solutions from the constraint IDD.

# Example $(x1 \geqslant 8) \vee (x1 \in [6,8) \wedge x2>0)$

- Variable ordering



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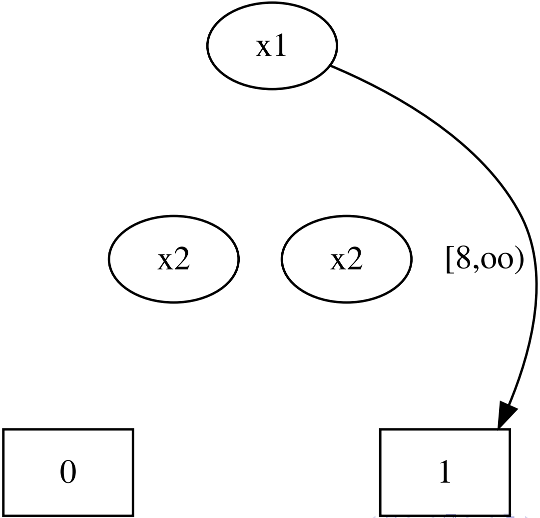
- x1 ⩾ 8.



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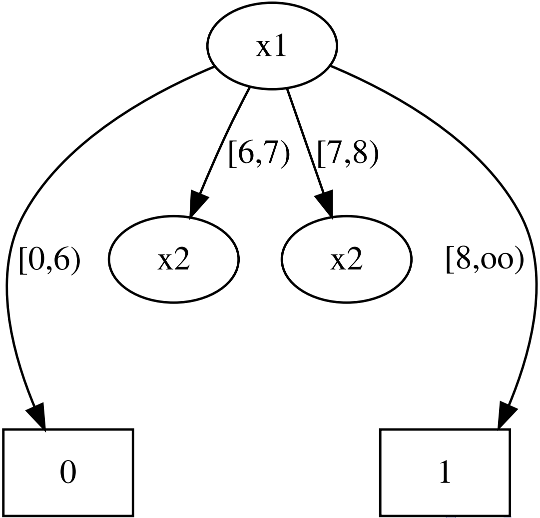
- some intermediate screenshots.



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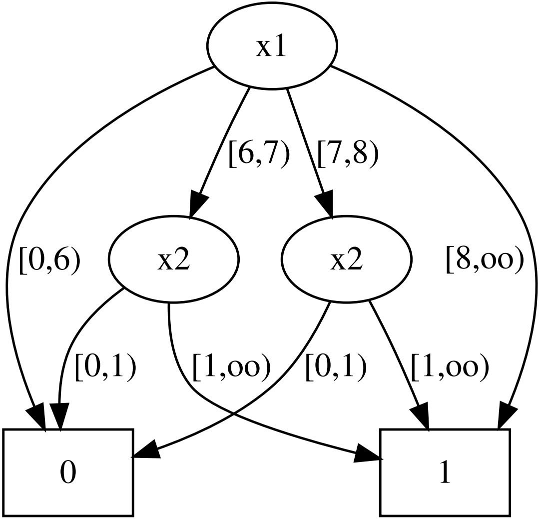
- One final solution.



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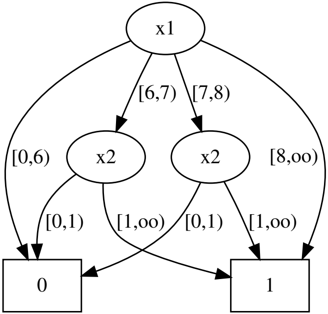
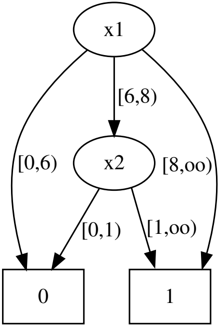
- Each non-terminal node has at least two different children.
- There exist no two nodes with isomorphic subgraphs.




not Reduced




Reduced

# IDD-based unfolding algorithm

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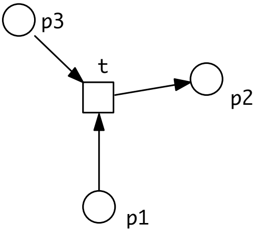
- The entire algorithm is given in the paper as pseudo code.

# Example




**Color definitions:**

cs = {1,8,3..6,10,9,11,20..23};
enum ab = {A,C,D};
variables:
cs : x;
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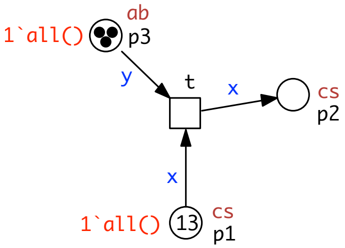
# Example (no constraints)

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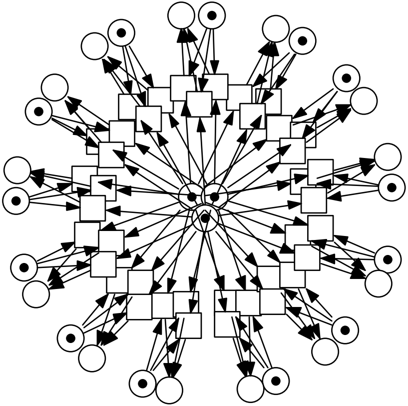
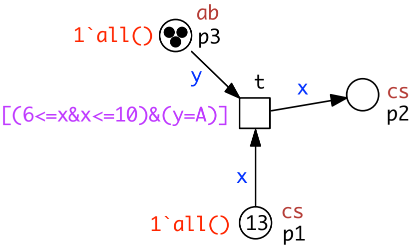


# Example (with Guard)

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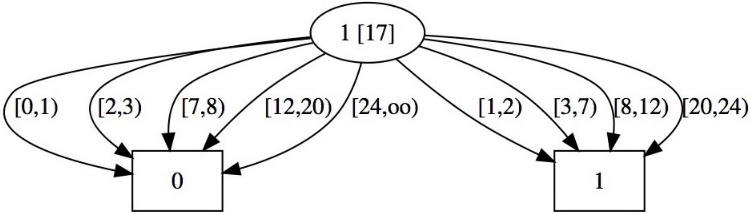
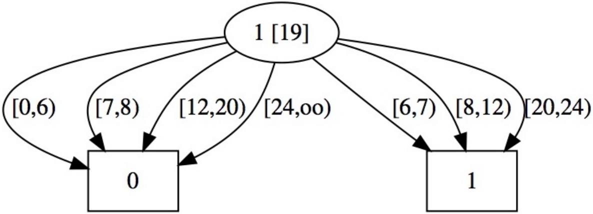
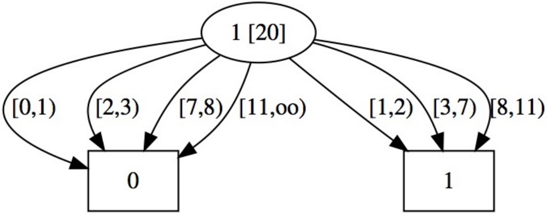
# Encoding the entire cs = {1,8,3..6,10,9,11,20..23}



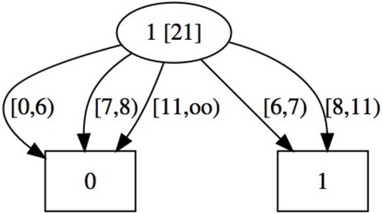
# Constraining the color set cs to $6 \leqslant x$



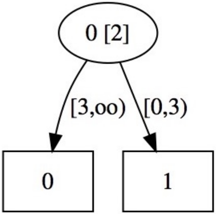
# Constraining the color set cs to $x \leqslant 10$



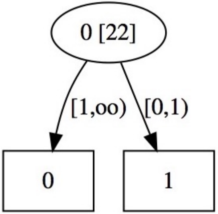
# Combining 6 ⩽ x and x ⩽ 10 using & operator



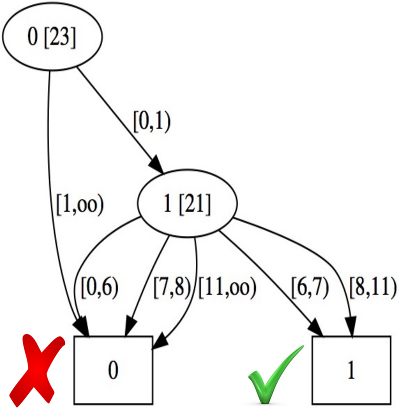
# Encoding the entire color set ab ={A,C,D}



# Constraining the color set ab to y = A



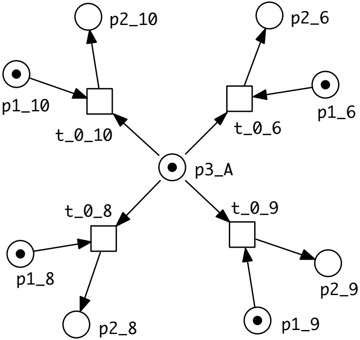
# Merging the result of ( $6 \leqslant x$ and $x \leqslant 10$) and ( $y = A$) using & operator




**Two-path solution:**

1st path : $(y = A, x = 6)$
2nd path : $(y = A,\ x = 8\ ..10)$

# Unfolded Net



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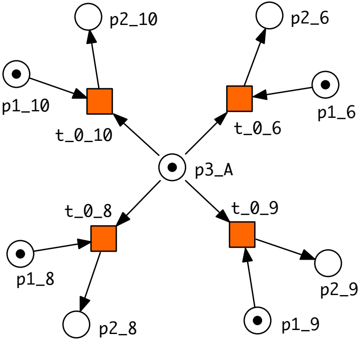

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  - 2nd group: requires substantial unfolding time.
- We used also two biological test cases from our own collection: https://www-dssz.informatik.tu-cottbus.de/DSSZ/Software/Examples?dir=IddUnfolding
  - 3D Diffusion.
  - Brusselator.

  serving as representatives for many more biological case studies, we have collected over the years.

# Bridges and Vehicles (MCC)

## the model

- a lane bridge with limited capacity.
- used by two types of vehicles.
- coloured model has 15 places, 11 transition and 57 arcs.

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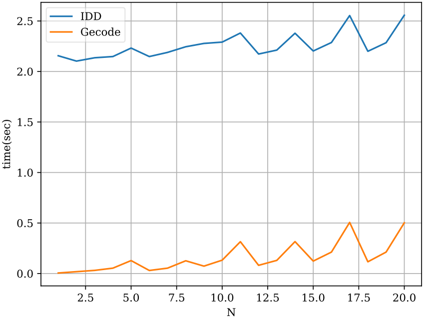



| $N$ | $\mid P \mid$ | $\mid T \mid$ | $\mid A \mid$ |
|---|---|---|---|
| 1 | 28 | 52 | 326 |
| 2 | 48 | 288 | 2090 |
| 4 | 78 | 968 | 7350 |
| 5 | 108 | 2228 | 17 190 |
| 9 | 128 | 1328 | 10 010 |
| 10 | 138 | 2348 | 18 090 |
| 11 | 168 | 5408 | 42 330 |
| 15 | 188 | 2108 | 15 950 |
| 16 | 198 | 3728 | 28 830 |
| 20 | 228 | 8588 | 67 470 |

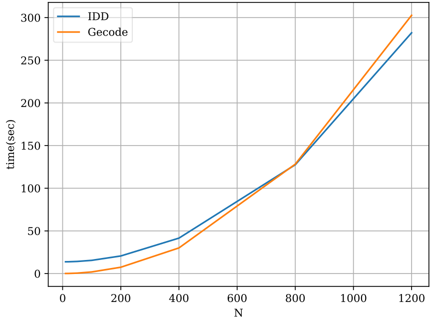
Figure: Bridges and vehicles (MCC); requires no substantial unfolding time

# Family reunion (MCC)

## the model

- reunification process.
- the coloured model has 104 places, 66 transition and198 arcs.
- it is scaled by the number of legal residents.

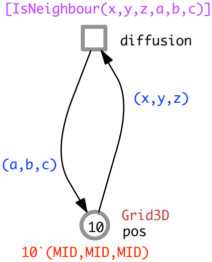
# Family reunion (MCC)




| N | \| P \| | \| T \| | \| A \| |
|---|---|---|---|
| 10 | 1475 | 1234 | 3799 |
| 20 | 3271 | 2753 | 8446 |
| 50 | 12 194 | 10 560 | 32 238 |
| 100 | 40 605 | 36 871 | 112 728 |
| 200 | 143 908 | 134 279 | 411 469 |
| 400 | 537 708 | 508 489 | 1 558 729 |
| 800 | 2 075 308 | 1 976 909 | 6 061 249 |
| 1200 | 4 612 908 | 4 405 329 | 13 507 769 |

Figure: Family Reunion (MCC); requires substantial unfolding time

# Diffusion in space (3D)



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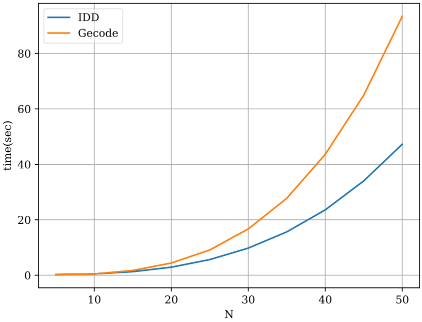
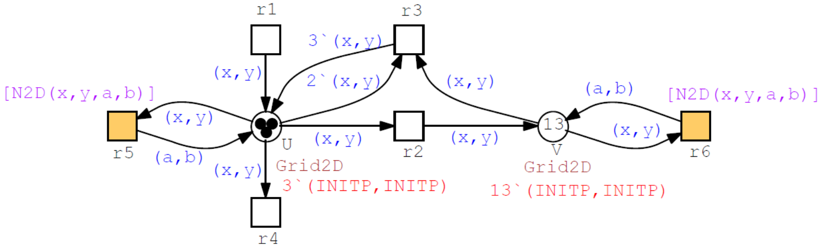


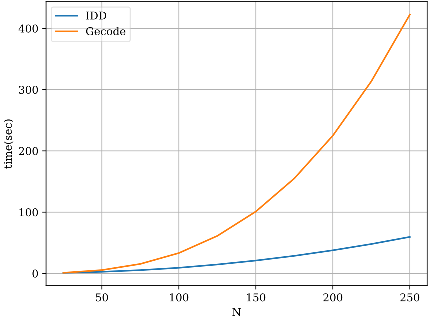

| $N$ | $\mid P \mid$ | $\mid T \mid$ | $\mid A \mid$ |
|---|---|---|---|
| 5 | 125 | 600 | 1200 |
| 10 | 1000 | 5400 | 10 800 |
| 15 | 3375 | 18 900 | 37 800 |
| 20 | 8000 | 72 998 | 145 996 |
| 25 | 15 625 | 90 000 | 180 000 |
| 30 | 27 000 | 156 600 | 313 200 |
| 35 | 42 875 | 249 900 | 499 800 |
| 40 | 64 000 | 374 400 | 748 800 |
| 45 | 91 125 | 534 600 | 1 069 200 |
| 50 | 125 000 | 735 000 | 1 470 000 |

Figure: Diffusion (3D); N – Grid size

# Brusselator



# Brusselator




| $N$ | $\lvert P \rvert$ | $\lvert T \rvert$ | $\lvert A \rvert$ |
|---|---|---|---|
| 25 | 1250 | 11 908 | 23 191 |
| 50 | 5000 | 48 808 | 95 116 |
| 75 | 11 250 | 110 708 | 215 791 |
| 100 | 20 000 | 197 608 | 385 216 |
| 125 | 31 250 | 309 508 | 603 391 |
| 150 | 45 000 | 446 408 | 870 316 |
| 175 | 61 250 | 608 308 | 1 185 991 |
| 200 | 80 000 | 795 208 | 1 550 416 |
| 225 | 101 250 | 1 007 108 | 1 963 591 |
| 250 | 125 000 | 1 244 008 | 2 425 516 |

Figure: Brusselator ; N – Grid size of a 2D square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  - models with a few guards or no guards.
  - models with simple colour sets.
  - e.g., 12 MCC models (no substantial time).

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- The complete performance report is available: https://www-dssz.informatik.tu-cottbus.de/DSSZ/Software/Examples?dir=IddUnfolding

# Our tools

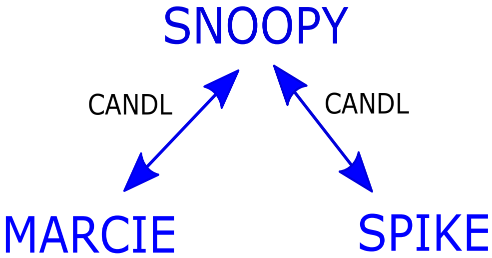
SNOOPY

MARCIE

SPIKE

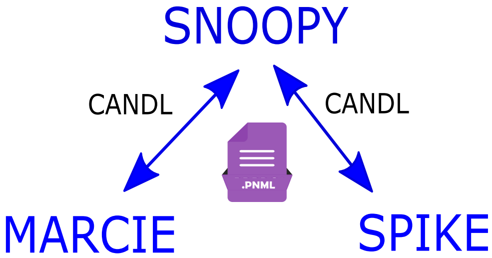
https://www-dssz.informatik.tu-cottbus.de/DSSZ/Software/

# Our tools




https://www-dssz.informatik.tu-cottbus.de/DSSZ/Software/

# Our tools

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# Future work

- Performance comparison considering:
  - memory consumption,
  - power consumption.

# Future work

- Performance comparison considering:
  - memory consumption,
  - power consumption.
- Implementation efficiency:
  - multi-threading: unfolding the coloured places and transitions is currently done sequentially,
  - reuse of already computed solutions,
  - choosing among several variable order strategies.

*Thank You For Your Attention*